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A Congested Double Path Approach for a Hub Location-allocation Problem with Service Level at Hubs

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ABSTRACT

In this paper an uncapacitated multiple allocation p-hub median problem is discussed. The model minimizes the transportation cost based on inventory service level at hubs through double paths between hubs. Inventory service level is defined as the percent of inventory shortage of vehicles at hubs according to passengers demand and replenishment time of vehicles. A real data from 25 nodes of municipality district 14 of Tehran is used to evaluate the performance of inventory service level in hub networks.

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Sets:	
N	Set of all nodes indexed by i or j.
K, l	Origin and destination hub nodes, respectively.
M	Set of paths between hubs in whole network indexed by m.
Parameters:	
W_{ij}	The yearly flows from origin i to destination j.
d_{ik}	Distance from origin i to origin hub k.
d_{kml}	Distance from origin hub k to destination hub l via route m.
d_{lj}	Distance from destination hub l to destination j.
c_{ik}	Unit transportation cost from origin i to origin hub k.
c_{kml}	Unit transportation cost from origin hub k to destination hub l via route m.
c_{lj}	Unit transportation cost from destination hub l to destination j.
α	Cost discount factor $0<\alpha<1$ (applied for hub links).
β	Time delay factor $\beta>1$ (applied for hub links).
ω_k	Replenishment time of vehicles at hub k.
t_{ik}	Time of transit from origin i to origin hub k.
t_{kml}	Time of transit from hub k to hub l via route m.
t_{lj}	Time of transit from destination hub l to destination j.
F_k	Fixed cost of opening a hub at node k.
TW_{ij}	Service time window from node i to node j.
Variables:	
X_{ikmlj}	Equals one if flow from i to j is assigned to hub pair (k,l) and route m and otherwise 0.
H_k	Equals one if node k is a hub and otherwise 0.
S_k	Probability of inventory shortage at hub k.
g_m	Total flow in each kind of routes in hub links in whole network.

1. INTRODUCTION

In recent years much researches in design and performance of transportation systems have been carried out to investigate the least expensive methods. There are two kinds of transportation systems: direct sending and hub network. In direct systems, flow from origin to destination will be sent directly and without considering other destinations. So every vehicle in each trip is faced with only one destination. This system is used especially when the volume of cargo is large or it is customize. On the other hand, when direct communication between the origin and destination in a

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network is impossible or too costly, the second type of system, the hub network is used. In this system the flows are collected in a point which is called hub, and then distributed between destinations. So in designing of hub networks, strategic decision making like locating hubs and assigning points of origins and destinations should be adopted. Hub networks have the advantage that the size of the vehicles will be determined by economy, thus the economic benefits from the use of vehicles and service level will be improved in this system [1].

Taxi service is one of the applications of hub location allocation problems. There are a lot of terminals within the urban logistic network and paths between each pair of origins and destinations should be determined in such way that passengers reach their final destination with the lowest cost and within a certain time period. Public transportation were investigated in hub location problems by Gelare and Nickel [2] and Yaman et al. [3]. Moreover, an integrated incomplete hub location-routing network is recently investigated by Karimi and Setak [4], in a similar research. One of the main applications of their model is the transportation system, specially public and urban transportation.

Increased traffic and congestion in some routes (especially connections between hub facilities), are factors which may cause pauses and a lot of problems in urban logistic systems. Because the volumes of daily travel between hub facilities are greater than those between other points in the network, this may cause some disorders in urban transportation systems. In such circumstances, using alternative paths to send flows between hubs will improve the performance of the network [5]. In this paper, a hub location allocation problem for an urban logistic system is investigated which contains two kinds of paths between hubs that differ in terms of travel time and expense, such that routes with a lower cost yield more travel time and vice versa. Additionally, this paper also considers the inventory service level at hubs and congestion on routes between hub facilities in the whole network. The contributions will be helpful in designating hub networks with minimum vehicles at hubs to give good services. According to congestion, this part of contribution can be helpful to balance and manage existing alternative paths between hub nodes and this is helpfully applicable for limited zone by using traffic cameras or some other related tools to control traffic in real world. All of these applications are considered in this work.

The rest of this paper is organized as follows: section 2 presents and identifies the contribution of relevant prior researches. The third section describes a mixed integer nonlinear mathematical model. Computational results and sensitivity analysis are presented in section 4 and lastly, conclusions and further research directions are discussed in section 5.

