



## Effect of Magnetic Field on the Rotating Flow in a Similar Czochralski Configuration

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## P A P E R I N F O

## Paper history:

Received 02 November 2015

Received in revised form 30 March 2016

Accepted 14 April 2016

## Keywords:

Rotating Flow

Czochralski

Co-Counter-Rotating Mixed Convection

Magnetic Field

## A B S T R A C T

We present a numerical study of the rotating flow generated by two rotating disks in co-/counter-rotating, inside a fixed cylindrical enclosure similar to the Czochralski configuration (Cz). The enclosure having an aspect ratio  $A = H/R_c$  equal to 2, filled with a low Prandtl number fluid ( $Pr = 0.011$ ), which is submitted to a vertical temperature gradient. The finite volume method has been used to solve numerically the governing equations of the studied phenomenon. We present the steady state flow; and make a comparison between the flow generated by the co-/counter-rotating end disks. This study was carried out for different Richardson numbers;  $Ri = 0.01, 0.1, 0.5, 1, 2, 3, 5$  and  $10$ . The effect of orientation of the magnetic field is also taken into account for different values of the Hartmann number ( $Ha = 0, 5, 10, 20, 30$  and  $50$ ). The obtained results show that the strongest stabilisation of the velocity field and heat transfer occurs when the flow generated by co-rotating end disks and the applied of magnetic field in radial direction provided a more stabilisation of the convective flow.

doi: 10.5829/idosi.ije.2016.29.04a.16

## NOMENCLATURE

$A$	aspect ratio $A = H/R_c$	$Ri$	Richardson number
$B$	intensity of magnetic field, Tesla	$T$	temperature, K
$B_r$	radial magnetic field, Tesla	$U_r$	dimensionless radial velocity
$B_z$	axial magnetic field, Tesla	$U_z$	dimensionless axial velocity
$C_p$	specific heat of liquid, $J \cdot kg^{-1} \cdot K^{-1}$	$U_\theta$	dimensionless tangential velocity
$F_{LR}$	dimensionless Lorentz force in $R$ -direction	<b>Greek Symbols</b>	
$F_{LZ}$	dimensionless Lorentz force in $Z$ -direction	$\alpha$	thermal diffusivity, $m^2 \cdot s^{-1}$
$F_{L\theta}$	dimensionless Lorentz force in $\theta$ -direction	$\beta$	thermal expansion coefficient, $K^{-1}$
$g$	gravitational acceleration, $m \cdot s^{-2}$	$\delta$	orientation of the magnetic field, $^\circ$
$Gr$	Grashof number	$\Theta$	dimensionless temperature
$H$	height of the cylinder, m	$\lambda$	thermal conductivity, $W \cdot m^{-1} \cdot K^{-1}$
$Ha$	Hartmann number	$\rho$	density of the fluid, $kg \cdot m^{-3}$
$Nu$	local Nusselt number	$\sigma$	electric conductivity, $\Omega^{-1} \cdot m^{-1}$
$Nu_{moy}$	average Nusselt number	$\Omega$	angular velocity, $rad \cdot s^{-1}$
$P$	dimensionless pressure	$\Phi$	dimensionless electric potential
$Pr$	Prandtl number	$\nu$	kinematic viscosity, $m^2 \cdot s^{-1}$
$R, Z$	dimensionless meridional coordinates	<b>Subscripts</b>	
$R_c$	radius of the cylinder, m	c	cold
$Re$	Reynolds number	h	hot

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## 1. INTRODUCTION

Rotating flows with heat transfer in an enclosed cylinder driven by the rotation of the end wall and vertical gradient of temperature have been investigated by researchers for many years. This type of flows is presented in many practical situations as rotational viscosimeters, centrifugal machinery, pumping of liquid metals at high melting point and crystal growth from molten silicon in Cz crystal pullers [1]. The state of the art in this field has been summarized in [2, 3]. The first experiments by Vogel [4] and later by Escudier [5] showed the formation of a concentrated vortex core along the center axis. Based on their pioneering works, a lot of experimental and numerical studies have later been carried out. Among those, Spohn et al. [6] used the visualization techniques of electrolytic precipitation and showed that vortex breakdown bubbles in the container flow are open and asymmetric at their downstream end in the regime of axisymmetric bubbles. Recently, Sotiropoulos et al. [7] used planar laser induced fluorescence technique and showed the existence of chaotic behavior in the Lagrangian transport within the bubbles for flows that are steady from the Eulerian standpoint. In computations, there are numerous investigations of the container problem, for example, Lopez [8], Brown and Lopez [9]. All these studies were able to capture the flow evolution observed in the laboratory. They found that, the heat transfer rate increases with an increase of  $Re$  but it decreases with an increase of  $Ha$ . Rahimi and Yousefi [10] presented the transient laminar fluid motion and heat transfer resulting from the impulsive heating or cooling of a vertical surface in a quiescent fluid. Sheikhzadeh et al. [11] investigated numerically the effect of magnetic field on natural convection. Magnetic fields are crucial in non-intrusive flow control of electrically conducting melts in various physics of industrial significance, MHD pumps, and plasma and fusion technology. Rahmamezhad et al. [12] investigated numerically the effects of a magnetic field on mixed convection of  $Al_2O_3$ -water nanofluid in a square lid-driven cavity. Continuous casting is this kind of process using different magnetic field configurations to control the flow of steel in the mold and further to promote efficiency and minimize defects [13]. For example, static [14], rotating and travelling magnetic fields [15, 16] provide attractive means for braking, stirring and mixing. When a magnetic field is applied on flow field, it induces a current and the interaction between this current and the magnetic field will generate a Lorentz force, this latter alters the flow field. In case of swirling flow under a magnetic field, the phenomena are commonly seen in metallurgical MHD, such as in solidified ingot. The swirling flows usually maintain a stable laminar flow and a second flow, and are named

