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Free Vibration Analysis of a Six-degree-of-freedom Mass-spring System Suitable for Dynamic Vibration Absorbing of Space Frames

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ABSTRACT

This study is concentrated on the natural frequencies and mode shapes of a simple three-member space frame coupled with a dynamic vibration absorber. The dynamic vibration absorber is modeled as a six-degree-of-freedom mass-spring system. For the first time, the free vibration of an elastic structure with a six-degree-of-freedom mass-spring system is found. Each member of the space frame has uniform mechanical and geometrical properties, but these may differ from one member to another. The exact result of this complex problem is obtained via analytical scheme and finite element method. All effects of the axial and torsional deformations and also in- and out-of-plane bending are taken into account. This formulation includes eighteen differential equations along with thirty six boundary and compatibility conditions. Comparison of the results by both approaches mentioned above, illustrates the accuracy of our solutions. Findings are useful benchmarks for the natural frequencies and mode shapes of the space frame coupled with a dynamic vibration absorber.

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1. INTRODUCTION

Frame systems are important structures which are used in many branches of engineering such as civil, mechanical and aerospace engineering. Vibrations of planar frames have received considerable attention in the literature [1-5]. On the other hand, space frame has received much less attention. This fact may be due to difficulty involved in the analysis of three-dimensional structures.

Dumir et al., used dynamic stiffness method for dynamic analysis of space frames under distributed harmonic loads [6]. Noorzaei presented a numerical solution for the modelling of the space frame-raft-soil system by means of finite element method [7]. Moon and Choi employed the finite element method for dynamic analysis of space frames [8]. Guo et al., proposed the formulation of dynamic reverberation-ray matrix analysis to study wave propagation of frames [9]. Tu et al., utilized the transfer dynamic stiffness matrix method to study free vibration of space frames [10]. Minghini et al. used two-node locking-free Hermitian finite elements for analyzing vibration frequencies and mode shapes of pultruded FRP plane and space frames with semi-rigid joints [11]. Mei and Sha used wave-based as well as experimental methods to study vibrations in simple space frames [12]. Mei and Sha employed a wave-based method to investigate vibrations in built-up multi-story space frames [13].

A vast number of publications are available in literature regarding vibration analysis of continuous structures such as beams and plates carrying mass-spring systems. The importance of this subject is well-recognized for its engineering applications. The design of tuned dynamic vibration absorber relies on this concept that by adding one or more spring-mass systems to a structure, natural frequencies and mode shapes can be alerted significantly to avoid resonance and other undesirable dynamic phenomena. Furthermore, in the design of robotics, and also when dealing with human structure interaction, structural systems are often modeled as a combination of beam and mass-spring systems [14].

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All mentioned literature reviews showed that the free vibration of an elastic continuous structure joined by a mass-spring system up to three degrees of freedom has been investigated so far. To the authors' best knowledge, the free vibration analysis of space frame including mass-spring system and having six-degree-of-freedom has not been treated yet. This mass-spring system can be used as a realistic model for a dynamic vibration absorber. Therefore, the main objective of this paper is to fill this gap and present a simple straightforward method for analyzing the free vibration of space frames with an attached six-degree-of-freedom mass-spring system. As a result, this study is devoted to the free vibration of a space frame with three orthogonal members having a dynamic vibration absorber, which is shown in Figure 1. The dynamic vibration absorber is modeled as a six-degree-of-freedom mass-spring system. Each member of the frame has uniform mechanical and geometrical properties. To generalize the solution, these properties may differ from one member to another. The governing eigenvalue problem includes eighteen differential equations and thirty six boundary and compatibility conditions. In the presented formulation, all interactions of axial deformation, torsional deformation and in- and out-of-plane bending are considered. After solving the related eigenvalue problem, natural frequencies and mode shapes of the system are found. To verify the correctness of the authors' formulation, the exact solution of the problem is also obtained by the finite element method.

2. GOVERNING DIFFERENTIAL EQUATIONS OF SPACE FRAMES

As mentioned previously, for free vibration analysis of space frames, in the most general case, the interaction of in-plane bending, out-of-plane bending, axial deformation and torsional deformation could be considered. Therefore, for a space frame member, the following differential equations describe the vibratory behavior of the member [13]:

$$EA\frac{\partial^2 u_x}{\partial x^2}+\rho A\omega^2 u_x=0 \tag{1}$$

$$GJ\frac{\partial^2 \theta_x}{\partial x^2}+I_0\omega^2\theta_x=0 \tag{2}$$

$$EI_z\frac{\partial^4 u_y}{\partial x^4}-\rho A\omega^2 u_y=0 \tag{3}$$

$$EI_y\frac{\partial^4 u_z}{\partial x^4}-\rho A\omega^2 u_z=0 \tag{4}$$

where u_x, θ_x, u_y and u_z are axial displacement, angular displacement, *xy*-plane transverse displacement and *xz*-plane transverse displacement, respectively. In addition, x is the position along the member axis, t time, E Young's modulus, A cross-sectional area, G the shear modulus, J the polar moment of the circular member, I_0 the mass moment of inertia per unit length of the member, ρ volume mass density, I_z the area moment of inertia of cross-section of the section about *z*-axis and I_y the area moment of inertia of cross-section of the section about the y-axis. For circular members, the following relations hold:

$$I_z=I_y=I \quad ; \quad I=\frac{\pi R^4}{4}$$
$$I_0=\rho J \quad ; \quad J=\frac{\pi R^4}{2} \tag{5}$$