2. LITERATURE REVIEW

The literature related to this research can be found in these main areas: p-hub median problems, hub problems under congestion at hubs and location problems considering inventory cost using the (S, S-1). The mentioned areas are discussed below:

2. 1. pHMP Literature Although a variety of models have been developed and investigated in the context of hub related problems, they can be assigned to the following main classes: the hub covering location problem (HCLP) [6], the p-hub center problem (pHCP) [6] and the p-hub median problem (pHMP) which is related to this research. In a pHMP, the objective is to locate “p” hubs in the network so that the total costs of transporting flows through the network are minimized [7]. Campbell [6] proposed pHCP based on the Mini-Max criterion by which the maximum transportation cost between all origin and destination pairs is minimized. This type of problem is useful for emergency facility locations. He also defined HCLP to locate hubs in a way that each none-hub node is covered by a pair of hub nodes. Karimi and Bashiri [8] presented an adequate literature review of this problem and developed a model with different coverage types.

There are two kinds of allocations: single and multiple. In single allocation hub location problems, each none-hub node is allocated to exactly one hub [9] and in multiple allocations a none-hub node can be allocated to more than one hub [10]. In this paper the multiple allocation version of the p-hub median problem is considered. The hub location problem was first presented with a quadratic integer formulation by O’Kelly [11]. He also proved that this is a NP-hard problem. Klincewicz [12] presented two heuristics, one based on exchanges of spoke assignments and the other one based on clustering nodes. He also proposed the tabu search and greedy randomized heuristic search. Campbell [7] for the first time formulated pHMP with multiple allocations as an integer programming model. He also formulated an integer programming model considering the threshold flow [13]. Skorin Kapov and Skorin Kapov [14] presented a tabu search heuristic to solve pHMP. Skorin Kapov et al. [15] formulated a mixed integer pHMP and used an exact solution to solve presented model. Ernest and Krishnamoorty [16] proposed a linear programming formulation to solve the pHMP and used simulated annealing to reach the upper bound for the LP based branch and bound. Smith et al. [17] solved the quadratic integer formulation using neural networks. For more detailed information of hub location problems the reader is referred to other works [18, 19].

2. 2. Congestion Literature Elhedheli and Hu [20] were the first who considered congestion issues in the context of hub location problems. They considered the congestion in an objective function using a convex nonlinear cost function to balance congestion at hubs of the network. Camargo et al. [21] also used the same congestion function to establish a compromise between the transportation cost savings induced by the economies of scale exploitation and the costs associated with the congestion effects. This paper also uses the same function as a leverage tool to balance and calculate congestion in any kind of path existing between hubs in the whole network. The justification to use such a function is discussed elsewhere [22] to estimate delay cost and for airport management. In particular, in the work of Palma and Marchal [23] it is stated for traffic models which assume that congestion cost is a power-law function. The same such function is used in this paper. The congestion cost function is aimed at balancing flow between existing paths in hub to hub links, avoiding the case where one path is over-utilized compared to other one. Over-utilized paths with lower cost or paths with lower travel time will never happen using the power-law congestion function. Nowadays, congestion costs in real world is used a lot, for example you can see in some crowded cities like Tehran, to control traffic in specific routes, limited zone policy is used. This policy by installing related tools like traffic cameras is used to control the flows. All these cases are considered as applications of congestion function.

2. 3. Inventory Literature There is a lot of research in the literature about the scope of location problems considering inventory approaches that use performance constraints to increase the efficiency of networks, some are mentioned below: Cole [24]; Nozick and Turnquist [25-27]; Daskin et al. [28]; Shen et al. [29]; Miranda and Garrido [30]; Shu et al. [31]; Lin et al. [32]. Although there is extensive literature about jointed inventory and location, there are few studies that have focused on the combined approaches of inventory cost and the case of hub location problems. A modulated hub inventory model applied in a bicycle sharing system is included in the work by Lin et al. [33] which considers a single hub covering problem for bicycle sharing system where the bike stations play the role of hubs in the network, and the origins, destinations and hubs are considered in separated sets of nodes. They use performance constraints to calculate the safety stock of every hub. In this paper, due to the random nature of demand, the Poisson distribution is utilized as a performance constraint, and the service level inventory shortage probability is evaluated at each hub. It's notable that origins, hubs and destinations are in the same set of nodes.