Ekman pumping [17, 18]. The second flow is superimposed on the primary swirling flow. Ben Hadid et al. [19] studied the 3D oscillatory flow of a conducting molten metal confined in a cylindrical cavity. They examined the temporal signals, the properties of symmetry and the assessments of energy characterizing oscillatory flow, and the damping of the flow oscillatory by a vertical magnetic field until the stabilization of this flow. Talmage et al. [20] studied the rotational movement of the molten metal during the production of crystals of silicon by the Cz technique with a high, uniform, and vertical magnetic field. MHD flow of molten Gallium during phase change in 3D enclosure was investigated [21]. In an enclosed cylinder [22] they simulated and studied the influences of conductivity of the upper lid, lower base and side wall were investigated at  $Ha = 100$  and  $Re = 100$ . The analytical solutions of the MHD flows for different electrical conductivity were presented and compared with the numerical results in [23]. Davidson et al. [24] studied Ekman pumping and Bödewadt-Hartmann layers in an axisymmetric pool and an open axisymmetric system, respectively. To the best of our knowledge, the effect of axial and radial magnetic field applied separately in rotating flow generated by the co/counter rotating end disks of the enclosed cylinder have never been studied, except for the case treated recently by Mahfoud and Bessaih [25], but with axial magnetic field applied. Therefore, the objective of this work is to study the effect of magnetic field orientations on rotating flow. This study was carried out for  $Ri \leq 10$ , and for different intensities of magnetic field ( $Ha = 0, 5, 10, 20, 30$  and  $50$ ).

## 2. PROBLEM DESCRIPTION AND MATHEMATICAL FORMULATION

The physical system under consideration Fig. 1, is a cylindrical enclosure has a radius  $R_c$  and height  $H$ , the aspect ratio is fixed ( $A = H/R_c = 2$ ). The enclosure contains a liquid silicon ( $Pr = 0.011$ ), the bottom disk is rotating with a constant angular velocity ( $+\Omega$ ), and maintained at a hot temperature  $T_h$ , while the top disk is in co-/counter-rotating and maintained at a cold temperature  $T_c$  (we called co-rotating flow when the two end disks rotate in the same direction with  $\Omega$ , and counter-rotating flow for the opposite directions). The side wall of the cylinder is adiabatic. The system may be subjected to a magnetic field of constant magnitude  $B$ , oriented in both axial and radial directions, separately. Adopting the assumptions of [25], and introducing the scales  $R_c$  for lengths,  $\Omega R_c$  for velocities,  $\rho(\Omega R_c)^2$  for pressure,  $(T_h - T_c)$  for temperature, and  $\Omega R_c^2 B$  for electric potential, the dimensionless equations of the system become:

$$\frac{1}{R} \frac{\partial}{\partial R}(RU_r) + \frac{\partial U_z}{\partial Z} = 0 \tag{1}$$

$$\frac{1}{R} \frac{\partial}{\partial R}(RU_r^2) + \frac{\partial}{\partial Z}(U_r U_z) + \frac{U_\theta^2}{R} = -\frac{\partial P}{\partial R} + \tag{2}$$

$$\frac{1}{\text{Re}} \left[ \frac{1}{R} \frac{\partial}{\partial R} (R \frac{\partial U_r}{\partial R}) + \frac{\partial^2 U_r}{\partial Z^2} - \frac{U_r}{R^2} \right] + \frac{Ha^2}{\text{Re}} F_{LR}$$

$$\frac{1}{R} \frac{\partial}{\partial R}(RU_r U_z) + \frac{\partial}{\partial Z}(U_z^2) = -\frac{\partial P}{\partial Z} + \tag{3}$$

$$\frac{1}{\text{Re}} \left[ \frac{1}{R} \frac{\partial}{\partial R} (R \frac{\partial U_z}{\partial R}) + \frac{\partial^2 U_z}{\partial Z^2} \right] + \text{Ri} \Theta + \frac{Ha^2}{\text{Re}} F_{LZ}$$

$$\frac{1}{R} \frac{\partial}{\partial R}(RU_r U_\theta) + \frac{\partial}{\partial Z}(U_z U_\theta) + \frac{U_r U_\theta}{R} = \frac{Ha^2}{\text{Re}} F_{L\theta} + \tag{4}$$

$$\frac{1}{\text{Re}} \left[ \frac{1}{R} \frac{\partial}{\partial R} (R \frac{\partial U_\theta}{\partial R}) + \frac{\partial^2 U_\theta}{\partial Z^2} - \frac{U_\theta}{R^2} \right]$$

$$\frac{1}{R} \frac{\partial}{\partial R}(RU_r \Theta) + \frac{\partial}{\partial Z}(U_z \Theta) = \frac{1}{\text{Pr}} \left[ \frac{1}{R} \frac{\partial}{\partial R} (R \frac{\partial \Theta}{\partial R}) + \frac{\partial^2 \Theta}{\partial Z^2} \right] \tag{5}$$

where,  $F_{LR}$ ,  $F_{LZ}$  and  $F_{L\theta}$  are the Lorentz force component in  $R$ ,  $Z$  and  $\theta$  directions, respectively. Thier expressions were given as following:

- Case of axial magnetic field ( $B_z$ ) :

$$F_{LR} = -U_r, F_{LZ} = 0 \text{ and } F_{L\theta} = \frac{\partial \Phi}{\partial Z} - U_\theta \tag{6a}$$

- Case of radial magnetic field ( $B_r$ ) :

$$F_{LR} = 0, F_{LZ} = -U_z \text{ and } F_{L\theta} = -\frac{\partial \Phi}{\partial Z} - U_\theta \tag{6b}$$

The appear flow parameter in the dimensionless mathematical model are: Reynolds number  $\text{Re} = \Omega R^2 / \nu$ , Richardson number  $\text{Ri} = Gr / \text{Re}^2$ , Grashof number  $Gr = g\beta(T_h - T_c)R^3 / \nu^2$ , Prandtl number  $\text{Pr} = \nu / \alpha$  and Hartmann number  $Ha = BR\sqrt{\sigma / \rho\nu}$  which indicate the ratio between the electromagnetic force and viscosity force.

The boundary conditions are given as following:

At:  $R = 0$  and  $0 \leq Z \leq 2$  :

$$U_r = U_\theta = \frac{\partial U_z}{\partial R} = 0, \frac{\partial \Theta}{\partial R} = 0, \frac{\partial \Phi}{\partial R} = 0 \tag{7a}$$

At:  $R = 1$  and  $0 \leq Z \leq 2$  :

$$U_r = U_\theta = U_z = 0, \frac{\partial \Theta}{\partial R} = 0, \frac{\partial \Phi}{\partial R} = 0 \tag{7b}$$

At:  $Z = 0$  and  $0 \leq R \leq 1$  :

$$U_r = U_z = 0, U_\theta = R, \Theta = 1, \frac{\partial \Phi}{\partial R} = 0 \tag{7c}$$

- Co-rotation case

At:  $Z = 2$  and  $0 \leq R \leq 1$  :

$$U_r = U_z = 0, U_\theta = R, \Theta = 0, \frac{\partial \Phi}{\partial R} = 0 \tag{7d}$$

- Counter-rotation case

At:  $Z = 2$  and  $0 \leq R \leq 1$  :

$$U_r = U_z = 0, U_\theta = -R, \Theta = 0, \frac{\partial \Phi}{\partial R} = 0 \tag{7e}$$

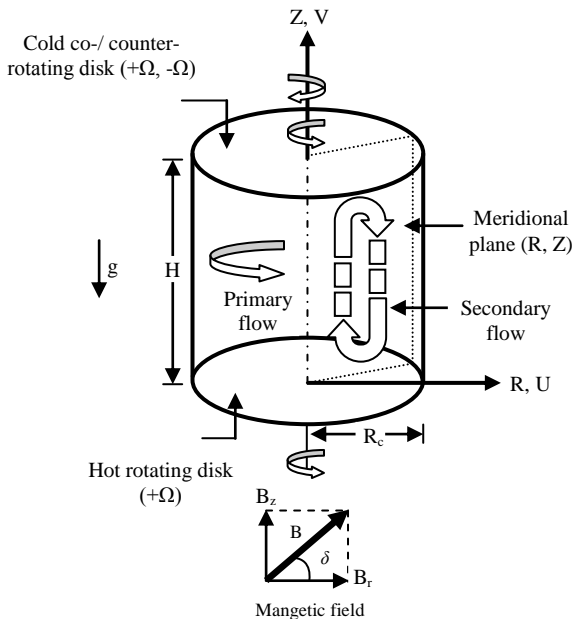


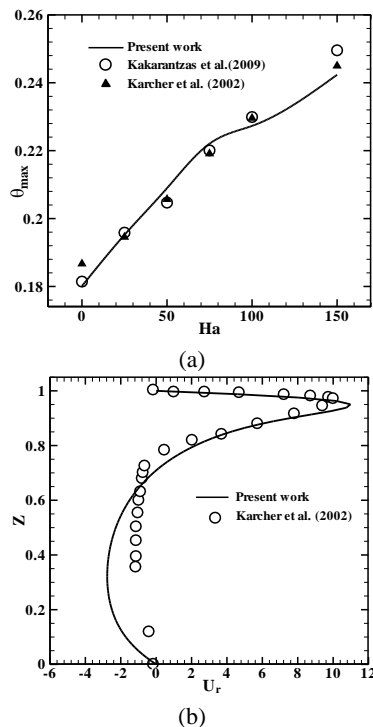
Figure 1. Sketch of physical problem.

