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RESEARCH NOTE

Stabilization and Walking Control for a Simple Passive Walker Using Computed Torque Method

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ABSTRACT

The simple passive dynamic walker can walk down a shallow downhill slope with no external control or energy input. Nevertheless, the period-one gait stability is only possible over a very narrow range of slopes. Since the passive gaits are extremely sensitive to slope angles, it is important to use a control strategy in order to achieve a wide range of stable walking. The computed torque method is proposed here to produce stable period-one gait cycles for different slopes. In present method, the unstable walking gait is stabilized by a stable period-one gait pattern on a small specific slope. The proposed approach is illustrated by the simplest passive walkers with point and curved feet. Simulation results reveal the usefulness of this control method for improvement in stability properties of the models.

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1. INTRODUCTION

Humans can walk easily on different surfaces with the lowest effort to keep the stability of walking motion. However, walking is a very complicated dynamic phenomenon and not very well understood. Designing biped robots has developed remarkably in recent years to better understand human walking, and many control algorithms have been presented for this purpose. Although most of biped robots are controlled based on Zero Moment Point criterion [1], many researches are interested in higher energy efficient gait generation. Passive dynamic walking proposed by McGeer [2] has been thought as a good example of efficient bipedal locomotion. He showed that a simple two-link mechanism can perform an indefinitely stable walk on a range of shallow slopes [2, 3]. But passive dynamic walking is sensitive to the robot's initial posture and the slope angle is narrow. To increase performance and to realize a practical biped, further studies has been done on passive walking [4-16]. The most popular ones are the compass-like biped robot which is introduced by Goswami et al. [4-6] and the point-foot walker which has been studied first by Garcia et al. [7, 8]. The main objective of these works is to produce a stable periodic walking from some initial conditions, and to analyze the stability properties of the resulted gait limit cycles using the Poincare map. Also, both of these two popular models are studied in the view of local stability and period doubling bifurcation leading to chaotic gaits with respect to the structural parameters of the systems such as the slope angle. Furthermore, the role of foot shape of these models in walking properties has been studied through simulations and experiments [17-19]. Passive dynamic walking is useful to generate quite natural gaits like human, but it depends on the slope angle. These walkers cannot walk on level ground and their motion on small slopes also implies low speeds [20]. To overcome this sensitivity problem, researchers have proposed semi-passive robots, also called under actuated robots, in order to avoid the use of inclined planes [21-23]. On the other hand, passive walking exhibits chaotic gaits as the slope angle steadily increasing and there are only unstable gaits in high speed region [7]. The chaotic gait is sensitive to the initial state and is not periodic; therefore, the biped robot may fall down easily. A periodic gait exists if the passive walker starts walking from the proper initial

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posture. Indeed, favorable initial conditions of these bipeds usually lead to falling forward or backward. Using linearized equations of motion [7], search algorithm [24] and energy balance method [19] are conventional approaches in finding proper initial condition for stable walking on small specific slopes. On the other, the design of control scheme to make the chaotic gait converge to a stable periodic gait on steeper slopes is very important and in this regard, some biped robot control algorithms have been presented [25-33]. Liu et al. [34] used adaptive excitation control method to increase the gait stability of an under-actuated biped robot with knees. Using the linearized controlled Poincare map and designing a state feedback controller to stabilize unstable limit cycles of the compass-gait model is a recent approach in passive walking control [35].

Goswami et al. [25], investigated two different control laws for enlarge the basin of attraction of passive limit cycles and can create new gaits, one law tracks a given mechanical energy of the robot and the other tracks, in addition, a specified average progression speed. The principle of their first control scheme is: as the robot walks down on a slope its support point also shifts downward at every touchdown. As it loses gravitational potential energy in this way its kinetic energy increases accordingly. In a steady walk this is exactly the amount of kinetic energy that is to be absorbed at the end of each step by the impact. If, at every touchdown we reset our potential energy reference line to the point of touchdown, the total energy of the robot appears constant regardless of its downward descent. The performance of this scheme is limited by the fact that the generated gaits are still close to the passive gait. In order to improve the robot performance, in the second control scheme, they proposed a control law which attempts to maintain, in addition, a specified average speed of progression.