3. MODEL FORMULATION

This problem is designated as a set of nodes with a given travel time and demand of flow between nodes. This model assumes that the network connections are in place, so fixed costs for establishing network connections are not considered. The objective is to determine p hub nodes among all existing nodes in the network such that the total costs are minimized and the inventory service level according to passenger demand and the replenishment time of vehicles is calculated at each hub. In this paper, also congestion on existing paths among backbone network (i.e., backbone network included hubs and their links) is investigated.

The novelties of the model are highlighted as blow:

- Considering a double path approach between hubs
- Considering inventory service level at hubs
- Considering congestion in existing paths

3. 1. Inventory Service Level Constraint To ensure that the designed hub network is efficient enough to hold passenger traffic at all hubs, many performance constraints can be used in the hub location problems. In the design of hub networks with taxi services, each hub should have enough vehicles to give suitable services. In hub network discussions, inventory level can be defined as the existing inventory service level for users of hub at each hub. So in order to investigate the ability of inventory service level we calculate the lack probability of the inventory service level at hubs according to passenger demands and replenishment time of vehicles. It's notable that the inventory service level includes the flows that use hub links to reach their destinations. For example in Figure 1, the inventory service level is calculated at hub k in scenarios "a", "d", "f" and "g". It is not investigated for the flows with a single hub from an origin to a destination like scenarios "b", "c" and "e". Assume that p hubs are located and assigned to none-hub nodes in the whole network. So each hub will face travel demands that follows the Poisson distribution. In this case the demand rate at hub k (A_k) according to travel demand from origin i to destination j via hubs k and l will be calculated as below, where X_{ikmlj} as a decision variable takes value 1 if the flows from i to j are assigned to hub k and l through path m. We have W_{ij}, the flow from origin i to destination j, where usually $W_{ij} \neq W_{ji}$. T number of days per month, B number of hours per day and v capacity of vehicles are used to convert hourly demand to become proportional with replenishment time of vehicles.

$$A_k = \frac{1}{TNv} \sum_i \sum_m \sum_{l \neq k} \sum_{j \neq i} X_{ikmlj} W_{ij} \qquad \forall k \in N \qquad (1)$$

Assume that hub k has $p-1$ vehicles (number of other hubs). If the demand rate at each hub A_k and the replenishment time of vehicles at each hub is considered as ω_k, then D the potential rate of demand that will face the lack of vehicles at hub k has a Poisson distribution with the mean rate of $(A_k \omega_k)$. In this case the probability of inventory shortage at hub k (S_k) will be calculated as below based on Palm's theorem. Palm's theorem states that number of demands in replenishment time (re-supply) follows a Poisson distribution with parameter $(A_k \omega_k)$ [34].

$$\Pr obability\,(D > p-1) = s_k = \sum_{q=p}^{\infty} \frac{e^{-A_k \omega_k} (A_k \omega_k)^q}{q!} = 1 - \sum_{q=0}^{p-1} \frac{e^{-A_k \omega_k} (A_k \omega_k)^q}{q!} \qquad \forall k \in N \qquad (2)$$

Here the inventory service level will be calculated as follows: first, we consider $p-1$ initial vehicles for each hub, then according to passenger demands and the replenishment time of vehicles, inventory shortage probability will be calculated by Equations (1) and (2). According to Table 1 and achieved inventory shortage probability, an additional inventory will be assigned to each hub.

And in the last step, by using h as the inventory holding cost, the total inventory cost will be calculated and applied in an objective function by inventory function (θ_k) which is defined as below:

$$\theta_k(s_k) = (p-1)([10 * s_k] + 1)h \qquad \forall k \in N \qquad (3)$$

3. 2. Congestion Constraint

Because of the responsibility of distributing flows between existing paths between hubs, and to reduce total congestion of vehicles and balance the whole network, monetary values must be expressed. In this case to calculate congestion on routes, we use congestion function τ_m and define congestion decision variable g_m, to apply its cost in the objective function as Equation (4) where $m \in M$, and M is a set of routes and includes two kinds of paths. One path, namely path 1, is based on less cost and more travel time while, path 2 is based on less traveling time and more cost.

$$g_m = \sum_i \sum_k \sum_{l \neq k} \sum_{j \neq i} X_{ikmlj} W_{ij} \qquad \forall m \in M \qquad (4)$$

The congestion cost function $\tau_m(g_m)$ is assumed to be given for each kind of paths existing between hubs in the whole network. This function is considered increasing on $[0, +\infty)$, proper convex and smooth. The used congestion cost function is a power law and is used as a penalty cost in the objective function where $\tau_m(g_m) = e \times (g_m)^b$, e and b take positive values, and $b > 1$ to balance the flow rate in each path. This class of function has been used by Elhedheli and Hu [20] and Camargo et al. [21] for single and multiple allocations to investigate hub congestion, respectively, while in this study the function is used to investigate congestion on paths between hubs in the whole multiple allocations hub network. Figure 2 shows the congestion cost function for different values of b. It's visible that by increasing b, congestion function will have more power to balance the flow between paths.