### 3. NUMERICAL METHOD AND CODE VALIDATION

The mathematical model constructed by the Equations (1) to (5) and associated boundary conditions (Equations (7a) to (7e)) are resolved using the finite volume method by Ziapour et al. [26]. The scalar variables ( $\Theta$  and  $P$ ) are stored at the middle of the volume; moreover the vector variables are stored at the faces of the control volume. The diffusion and convection terms are discretized by a second-order central difference scheme, and the SIMPLER Algorithm [27] is used to solve the coupled velocity-pressure. The Acquired algebraic equations are solved by TDMA. The convergence of computation was checked by the differences in flow parameter between two consecutive iteration that must be less than  $10^{-5}$ . The used grid is not regular, we chosen the space increment according to geometric progressions as the same to the work of Atia et al. [28]. This grid refinement allows controlling the Hartmann layer developed at near the walls caused by

strong gradients of velocity and temperature. So, thus requiring a great number of nodes to solve the specific characteristics of the MHD flow, also in order to reduce the numerical errors [29]. To examine the effect of grid size on the numerical results, several grids were studied such as:  $50 \times 100$ ,  $60 \times 120$ ,  $80 \times 160$  and  $100 \times 200$ . We consider the case of a mixed convection flow with  $Ri = 2$ . The computational results we directing to choice the grids  $80 \times 160$ , since the time of program execution for those of the grid ( $100 \times 200$ ) high than the chosen grid (the same computation domain was made in[25]).

To develop a constructive and objective comparison of the obtained results by our numerical simulations with experimental data, and to give interpretations to the observed phenomena, it is useful to validate our work. Firstly, a comparison with the results of Kakarantzas et al. [30] and Karcher et al. [31] was made. The author used a cylindrical enclosure have an aspect ratio equal to  $A = 4.125$ , and a fluid have  $Pr = 0.0203$ . The upper wall is heated electrically, moreover the bottom and lateral wall cooled by cycle water. The maximum value of the temperature versus  $Ha$  is presented in Fig. 2a. A good agreement between the obtained and reported results was observed. Secondly, a comparison of the radial velocity component  $U_r$  at  $R = 0.25$ , with experimental measurements obtained by Karcher et al. [31] for  $Ha = 0$  (Figure 2b) has been carried out. It is clear that the computed values can be seen to be in excellent agreement with the data measurements.



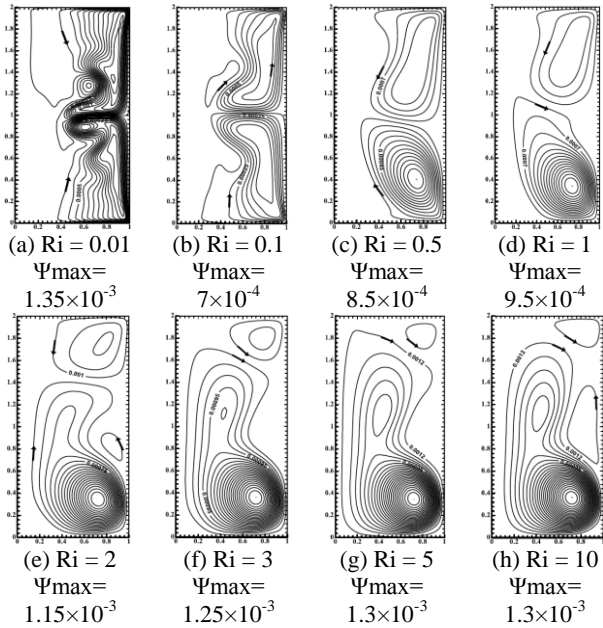
**Figure 2.** Comparison of our results with those of (a) Kakarantzas et al. [30] and (b) Karcher et al. [31].

## 4. RESULTS AND DISCUSSION

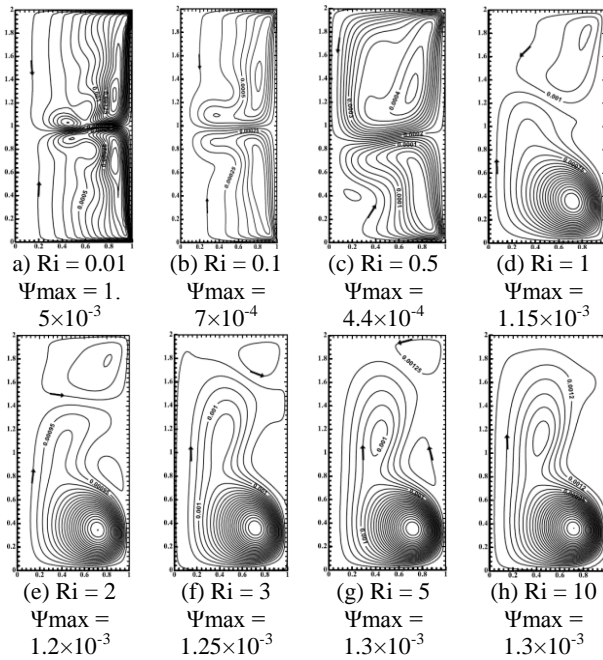
**4. 1. Rotating Flow Without Magnetic Field** We are interested herein to the behavior of the flow without magnetic field ( $Ha = 0$ ). Equations (1) to (5), are solved numerically, without taking into account the Lorentz forces  $F_{LR}$ ,  $F_{LZ}$ ,  $F_{L\theta}$  in the Navier–Stokes equations.

**4. 1. 1. Co-rotation Case** By carefully examining the results presented in Figure 3, it is noted that for all  $Ri$  the flow is multicellular. Particularly for  $Ri \leq 1$ , the flow is bi-cellular, we note also the presence of the Eckman layer in the area under the rotating disks is characterized by the large mass flow. These cells form opposing structures and the fluid in the bulk of the cylinder rotates almost rigidly with an intermediate velocity of the end disks, and due to the forced convection created by the rotation of the disks which dominates in this range. An inter-disk shear flow is created. These results are in good agreement with those of Omi and Iwatsu [32] with the creation of the meridional circulation. We note also that for the co-rotating case the stream function contours are nearly symmetrical about the mid-plane  $Z=1$ . When increasing  $Ri$  ( $Ri > 1$ ) we can see that the cell background is increasing in size, either in intensity until the appearance of a secondary cell rotating within the main flow, by against the cell at the top is reduced until the occurrence of another recirculation zone adjacent to the side wall.