Freidovich et al. [36], used a virtual holonomic constraint for obtaining stable gaits of passive 2 degrees of freedom robot. For this purpose, they used a searching algorithm, and changed the problem to an optimization problem to find this stable gait. Ames [37] presents the process of formally achieving bipedal robotic walking through controller synthesis inspired by human locomotion. He considered a humanoid robot, and tries to design a control based on Lyapunov function and quadratic programing to obtain stable walking. Asano et. al. [26] proposed virtual passive dynamic walking utilizing modified gravity condition with virtual gravity field for obtaining stable walking of biped robot. They proposed virtual passive walk and a virtual passivity mimicking control law for this purpose. They proposed this control strategy for generating the steady walking pattern even if the physical parameters are not suitable.

In this work, a numerical algorithm is used to get initial conditions for stable and unstable period-one gait limit cycles of the simplest passive walker with point and curved feet. Then, a control method based on computed torque is presented in order to control unstable gaits on steeper slopes. Since biped robots have stable gaits for small slope angles, whose gait pattern and stability characteristics are know, they can be used to generate stable gait of higher slopes with unstable gait. Then, with using a controller, it is possible to generate stable gait at higher slopes, or for biped parameters with unstable gaits. This paper is organized as follows. Section 2 describes the hybrid dynamic of the models. Finding period-one limit cycles and stability analyes are explained in Section 3. The control approach is presented in Section 4. Section 5 provides some numerical simulations to verify the performance of the control method proposed in this work. Finally, conclusions are presented in Section 6.

2. WALKING DYNAMICS

2. 1. Models The model which are based on the same prototype as considered by Garcia et al. [7] and its modified model with curved feet are illustrated in Figure 1. Also, this figure identifies the physical parameters in description of the system dynamics. Models have only two symmetric legs which are rigidly connected by a frictionless hinge joint at the hip, allowing them to swing freely.

There are three point-masses, one for the hip and two for the legs. At the start of a step, both legs are in contact with the surface. These two-dimensional walkers go down an inclined plane with suitable initial conditions. During walking, the entire stance leg is fixed on the surface while the swing leg acts as a free pendulum pivoting around the hip until its foot touches the slope surface. At this instantaneous plastic collision, the roles of the two legs will be changed. The scuffing problem of the swing leg is neglected at the mid-stance. To describe the dynamic behavior of the models, the equations of motion of the swing phase together with transition rule are needed. Transition rule is obtained by using conservation law of angular momentum and geometric collision condition.

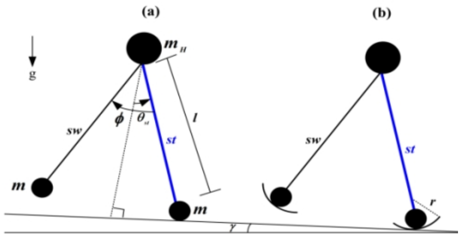

Figure 1. Schematic of the simplest walking model (a) with point feet, (b) with curved feet.

2. 2. Dynamics of the Swing Phase Continuous dynamics of the models defines the motion of the stance and swing leg between collisions. The models have two degrees of freedom, θ_{st} and ϕ which are functions of dimensionless time $\tau=(g/l)^{1/2}t$. θ_{st} is the angle of the stance leg relative to the slope normal and ϕ is the angle between the stance leg and the swing leg. Under some assumptions noted before, the two coupled second-order differential equations of swing phase can be determined by using the well-known method of Lagrange- Euler equation [5]:

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + G(\theta) = 0 \quad (1)$$

where $\theta=[\theta_{st}\ \ \phi]^T$, M is the inertia matrix, C includes the centrifugal and coriolis terms, and G is a vector of gravity forces. To simplify the governing equations, dimensionless parameters such as $\beta=m/m_H$ and $\bar{r}=r/l$ are introduced.

For the simplest walking model with point feet, as presented in [7], the matrices in Equation (1) are:

$$M(\theta) = \begin{bmatrix} 1+2\beta(1-\cos\phi) & -\beta(1-\cos\phi) \\ 1-\cos\phi & -1 \end{bmatrix} \quad (2)$$

$$C(\theta,\dot{\theta}) = \begin{bmatrix} -\beta\sin\phi\ (\dot{\phi}^2 - 2\dot{\theta}_{st}\dot{\phi}) \\ \dot{\theta}_{st}^{\ 2}\sin\phi \end{bmatrix} \quad (3)$$

$$G(\theta) = \begin{bmatrix} \beta[\sin(\theta_{st}-\phi-\gamma) - \sin(\theta_{st}-\gamma)] - \sin(\theta_{st}-\gamma) \\ \sin(\theta_{st}-\phi-\gamma) \end{bmatrix} \quad (4)$$

