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A New Model for Fleet Assignment Problem, Case Study of Iran Air Network at Vision 2036

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ABSTRACT

The fleet assignment problem (FAP) assigns the type of airplane at each flight segment. Since airplane is an expensive resource, poor fleet assignment can cause a great increase in costs of airlines. There are so many consideration which should be tackled in formulation of a FAP problem, especially the parameters regarding to the airplanes. This paper presents a novel, mixed integer programing for formulation of FAP problem. The aim of this paper is to assign most appropriate fleet type to flight while minimizing the cost and determining optimal number of plane for each fleet. The model guarantees that each pair of nodes is served in a limit of planes utilization time constraint. The model was applied in a real example. In this example the demand matrix between 69 airports in Iran was estimated for year 2036. Finally, the most appropriate fleet was determined in the outlook 2036. The results show that the airplane assignment by the proposed model will need less purchasing cost comparing to current fleet needs and total investment cost in air industry will be decreased so much.

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1. INTRODUCTION

The increase in transportation demand had made the public transportation more important. It is therefore vital to improve planning and scheduling of transportation in order to decrease costs. Air travel is a major and expensive mode of transportation which needs large investment. Due to existing of many specifications for every airplane, formulation of FAP problem faces with a high degree of complexity.

Planning of the airlines services have three steps including, Demand forecasting, Fleet assignment and Crew planning. Demand forecasting is to predict the volume of trips between two airports in a network. There are so many techniques to forecast, for example regression, time series and system dynamics [1].

The fleet assignment problem (FAP) is to determine the type of plane to assign to each flight segment based on equipment capabilities and availabilities, operational costs, and potential revenues [2]. In the FAP problem, the number of airplanes and utilization time are the limitation which should be considered. The FAP is crucial in airline planning because it reduces costs and investments. The crew planning includes strategies of airlines to engage and assign employment to airplane and airports [3]. This step is very important because the costs of crew is the second item of the airline cost profile after the fuel cost.

The problem of fleet assignment is one of the drastic and most comprehensive problems faced in airline planning. There are so many considerations which an airline should pay attention such as the variety of fleet with different purchasing cost, operating cost, capacity, speed, utilization time and so on. Hence a model with considering of parameters associated to the fleet is so useful and important for airplane scheduling and

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planning. In this study, we try to present a model which consider the airline concerns about fleet characteristics.

The all parameters which are applied in the proposed model did not used by researches in the literature. The proposed model selects appropriate number of airplane of each fleet and assigns them to flight legs in the network. It permits assignment of two or more fleet to a flight legs simultaneously. In this paper, main contributions can be mentioned as follows:

-Developing a new model for the FAP which considers fleet characteristics while in previous researches, fleet assignment is done inside of a predefined network with considering of less characteristics.

-Optimization of network structure and fleet assignment in an integrated approach.

Also, the strengths of this study can be mentioned as follows:

There are some concerns for the airlines for example selecting appropriate and optimum location of hub (connection) airports and considering fleet characteristics. Because the purchasing cost is a major element in the cost profile of airlines, characteristics associated to the fleet should be employed to assigning. Introducing of a decision method to be a useful tool for airlines scheduling and planning is major strength of this work.

Analysis of the work for a real case and its related analysis is another strength of this study.

The rest of this paper is organized as follow: A literature of research is discussed in section 2. Detailed description of the proposed model is described in Section 3. Numerical example are reported in Section 4. Finally, conclusion and direction for the future research are presented in the last section.

2. LITERATURE REVIEW

Efficient and appropriate formulation and solution for FAP have been discussed in the literature, and many researchers focused on this scope. Abara [4] was one of the first researchers to address realistically sized fleet assignment problems using a connection-based network structure. In his network, arrived and departed time of flight were considered. Then, Berge and Hopperstad [5] and Hane et al. [6], used this network structure in formulating the fleet assignment problem. Hane et al. [6] have proposed a series of preprocessing steps that aim to reduce the size of the network as well as the computational effort.