**4. 1. 2. Counter-rotation Case** Figure 4 shows the dimensionless streamlines for various  $Ri$  for the case of counter-rotating end disks. Almost the same comment as the disks rotates in co-rotation. We note that for all  $Ri$  the flow is multicellular, there is also an opposite flow than the first case of co-rotating, wherein for  $Ri \leq 0.5$ , we can see a symmetric situation. In which, each disk tends to cause a half thickness of fluid relative angular velocity ( $+\Omega$ ,  $-\Omega$ ). For  $Ri > 0.5$ , we note that the symmetry of the base flow is gone, and Ekman layers formed on the two rotary disks, a cell high capacity located near bottom wall, followed by a small cell near upper wall with the appearance of a side of the side wall cell (cells detachment) for  $Ri = 2$  and  $Ri = 5$ , these emerging cells due to the acceleration of velocity of the fluid particles. For  $Ri > 0.5$ , we note that the symmetry of the base flow is gone, and the Ekman layers formed on the two rotary disks, a cell high capacity located near bottom wall, followed by a small cell near upper wall with the appearance of a side of the side wall cell (cells detachment) for  $Ri = 2$  and  $Ri = 5$ , these emerging cells due to the acceleration of velocity of the fluid particles.



**Figure 3.** Streamlines in R-Z plane, for different values of Ri. Case of co-rotating disks (without magnetic field).



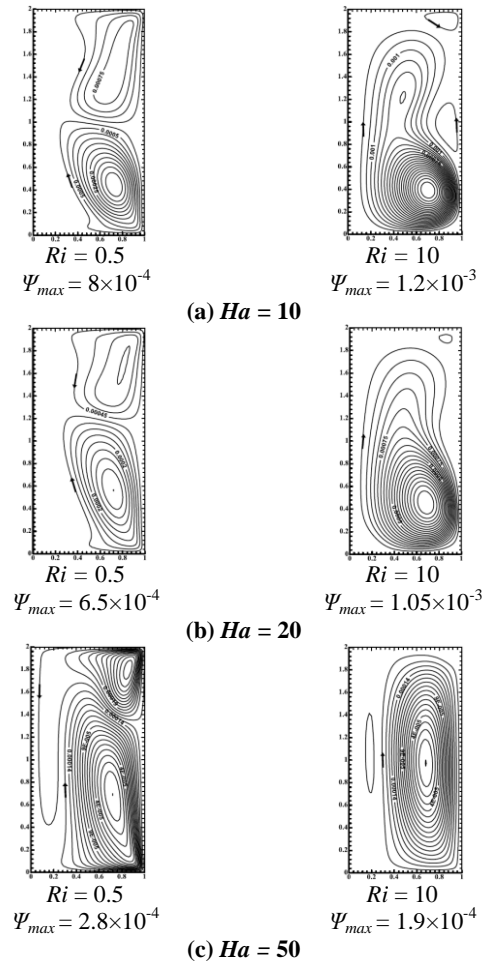
**Figure 4.** Streamlines in R-Z plane, for different values of Ri. Case of counter-rotating disks (without magnetic field).

**4. 2. Rotating Flow With Magnetic Field** The application of a magnetic field is known about the stability of convective flows [19, 33]. If the electrical conducting fluid subjected to the magnetic field, the Lorentz force also was activated (the heat convection was provided by the combined action of buoyancy and the Lorenz force). This letters interrupted with the buoyancy force and change the heat and flow field. The

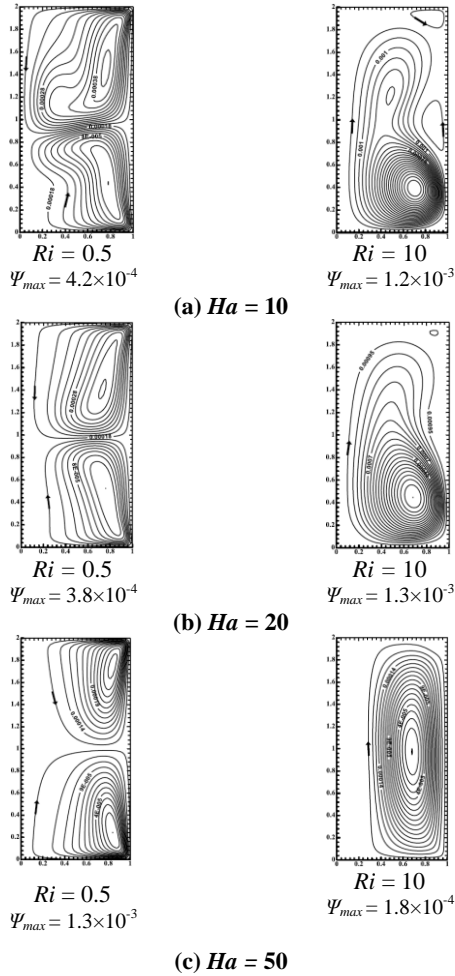
magnetic field intensity and its orientation are the important factor they be affected on the velocity flow and the heat transfer rate.