For curved feet model, the terms of matrices are as follows:

$$\begin{aligned} M_{11}(\theta) &= (1+\beta)[1-2\bar{r}(1-\bar{r})(1-\cos\theta_{st})] \\ &+2\beta\bar{r}^2(1-\cos\theta_{st}) \\ &+\beta\{1-2[\bar{r}\cos(\theta_{st}-\phi)+(1-\bar{r})\cos\phi]\} \\ M_{12}(\theta) &= \beta[-1+\bar{r}\cos(\theta_{st}-\phi)+(1-\bar{r})\cos\phi] \\ M_{21}(\theta) &= 1-\bar{r}\cos(\theta_{st}-\phi)-(1-\bar{r})\cos\phi \\ M_{22}(\theta) &= -1 \end{aligned} \quad (5)$$

$$\begin{aligned} C_{11}(\theta,\dot{\theta}) &= \beta\bar{r}(\dot{\theta}_{st}-\dot{\phi})^2\sin(\theta_{st}-\phi) \\ &+\beta(1-\bar{r})(2\dot{\theta}_{st}-\dot{\phi})\dot{\phi}\sin\phi \\ &+[(1+\beta)\bar{r}(1-\bar{r})-\beta\bar{r}^2]\dot{\theta}_{st}^{\ 2}\sin\theta_{st} \\ C_{21}(\theta,\dot{\theta}) &= (1-\bar{r})\dot{\theta}_{st}^{\ 2}\sin\phi \end{aligned} \quad (6)$$

$$\begin{aligned} G_{11}(\theta) &= [\bar{r}\sin\gamma-(1-\bar{r})\sin(\theta_{st}-\gamma) \\ &+\beta[2\bar{r}\sin\gamma-(1-2\bar{r})\sin(\theta_{st}-\gamma)+\sin(\theta_{st}-\phi-\gamma)] \\ G_{21}(\theta) &= \sin(\theta_{st}-\phi-\gamma) \end{aligned} \quad (7)$$

2. 3. Transition Rules at Collision Foot collision occurs when the swing leg is coincident with the ramp surface during moving forward and the former stance leg leaves the slope in the same instant. In this situation, there is a double-support phase and both legs are in contact with the surface. Then, the swing leg becomes the new stance leg and vice versa. This instantaneous impact is assumed to be plastic and without slipping. The geometric collision condition of this impact is given by $\phi=2\theta_{st}$. The effect of collision which appears in initial values including angular positions and their velocities is called transition rules. These algebraic equations relate the state vector $q=[\theta_{st}\ \ \phi\ \ \dot{\theta}_{st}\ \ \dot{\phi}]^T$ before foot collision to the same vector after collision. Transition rules of angular positions are determined by employing geometric collision conditions. Transition rules of angular velocities can be derived from angular momentum conservation around the impact point and the hip. Thus, the impact equations are given as follows:

$$\theta^+ = J\theta^-,\ \dot{\theta}^+ = (Q^+)^{-1}Q^-\dot{\theta}^- \quad (8)$$

The superscripts '–' and '+' respectively denote just before and after collision. At every step, both of the models have just one foot collision. The simplest walking models with point and curved feet have a similar matrix J as the following form:

$$J = \begin{bmatrix} -1 & 0 \\ -2 & 0 \end{bmatrix} \quad (9)$$

The matrices, Q^- and Q^+ of the basic model are specified below:

$$Q^- = \begin{bmatrix} \cos\phi^- & 0 \\ 0 & 0 \end{bmatrix}$$
$$Q^+ = \begin{bmatrix} 1+2\beta(1-\cos\phi^+) & -\beta(1-\cos\phi^+) \\ 1-\cos\phi^+ & -1 \end{bmatrix} \quad (10)$$

For the curved feet model, the elements of these square matrices are given by:

$$\begin{aligned} Q_{11}^- &= \bar{r}^2 + (1-\bar{r})^2\cos\phi^- + \bar{r}(1-\bar{r})[\cos\theta_{st}^- \\ &+\cos(\phi^- - \theta_{st}^-)] + \beta\bar{r}\{1+2[1+\bar{r}-(1-\bar{r})\cos\phi^- \\ &-\bar{r}\cos(\phi^- - \theta_{st}^-) - \bar{r}\cos\theta_{st}^-]\} \\ Q_{12}^- &= -\beta\bar{r}[1-\cos(\phi^- - \theta_{st}^-)] \\ Q_{21}^- &= \bar{r}(1-\cos\theta_{st}^-) \\ Q_{22}^- &= 0 \end{aligned} \quad (11)$$