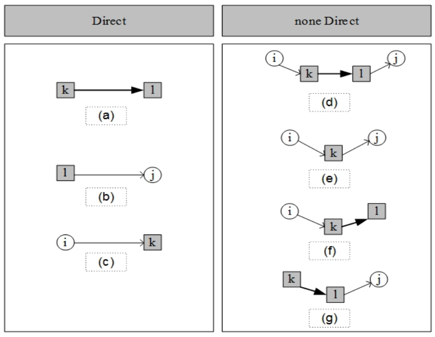


Figure 1. Different conditions of transporting flows

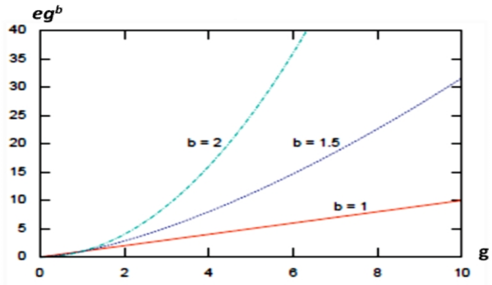


Figure 2. Congestion cost function for different values of *b*

TABLE 1. Inventory shortage scenarios

Probability of inventory shortage	Initial inventory	Additional inventory	Total inventory
[0-0.1)	p-1	0	p-1
[0.1-0.2)	p-1	1*(p-1)	2*(p-1)
[0.2-0.3)	p-1	2*(p-1)	3*(p-1)
[0.3-0.4)	p-1	3*(p-1)	4*(p-1)
[0.4-0.5)	p-1	4*(p-1)	5*(p-1)
[0.5-0.6)	p-1	5*(p-1)	6*(p-1)
[0.6-0.7)	p-1	6*(p-1)	7*(p-1)
[0.7-0.8)	p-1	7*(p-1)	8*(p-1)
[0.8-0.9)	p-1	8*(p-1)	9*(p-1)
[0.9-1)	p-1	9*(p-1)	10*(p-1)

3. 3. Mathematical Model To present the model formulation the following symbols, variables and parameters are introduced. The congested multiple allocations p-hub median problem considering inventory service level at hubs now can be stated as follows:

$$Minimum\sum_i\sum_k\sum_m\sum_l\sum_j W_{ij}X_{ikmlj}(d_{ik}c_{ik}+\alpha d_{kml}c_{kml}+d_{lj}c_{lj}) + \sum_k F_k H_k + \sum_k \theta_k(s_k) + \sum_m \tau_m(g_m) \quad (5)$$

Subject to:

$$\sum_k H_k = p \quad (6)$$

$$\sum_k\sum_m\sum_l X_{ikmlj} = 1 \qquad \forall i,j \in N \quad (7)$$

$$\sum_k X_{ikmlj} \le H_l \qquad \forall i,j,l \in N \quad \forall m \in M \quad (8)$$

$$\sum_l X_{ikmlj} \le H_k \qquad \forall i,j,k \in N \quad \forall m \in M \quad (9)$$

$$A_k = \frac{1}{TNv}\sum_i\sum_m\sum_{l\ne k}\sum_{j\ne i} X_{ikmlj}W_{ij} \qquad \forall k \in N \quad (10)$$

$$S_k = 1-\sum_{q=0}^{p-1}\frac{e^{-(A_k\omega_k)}(A_k\omega_k)^q}{q!} \qquad \forall k \in N \quad (11)$$

$$g_m = \sum_i\sum_k\sum_{l\ne k}\sum_{j\ne i} X_{ikmlj}W_{ij} \qquad \forall m \in M \quad (12)$$

$$\sum_k\sum_m\sum_l X_{ikmlj}(t_{ik}+\beta * t_{kml}+t_{lj}) \le TW_{ij} \qquad \forall i,j \in N \quad (13)$$

$$X_{ikmlj} \in \{0,1\} \qquad \forall i,k,l,j \in N \quad \forall m \in M \quad (14)$$

$$X_{ikmlj} = 0 \qquad \forall (i=j),k,l,j \in N \quad \forall m \in M \quad (15)$$