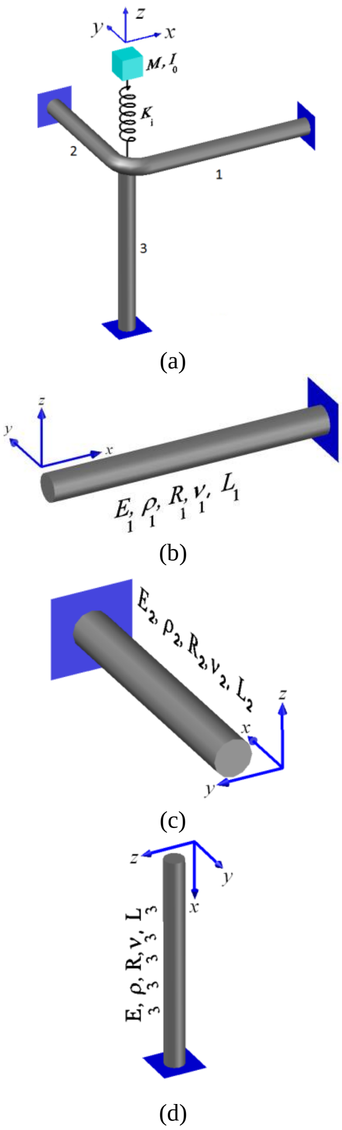


Figure 1. The studied space frame: (a) schematic model; (b) member one properties; (c) member two properties; (d) member three properties

in which R is the radius of the member. It should be mentioned that no warping occurs for the circular sections.
The solutions of Equations (1)-(4) have the next appearance:

$$\begin{aligned} u_x(x) &= c_1' \sin \lambda_a x + c_2' \cos \lambda_a x \\ \theta_x(x) &= c_3' \sin \lambda_t x + c_4' \cos \lambda_t x \\ u_y(x) &= c_5' \sin \lambda_b x + c_6' \cos \lambda_b x + c_7' \sinh \lambda_b x + c_8' \cosh \lambda_b x \\ u_z(x) &= c_9' \sin \lambda_b x + c_{10}' \cos \lambda_b x + c_{11}' \sinh \lambda_b x + c_{12}' \cosh \lambda_b x \end{aligned} \tag{6}$$

where c_1' to c_{12}' are unknown constants, which are to be determined. The parameters λ_a, λ_t and λ_b have the coming form:

$$\begin{aligned} \lambda_a &= \sqrt{\frac{\rho A \omega^2}{EA}} \\ \lambda_t &= \sqrt{\frac{\rho A R^2 \omega^2}{2GJ}} \\ \lambda_b &= \sqrt[4]{\frac{\rho A \omega^2}{EI}} \end{aligned} \tag{7}$$

In the next section, the eigenvalue problem governing the vibratory behavior of the mentioned space frame is formulated.

3. FORMULATION OF EIGENVALUE PROBLEM

This section presents the eigenvalue problem which governs the space frame with attached dynamic vibration absorber. The dynamic vibration absorber is modeled as a six-degree-of-freedom mass-spring system, which is connected to the space frame by means of six springs. This system has three translational springs of stiffness K_1, K_2 and K_3 in the x, y and z directions, respectively, and three rotational springs of stiffness K_4, K_5 and K_6 in the x, y and z directions, correspondingly. Figure 1 demonstrates a schematic diagram of the mechanical system under study.

The related eigenvalue problem includes governing differential equations and boundary and compatibility conditions. It should be noted that upon solution of this problem, the eigenvalues and eigenfunctions of the problem which are the frequency parameters and mode shapes of the system are in hand. In the subsequent subsections, the differential equations and boundary and compatibility conditions are given.

3. 1. DIFFERENTIAL EQUATIONS

As mentioned in Section 2, considering the interaction of in-plane bending, out-of-plane bending, axial deformation and torsional deformation, four differential equations are needed to study the behavior of a space frame member. These four differential equations are introduced by Equations (1)-(4). Totally, twelve differential equations exist for the three-member space frame and six differential equations govern the behavior of the mass-spring system. Using index notation, the differential equations of the space frame may be written in the shapes shown hereunder:

$$E_i A_i \frac{\partial^2 u_{xi}}{\partial x^2} + \rho_i A_i \omega^2 u_{xi} = 0 \tag{8}$$

$$G_i J_i \frac{\partial^2 \theta_{xi}}{\partial x^2} + I_{0i} \omega^2 \theta_{xi} = 0 \tag{9}$$

$$E_i I_i \frac{\partial^4 u_{yi}}{\partial x^4} - \rho_i A_i \omega^2 u_{yi} = 0 \tag{10}$$

$$E_i I_i \frac{\partial^4 u_{zi}}{\partial x^4} - \rho_i A_i \omega^2 u_{zi} = 0 \tag{11}$$

where $i = 1,2,3$ indicates the member number. The solutions of Equations (8) - (11) have the following forms:

$$\begin{aligned} u_{x1}(x) &= c_1 \sin \lambda_{a1} x + c_2 \cos \lambda_{a1} x \\ \theta_{x1}(x) &= c_3 \sin \lambda_{t1} x + c_4 \cos \lambda_{t1} x \\ u_{y1}(x) &= c_5 \sin \lambda_{b1} x + c_6 \cos \lambda_{b1} x + c_7 \sinh \lambda_{b1} x + c_8 \cosh \lambda_{b1} x \\ u_{z1}(x) &= c_9 \sin \lambda_{b1} x + c_{10} \cos \lambda_{b1} x + c_{11} \sinh \lambda_{b1} x + c_{12} \cosh \lambda_{b1} x \\ u_{x2}(x) &= c_{13} \sin \lambda_{a2} x + c_{14} \cos \lambda_{a2} x \\ \theta_{x2}(x) &= c_{15} \sin \lambda_{t2} x + c_{16} \cos \lambda_{t2} x \\ u_{y2}(x) &= c_{17} \sin \lambda_{b2} x + c_{18} \cos \lambda_{b2} x + c_{19} \sinh \lambda_{b2} x + c_{20} \cosh \lambda_{b2} x \\ u_{z2}(x) &= c_{21} \sin \lambda_{b2} x + c_{22} \cos \lambda_{b2} x + c_{23} \sinh \lambda_{b2} x + c_{24} \cosh \lambda_{b2} x \\ u_{x3}(x) &= c_{25} \sin \lambda_{a3} x + c_{26} \cos \lambda_{a3} x \\ \theta_{x3}(x) &= c_{27} \sin \lambda_{t3} x + c_{28} \cos \lambda_{t3} x \\ u_{y3}(x) &= c_{29} \sin \lambda_{b3} x + c_{30} \cos \lambda_{b3} x + c_{31} \sinh \lambda_{b3} x + c_{32} \cosh \lambda_{b3} x \\ u_{z3}(x) &= c_{33} \sin \lambda_{b3} x + c_{34} \cos \lambda_{b3} x + c_{35} \sinh \lambda_{b3} x + c_{36} \cosh \lambda_{b3} x \end{aligned} \tag{12}$$

in which

$$\begin{aligned} &\lambda_{a1} = \sqrt{\frac{\rho_1 \omega^2}{E_1}}; \quad \lambda_{t1} = \sqrt{\frac{\rho_1 A_1 R_1^2 \omega^2}{2G_1 J_1}}; \quad \lambda_{b1} = \sqrt[4]{\frac{\rho_1 A_1 \omega^2}{E_1 I_1}} \\ &\lambda_{a2} = \sqrt{\frac{\rho_2 \omega^2}{E_2}}; \quad \lambda_{t2} = \sqrt{\frac{\rho_2 A_2 R_2^2 \omega^2}{2G_2 J_2}}; \quad \lambda_{b2} = \sqrt[4]{\frac{\rho_2 A_2 \omega^2}{E_2 I_2}} \\ &\lambda_{a3} = \sqrt{\frac{\rho_3 \omega^2}{E_3}}; \quad \lambda_{t3} = \sqrt{\frac{\rho_3 A_3 R_3^2 \omega^2}{2G_3 J_3}}; \quad \lambda_{b3} = \sqrt[4]{\frac{\rho_3 A_3 \omega^2}{E_3 I_3}} \end{aligned} \tag{13}$$

It is observed that thirty six unknown constants are presented in Eq. (12). Therefore, thirty six boundary and compatibility conditions should be specified. The differential equations of the mass-spring system will be introduced in Section 4.

3. 2. BOUNDARY AND COMPATIBILITY CONDITIONS

The boundary and compatibility conditions of the problem may be expressed as follows.
1) Boundary conditions at each end support. At each end support, six boundary conditions exist. For the fixed end conditions, axial, angular, in-plane and out-of-plane

displacements along with in-plane and out-of-plane rotations are zero. These lead to the following mathematical relationships:

$$\begin{array}{lllll} u_{x1}(L_1)=0 & ; & u_{x2}(L_2)=0 & ; & u_{x3}(L_3)=0 \\ \theta_{x1}(L_1)=0 & ; & \theta_{x2}(L_2)=0 & ; & \theta_{x3}(L_3)=0 \\ u_{y1}(L_1)=0 & ; & u_{y2}(L_2)=0 & ; & u_{y3}(L_3)=0 \\ u_{z1}(L_1)=0 & ; & u_{z2}(L_2)=0 & ; & u_{z3}(L_3)=0 \\ u'_{y1}(L_1)=0 & ; & u'_{y2}(L_2)=0 & ; & u'_{y3}(L_3)=0 \\ u'_{z1}(L_1)=0 & ; & u'_{z2}(L_2)=0 & ; & u'_{z3}(L_3)=0 \end{array} \tag{14}$$

2) Equilibrium of forces at the intersecting joint. The shear forces and axial forces of the frame must be in equilibrium with the forces of the springs of the mass-spring system in the *x*, *y* and *z* directions. Using Newton's second law results in the following equations:

$$\begin{aligned} & E_1A_1u'_{x1}(0)+E_2I_2u'''_{y2}(0)-E_3I_3u'''_{y3}(0)-F_1=0 \\ & E_2A_2u'_{x2}(0)+E_3I_3u'''_{z3}(0)-E_1I_1u'''_{y1}(0)-F_2=0 \\ & E_3A_3u'_{x3}(0)+E_2I_2u'''_{z2}(0)+E_1I_1u'''_{z1}(0)+F_3=0 \end{aligned} \tag{15}$$

in which F_1, F_2 and F_3 are the forces of the translational springs of the mass-spring system in the *x*, *y* and *z* directions, respectively. These forces can be written as:

$$\begin{aligned} & F_1=K_1\left[u_{x1}(0)-u_x\right] \\ & F_2=K_2\left[u_{y1}(0)-u_y\right] \\ & F_3=K_3\left[u_{z1}(0)-u_z\right] \end{aligned} \tag{16}$$

where u_x, u_y and u_z are the displacements of the mass-spring system in the *x*, *y* and *z* directions, respectively. Furthermore, K_1, K_2 and K_3 indicate the stiffness of the translational springs of the mass-spring system in the *x*, *y* and *z* directions, respectively.