$$\begin{aligned} Q_{11}^+ &= 1-2\bar{r}(1-\bar{r})(1-\cos\theta_{st}^+) - \\ &2\beta[\bar{r}(1-2\bar{r})(1-\cos\theta_{st}^+) + \bar{r}\cos(\phi^+ - \theta_{st}^+) \\ &+(1-\bar{r})\cos\phi^+ - 1] \\ Q_{12}^+ &= -\beta[1-\bar{r}\cos(\phi^+ - \theta_{st}^+) - (1-\bar{r})\cos\phi^+] \\ Q_{21}^+ &= 1-(1-\bar{r})\cos\phi^+ - \bar{r}\cos(\phi^+ - \theta_{st}^+) \\ Q_{22}^+ &= -1 \end{aligned} \quad (12)$$

Consequently, by merging the transition rules for angular positions and their velocities, the jump equations are identified as following:

$$q^{+} = \begin{bmatrix} J & 0_{2\times 2} \\ 0_{2\times 2} & (Q^{+})^{-1}Q^{-} \end{bmatrix} q^{-} \qquad (13)$$

3. CYCLIC WALKING

3. 1. Proper Initial Conditions Walking pattern of a passive walker consists of two parts. The first part is the time-continuous gait including the motion of swing phase. The second part is a series of discrete collision events. Each step can be defined as a function which takes the values of the various angles and their rates just after a collision to just after the next collision and can be written as $\{q^{+}\}^{i+1}=F\{q^{+}\}^{i}$.

With regard to walking, this function is termed stride function or Poincare map by McGeer [2]. A period-one gait cycle corresponds to a set of initial conditions of the walker which returns to itself after one step and is called a fixed point of the stride function. To get a stable gait cycle, we need to solve equations of the swing phase and transition rules with appropriate initial conditions at the start of the step. These conditions which are the roots of the step function can be obtained numerically, since it is difficult to solve the governing equations analytically. On the other, constructing Poincare map and finding its fixed points can be difficult for complex models with various foot shapes. To overcome this shortcoming , this work looks at this function as a new process. First, an iterative technique is used to approximate the solution of the swing phase. Although there are several methods of numerically integrating differential equations of higher orders like Runge-Kutta method, the Finite Difference method is employed here to approximate the solution of nonlinear differential equations of the swing phase. This technique replaces the derivatives in the equation with finite difference approximations on a discrete time. It is well-known that, the central difference formula gives a better approximation to the first derivative among finite difference formulas of order O (h^2). For this discrete system, the range of step period [0, T] is divided into n sub intervals of width Δt as shown in Figure 2.

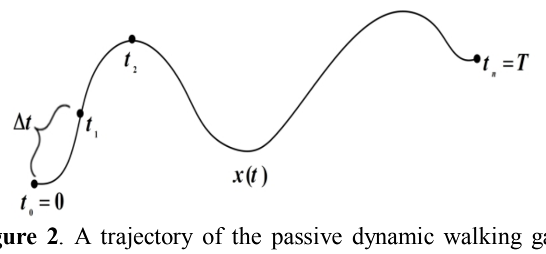

Figure 2. A trajectory of the passive dynamic walking gait. The gait starts at t_0= 0 and it is terminated at the end of a step at t_n= T.

After deriving the differential equations of the swing phase in discrete form for n time intervals in a step period, transition rules and collision condition are applied and then our step function is constructed. Now, it is necessary to find roots of the step function for this discrete system. Although there exist several efficient numerical methods like iterative Newton-Raphson method, root finding of a function is one of the most common numerical problems that requires the capabilities of MATLAB or other numerical software. For solving systems of nonlinear equations F(x) =0, MATLAB provides a function in the Optimization Toolbox. By supplying a good starting point, the zeros of the step function that includes proper initial conditions and step period can be calculated. For a passive biped robot there may be several walking pattern and step periods, among them walking pattern with stable periodic gait is desirable. Depending on starting guess, two initial conditions and step periods (long and short period) can be found to simulate the walking motion for specified slope angle. According to important role of starting point in root finding and convergence to the proper initial conditions, the initial conditions of the basic model can be used as the starting guess for the modified model. Next subsection will explain the procedure for finding periodic gaits from the resulted initial conditions.