$$A_k, g_m \ge 0 \qquad \forall k \in N \quad \forall m \in M \quad (16)$$

$$0 \le S_k \le 1 \qquad \forall k \in N \quad (17)$$

$$H_k \in \{0,1\} \qquad \forall k \in N \quad (18)$$

In the above mathematical model the objective function (5) minimizes the sum of total transportation costs for all origins to destination flows, sum of installation costs of hub facilities, congestion costs on routes in the whole network and inventory costs at hubs. The exact number of hubs is p due to Constraint (6). Constraint (7) ensures that every origin-destination node pair is assigned to a hub pair and at least one kind of route exists between hubs. It is notable that the choice of hub assignments and route is based on the best economic option. An origin-destination flow W_{ij} can be assigned to node pair (k,l) only if k and l are hubs. Constraints (8) and (9) express these requirements. Constraints (10) and (11) define the calculation of the probability of inventory shortage at node k. Constraint (12) determines the total amount of flow on existing routes between hubs in the whole network. Constraint (13) guarantees that all inter-urban travel time from origin i to destination j should be done within the service time window. Constraints (14) to (18) express the type and the range of decision variables of the model.

4. COMPUTIONAL RESULTS

4. 1. Input Data and Parameters Datasets of district 14 in Tehran are used in order to evaluate the proposed model. In this dataset, 25 parishes of district 14 have been considered as the set of nodes. The distance matrix of these nodes and time dataset are gathered from Google maps. In order to generate the demand matrix, first the population of each node was considered and according to phrase (19) and (20), the demand matrix between nodes was provided [35].

$$W_{ij} = Q \times p_i \frac{p_j}{\left[\sum_{k=1}^{25} p_k\right] - p_i} \qquad \forall i \ne j; i,j = 1,\dots,25 \quad (19)$$

$$W_{ij} = 0 \qquad \forall i=j; i,j=1,\dots,25 \quad (20)$$

in which P_i is considered as the population of node i, and W_{ij} is the demand flow from node i to node j. According to this, it is obvious that the demand matrix is an asymmetric matrix in which $W_{ij} \ne W_{ji}$. Q is used as a normalization factor. The replenishment time for node k was calculated by adding the maximum existing time from node k to other nodes and the maximum existing time from other nodes to node k was calculated and then the calculated phrase in each node was multiplied with β factor $(\beta \ge 1)$ to account for delay time. This procedure will be carried out for each one of the routes, (Replenishment time for node k based on route 1 and route 2) and the average will be considered as the replenishment time of each node. Phrase (21) to (23) show the procedure used to calculate the replenishment time for node k.

$$\omega_k(Route1) = (\max\{t_{kl} \mid l \in N\} + \max\{t_{lk} \mid l \in N\}) \times \beta \qquad \forall k \in N \quad (21)$$

$$\omega_k(Route2) = (\max\{t_{kl} \mid l \in N\} + \max\{t_{lk} \mid l \in N\}) \times \beta \qquad \forall k \in N \quad (22)$$

$$\omega_k(average) = \frac{\omega_k(Route1) + \omega_k(Route2)}{2} \qquad \forall k \in N \qquad (23)$$

Other input parameters of the model are listed in Table 2. Parameters which are not listed in Table 2, are available in: http://wp.kntu.ac.ir/hkarimi/files/Dataset.rar

4. 2. Numerical Results Testing of the mathematical formulations of the gathered dataset was done using GAMS 23.1 for BONMIN solver. These 25 nodes datasets are related to taxi services in district 14 of Tehran. Table 3 shows the result of the presented exact solution for two different values of the discount factor $(\alpha = 0.5, \alpha = 0.9)$. The optimal solution was obtained for the node sizes 8, 12, 15, 20 and 25 in which each node size is 3, 4, and 5 number of hubs, respectively. The second column of solution shows the nodes which are selected as hub facilities in the whole network at each scale of the problem. The best objective function value and the CPU time are reported in column one and three.

4. 3. Analysis of Numerical Results According to the results presented in Tables 3, the total cost of the objective function increased as the scale of the problem increased. Figure 3 illustrates the costs distribution of the results reported in Table 3, at a scale of 12 and 15, for two different values of the discount factor $(\alpha = 0.5, \alpha = 0.9)$ and differing number of hub facilities $(p = 3,4\&5)$. The results show that the economy of scale reduces the transportation costs and thus the total costs of objective function. Figure 3 also shows that the inventory costs increase as the number of hub facilities increases in each economy of scale, it is justifiable, because according to Table 1, for each value of (p), each hub will have $p-1$ initial inventory to service other hubs. The results also show that the transportation costs are in an inverse ratio in relation with the number of hub facilities. This becomes apparent when the number of hub facilities increases benefiting from economy of scale in hub to hub links, and thereby reducing the transportation cost .

