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TECHNICAL NOTE

Modelling of Resonance Frequency of MEMS Corrugated Diaphragm for Capacitive Acoustic Sensors

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ABSTRACT

In this paper, a new model for resonance frequency of clamped circular corrugated diaphragm has been presented. First, an analytical analyzes has been carried out to derive mathematic expressions for mechanical sensitivity of diaphragm with residual stress. Next, by using Rayleigh's method we present mathematical model to calculate the resonance frequency of corrugated diaphragm and investigate the effect of residual stress, Young modulus and corrugation numbers on resonance frequency of diaphragm. In this work, MATLAB software for analytical analyzes and Intellisuite MEMS tool for finite element simulation have been used. The analytical and simulated results have been compared together and there is good agreement between them.

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1. INTRODUCTION

The capacitive acoustic sensors generally consist of a diaphragm, which is vibrated by impinging of acoustic wave pressure, a back plate and air gap. High residual stress in diaphragm causes undesirable effects such as high actuation voltage, film buckling and diaphragm cracking [1, 2]. The residual stress has also influence on the resonant frequency, cut-off frequency and sensitivity [3]. A possible method to reduce the effect of residual stress in diaphragm is utilization of corrugation technique [4]. The shape of the frequency response of the MEMS acoustic sensor is determined by resonance frequency of the diaphragm. The resonance frequency depends on geometric parameters and material properties such as Young's modulus, residual stress, mass and density [5-7].

In previous works, the resonance frequency of the flat diaphragm is investigated and modelled with different methods. The result shows that the film stress has strong influence on resonance frequency of diaphragm [8]. It is clear that the corrugations decrease the effect of residual stress on diaphragm, but corrugations shift natural frequency of diaphragm. The dynamic properties of corrugated diaphragm were not investigated in previous works due to complexity of structure. In this paper, a new and accurate model of resonance frequency is presented for MEMS corrugated diaphragm.

2. MECHANICAL SENSITIVITY OF CORRUGATED MEMBRANE AND PLATE

The central deflection (w) of a flat circular plate with clamped edges and residual stress due to a homogeneous pressure (P) can be calculated from [9]:

$$\frac{PR^4}{Et^4}=\frac{5.33}{1-v^2}\cdot\frac{w}{t}+\frac{2.83}{1-v^2}\left(\frac{w}{t}\right)^3 \tag{1}$$

where, E, v, R and t are Young's modulus, Poisson's ratio, radius and thickness of the diaphragm, respectively. The mechanical sensitivity of plate is defined as [10]:

$$S_{fp}=\frac{dw}{dp} \tag{2}$$

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For small deflection, the mechanical sensitivity of a flat circular plate, by neglecting the 3rd order term, can be expressed as:

$$S_{fp} = \frac{R^4 3(1-v^2)}{16Et^3} \tag{3}$$

The diaphragm with high residual stress behaves as membrane. The central deflection, w, of a flat circular membrane with clamped edges, due to a homogeneous pressure, P, can be presented as [9]:

$$\frac{PR^4}{Et^4} = \frac{4\sigma_0 R^2}{Et^2}(\frac{w}{t}) \tag{4}$$

where, σ_o is residual stress of the diaphragm. The mechanical sensitivity of a flat circular membrane can be expressed as:

$$S_{fm} = \frac{R^2}{4.t.\sigma_0} \tag{5}$$

The residual stress of diaphragm depends on fabrication process, but it can be controlled within certain limits by the parameters of the deposition process. Since accurate control of thin film stress in processing are rather difficult, therefore shallow corrugated diaphragm has been verified to decrease the effect of residual stress. In corrugated membrane, the equilibrium stress of a membrane can be calculated as:

$$\sigma_e = \eta\sigma_0 \quad \eta < 1 \tag{6}$$

where, σ_e is equilibrium stress of corrugation diaphragm and η is attenuation coefficient of stress. The attenuation coefficient of stress for corrugated diaphragm, which is shown in Figure 1 that consists of flat and corrugated zone is given by [11]:

$$\eta = \frac{\sigma_e}{\sigma_0} = \frac{R.t^2}{R.t^2 + 6N_c.h_c^2.w_c.\sin\beta + 8N_c.h_c^3.\sin^2\beta} \tag{7}$$

where, w_c, h_c, N_c and β are width, height, corrugation numbers and angle between width and height, respectively. The central deflection of a corrugated circular plate with clamped edge expressed as [12]:

$$P = a_p E \frac{t^4}{R^4}\frac{w}{t} + b_p \frac{E}{(1-v^2)}.\frac{t^4}{R^4}.\frac{w^3}{t^3} \tag{8}$$

where, a_p is the dimensionless linear coefficient, b_p is the dimensionless non-linear coefficient [13].