3. 2. Period-one Gait Limit Cycle and Stability Analysis After finding initial conditions and corresponding step periods, periodic walking solutions must be simulated. Solving equations of the swing phase for a period of time and corresponding to one step, for a given set of resulted initial conditions, yields an Initial Value Problem (IVP). Using function ODE45 in MATLAB, numerical calculations found two period-one solutions to this IVP as short and long period gait cycles with respect to different initial conditions and step periods.

The cyclic stability of the periodic gait is analyzed around fixed points on Poincare map by evaluating the eigenvalues of the Jacobian. If the eigenvalues of Jacobian matrix are inside the unit circle in complex plane, the limit cycle is stable. Garcia et al. found the Jacobian by both numerical and analytic procedure [7]. In this paper, the Jacobian matrix is calculated by simulating one step motion for a small perturbation on each of the states of the initial conditions. First, a small perturbation is considered which applied to each of the variables of the initial conditions for finding periodic solution. Then, this solution is obtained by the intersection of the solution of IVP with the impact event for each state. This process is repeated for all the perturbed variables of the initial conditions. Finally, the Jacobian matrix and its eigenvalues are determined in a small neighborhood of the initial conditions.

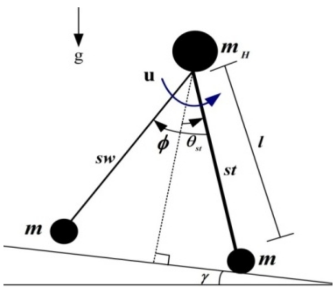

Figure 3. Applied control torque to the hip of the simplest walking model.

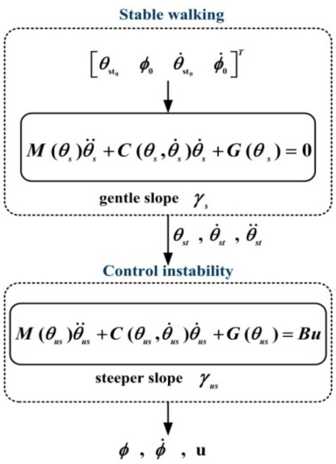

Figure 4. The outline of the designed control scheme based on computed torque method.

4. CONTROL APPREOACH

As a result, it is revealed that the passive walker has the self-stabilization property; that is, the walking motion converges to a periodic gait. But unstable period-one gait cycles and then chaotic gaits appear by changing the structural parameters of the systems such as slope angle. We stress that for the control of the simplest passive walker, we apply only one actuator torque u at the hip to restore kinetic energy dissipating on impact as seen in Figure 3.

The equation of motion for robot with control input can be obtained from the Lagrange- Euler equations. Then Equation (1) an be rewritten as:

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + G(\theta) = Bu \tag{14}$$

The input matrix B in the above equation is defined by $[-1\ 1]^T$. If the actuator torque u always remains zero, the simplest walking model is said to be completely passive. Expression in (14) designs the dynamics of the passive walker during the swing phase; therefore, the angular momentum is conserved. Since the main objective in the control problem of biped robots is to have stable periodic gaits, the proposed control scheme based on computed torque method computes the required torque using the stable dynamic walking of the robot as a desired trajectory. Indeed, the values of angular position, velocity and acceleration of the stance leg over a stable gait on a shallow slope γ_s are used as input values for stabilization an unstable gait on a steeper slope γ_{us}. In this situation, the walker will be able to walk on the slope γ_{us} by using stance leg angle and its angular velocity for the slope γ_s. Thus, the proper values of the swing leg angle, its velocity and control torque related to the input values are calculated. In fact, this control method implies that stable walking can be obtained by applying a control torque and tracking the desired stable trajectory on a gentle slope. As mentioned previously, nonlinear dynamic equations of the walker on the slope γ_s can be solved as an IVP by defining the resulted proper initial conditions as initial values. Now, using the solution of the IVP related to the stance leg during stable walking, control torque and the angle of the swing leg in Equation (15) for the slope γ_{us} are determined.

$$\begin{bmatrix} M_{n_{11}} & M_{n_{12}} \\ M_{n_{21}} & M_{n_{22}} \end{bmatrix}\begin{pmatrix} \ddot{\theta}_{st_{\gamma_s}} \\ \ddot{\phi}_{\gamma_{us}} \end{pmatrix} + \begin{bmatrix} C_{n_{11}} & C_{n_{12}} \\ C_{n_{21}} & C_{n_{22}} \end{bmatrix}\begin{pmatrix} \dot{\theta}_{st_{\gamma_s}} \\ \dot{\phi}_{\gamma_{us}} \end{pmatrix} + \begin{bmatrix} G_{n_{11}} \\ G_{n_{21}} \end{bmatrix} = \begin{bmatrix} -\beta \\ 1 \end{bmatrix}\bar{u} \tag{15}$$