According to the results shown in Figure 3, Table 4 shows how the inventory costs are calculated for the case problem $n = 12$, $\alpha = 0.9$ and $p = 4$. The results indicate that the probability of inventory shortage in the obtained optimal hub facilities 3, 6, 7 and 10 are 0.27, 0.22, 0.31 and 0.26, respectively. Results reported that the total inventory cost according to inventory shortages in hub facilities with inventory rate cost (0.85 million Toman per month) is equal to 33,150,000 Toman as illustrated in Figure 3.

4. 4. Sensitivity Analysis Increasing the frequency of inventory replenishment or reducing the replenishment time of vehicles in hub facilities according to passengers demand will reduce the percentage of inventory shortage in hub facilities and this will decrease total inventory costs in the whole network.

TABLE 2. The model parameters.

Input parameters	Values	Unit
α	0.5 & 0.9	-
β	1.2	-
c	0.8	Toman-taxi meter
h	0.85	Milion toman per month
T	30	Days per month
N	8	Hours per day
p	3, 4 & 5	-
v	4 & 10	people
TW	20-25-30	minutes

TABLE 3. Computinal results for different values of α and p.

α	n	p	GAMS – BONMIN solver		
			Opt. Cost	**Opt. Hubs**	**CPU (s)**
0.9	8	3	37891631	3, 5, 7	45.59
		4	52188872	3, 4, 6, 7	96.25
		5	68247584	2, 3, 4, 6, 7	93.4
	12	3	81714689	9, 10, 12	1285.68
		4	96149586	3, 6, 7, 10	1412.3
		5	106106605	3, 4, 6, 7, 10	991.84
	15	3	102577604	3, 7, 13	12177.1
		4	119650320	6, 7, 11, 13	12945.9
		5	133765810	3.7.10,11,13	12612.1
	20	3	127964315	3, 7, 19	22282.65
		4	145304723	3, 10, 17, 20	24314.3
		5	166819714	3, 6, 10, 18, 20	23231.5
	25	3	158651320	4, 10, 13	53652.2
		4	172543991	3, 7, 13, 17	54844.23
		5	185396205	4, 10, 15, 17, 21	54012.19
0.5	8	3	37010158	3, 6, 7	74.44
		4	51864518	3, 4, 6, 7	124.34
		5	67615539	1, 2, 3, 4, 7	117.59
	12	3	73664950	9, 10, 12	1191.7
		4	86075220	4, 6, 7, 10	1652.3
		5	95831987	1, 3, 4, 6, 9	1560.3
	15	3	95316491	3, 10, 13	11153.4
		4	108967818	6, 7, 11, 13	12965.7
		5	122659113	4, 6, 10, 11, 13	12645.0
	20	3	119967854	4, 10, 17	24134.7
		4	129674830	1, 3, 10, 20	27348.1
		5	138876316	1, 3, 7, 10, 17	24930.3
	25	3	147068753	3, 7, 21	54902.12
		4	158514070	3, 10, 17, 25	55904.90
		5	170118500	5, 3, 11, 19, 21	54872.16

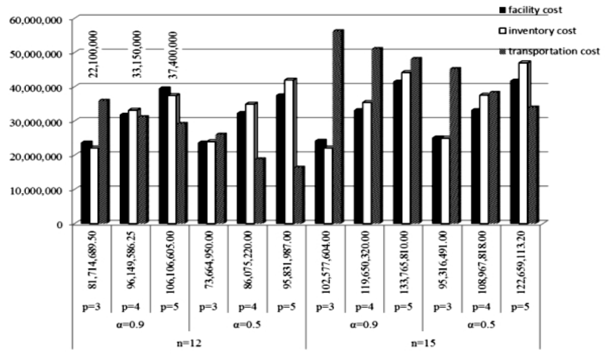


Figure 3. The model cost distribution $(v = 4; b = 0; TW = 25)$.