3) Equilibrium of moments at the intersecting joint. The bending moments and torques of the space frame members and moments of the rotational springs of the mass-spring system must be self-equilibrated in the *x*, *y* and *z* directions. Writing Newton's second law gives the coming equalities:

$$\begin{aligned} & G_1J_1\theta'_{x1}(0)+E_2I_2u''_{y2}(0)-E_3I_3u''_{y3}(0)-M_1=0 \\ & G_2J_2\theta'_{x2}(0)+E_3I_3u''_{z3}(0)-E_1I_1u''_{y1}(0)-M_2=0 \\ & G_3J_3\theta'_{x3}(0)+E_1I_1u''_{z1}(0)+E_2I_2u''_{z2}(0)+M_3=0 \end{aligned} \tag{17}$$

in which, M_1, M_2 and M_3 are the moments of the rotational springs of the mass-spring system in the *x*, *y* and *z* directions, respectively. They are defined as:

$$\begin{aligned} & M_1=K_4\left[\theta_{x1}(0)-\theta_x\right] \\ & M_2=K_5\left[\theta_{y1}(0)-\theta_y\right] \\ & M_3=K_6\left[\theta_{z1}(0)-\theta_z\right] \end{aligned} \tag{18}$$

where K_4, K_5 and K_6 demonstrate the stiffness of the rotational springs of the mass-spring system in the *x*, *y* and *z* directions, respectively.

4) Compatibility of rotations and angular displacement at the intersecting joint can be expressed as:

$$\begin{array}{lll} u'_{y1}(0)=u'_{y2}(0) & ; & u'_{y1}(0)=\theta_{x3}(0) \\ u'_{y3}(0)=-u'_{z1}(0) & ; & u'_{z1}(0)=\theta_{x2}(0) \\ u'_{z2}(0)=-u'_{z3}(0) & ; & u'_{z2}(0)=-\theta_{x1}(0) \end{array} \tag{19}$$

5) Compatibilities of displacements at the intersecting joints are satisfied by:

$$\begin{array}{lll} u_{x1}(0)=-u_{y2}(0) & ; & u_{x1}(0)=u_{y3}(0) \\ u_{x2}(0)=u_{y1}(0) & ; & u_{y1}(0)=-u'_{z3}(0) \\ u_{x3}(0)=-u_{z1}(0) & ; & u_{z1}(0)=u'_{z2}(0) \end{array} \tag{20}$$

All thirty six boundary and compatibility conditions given by Equations (14), (15), (17) and (19), (20), along with twelve differential equations expressed in Equations (8)-(11) form the governing eigenvalue problem for this space frame. In order to use the compatibility conditions of the forces and moments of the intersecting structural joint, given by Equations (15) and (17), the values of F_1, F_2 and F_3 along with M_1, M_2 and M_3 should be found. In the next section, these forces and moments are obtained, considering the behavior of the mass-spring system.

4. MASS-SPRING SYSTEM FORMULATION

The mentioned oscillator is modeled as a six-degree-of freedom mass-spring system. In this section, six differential equations governing the vibratory behavior of the mass-spring system are investigated. To formulate the mass-spring system, the Newton's second law of motion is employed. Writing the translational Newton's second law of motion in the x direction, i.e., $\sum F_x = Ma_x$, gives the following equality:

$$F_1 = M\ddot{u}_x \tag{21}$$

in which, M is the mass of the dynamic vibration absorber. Substituting Equations (16) into Equation (21) yields:

$$K_1\left[u_{x1}(0)-u_x\right]=M\ddot{u}_x \tag{22}$$

or

$$M\ddot{u}_x + K_1u_x = K_1u_{x1}(0) \tag{23}$$

Using $u_x = Ue^{i\omega t}$ results in:

$$-M\omega^2u_x + K_1u_x = K_1u_{x1}(0) \tag{24}$$

which gives

$$u_x = \frac{K_1u_{x1}(0)}{K_1 - M\omega^2} \tag{25}$$

Defining

$$K_1 = k_1 \frac{E_1 I_1}{L_1^3} \quad ; \quad M = \alpha \rho_1 A_1 L_1 \tag{26}$$

Equation (25) becomes:

$$u_x = \frac{k_1 \frac{E_1 I_1}{L_1^3} u_{x1}(0)}{k_1 \frac{E_1 I_1}{L_1^3} - \alpha \rho_1 A_1 L_1 \omega^2} \tag{27}$$

where k_1 denotes the stiffness of the translational spring of the dynamic vibration absorber in the x direction. It should be added that α is a constant. Equation (27) can be simplified as:

$$u_x = \frac{k_1 u_{x1}(0)}{k_1 - \alpha (\lambda_{b1} L_1)^4} \tag{28}$$

Likewise, it can be shown that the following relations hold for u_y and u_z :

$$u_y = \frac{K_2 u_{y1}(0)}{K_2 - M \omega^2} \quad ; \quad u_z = \frac{K_3 u_{z1}(0)}{K_3 - M \omega^2} \tag{29}$$