Rushmeier and Kontogiorgis [7] employed some preprocessing techniques to solve the problem more effectively without having to specify feasible connections for each flight .They also considered some additional crew-based side-constraints in their model. They designed a heuristic method to solve the problem in which the LP (linear programming) relaxation is first solved and the resulting solution is rounded to obtain an initial solution which is fed into a depth-first branch-and-bound process. Chung and Chung [8] attempted to solve the fleet assignment problem using genetic algorithm. They introduced modified operators to apply in genetic algorithm. A weekly fleet assignment model is presented by Kliwer and Tschoke [9].

They used a simulated annealing (SA) approach to deal with higher complexity. Belanger et al. [10] proposed a model for the periodic fleet assignment problem with time windows in which departure times are also determined.

In fleet assignment, profit is maximized by minimizing two types of costs: operational and spill costs [11]. Operational costs are those for flying the flight leg with the assigned aircraft type and usually include factors such as fuel and landing fees. Spill costs represent lost opportunity costs that arise if passenger demand exceeds the aircraft capacity and thus, potential revenue is lost [12]. Cacchiani and Salazar-González [13] considered fleet-assignment, aircraft-routing and crew-pairing problems of an airline flying and introduced a mathematical formulation based on binary variables. Finally, they applied a heuristic method to solve a real case. Ozdemier et al. [14] applied the general mathematical formulation of FAP to Turkish airlines and prepared an optimized set of airplanes for using in the Turkish network. Tavakkoli moghadam et al. [15] presented a new multi-period mathematical model and a solution procedure to optimize the railcar fleet size and freight car allocation, wherein car demands, and travel times, are assumed to be deterministic, and unmet demands are backordered.

Vaziri [16] assessed 123 variables to identify key characteristics reflecting air transport potentials in five sets namely, air transport, socio-economic status, population demography, geographical and environmental features and political features. Bazargan [17]introduced operating costs in the fleet assignment model, with passenger-spill costs, recapture rate flight cover and etc. An optimum solution is found in Bazargan`s study.

We refer the interested reader to several survey papers for more information on these problems; for instance, Gopalan and Talluri [18] for the general airline scheduling process; Barnhart et al. [19] for the crew scheduling problem; and Sherali and Bish [20] for concepts, models, and algorithms in FAP problems.

Table 1 presents previous researches on FAP model. In this table, researches on FAP models are categorized according to the models which have been introduced by previous researchers. It is worth to mention that the current study can be placed in the third category.

TABLE 1. A review on main categories, subjects and models on FAP

Main category	Subject	Model	Researchers
Basic fleet assignment models	Basic FAM using a connection network structure	Basic fleet assignment model using a connection network	Abara [4]
	Basic FAM using a time–space network structure	Basic fleet assignment model based on time–space network structure	Hane et al. [6]
Integrated fleet assignment models	FAM integrated with schedule design	Integrated leg selection and fleet assignment model	Lohatepanont and Barnhart [21]
	FAM integrated with maintenance, routing, and crew considerations	String-based fleet assignment and routing model	Barnhart et al. [22]
Fleet assignment models with additional considerations	FAM including passenger considerations	Path-based fleet assignment model	Barnhart et al. [23]
		Integrated leg selection and path-based fleet assignment model	Lohatepanont and Barnhart [21]
		Integrated scheduling and FAM with passenger demand considerations	Yan and Tseng [24]
	A weekly fleeting model	A weekly fleet assignment model with homogeneity	Be´langer et al. [10, 25]
Dynamic re-fleeting mechanisms based on updated demands		Integrated path-based re-fleeting model for family k	Sherali et al. [26]

3. PROBLEM FORMULATION

In this section, to assign fleet to the network, we propose a two steps procedure. Step 1 determines the network structure and step 2 determines the assigned fleet (FAP) for extracted routes.

Suppose the demand between nodes in a network (demand matrix) are given and the goal is to serve demands with minimum cost. We should select the appropriate fleet and the number of airplanes for each fleet to serve all demands in the each network. Airline should determine the arcs between nodes to build a new network with redistributed demands. So, as mentioned before the proposed method of this paper consists of two steps. First, we should construct the optimum arcs between demand points and as a second step we should assign airplanes for each arc. Figure 1 presents the procedure discussed above. In step 1 the airline determines the routes in the network, finally in the step 2 the efficient airplane assignment is associated to the demand arcs. The model which used for constructing network is P- hub median model while total transportation costs should be minimized. Also a proposed model is presented for the second step of the proposed approach.