**4. 2. 1. Case of Co-rotating Disks Under the Axial Magnetic Field ( $\delta = 90^\circ$ )**

We fixed the magnetic field orientation at the angle  $\delta = 90^\circ$ . In Figure 5 (a-c), we present the streamlines for  $Ri = 0.5$  and 10 respectively, and for different Ha. For low magnetic field ( $Ha = 10$ ), it is clear that the relevant figure shows no change in the flow structure. But from  $Ha = 20$ , the latter starts to change, by the expansion of the cells which is a clear and significant change gradually as Ha increases. For  $Ri = 0.5$ , we note the presence of the Ekman layer on the two rotating disks, such that the main cell near hot lower rotary disk occupies almost all of the cylinder volume. For  $Ri = 10$ , the flow occurs more organized than the increasing of magnetic field intensity eliminates the small cells which nascent recirculation due to the acceleration of fluid particles. These changes due to electromagnetic forces .



**Figure 5.** Streamlines in R-Z plane, for different values of Hartmann number ( $Ri = 0.5$  and 10). Case of co-rotating disks (The magnetic field is applied in axial direction).



**Figure 6.** Streamlines in  $R$ - $Z$  plane, for different values of Hartmann ( $Ri = 0.5$  and  $10$ ). Case of counter-rotating disks (The magnetic field is applied in axial direction).

#### 4. 2. 2. Case of Counter-rotating Disks Under the Axial Magnetic Field ( $\delta = 90^\circ$ )

In Figure 6 (a-c), we presented the streamlines for  $Ri = 0.5$  and  $10$  under the axial magnetic field,  $Ha = 10$ . We note that the flow structure has undergone a slight variation despite that we have increased  $Ha$  of at  $10$ , the flow structure begins to change from  $Ha$  is set  $Ha = 20$ , by the expansion of the cells which is a clear and significant change gradually as  $Ha$  increases, Indeed,  $Ha = 50$  the flow is more regular. We can connect this deformation to increasing of  $Ha$ , so the flow becomes more organized by the electromagnetic force which affects the fluid particles. It is notified also the presence of the Ekman layer for  $Ri = 0.5$  located on the two rotating disks and the fluid in which each disk tends to cause a fluid to split-thickness relative angular velocity, at sufficiently high velocity the shear layer between the disk may become unstable, and wrap according vortex lines oriented radially the magnetic fields will be able to absorb the disturbance (Omi and Iwatsu[32]). To see a

more information about the intensity of axial magnetic field  $B_z$  on the co-/counter-rotating flow, we have summarized in the Table 1.

#### 4. 2. 3. Case of Co-rotating Disks Under the Radial Magnetic Field ( $\delta = 0^\circ$ )

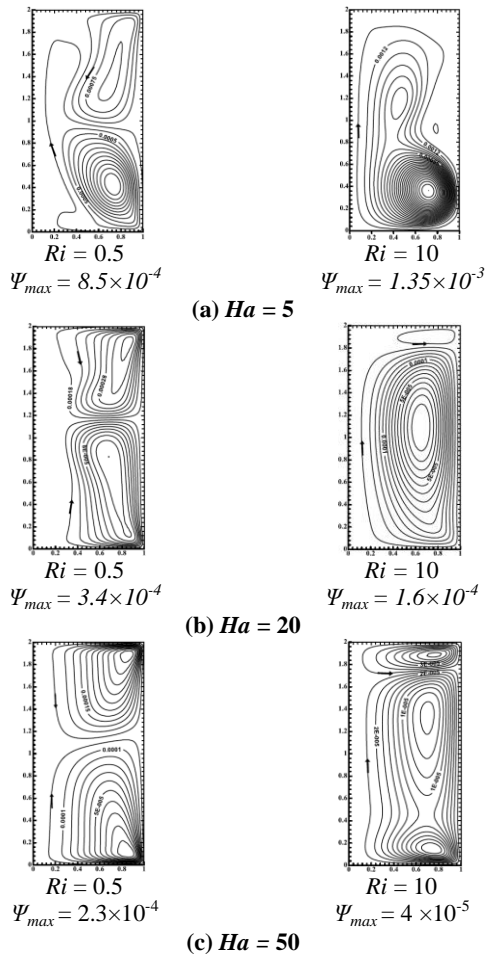
In Figure 7 (a-c), we present the streamline for  $Ri = 0.5$  and  $10$ , and for different  $Ha$  (radial magnetic field). The change in the flow structures is started from  $Ha = 5$ ; the cells are expanded gradually with increasing  $Ha$ . Then, for  $Ha \geq 20$  and  $Ri = 0.5$  the flow becomes symmetric with respect to the mid-plane  $Z = 1$ . However, for  $Ha \geq 20$  and  $Ri = 10$  we can see a reverse flow where the lower main cell increases gradually as higher cell and occupied the most volume of the cavity at  $Ha = 50$ . These changes show the magnetic stability. In addition, the effect of the radial magnetic field on the flow is more reliable than the axial magnetic field where we see a more different structure stable.