The subscript n in the above expression indicates the dimensionless terms of matrices. Using the expression in the second row of the Equation in (15), the dimensionless control torque $\bar{u}=u/ml^2$ is given by:

$$\bar{u} = M_{n_{21}}\ddot{\theta}_{st_{\gamma_s}} + M_{n_{22}}\ddot{\phi}_{\gamma_{us}} + C_{n_{21}}\dot{\theta}_{st_{\gamma_s}} + C_{n_{22}}\dot{\phi}_{\gamma_{us}} + G_{n_{21}} \tag{16}$$

Substituting the expression in (16) into the first row of (15) and introducing $N_n=C_n(\theta,\dot{\theta})\dot{\theta}+G_n(\theta)$, we obtain:

$$\ddot{\phi}_{\gamma_{us}} = \frac{(M_{n_{11}} + \beta M_{n_{21}})\ddot{\theta}_{st_{\gamma_s}} + N_{n_{11}} + \beta N_{n_{21}}}{-(M_{n_{12}} + \beta M_{n_{22}})} \tag{17}$$

Then, Equation (17) can be solved as an IVP with initial values $\begin{bmatrix}\phi_{\gamma_{us0}} & \dot{\phi}_{\gamma_{us0}}\end{bmatrix}^T$. Due to the importance of appropriate initial values for the solution of an IVP, two values $\phi_{0_{\gamma_s}}$ and $\dot{\phi}_{0_{\gamma_s}}$ obtained from the solution of the IVP for a desired slope γs can be the best possible choice. Figure 4 provides a schematic representation of the proposed control approach.

After finding proper value of the swing leg angle and its angular velocity for stable walking on a steeper slope, cyclic stability is investigated to ensure the effectiveness of the presented control method. It is important to note that the computed torque method changes walking pattern or initial conditions in order to stabilize unstable period-one gait cycles. An excellent advantage of this approach is that, it is easily accessible in comparison with the other conventional methods.

5. NUMERICAL SIMULATION

Our interest in this section is to determine period-one gait cycles with proper initial conditions. Then, we will verify the effectiveness of our control method for the stabilization of the unstable passive dynamic walking of the simplest walker with point and curved feet. It is assumed that the hip mass mH is much larger than the foot mass m (β=0) to prevent the effect of the motion of the swinging foot on the motion of the hip. Simulation results of these studied models will be discussed as follows.

5. 1. Point Feet Model By applying the proposed method based on finite difference approximation to find proper initial conditions, simulation results reveal that there are long and short gait cycles for the point feet model. The long period-one limit cycle with initial condition as [0.2003, 0.4006, -0.1998, -0.0158] for the slope angle 0.009 rad is shown in Figure 5. As seen, this limit cycle is stable because the corresponding eigenvalues of the Jacobian matrix at the resulted initial conditions are in the unit circle.

The short period-one gait cycle for the same slope angle is shown in Figure 6. Its initial conditions are found as [0.1939, 0.3878,-0.2038, -0.0151]. Because one of the eigenvalues is larger than 1 in magnitude in Figure 6.b, this result implies that the short period-one limit cycle is unstable. Because there exist no stable desired walking for tracking, control of instability is not possible. The stability analysis imply that the short period-one gait cycles are always unstable, whereas long period-one gait cycles are stable for a range of slopes lying between 0 rad and 0.015 rad. The loci of eigenvalues in Figure 7 exhibits unstable long period-one gait cycle of the simplest walking model with point feet for slope angle 0.015 rad.

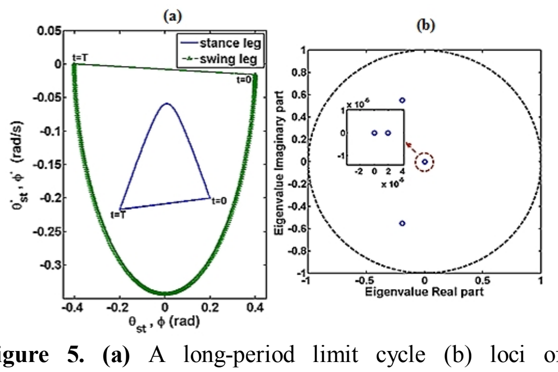

Figure 5. (a) A long-period limit cycle (b) loci of eigenvalues for the point feet model (γ=0.009 rad).