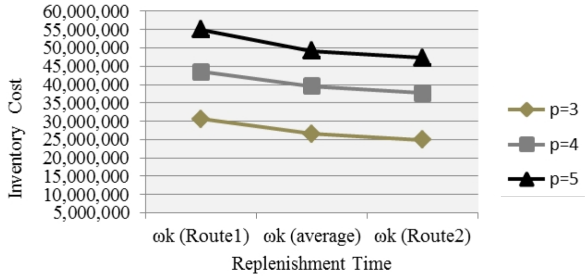


Figure 4. Inventory cost plots $(n = 15, \alpha = 0.9, TW = 25, b = 0, v = 4)$.

TABLE 4. Total inventory (invt.) cost

Hubs	Initial invt.	Invt. shortage	Additioal invt.	Total invt.	Total invt. costs
3	3	0.27	6	9	7650000
6	3	0.22	6	9	7650000
7	3	0.31	9	12	10200000
10	3	0.26	6	9	7650000
Total	12	-	21	33	33150000

TABLE 5. Flow distribution for $n = 15$, $\alpha = 0.5$, $e = 1$ and b=(0, 1.1, 1.3, 1.5).

b values	Rate of flow in path 1	Rate of flow in path 2	Rate of imbalance
0	16.62	83.37	5.01
1.1	24.26	75.74	3.12
1.3	36.72	63.27	1.72
1.5	47.09	52.90	1.12

Figure 4 shows the effect of replenishment time of vehicles on total inventory costs. This figure also shows that the total inventory cost increases as the number of hub facilities (p) increases. The inventory costs shown with average replenishment time are related to the case problem $n = 15$ and $\alpha = 0.9$ which is depicted in Figure 3. Use of routes with higher costs and less travel time reduces the replenishment time of vehicles at stations and the percentage of inventory shortage because passengers face less inventory failure and the period of replenishment time is shorter, consequently this will decrease total inventory costs in the whole network even though transportation costs increase by using these routes.The results show that total inventory cost will increase when considering more hub facilities. This increased cost is because of the initial and additional inventory according to the lack of inventory in each station which is related to the number of other hubs.

Table 5 shows the congestion and rate of flow of existing paths between hub facilities. The first column contains different values of b and the first row shows results without considering congestion effects $(b = 0)$. The second and third columns show rate of flow through path 1 and path 2, respectively, in the whole network. The rate of flow in each path is equal to the ratio of flow in each path to the total amount flow. For example, the rate of flow in path 1 is equal to: $\frac{g(\text{path1})}{g(\text{path1})+g(\text{path2})} \times 100$.

Column four shows the imbalance rate of flow which is the ratio of column two to column three. For example, the imbalance rate of flow in row one is equal to: $\frac{83.37}{16.62} = 5.01$. According to the results reported in Table 5, the imbalance rate of flow will take the maximum value of itself when the effect of congestion is not considered $(b = 0)$. On the other hand, by increasing the effect of congestion the imbalance ratio tends to reach its minimum value 1. Reported results show that by default and without considering congestion effects, the flow tends to cross from paths with lower cost, but in reality the increasing congestion effects will balance out this tendency, so increasing congestion effects increase transportation costs.

Figure 5 shows the rate of flow in different paths compared with different travel time windows for different values of congestion issue. Results show that by decreasing the internal travel time windows, the rate of flow in path 2 will be more than path 1. This increases transportation costs in the whole network. Results illustrate that by applying a 10 minutes reduction in travel time windows, transportation costs will rise by nearly 20 percent, on average. Moreover, results show that the congestion effects are more effective in problem cases with travel time window (TW=25&30), than those with $(TW=20)$. So by increasing congestion effects, the rate of flow tends to be equal in path 1 & 2 in problem cases with (TW=25&30).

Table 6 investigates the inventory cost according to the capacity of vehicles and considering constant values of replenishment times for the case problem $n = 12$ and $\alpha = 0.9$. Results show that increasing the capacity of vehicles decreases the probability of inventory shortage, considering the fixed replenishment time of vehicles, which causes a reduction in total inventory costs. It is notable that the replenishment time of vehicles with more capacity will be more than those with lower capacity in the real world .