These relations can be written in the form:

$$\begin{aligned} u_y &= \frac{k_2 u_{y1}(0)}{k_2 - \alpha \frac{E_1 I_1 L_2^3}{E_2 I_2 L_1^3} (\lambda_{b1} L_1)^4} \\ u_z &= \frac{k_3 u_{z1}(0)}{k_3 - \alpha \frac{E_1 I_1 L_3^3}{E_3 I_3 L_1^3} (\lambda_{b1} L_1)^4} \end{aligned} \tag{30}$$

in which:

$$K_2 = k_2 \frac{E_2 I_2}{L_2^3} \quad ; \quad K_3 = k_3 \frac{E_3 I_3}{L_3^3} \tag{31}$$

In Equation (31), k_2 and k_2 are the stiffness of the translational springs of the dynamic vibration absorber in the y and z directions, respectively. The three remaining equations are obtained using the rotational form of Newton's second law of motion, i.e., $\sum M = I_0 \ddot{\theta}$, where I_0 is mass moment of inertia. Findings are:

$$\begin{aligned} \theta_x &= \frac{K_4 \theta_{x1}(0)}{K_4 - I_0 \omega^2} \\ \theta_y &= \frac{K_5 \theta_{y1}(0)}{K_5 - I_0 \omega^2} \\ \theta_z &= \frac{K_6 \theta_{z1}(0)}{K_6 - I_0 \omega^2} \end{aligned} \tag{32}$$

Using:

$$\begin{aligned} K_4 &= k_4 \frac{E_1 I_1}{L_1} \quad ; \quad K_5 = k_5 \frac{E_2 I_2}{L_2} \\ K_6 &= k_6 \frac{E_3 I_3}{L_3} \quad ; \quad I_0 = \beta \rho_1 A_1 L_1^3 \end{aligned} \tag{33}$$

The values of θ_x , θ_y and θ_z take the following form:

$$\begin{aligned} \theta_x &= \frac{k_4 \theta_{x1}(0)}{k_4 - \beta (\lambda_{b1} L_1)^4} \\ \theta_y &= \frac{k_5 \theta_{y1}(0)}{k_5 - \beta \frac{E_1 I_1 L_2}{E_2 I_2 L_1} (\lambda_{b1} L_1)^4} \\ \theta_z &= \frac{k_6 \theta_{z1}(0)}{k_6 - \beta \frac{E_1 I_1 L_3}{E_3 I_3 L_1} (\lambda_{b1} L_1)^4} \end{aligned} \tag{34}$$

where k_4, k_5 and k_6 are the stiffness of the rotational springs of the dynamic vibration absorber. Furthermore, β is a constant. The expressions obtained for degrees of freedom of the mass-spring system can be substituted into Equations (16) and (18). Consequently, the boundary and compatibility conditions introduced previously can be utilized in the following sections. It is worth mentioning that the six natural circular frequencies of the mass-spring system, when it is not coupled with the space frame, have the following shapes:

$$\begin{cases} \omega_1 = \sqrt{\frac{K_1}{M}} \\ \omega_2 = \sqrt{\frac{K_2}{M}} \\ \omega_3 = \sqrt{\frac{K_3}{M}} \end{cases} \quad ; \quad \begin{cases} \omega_4 = \sqrt{\frac{K_4}{I_0}} \\ \omega_5 = \sqrt{\frac{K_5}{I_0}} \\ \omega_6 = \sqrt{\frac{K_6}{I_0}} \end{cases} \tag{35}$$

5. EIGENVALUES AND EIGENFUNCTIONS

In order to obtain the eigenvalues and eigenfunctions of the problem under study, the following strategy is employed. According to the boundary and compatibility conditions, the required derivatives of functions u_{xi}, θ_{xi}, u_{yi} and u_{zi} should be calculated. Afterward, these derivatives, as well as functions u_{xi}, θ_{xi}, u_{yi} and u_{zi}, are substituted in the thirty six boundary and compatibility conditions, which are expressed by Equations (14), (15), (17), (19) and (20). This action leads to form the coming homogeneous system of algebraic equations with c_1 to c_{36} as unknowns:

$$[\mathbf{A}]\{\mathbf{C}\} = \{\mathbf{0}\} \tag{36}$$

To have a nontrivial solution, the determinant of the coefficient matrix must be equal to zero, as it is demonstrated in the following relationship:

$$|\mathbf{A}| = 0 \tag{37}$$

Calculating the determinant yields the frequency equation of the problem under study. Close attention should be paid to the fact that the resulting frequency equation has nine unknowns, i.e., λ_{a1}, λ_{t1}, λ_{b1}, λ_{a2}, λ_{t2}, λ_{b2}, λ_{a3}, λ_{t3} and λ_{b3}. In order to have the frequency equation with a single unknown, i.e., λ_{b1}, the following relations are used:

$$\begin{aligned}
&\lambda_{a1} = \frac{R_1^2}{2}\lambda_{b1}^2; \quad \lambda_{t1} = \sqrt{\frac{E_1}{G_1}}\frac{R_1^2}{2}\lambda_{b1}^2 \\
&\lambda_{a2} = \frac{R_2^2}{2}\lambda_{b2}^2; \quad \lambda_{t2} = \sqrt{\frac{E_2}{G_2}}\frac{R_2^2}{2}\lambda_{b2}^2; \quad \lambda_{b2} = \sqrt[4]{\frac{E_2\rho_1}{E_1\rho_2}}\sqrt{\frac{R_2}{R_1}}\lambda_{b1} \\
&\lambda_{a3} = \frac{R_3^2}{2}\lambda_{b3}^2; \quad \lambda_{t3} = \sqrt{\frac{E_3}{G_3}}\frac{R_3^2}{2}\lambda_{b3}^2; \quad \lambda_{b3} = \sqrt[4]{\frac{E_3\rho_1}{E_1\rho_3}}\sqrt{\frac{R_3}{R_1}}\lambda_{b1}
\end{aligned} \tag{38}$$