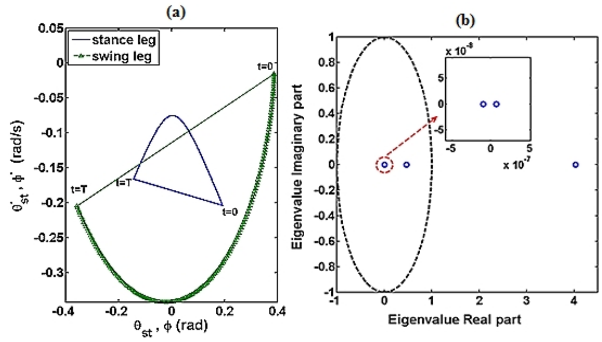

Figure 6. (a) A short-period limit cycle (b) loci of eigenvalues for the point feet model (γ=0.009 rad).

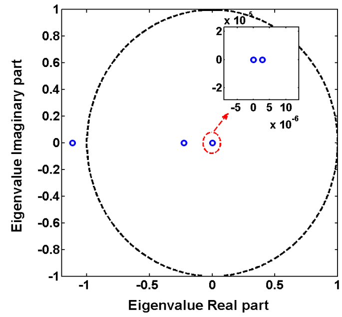

Figure 7. Loci of eigenvalues for the point feet model at long period-one gait cycle (γ=0.015 rad).

From the result of Garcia et al. [7], the simplest walking model has stable period-one gaits for slopes up to 0.0151 rad. With increasing slope angle, unstable cycles of long period-one appear and the walking-like motions become chaotic through a sequence of period doublings. To stabilize unstable cycles of long period-one on steeper slope angles and prevent falling, we use the control method based on computed torque. It is observed that under the control torque in Figure 8, all of the eigenvalues of the Jacobian matrix for the slope angle 0.015 rad lie in the unit circle as seen in Figure 9. Though the control torque is needed continuously, its value is very small. In this control method, we use the angle of stance leg and its velocity over a stable long period-one gait cycle on the slope 0.009 rad as a desired trajectory. Therefore, this unstable gait cycle becomes stable. It is reasonable to expect that if the value of the gentle slope of the stable desired trajectory is close to the steeper slope, a very small control torque is required for motion control. This expected result is shown in Figure 10, and it is clear that as the slope angle increases, the maximum value of the control torque decreases. Thus, choice of the gentle slope angle and desired trajectory due to the low energy consumption in passive dynamic walking has an important role in this control method.

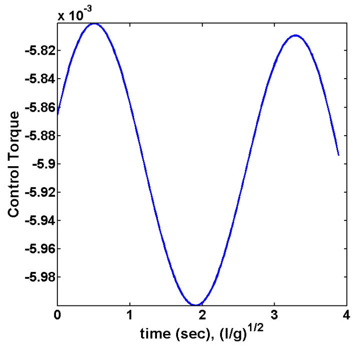

Figure 8. Control torque for stabilization the long period-one gait cycle of the point feet model at γ=0.015 rad.

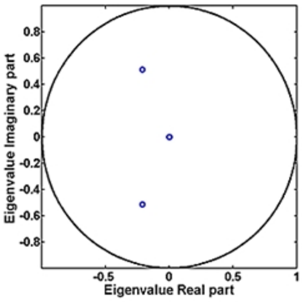

Figure 9. Loci of eigenvalues for the controlled point feet model at long period-one gait cycle (γ=0.015 rad).

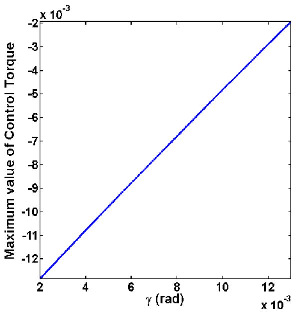

Figure 10. Control torque variation with increasing the slope angle for the point feet model.