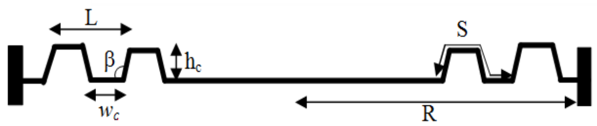

Figure 1. Cross section of corrugated diaphragm

$$a_p = \frac{2(q+1)(q+3)}{3(1-\frac{v^2}{q^2})} \tag{9}$$

$$b_p = 32\frac{1-v^2}{q^2-9}(\frac{1}{6} - \frac{3-v}{(q-v)(q+3)}) \tag{10}$$

where, q is corrugated profile factor which is given by the following equations [13]:

$$q^2 = \frac{S}{L}(1+1.5\frac{h_c^2}{t^2}) \tag{11}$$

where, S, L and h_c are the spatial period, arc length and depth of corrugations diaphragm, respectively (see Figure 1). For small deflection, the mechanical sensitivity of a corrugated circular plate, by neglecting the 3rd order term, expressed as:

$$S_{cp} = \frac{R^4}{t^3.E.ap} \tag{12}$$

The central deflection, w, of a high residual stress corrugated circular membrane with clamped edges, due to a homogeneous pressure, P, can be given as [13]:

$$\frac{PR^4}{Et^4} = \frac{4\sigma_e R^2}{Et^2}(\frac{w}{t}) \tag{13}$$

The mechanical sensitivity of a corrugated circular membrane can be expressed as:

$$S_{cm} = \frac{R^2}{4.t.\sigma_e} \tag{14}$$

3. MODELLING OF RESONANCE FREQUENCY FOR CORRUGATED DIAPHRAGM

The diaphragm is chosen circular. It is well-known that the circular diaphragm can behave such as membrane or plate. The difference between them depends on the source of the restoring force during the vibration. For a membrane, the restoring force is created from the membrane tension. However, in plates the restoring force is due to the Young's modulus. For the resonance behaviour of diaphragm we should distinguish two limiting cases: Disk approximation and Membrane approximation.

1) Disk approximation: in this case the diaphragm is thick and the residual stress is zero. The diaphragm is defined to be a free vibrating edge-clamped circular disc. Thus, the first natural frequency of flat circular plate is expressed as [14]:

$$f_{fp} = \frac{\alpha_{mn}}{2\pi R^2}\sqrt{\frac{E.t^2}{12.\rho.(1-v^2)}} \tag{15}$$

where, f_{fp} is the first resonance frequency of flat circular plate, ρ is the density of the plate and α_{mn} is the vibration constant determined by the vibration mode (for first resonance frequency α_{mn} is 10.21).

2) Membrane approximation: the diaphragm is thin and the stretch forces are created by film residual stress. The resonance frequency of a flat circular membrane with clamped edge is expressed by [14]:

$$f_{fm} = \frac{\gamma_{mn}}{2\pi R}\sqrt{\frac{\sigma_0}{\rho}} \tag{16}$$

where, f_{fm} is the first resonance frequency of flat circular membrane and γ_{mn} is a constant that depends on the vibration modes (for first resonance frequency γ_{mn} is 2.404).

In order to obtain an expression for resonance frequency of flat diaphragm, Muralt et al. combined Equations (15) and (16) using Rayleigh-Ritz method. Thus, the first resonance frequency of flat diaphragm (f_{fd}) with residual stress is defined as [15]:

$$f_{fd} = \sqrt{f_{fm}^2 + f_{fp}^2} = \sqrt{\frac{1}{(2\pi R)^2.\rho.t}[\frac{\alpha_{mn}^2}{R^2}.D + \gamma_{mn}^2\sigma_0]} \tag{17}$$

Using Equations (3) and (5), the Equation (17) can be rewritten as:

$$f_{fd} = \sqrt{\frac{1}{(2\pi)^2.\rho.t}[\frac{\alpha_{mn}^2}{64.s_{fp}} + \frac{\gamma_{mn}^2}{4.s_{fm}}]} \tag{18}$$

Equation (18) shows that the resonance frequency of the diaphragm is inversely proportional to the sensitivity and the thickness of diaphragm. Thus the resonance frequency of the corrugated diaphragm with residual stress can be defined as:

$$f_{cd} = \sqrt{f_{cm}^2 + f_{cp}^2} = \sqrt{\frac{1}{(2\pi)^2.\rho.t}[\frac{\alpha_{mn}^2}{64.s_{cp}} + \frac{\gamma_{mn}^2}{4.s_{cm}}]} \tag{19}$$

By replacing Equations (12) and (14) in (19) and rearranging, we have:

$$f_{cd} = \frac{1}{2\pi.R}\left(\frac{\alpha_{mn}^2.E.ap.t^2}{64.\rho.R^2} + \frac{\gamma_{mn}^2.\eta.\sigma_0}{\rho}\right)^{\frac{1}{2}} \tag{20}$$

According to Equation (20), the resonance frequency of corrugated diaphragm depends on the thickness, residual stress and radius of diaphragm.