#### 4. 2. 4. Case of Counter-rotating Disks Under the Radial Magnetic Field ( $\delta = 0^\circ$ )

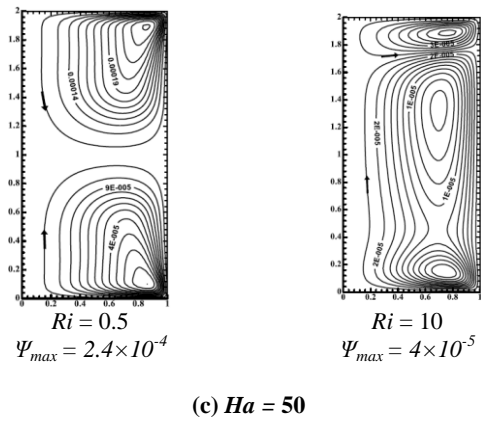
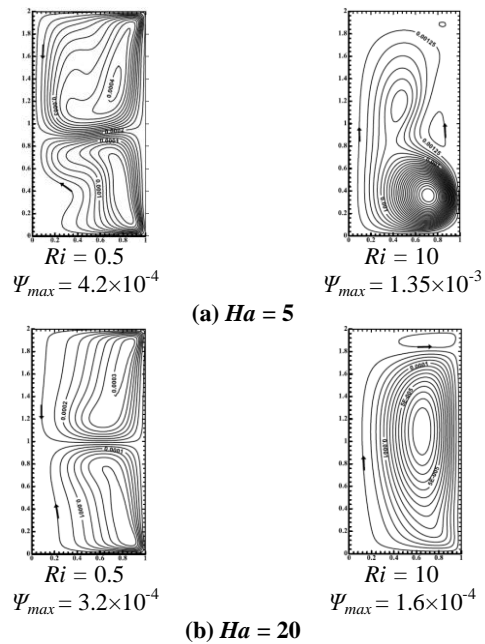
For the case of counter-rotating disks in the presence of radial magnetic field, we present the streamlines for  $Ri = 0.5$  and  $10$ , and for different  $Ha$  (Figure 8 (a-c)). It is notified that a small variation on the flow structure is done when increases  $Ha$  at  $5$  the clear change on the flow structure starts from  $Ha = 20$  by expansion of cells (size/intensity) which is a significant change with successive increase of  $Ha$ . For  $Ri = 10$ , these cells are occupied almost the entire volume of the cavity. Noting that the effect of the radial magnetic field on the flow is lower for the  $Ri$  increases ( $Ri = 10$ ) which shows an important effect of the magnetic field on flows dominated by natural convection ( $Ri > 1$ ). Table 2, present the flow characteristic for different  $Ha$ . The magnetic field is applied in radial direction  $B_r$  ( $Ri = 0.5$  and  $10$ ). We can conclude that the increase of  $Ha$  provides the decrease in the stream function and also for average Nusselt number. So, the rotating flow stabilizes when the intensity of radial magnetic field increased.

**TABLE. 1.** Flow characteristic for different  $Ha$  in two case co-/ counter-rotating disks (axial magnetic field  $B_z$ ).

Parameters		$\Psi_{\max}$		$Nu_{\text{moy}}$	
$Ha$	$Ri$	Co-rotating	Counter-rotating	Co-rotating	Counter-rotating
0	0.5	$4.4 \times 10^{-4}$	$8.5 \times 10^{-5}$	1.419	1.422
	10	$1.3 \times 10^{-3}$	$1.3 \times 10^{-3}$	1.292	1.296
5	0.5	$8.5 \times 10^{-4}$	$4.2 \times 10^{-4}$	1.412	1.420
	10	$1.3 \times 10^{-3}$	$1.3 \times 10^{-3}$	1.290	1.292
10	0.5	$8 \times 10^{-4}$	$4.2 \times 10^{-3}$	1.404	1.416
	10	$1.2 \times 10^{-3}$	$1.2 \times 10^{-3}$	1.281	1.289
20	0.5	$6.5 \times 10^{-4}$	$3.8 \times 10^{-4}$	1.397	1.411
	10	$1.05 \times 10^{-3}$	$1.05 \times 10^{-3}$	1.276	1.285
50	0.5	$2.8 \times 10^{-4}$	$2.5 \times 10^{-4}$	1.367	1.399
	10	$9 \times 10^{-4}$	$1.8 \times 10^{-4}$	1.250	1.269



**Figure 7.** Streamlines in  $R$ - $Z$  plane, for different values of the Hartmann number ( $Ri = 0.5$  and  $10$ ). Case of co-rotating disks (The magnetic field is applied in radial direction).



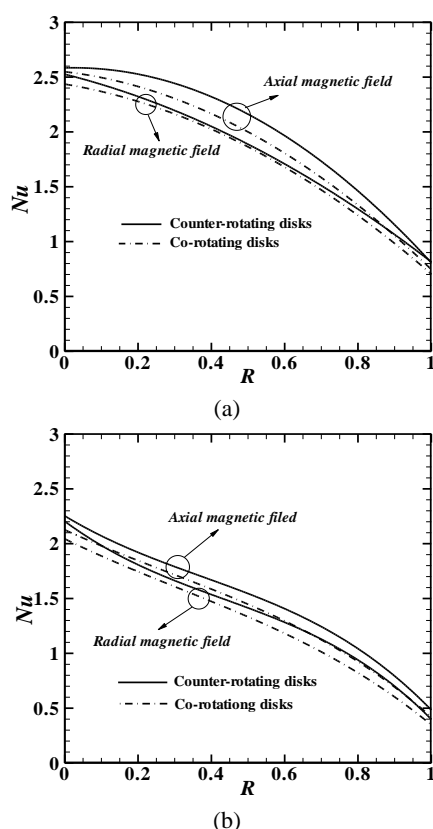
**Figure 8.** Streamlines in  $R$ - $Z$  plane, for different values of the Hartmann ( $Ri = 0.5$  and  $10$ ). Case of counter-rotating disks (The magnetic field is applied in radial direction).