Having these, the resulting complicated frequency equation can be numerically solved using the well-known Newton-Raphson method. Consequently, upon solution of the frequency equation, the values of λ_{b1}s are in hand. Then, Equation (13) is utilized to find the natural circular frequencies of the space frame, i.e., ω. Finally, substituting the λ_{b1}s into the matrix Equation (29), the mode shapes of the mechanical system under study are obtained. As a second way, a finite element solution is developed. Comparing the results obtained by both exact and numerical solutions demonstrates the accuracy of our formulation.

6. Finite Element Formulation

A finite element solution is presented in this section for the complex problem under study. Figure 2 shows the degree of freedom of elements for each member of the space frame. It is observed that each node has six degrees of freedom, namely, three translational and three rotational. Therefore, the total nodal displacements are given in the equation below:

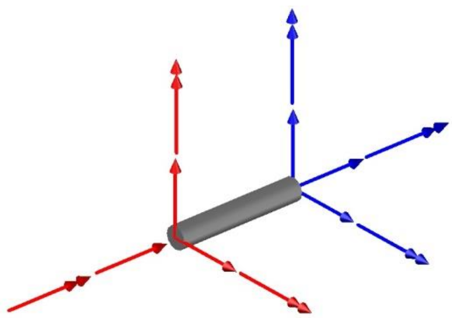
$$\{\mathbf{D}\} = \{u_{xi} \ \ u_{yi} \ \ u_{zi} \ \ \theta_{xi} \ \ \theta_{yi} \ \ \theta_{zi} \ \ u_{xj} \ \ u_{yj} \ \ u_{zj} \ \ \theta_{xj} \ \ \theta_{yj} \ \ \theta_{zj}\}^{\mathrm{T}} \tag{39}$$

All displacement functions can be written using the shape functions:

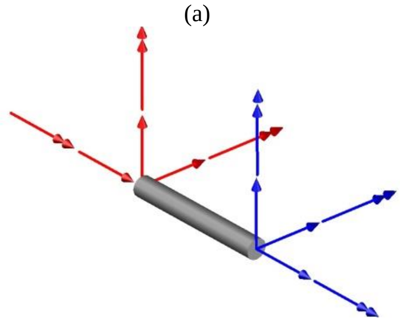
$$\begin{aligned}
&\{u_x\} = [\mathbf{N}_a]\{u_{xi} \ \ u_{xj}\} \\
&\{\theta_x\} = [\mathbf{N}_t]\{\theta_{xi} \ \ \theta_{yi}\} \\
&\{u_y\} = [\mathbf{N}_b]\{u_{yi} \ \ \theta_{yi} \ \ u_{yj} \ \ \theta_{yj}\} \\
&\{u_z\} = [\mathbf{N}_b]\{u_{zi} \ \ \theta_{zi} \ \ u_{zj} \ \ \theta_{zj}\}
\end{aligned} \tag{40}$$

where $[\mathbf{N}_a]$, $[\mathbf{N}_t]$ and $[\mathbf{N}_b]$ are the shape functions for axial displacement, torsional displacement and in- and out-of-plane bending. Herein, the well-known Hermit shape functions are used with following appearance:

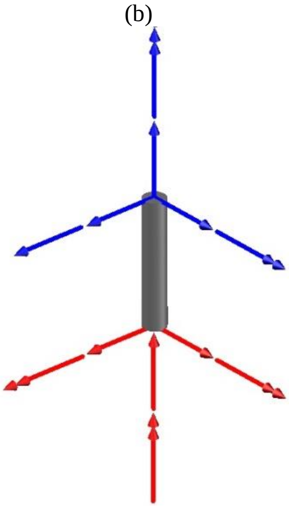
$$\begin{aligned}
&[\mathbf{N}_a] = [1-\frac{x}{L} \quad \frac{x}{L}] \\
&[\mathbf{N}_t] = [1-\frac{x}{L} \quad \frac{x}{L}] \\
&[\mathbf{N}_b] = [1-3\frac{x^2}{L^2}+2\frac{x^3}{L^3} \quad x-2\frac{x^2}{L}+\frac{x^3}{L^2} \\
&\qquad 3\frac{x^2}{L^2}-2\frac{x^3}{L^3} \quad -\frac{x^2}{L}+\frac{x^3}{L^2}]
\end{aligned} \tag{41}$$



(a)



(b)



(c)

Figure 2. Degrees of freedom for elements in each member: (a) member one; (b) member two; (c) member three

Using the shape functions and their needed derivatives, the stiffness and mass matrices for each member are found. It should be pointed out that the final stiffness and mass matrices have $(ndof\ +6)\times(ndof\ +6)$ entries, where *ndof* indicates the number of degrees of freedom of the space frame. This modification is due to the interaction of the six-degree-of-freedom mass-spring system and the space frame. In order to obtain the natural frequencies of the space frame, the following equation of motion should be solved:

$$[\mathbf{M}]\{\ddot{\mathbf{D}}\}+[\mathbf{K}]\{\mathbf{D}\}=\{\mathbf{0}\} \tag{42}$$

Assuming harmonic motion, one obtains:

$$\{\ddot{\mathbf{D}}\}=\{\bar{\mathbf{D}}\}e^{i\omega t} \tag{43}$$

Substituting this equation into Eq. (36) esults in:

$$\left([\mathbf{K}]-\omega^2[\mathbf{M}]\right)\{\bar{\mathbf{D}}\}=\{\mathbf{0}\} \tag{44}$$

For a nontrivial solution:

$$\left|[\mathbf{K}]-\omega^2[\mathbf{M}]\right|=0 \tag{45}$$

From the last system of equations, the natural frequencies of the space frame under study may be calculated.