4. RESULTS AND DISCUSSION

In this section, we present first resonance frequency of corrugated diaphragm and investigate the effect of some parameters such as Yang's modulus, residual stress and corrugation numbers on resonance frequency of diaphragm. We used MATLAB software for mathematical analysis and IntelliSuite MEMS tool for finite element analysis of corrugated diaphragm. Figure 2 shows the first resonance frequency of corrugated diaphragm versus residual stress. In this example, the corrugated diaphragm has been assumed with Poisson's ratio of 0.22, Yung's modulus of 169 GPa, diaphragm radius of 0.31 mm, diaphragm thickness of 1μm, h_c = 2.4 μm, $N_c = 5$ and $\beta = 90°$.

In Figure 2, to demonstrate the accuracy of the new model, the mathematical result from Equation (19) is compared with simulation result. The good agreement between simulation and analytical results shows the accuracy of the new model for resonance frequency of corrugated diaphragm.

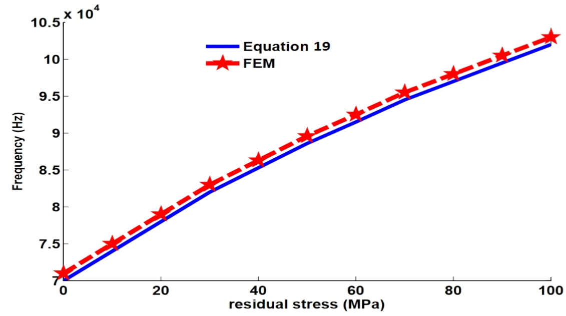

Figure 2. First resonance frequency of corrugated diaphragm versus residual stress

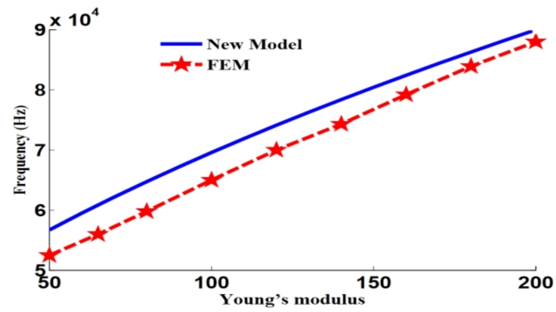

Figure 3. First resonance frequency of diaphragm versus Young's modulus

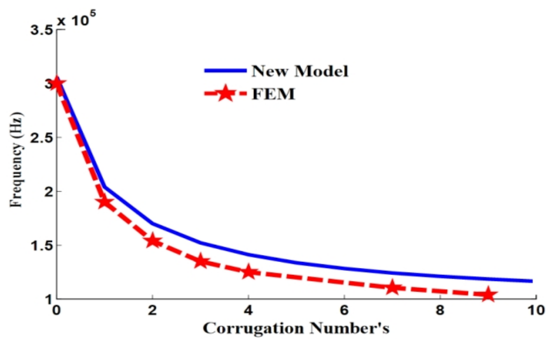

Figure 4. First resonance frequency of diaphragm versus corrugation numbers

Young's modulus is an important parameter and has significant effect on diaphragm performance. Figure 3 shows the first resonance frequency of the corrugated diaphragm versus Young's modulus. It can be seen that, the resonance frequency of diaphragm is proportional to Young's modulus. Figure 4 shows the resonance frequency of corrugated diaphragm versus number of corrugations. It is clear that the resonance frequency is approximately inversely proportional to corrugation numbers. As can be seen, the resonance frequency of corrugated diaphragms depends on corrugation numbers.

5. CONCLUSION

In this paper, a new mathematical model is presented to calculate the resonance frequency of the clamped circular corrugated diaphragm for MEMS capacitive acoustic sensor. The results show that the first resonance frequency of corrugated diaphragm is proportional to Young's modulus and residual stress and inversely proportional to corrugation numbers. Analytical results using MATLAB and simulated results using Intellisuite MEMS tool have been compared together and a good agreement between them shows the accuracy of the new model.

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چکیده

در این مقاله مدل جدیدی برای فرکانس رزونانس دیافراگم موجدار دایره ای چهار طرف ثابت ارائه شده است. در ابتدا آنالیز تحلیلی برای استخراج معادلات ریاضی حساسیت مکانیکی دیافراگم که دارای استرس پسماند است صورت می گیرد. سپس با استفاده از روش رایلی یک مدل ریاضی برای محاسبه فرکانس رزونانس دیافراگم موجدار ارائه می شود و اثر استرس پسماند ، مدول یانگ و تعداد موج بر روی این فرکانس بررسی می شود. در این مقاله از نرم افزار متلب برای آنالیز تحلیلی و نرم افزار Intellisuite برای شبیه سازی استفاده شده است. نتایج حاصل از آنالیز تحلیلی و شبیه سازی همدیگر را به خوبی تصدیق می کنند.

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