3.1 P-hub median problem

After 1978 and simultaneous with liberalization in air industry of America, Hub network application became familiar for airlines. Before 1978, a network with the point to point links was main network in the world, because the all of the itineraries were determined by government and airlines had no option to select itineraries. After liberalization, the hub network was used by airlines that had drawn their own network according to the appropriate hubs. In this paper, according to the P-Hub median model, we select routes in the network. Table 2 presents characteristics of P-hub median problem. P-Hub median formulation is presented as follow [20]:

$$\min \sum_k F_k y_k + \sum_i \sum_j \sum_k \sum_m cc_{ijkm} w_{ij} x_{ijkm} + \sum_i \sum_j c_{ij} w_{ij} z_{ij} \tag{7}$$

$$S.T: \; z_{ij} + \sum_k \sum_m x_{ijkm} = 1 \qquad \forall i, j \quad i \neq j \tag{8}$$

$$\sum_k x_{ijkm} \le y_m \qquad \forall i, j, m \tag{9}$$

$$\sum_m x_{ijkm} \le y_k \qquad \forall i, j, k \tag{10}$$

$$\sum_{i} y_i = p \tag{11}$$

$$cc_{ijkm} = c_{ik} + \alpha c_{km} + c_{mj} \qquad \forall i, j, k, m \tag{12}$$

$$x_{ijkm}, y_k, z_{ij} \in \{0.1\} \tag{13}$$

Equation 7 as an objective function minimizes total cost of traveling on the network and hubs establishment costs. Equation 8 guarantees that each two cities either have a direct link or a link via hubs. In Equation 9, if city *m* was not selected as a hub, then there was not any link via it. Equation 10 specifies same limitation of Equation 3 for city *k*. Equation 11 ensures that the number of selected hubs by the model be equal to P. Equation 12 computes cost of travelling between cities *i* and *j* via hubs *k* and *m*. Equation 13 introduces binary variables.

TABLE 2. Characteristics of P-hub median problem

Sets	Indices	Parameters	Decision Variables
N includes all of the cities.	*i and j* = Index for cities, $i \,\&\, j \subseteq N$	c_{ij} = Cost of travel between cities *i* and *j*.	$x_{ijkm} = 1$, If city *i* connected to the city *j* across hubs *k* and *m*.
H includes potential hub in the network, $H \subseteq N$.	*k and m* = Index for hubs $k \,\&\, m \subseteq H$	w_{ij} =Demand between *i* and *j*.	y_k (y_m)= binary variables which are 1 if city *k (m)* was selected as a hub city and is 0 otherwise.
	P = Index for number of hubs.	cc_{ijkm} = Cost of travel between cities *i* and *j* which across from hubs *k* and *m*.	F_k = fixed cost for establishing of hub k.
		α = Discount factor for traveling between two hubs.	$Z_{ij} = 1$, If city *i* connected to the city j directly.

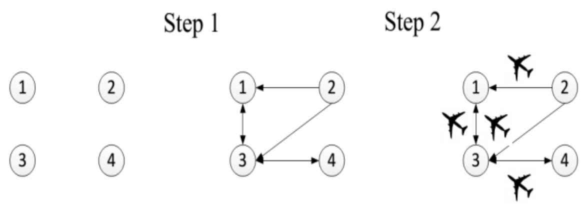

Figure 1. Proposed steps for fleet assignment

According to the designed network by P-hub median model, redistributed demands are determined because of hub points stops, now the fleet assignment should apply in the constructed network.

3. 2. Proposed FAP model In this section, we assign proper airplanes at each fleet with proposed fleet assigned model, the notation and variables of the proposed model are described as follow:

Indices:

i and *j* = City index.

p = Fleet index.

k = Number of fleet in each type.

Parameters:

Z_{ij}: The strategy matrix of airline, airline determinates whether *ij* path must be served or not.

D_{ij}: The distance between city *i* and city *j*.

W_{ij}: The demand between city *i* and city *j*.

C_p: Capacity of fleet *p*., S_p: Speed of fleet *p*.

U_p: Maximum utilization time of fleet *p* per day.

L_p: cost of purchasing airplanes of fleet *p*.

FC_p: Fixed cost of fleet *p* for every flight.