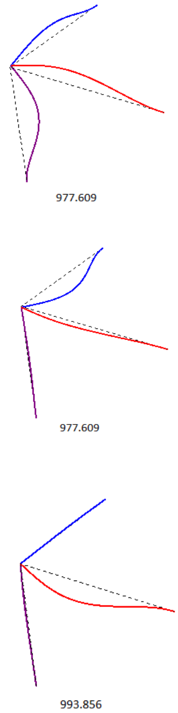


Figure 3. The first three natural frequencies and mode shapes of the bare space frame

7. NUMERICAL RESULTS

This section is devoted to solve sample problems of the free vibration of the space frame with and without attached mass-spring system. Figure 3 indicates the first three circular frequencies, i.e., ω and three-dimensional mode shapes of the space frame without mass-spring system, with steel members, having properties of $R_1=R_2=R_3=0.1$ m, $L_1=L_2=L_3=2$ m, $E_1=E_2=E_3=2\times10^{11}$ Pa, $\rho_1=\rho_2=\rho_3=7850\,\text{kg/m}^3$ and $\nu_1=\nu_2=\nu_3=0.3$. It should be noted that the ratio $L/R=20$ is selected for each member. This limitation is usually required for using the Euler-Bernoulli beam theory.

In order to investigate the effect of the member lengths on the natural frequencies of the system, the length of the third member is assumed to be $L_3=6$ m. The first eight circular frequencies and three-dimensional mode shapes of the space frame in this case for $R_1=R_2=R_3=0.1$ m, $L_1=L_2=2$ m, $L_3=6$ m, $E_1=E_2=E_3=2\times10^{11}$ Pa, $\rho_1=\rho_2=\rho_3=7850\,\text{kg/m}^3$ and $\nu_1=\nu_2=\nu_3=0.3$ are indicated in Figure 4. It is observed that by increasing the length of the third member from 2m to 6m, the natural frequencies of the space frame is dramatically decreased. For instance, the first natural frequency decreases from 977.609 to 141.101 when L_3 increases from $2\,\text{m}$ to $6\,\text{m}$.

Figure 5 may be advantageous for studying the effect of the radius of members on the frequencies of the space frame. In this figure, the first eight natural frequencies and mode shapes of the mechanical system under study for $R_1=R_2=0.1$ m, $R_3=0.2$ m $L_1=L_2=L_3=2$ m, $E_1=E_2=E_3=2\times10^{11}$ Pa, $\rho_1=\rho_2=\rho_3=7850\,\text{kg/m}^3$ and $\nu_1=\nu_2=\nu_3=0.3$ are illustrated. As expected, increasing the stiffness of the space frame, by increasing the radius of the third member, results in a boost in the natural frequencies of the frame. For instance, the fundamental natural frequency of the space frame increases from 977.609 to 1135.250 when the radius of the third member increases from 0.1m to 0.2m.

At this stage, a space frame with clamped ends and general properties of $R_1=0.1\,\text{m}$, $R_2=0.2\,\text{m}$, $R_3=0.3\,\text{m}$, $L_1=2\,\text{m}$, $L_2=4\,\text{m}$, $L_3=5$ m, $E_1=E_2=E_3=2\times10^{11}$ Pa, $\rho_1=\rho_2=\rho_3=7850\,\text{kg/m}^3$, $\nu_1=0.2$ and $\nu_2=\nu_3=0.3$ is considered. The first eight natural frequencies and mode shapes of the frame are presented in Figure 6.

Next, the space frame with attached dynamic vibration absorber is taken into account. Table 1 represents the values of the first ten natural frequencies of the coupled system when the dynamic vibration absorber is connected to the frame via just one translational spring, i.e, K_3.

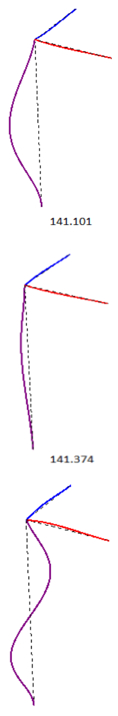


Figure 4. The first three natural frequencies and mode shapes of the bare space frame with different lengths

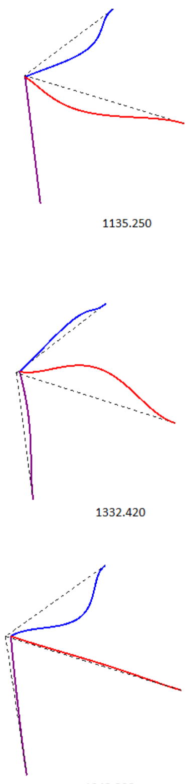


Figure 5. The first three natural frequencies and mode shapes of the bare space frame with different raduis

The properties of the frame are the same as the one in Figure 6. The properties of the dynamic vibration absorber are $\alpha = 10$, $\beta = 10$, and k_3 has different values. It should be noted that the first five natural frequencies of the system are zero. This is because the stiffness of five springs of the dynamic vibration absorber are assumed zero. Furthermore, the value of the sixth natural frequency, which can be considered as the fundamental one, has decreased from 447.034 to 287.177 for $k_3 = 1$.