**TABLE 2.** Flow characteristic for different  $Ha$  in two case co-/ counter-rotating case (radial magnetic field  $B_r$ ).

Parameters		$\Psi_{max}$		$Nu_{moy}$	
$Ha$	$Ri$	Co-rotating	counter-rotating	Co-rotating	counter-rotating
0	0.5	$4.4 \times 10^{-4}$	$8.5 \times 10^{-5}$	1.419	1.422
	10	$1.3 \times 10^{-3}$	$1.3 \times 10^{-3}$	1.292	1.296
5	0.5	$8.5 \times 10^{-4}$	$4.2 \times 10^{-4}$	1.411	1.420
	10	$1.35 \times 10^{-3}$	$1.35 \times 10^{-3}$	1.290	1.291
10	0.5	$6.5 \times 10^{-4}$	$3.8 \times 10^{-4}$	1.402	1.415
	10	$1.1 \times 10^{-3}$	$1.1 \times 10^{-3}$	1.278	1.287
20	0.5	$3.4 \times 10^{-4}$	$3.2 \times 10^{-4}$	1.391	1.408
	10	$1.6 \times 10^{-4}$	$1.6 \times 10^{-4}$	1.272	1.280
50	0.5	$2.3 \times 10^{-4}$	$2.4 \times 10^{-4}$	1.360	1.390
	10	$4 \times 10^{-5}$	$4 \times 10^{-5}$	1.243	1.261

**4. 2. 5. Comparison Between Two Cases Co-/Counter-rotating Disks With Magnetic Field**

In Table 2, we made comparison between two cases. We can see that, the increase of  $Ha$  provides the decrease of the flow intensity in the two cases, but more important in the counter-rotating disks. We can also see the application of  $B_r$  can slow better than the application of  $B_z$  (Figure 9). This is for  $Ri = 0.5$  and  $10$ , and for two orientations of magnetic field ( $Ha = 50$ ). We can see that the application of the magnetic field causes a decrease in the local Nusselt number, and the best stabilization is found for  $B_r$ .



**Figure 9.** Effect of magnetic field orientation ( $Ha = 30$ ) on the local Nusselt number: (a)  $Ri = 0.5$ , (b)  $Ri = 10$ .

## 5. CONCLUSIONS

A numerical study of the rotating flow generated by two rotating disks in co-/counter-rotating direction, inside a cylindrical enclosure similar to Cz configuration was carried out. Comparisons with published work on specific cases are performed and found to be in good agreement. From the obtained results, the following conclusions are drawn:

- The increase of  $Ri$  affects straightly on the structure of the flow in both cases co-/counter-rotation, wherever the velocity field and heat transfer rate are destabilized.
- The comparison between the flow generated by the co-rotating and counter-rotating disks shows that the strongest stabilization of the velocity field and heat transfer occurs when the flow generated by co-rotating end disks.
- In the presence of the magnetic field, the flow is more regular. An increase in the intensity of the magnetic field causes a change in the magnitude and flow structure, where is stabilize the velocity field and heat transfer rate.
- The magnetic field stabilization is important for the case of dominated natural convection ( $Ri > 1$ ).

- A more reduction of the velocity field and heat transfer rate is important when the radial magnetic field is applied ( $B_r$ ).

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## Effect of Magnetic Field on the Rotating Flow in a Similar Czochralski Configuration

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### P A P E R I N F O

چکیده

#### Paper history:

Received 02 November 2015  
Received in revised form 30 March 2016  
Accepted 14 April 2016

#### Keywords:

Rotating Flow  
Czochralski  
Co-Counter-Rotating Mixed Convection  
Magnetic Field

ما در حال حاضر، مطالعه عددی جریان چرخشی تولید شده را توسط دو دیسک چرخشی در چرخش همسو/ناهمسو، در داخل محوطه یک استوانه ثابت مشابه پیکربندی Czochralski (Cz) نشان می دهیم. محوطه دارای نسبت  $A = H / RC$  برابر با ۲، پر شده با سیالی با عدد پراوتل کوچک ( $Pr = 0.1$ ) می باشد که به گرادیان دما عمودی نسبت داده می شود. روش حجم محدود برای حل عددی معادلات حاکم پدیده تحت مطالعه استفاده شده است. ما جریان حالت پایدار را ارائه می دهیم؛ و مقایسه ای بین جریان تولید شده توسط دیسک های انتهایی با چرخش همسو/ناهمسو نشان می دهیم. این مطالعه برای عددهای مختلف ریچاردسون  $Ri$  برابر با ۰.۱، ۰.۱، ۰.۵، ۱، ۲، ۳، ۵ و ۱۰ انجام شد. اثر جهت میدان مغناطیسی نیز برای مقادیر مختلف عدد هارتمن  $Ha$  برابر با ۰، ۵، ۱۰، ۲۰، ۳۰ و ۵۰ به حساب می آید. نتایج به دست آمده نشان می دهد که قوی ترین تثبیت میدان سرعت و انتقال حرارت هنگامی رخ میدهد که جریان، دیسک های انتهایی چرخش byco تولید می کند و اعمال میدان مغناطیسی در جهت شعاعی، ثبات بیشتری از جریان همرفتی را ایجاد می کند.

doi: 10.5829/idosi.ije.2016.29.04a.16