VC_p: Variable cost of fleet *p* per kilometer.

TC_{pij}: Total flight cost in path *ij*

$(TC_{pij} = FC_p + VC_p \times D_{ij})$.

Decision variables:

x_{pkij}: A binary variable which it is 1 if airplane number *k* of fleet *p* is assigned to route *ij* (from city *i* to city *j*), otherwise 0.

o_{pk}: A binary variable which counts required number of airplanes of each fleet.

a_{pkij}: A binary variable and it is 1 if capacity of airplane number *k* of fleet *p* is enough to serve demand of route *ij*, otherwise 0.

f_{pkijn}: A binary variable and it is 1 if airplane number *k* of fleet *p* is traveled from *i* to *j* and also it has enough capacity to serve demand of route *ij*, otherwise 0.

b_{pkij}: A binary variable and it is 1 if airplane number *k* of fleet *p* has enough time to serve demand of *ij*, otherwise 0. Note that each airplane has limited utilization time in every day.

g_{pkijn}: A binary variable and it is 1 if airplane number *k* of fleet *p* traveled from *i* to *j* has enough time to serve demand of *jn*, otherwise 0.

h_{pkijn}: A binary variable and it is 1 if airplane number *k* of fleet *p* traveled from *i* to *j* has enough capacity and time to serve demand of path *jn*, otherwise 0.

The variables *g* and *h* are binary variables which consider situation of an airplane to continue a route. According to the characteristics which are used for the model, the utilization time and capacity are two limitations for assigning an airplane. Hence, when an airplane traveled from city *i* to city *j* and then is scheduled to travel from city *j* to city *n*, it is necessary to check whether the airplane will have enough time and capacity. Variable *g* consider utilization time and variable *h* consider the available capacity for demand. If these variables get the value of zero, it means that the airplane has not enough time or capacity to continue the route, so the definition of variables *g* and *h* are correct.

t_{pk}: A continuous variable which calculates total service time of airplane number *k* of fleet *p*.

r_{ij}: A continuous variable which calculates assigned capacity of fleet to route *ij*.

According to the above mentioned definition, the objective function and constraints of the proposed model are given in the following equations.

$$Min \sum_p \sum_k \sum_i \sum_j Z_{ij} x_{pkij} TC_{pij} + \sum_p \sum_k o_{pk} L_p \tag{14}$$

$$o_{pk} \geq x_{pkij} \qquad \forall p,k,i,j \tag{15}$$

$$\sum_p \sum_k x_{pkij} C_p \geq W_{ij} \qquad \forall i,j \ \& \ i \neq j \tag{16}$$

$$t_{pk} = \sum_i \sum_j x_{pkij} \left(\frac{D_{ij}}{S_p} \right) \qquad \forall p,k \tag{17}$$

$$r_{ij} = \sum_p \sum_k x_{pkij} C_p \qquad \forall i,j \ \& \ i \neq j \tag{18}$$

$$r_{ij} \leq W_{ij} + \min_p \left(C_p \right) \qquad \forall i,j \tag{19}$$

$$t_{pk} \leq U_p \qquad \forall p,k \tag{20}$$

$$a_{pkij} = \begin{cases} 0 & if \quad W_{ij} \geq C_p \\ 1 & o.w \end{cases} \qquad \forall p,k,i,j \ , \ i \neq j \tag{21}$$

$$f_{pkijn} = \begin{cases} 1 & if \quad a_{pkij} = 0 \ \& \ x_{ij} = 1 \\ 0 & o.w \end{cases} \qquad \forall p,k,i,j,n \ , \ i \neq j \neq n \tag{22}$$

$$b_{pkij} = \begin{cases} 0 & if \quad U_p \geq t_{pk} + \left(\frac{D_{ij}}{S_p} \right) \\ 1 & o.w \end{cases} \qquad \forall p,k,i,j \ , \ i \neq j \tag{23}$$

$$g_{pkijn} = \begin{cases} 1 & if \ b_{pkij} = 0 \ \& \ x_{ij} = 1 \\ 0 & o.w \end{cases} \qquad \forall p,k,i,j,n \ , \ i \neq j \neq n \tag{24}$$