For $k_3 = 5$ and $k_3 = 10$ the fourth and fifth natural frequencies are changed. Other natural frequencies are almost the same as the bare space frame. It can be concluded that increasing the stiffness of the spring increases the value of the natural frequencies.

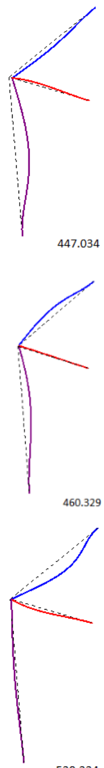


Figure 6. The first three natural frequencies and mode shapes of the bare space frame with different properties

TABLE 1. The values of natural frequencies of the coupled frame for different values of k_3

k_3	Mode number				
	1	**2**	**3**	**4**	**5**
1	287.177	447.034	460.329	538.334	680.248
5	447.034	460.327	538.334	640.338	680.724
10	447.034	460.328	538.334	680.089	902.86

In order to study the effect of the mass of the dynamic vibration absorber, Table 2 is used. The natural frequencies of the system are given for the properties of the previous example in Table 1, but for $k_3 = 5$ and different values of α and β. From Table 2, it is seen that increasing the mass of the dynamic vibration absorber decreases the value of the fundamental natural frequency. For instance, the value of the fundamental natural frequency decreases from 447.034 to 370.03, when α and β increase from 20 to 30.

Finally, the coupled frame with the properties of the bare frame in Figure 7 and $k_1 = 1$, $k_2 = 18$, $k_3 = 8$, $k_4 = 2$, $k_5 = 15$, $k_6 = 12$, $\alpha = 5$ and $\beta = 30$ are considered. The first eight mode shapes of the systems are depicted in Figure 7.

TABLE 2. The values of natural frequencies of the coupled frame $k_3 = 5$ and different values of α and β.

α and β	Mode number				
	1	2	3	4	5
10	447.034	460.327	538.334	640.338	680.724
20	447.034	453.11	460.372	538.334	680.335
30	370.03	447.034	460.333	538.334	680.314

Figure 7. The first three natural frequencies and mode shapes of the coupled space frame with different properties

Comparing Figures 6 and 7, it can be concluded that the natural frequencies of the coupled system are significantly changed. These results show that one can obtain the optimum value of the natural frequencies of the system by adjusting the values of the system parameters.

8. CONCLUDING REMARKS

Exact solutions for free vibration analysis of space frames with clamped and free ends joined by a dynamic vibration absorber are proposed in this article. A coupled three-dimensional formulation, including the effects of axial deformation, torsional deformation, in- and out-of-plane bending is carried out. This action leads to a governing boundary value or an eigenvalue problem. The mathematical model includes eighteen differential equations and thirty six boundary and compatibility conditions. All of mentioned formulations are derived in detail. Furthermore, a general finite element solution is presented and dynamic properties of the space frame are found. The natural frequencies and three-dimensional mode shapes of the system are calculated for different values of member mechanical and geometrical properties. The authors' results can be used as a benchmark problem to study the free vibration analysis of space frames. It is shown that increasing the stiffness of the dynamic vibration absorber increases the values of the natural frequencies of the coupled frame. Moreover, increasing the mass or mass moment of inertia of the dynamic vibration absorber decreases the values of natural frequencies of the space frame. The optimum value for the natural frequencies can be obtained by tuning the values of the properties of the dynamic vibration absorber, such as, stiffness of springs or mass.

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Free Vibration Analysis of a Six-degree-of-freedom Mass-spring System Suitable for Dynamic Vibration Absorbing of Space Frames

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چکیده

در این پژوهش، بسامدهای طبیعی و شکل‌های حالت نوسانی یک قاب فضایی ساده درگیر با یک جذب کننده‌ی نوسان پویا به‌دست می‌آیند. جذب کننده‌ی نوسان پویای با یک سامانه‌ی شش درجه‌ی آزادی جرم‌ـفنر الگو می‌شود. برای نخستین بار، نوسان آزاد یک سازه‌ی کش‌سان با یک سامانه‌ی شش درجه آزادی بررسی خواهد شد. هر عضو قاب ویژگی‌های مکانیکی و هندسی یک‌نواخت دارد، اما این ویژگی‌ها می‌تواند نسبت به دیگری متفاوت باشد. پاسخ دقیق این مسأله‌ی پیچیده با راه‌کار تحلیلی و شیوه‌ی جزءهای محدود در دسترس قرار می‌گیرد. اثرهای تغییرشکل‌های محوری و پیچشی به همراه خمش درون صفحه و برون صفحه به‌کار می‌روند. رابطه‌سازی مسأله دارای هجده معادله‌ی دیفرانسیل و سی و شش شرط مرزی و سازگاری است. با مقایسه‌ی پاسخ‌های دو روش، درستی شیوه‌های پیشنهادی نویسندگان آشکار می‌گردد. این نتیجه‌ها می‌توانند به عنوان سنگ نشانه برای بسامدهای طبیعی و شکل‌های حالت نوسانی قاب‌های فضایی با جذب کننده‌ی نوسان پویا استفاده شوند.

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