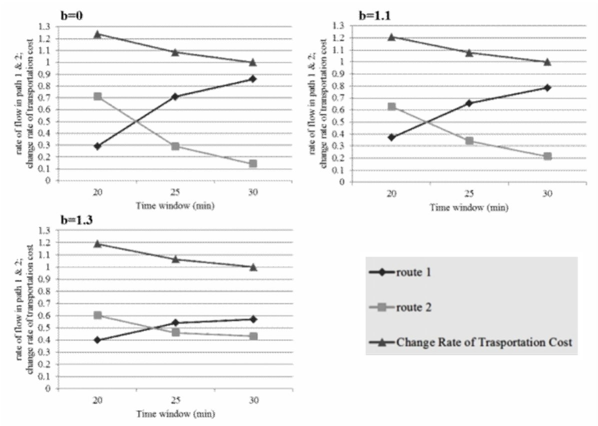
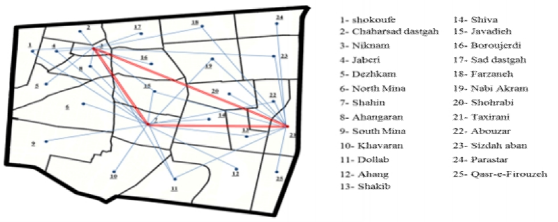


Figure 5. Time window, congestion and transportation cost plots (TW= 20, 25 & 30; b=0, 1.1 & 1.3) .



P=3	3		7		21		S(k)
	path 1	path 2	path 1	path 2	path 1	path 2	
3			0.67	0.33	0.54	0.46	51%
7	0.59	0.41			0.87	0.13	46%
21	0.69	0.31	0.74	0.26			38%

Total Cost: 147068753
Fixed Cost: 23054000
Inventory Cost: 25500000
Transportation Cost: 98514753

Figure 6. Network structure, $(n = 25, \alpha = 0.5, v = 10, p = 3)$.

TABLE 6. Computational example for inventory shortage with different vehicle capacities $(n = 12, \alpha = 0.9, TW = 25, b = 0)$

n=12, α=0.9		Percent of inventory shortage					Total inventory cost
P=3	v=4	0.31	0.41	0.32	-	-	22100000
	v=10	0.21	0.14	0.13	-	-	11900000
P=4	v=4	0.29	0.22	0.30	0.21	-	33150000
	v=10	0.15	0.11	0.21	0.13	-	22950000
P=5	v=4	0.21	0.12	0.18	0.17	0.19	37400000
	v=10	0.08	0.19	0.13	0.07	0.12	27200000

Table 6 reports an inventory shortage probability reduction in each hub because passenger demand is distributed between hubs when number of hubs increases. Table 6 shows, in brief, that increasing inventory service level (decreasing percent of inventory shortage) will increase total inventory costs. According to the map of district 14 of Tehran; Figure 6, shows hub network structure, distributing costs of a network, the rate of flow in backbone network and the percent of inventory shortage in each hub for *P*=3.

5. CONCLUSION

This paper investigated the effect of congestion on routes and inventory service level within the context of hub location allocation network. The contribution of this research lies in the development of a modeling framework. This paper expanded the basic model of hub location which incorporates service time requirements between nodes, fixed cost of operating a hub, the cost of considering inventory service level at hubs and congestion on routes which exist between hubs.
Results showed that increasing effective factors like the number of hub facilities, the entering frequency of vehicles at hubs and also increasing the capacity of vehicles in a short replenishment time will increase inventory service level (reduce the percent of inventory shortage) of the hub network that this caused an increase in inventory costs. Results ultimately lead to lower costs and increase the benefits of transportation system services for urban passengers.

This research can be extended by adding different modes of transportation like bus and subway. As another research, applying congestion at hubs with different methods can expend this research which is expected to have a direct effect on inventory service level. Moreover, considering models with approaches such as vehicle fuel consumption or CO_2 emissions from vehicles which integrate hub location problems with green supply chain can also make an improvement in contributions of this research. All of these ideas are compatible with taxi services and can have significant effects on improving the urban logistic systems.

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A Congested Double Path Approach for a Hub Location-allocation Problem with Service Level at Hubs

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چکیده

در این مقاله یک نوع از مساله مکان‌یابی و تخصیص چندگانه p-هاب میانه بدون محدودیت ظرفیت مورد بررسی قرار گرفته شده است. مدل ارائه شده در این مقاله، هزینه های حمل و نقل را براساس سطح خدمت موجود در هاب ها و مسیرهای دوگانه موجود بین هاب ها حداقل می کند. سطح خدمت‌دهی موجودی، میزان موجودی وسایل نقلیه در ایستگاه‌ها برای حمل‌ونقل مسافران با توجه به تقاضای مسافران و زمان بازپرسازی وسایل نقلیه در ایستگاه‌ها تعریف شده است. برای ارزیابی مدل ارائه شده از مجموعه داده جمع‌آوری شده ۲۵ محله منطقه ۱۴ شهر تهران استفاده شده است.

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