$$h_{pkijn} = \begin{cases} 1 & if \ f_{pkijn} = 1 \ \& \ g_{pkijn} = 1 \\ 0 & o.w \end{cases} \tag{25}$$
$$\forall p,k,i,j,n \ , \ i \neq j \neq n$$

$$h_{pkijn} \leq x_{pkjn} \qquad \forall p,k,i,j,n \ , \ i \neq j \tag{26}$$

$$x_{pkij}, o_{pk}, a_{pkij}, f_{pkijn}, b_{pkij}, g_{pkijn}, h_{pkijn} \in \{0,1\} \tag{27}$$

$$t_{pk}, r_{ij} \in R^+ \tag{28}$$

The objective function given in Equation 14 is a mixed integer linear equation consisting of two sub-functions. The first sub-function minimizes the total cost of assigning fleet to network and the second sub-function minimizes cost of fleet purchasing. Equation 15 computes number of airplanes for each fleet and saves it in o_{pk} variable.

Equation 16 guarantees that demand between cities are served by airplanes. Equation 17 computes time of serving for each airplane. Equation 18 computes dedicated capacity of different airplanes for route *ij*. Equation 19 guarantees that devoted capacity to each path, be less than sum of the demand of the route and minimum capacity of the fleet, so the extra devoted capacity will be minimum.
Equation 20 ensures that service time of an airplane be less than its utilization time. According to the Equation

21 a_{pkij} is an auxiliary variable to show that capacity of an airplane number *k* of fleet *p* is less than demand of the route ij or not. Based on Equation 22 $f_{pkijn}=1$ if $a_{pkij}=\cdot$ and $x_{ij}=1$. On the other hand, this equation guarantees that if the airplane number *k* of fleet *p* traveled path *ij* and also it has not empty capacity (opportunity cost), then $f_{pkijn}=1$. Equation 23 ensures $b_{pkij}=0$ if $U_p \geq t_{pk}+\left(\frac{D_{ij}}{S_p}\right)$. This inequality consider whether the airplane number k of fleet p has enough time to travel path ij? The predefined flight time limit for airplanes of fleet *p* is U_p. Also, t_{pk} is total traveled time of airplane number k of fleet p up to now. Hence, an airplane could travel path *ij* if summation of t_{pk} and needed time for travelling *ij* (which is $\frac{D_{ij}}{S_p}$) would be less than U_p.

According to the Equation 24, $g_{pkijn}=1$ if $b_{pkij}=0$ and $x_{ij}=1$. So, if the airplane number *k* of fleet *p* traveled path *ij* and also it has enough time to travel path *jn*, then $g_{pkijn}=1$.

Based on Equation 25 if $f_{pkijn}=1$ and $g_{pkijn}=1$ then $h_{pkijn}=1$. This equation set $h_{pkijn}=1$ if the airplane number *k* of fleet *p* traveled path *ij*, and has enough time to travel path *jn* and also it has not any unused capacity on travel *jn*. Equation 26 guarantees if $h_{pkijn}=1$, then $x_{pkijn}=1$ and it means that airplane number *k* from fleet *p* must travel from *j* to *n*. Equations 27 and 28 introduce binary and continuous variables.

3. 3. linearization In the proposed model, there are some conditional equations which should be appeared in a linear form.

There are two types of conditional constraint in the model. First one has a format of $a=\begin{cases}0 & if \quad b\geq c\\ 1 & o.w\end{cases}$.

To linearize this constraint, we introduce two inequalities as $\begin{matrix} b-c\geq -Ma \\ b-c\leq M(1-a)-\varepsilon\end{matrix}$. In these sets of inequality, M is a big number and ε is a very small one. By added two constraints conditional statements will be satisfied.

The second conditional constraint has a format as $a=\begin{cases}1 & if \quad b=0 \ \& \ c=1\\ 0 & o.w\end{cases}$. We introduce inequality set as $\begin{matrix} 2a\leq c+(1-b) \\ a\geq c+(1-b)-1\end{matrix}$. Note that *b* and *c* are binary variables. Again by added two mentioned constraints, conditional statement will be satisfied in a linear form. Table 4 presents the equations set for every constraint of the proposed model. As this table shows, for each conditional constraint of the proposed model, two linear equations have been introduced.

4. NUMERICAL EXAMPLE

In this section, we investigated a real data from air travel in Iran air network to predict the optimum basket of air plane for two next decades. According to the time series analysis of passenger movement, the traffic of Iran passenger will be doubled at the outlook 2036. The best time series model for Iran network with minimum MSE (Mean Squared Error) is ARIMA (1,0,0). According to this model, the total movement of Iran will be 50 millions in year 2036 (The employed data set will be available at: http://s3.picofile.com/file/8204636350/Data_set.xlsx.html). The model consists of 69 airports, 64 active airports plus 5 under construction ones. The demand matrix is distributed among airports according to the expected population in 2036. Now, as the first step, we select the most appropriate hub and then in the second step we assign proper fleet to the demand arcs. Also, according to the passenger facilities and airport specifications, 13 airport are selected as hub candidate. The Figure 3 presents the graph of cost of network according to the number of hubs. The question which considered is the number of hubs which should be selected. Obviously, by increasing number of the hubs, the cost of the network is decreasing. However, is the large number of hubs good? One of the most important consideration for the airlines, is the satisfaction of passengers. So, the number of the hubs should be moderated for not increasing abrupt during the travel for changing air plane. Table 6 shows number of the direct and indirect routes according to the number of the hubs. As it demonstrated, by increasing number of the hubs, the proportion of indirection routes (routes with stop point in hubs) will be increased. Hence, we propose 3 hubs for Iran air network. This leads to only 9 percent of the routes in the network becomes indirect. On the other hand, the tradeoff between cost of network (Figure 3) and proportion of the indirect routes (Table 6) shows that a proper number should be selected. In this study we assumed that at most 10 % of flights to be indirect. According to the following table we can see that considering of 3 hubs will lead to about 9 % of indirect flights with minimum costs, so 3 hubs were selected to be considered. As mentioned in the paper, after minimum cost, a critical consideration in planning air network is passenger satisfaction. Airlines should consider maximum hubs (to decrease cost of network) and minimum hubs (to increase passenger satisfaction). So, authors proposed three hubs to consider all of the two concerns.

TABLE 4. Set of equations to linearize conditional constraints of the proposed model

Conditional constraints		**Sets of linear equations**	
$a_{pkij}=\begin{cases}0 & if \quad W_{ij}\ge C_p\\ 1 & o.w\end{cases}$	$\forall p,k,i,j\ ,\ i\ne j$	$W_{ij}-C_p\ge -Ma_{pkij}$ $W_{ij}-C_p\le M\left(1-a_{pkij}\right)-\varepsilon$	$\forall p,k,i,j\ ,\ i\ne j$ $\forall p,k,i,j,\ i\ne j$
$f_{pkijn}=\begin{cases}1 & if \quad a_{pkij}=0\ \ \&\ \ x_{ij}=1\\ 0 & o.w\end{cases}$	$\forall p,k,i,j,n\ ,\ i\ne j\ne n$	$2f_{pkijn}\le x_{pkij}+\left(1-a_{pkjn}\right)$ $f_{pkijn}\ge x_{pkij}+\left(1-a_{pkjn}\right)-1$	$\forall p,k,i,j,n\ ,\ i\ne j\ne n$ $\forall p,k,i,j,n\ ,\ i\ne j\ne n$
$b_{pkij}=\begin{cases}0 & if \quad U_p\ge t_{pk}+\left(\dfrac{D_{ij}}{S_p}\right)\\ 1 & o.w\end{cases}$	$\forall p,k,i,j\ ,\ i\ne j$	$U\left(p\right)-t_{pk}-\left(\dfrac{D_{ij}}{S_p}\right)\ge -Mb_{pkij}$ $U\left(p\right)-t_{pk}-\left(\dfrac{D_{ij}}{S_p}\right)\le M\left(1-b_{pkij}\right)-\varepsilon$	$\forall p,k,i,j\ ,\ i\ne j$ $\forall p,k,i,j\ ,\ i\ne j$
$g_{pkijn}=\begin{cases}1 & if \quad b_{pkij}=0\ \ \&\ \ x_{ij}=1\\ 0 & o.w\end{cases}$	$\forall p,k,i,j,n\ ,\ i\ne j\ne n$	$2g_{pkijn}\le x_{pkij}+\left(1-b_{pkjn}\right)$ $g_{pkijn}\ge x_{pkij}+\left(1-b_{pkjn}\right)-1$	$\forall p,k,i,j,n\ ,\ i\ne j$ $\forall p,k,i,j,n\ ,\ i\ne j$
$h_{pkijn}=\begin{cases}1 & if \quad f_{pkijn}=1\ \ \&\ \ g_{pkijn}=1\\ 0 & o.w\end{cases}$	$\forall p,k,i,j,n\ ,\ i\ne j\ne n$	$2h_{pkijn}\le f_{pkijn}+g_{pkijn}$ $h_{pkijn}\ge f_{pkijn}+g_{pkijn}-1$	$\forall p,k,i,j,n\ ,\ i\ne j$ $\forall p,k,i,j,n\ ,\ i\ne j$

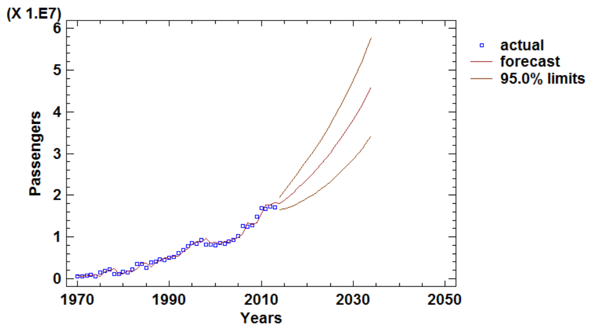

Figure 2. Plot of demand forecasting for Iran air travel at vision 2036

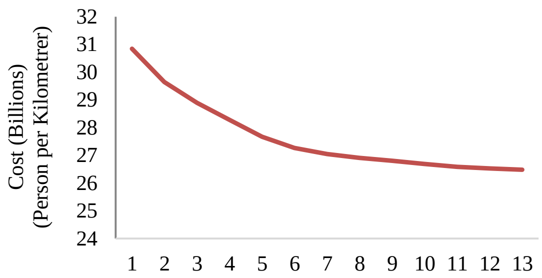

Figure 3. Cost of network versus number of hubs

TABLE 6. Number of direct and indirect routes according to the number of hubs

Number of hubs	Indirect routes	Direct routes	Total routes	Proportion of Indirect routes
1	2	4690	4692	0.00
2	90	4602	4692	0.02
3	414	4278	4692	0.09
4	642	4050	4692	0.14
5	928	3764	4692	0.20
6	1370	3322	4692	0.29
7	1536	3156	4692	0.33
8	1746	2946	4692	0.37
9	2000	2692	4692	0.43
10	2172	2520	4692	0.46
11	2248	2444	4692	0.48
12	2336	2356	4692	0.50
13	2400	2292	4692	0.51

The selected hubs for Iran air networks are Tehran, Mashhad and Shiraz as presented in Figure 4. Now as a second step, the demand matrix should be redistributed according to the proposed hub network. After redistributing demand between matrixes, we apply the proposed FLP model to select the appropriate number of the air plane. We introduce four types of fleet as shown in Table 7. This fleet was extracted from standard category of famous aviation company.

TABLE 7. Attributes of fleet

	Type 1	Type 2	Type 3	Type 4
Purchasing Cost ($)	4×10^{7}	10^{8}	25×10^{7}	4×10^{8}
Capacity	50	150	300	500
Speed (km/h)	400	500	600	700
Utilization Time (h)	8	8	8	8
Flight Fixed Cost ($)	1000	4000	10000	15000
Flight Variable Cost ($)	200	800	2000	3000

TABLE 8. Results of proposed model

	Needed Time at Day	Needed Number	Needed number with passenger load Factor	Number of Seats
Type 1	477.90	119.47	150	7500
Type 2	218.02	54.51	69	10312.5
Type 3	116.12	29.03	38	11250
Type 4	54.78	13.69	18	8750

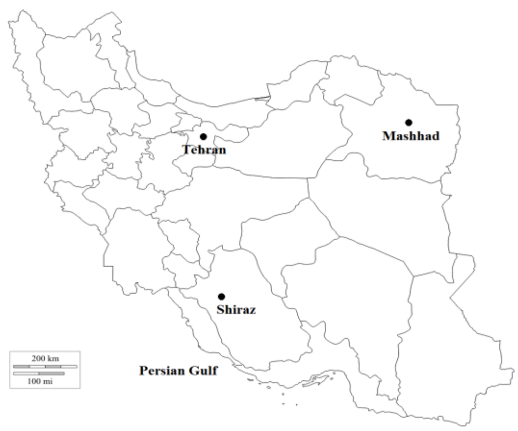

Figure 4. Three Hubs selected for Iran air travel network at outlook 2036

According to the mentioned types of airplane in Table 7, the needed service time for each fleet type was calculated as Table 8. The cost of assigning proposed airplane basket to the network is 69.936.000.000 passenger-kilometer. The load factor is supposed to be 0.8 for each airplane. Load factor is percentage of the filled seats and measures the capacity utilization of airplanes.

According to proposed assignment, purchasing cost for airplanes basket is 29.6 billion dollars. The purchasing cost for current fleet of Iran is about 20 billion dollars. By doubling this cost for next two decades, by supposing current network and assigning method, the cost of purchasing needed fleet for vision 2036 will be 40 billion dollars. So, the proposed method could decrease the cost of purchasing airplanes about 10.4 billion dollars.

On the other hand, if the proposed network and proposed assignment model will be used in designing of Iran air network, the cost of investment for purchasing airplane will be decreased about 10.4 billion dollars. In addition of purchasing cost, other related costs such as crew, maintenance and so on will decreased. Tradeoff between passenger satisfaction and cost of network is an advantages of proposed network and considering fleet parameters is a critical key of proposed FAP which is important for airlines.

5. CONCLUSION

In this paper, an FAP model based on airplane specifications has been proposed. The proposed model assigns the proper airplane to each demand flow according to the capacity, utilization time, purchasing cost and using cost of each fleet. To have an appropriate fleet assigning, we recommend the first network demand redistributed according to the hub model. This helps reorganizing the routes for decreasing the cost of traveling across the network. Hence, the proposed method of the paper includes two steps. At the first step, a network should be constructed according to the demand matrix and at the second one, the appropriate fleet are assigned to each demand leg (route) by using proposed model of FAP. One example were used to demonstrate performance of the proposed model.

Adding other specification such as crew limitation makes the model more realistic. Also, considering time window in the model to make easy daily scheduling for airlines is an attractive future research direction.

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A New Model for Fleet Assignment Problem, Case Study of Iran Air Network at Vision 2036

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چکیده

مسئله تخصیص ناوگان (FAP) به معنی تعیین نوع مناسب از یک ناوگان (هواپیما) برای تخصیص به هر مسیر است. هواپیما به عنوان یک منبع بسیار گران به شمار می رود و از این رو اختصاص نامناسب آن می تواند هزینه خطوط هوایی را افزایش دهد. ملاحظات بسیاری وجود دارد که در فرمول بندی یک مسئله تخصیص ناوگان باید مدنظر قرار گیرد خصوصا پارامترهای مرتبط با ویژگی های هواپیما. این مقاله مدلی جدید از برنامه ریزی عدد صحیح مختلط ارائه می دهد که به دنبال تخصیص ناوگان مناسب به هر پرواز به گونه ای است که هزینه جابه‌جایی و فعالیت ناوگان در شبکه کاهش یابد و تعداد بهینه هواپیما برای هر پرواز به دست آید. مدل به دست آمده تضمین می کند که تقاضای میان هر دو شهر لزوما برآورده شود و زمان فعالیت هر هواپیما در شبکه از زمان معین در نظر گرفته شده برای آن کمتر باشد. در ادامه در یک مثال واقعی، با پیش بینی تقاضای مسافرت هوایی در افق ۲۰۳۶ در ایران، ناوگان مناسب هوایی برای پوشش تقاضای داخلی ایران پیشنهاد شده است. نتایج این پژوهش نشان می دهد که مدل پیشنهادی با ارائه یک سبد بهینه برای ناوگان هوایی ایران، نسبت به ادامه وضعیت موجود، قابلیت کاهش قابل توجهی در هزینه ساخت یا خرید ناوگان داشته و می تواند بهره وری این صنعت را به میزان زیادی افزایش دهد.

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