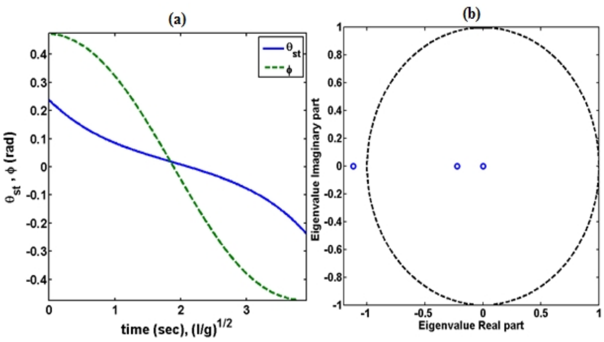

Figure 11. (a) Leg angles versus time over one step at a long-period gait cycle, (b) loci of eigenvalues for the point feet model (γ=0.015, β=0.0001).

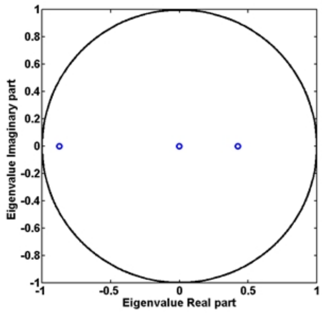

Figure 12. Loci of eigenvalues for the controlled point feet model at long period-one gait cycle with mass ratio disturbance (γ=0.015 rad).

In order to investigate the robustness of the control approach, small perturbations are added into the physical parameters of biped robot. These perturbations in biped parameters act as external disturbances. For simulation purposes, the amount of controller input is calculated based on parameters without disturbances, while dynamics of robot is disturbed. For example, the mass ratio (β) has a disturbance Δβ=0.0001 in the process of walking. A plot of θ_{st} and ∅ over one step at a long-period gait cycle and its loci of eigenvalues are shown in Figure 11 for γ=0.015 rad and Δβ=0.0001. As seen from this figure, without control, period one gait cycle is unstable. By applying the computed torque control, this unstable gait becomes stable as seen in Figure 12. Simulation result reveals that the presented control method has weak sensitiveness to the parameter disturbances.

5. 2. Curved Feet Model Similarly, when $\bar{r}$ =0.05, the curved feet model also exhibits two period-one limit cycles as long and short period. From the stability analysis, the short period-one gait cycles are always unstable such as the limit cycle and its loci of eigenvalues for γ=0.009 rad are shown in the Figure 13. The corresponding initial conditions are [0.2032, 0.4064, -0.2096, and -0.0162].

On the other hand, this model has stable long period-one gait cycles for slopes up to 0.0165 rad. Recently, this range of slope angles for the curved feet model has been found by [19]. This result implies that adding curved feet to a basic model can make walking more stable. In order to stabilize this unstable walking on the slope angle 0.0165 rad as shown in Figure 14, the control process is applied by using the stable desired trajectory on the slope 0.009 rad. Proper initial conditions for the limit cycle of the curved feet model with and γ=0.009 rad in Figure 15 are found as [0.2102, 0.4204, -0.2052, and -0.0161]. As mentioned, the loci of eigenvalues also show the stability of this limit cycle.

Then, the required control torque for stable walking related to the input values as the stance leg angle and its velocity is computed as shown in Figure 16. Clearly, the maximum value of the control torque for the curved feet model is increased compared to the point feet model. Under this control method, all the eigenvalues are smaller than 1 in magnitude as seen in Figure 17 and the robot with curved feet can walk stable.

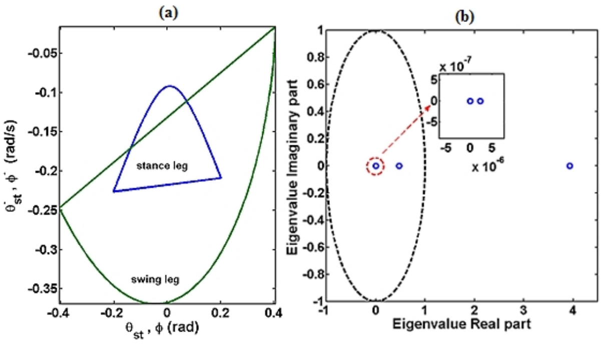

Figure 13. (a) A short-period limit cycle (b) loci of eigenvalues for the curved feet model ($\bar{r}$ =0.05, γ=0.009 rad).

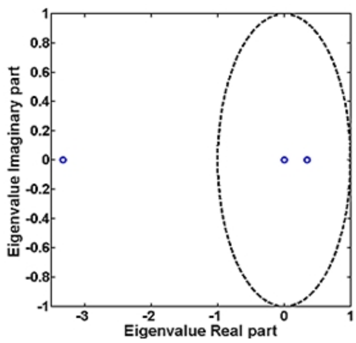

Figure 14. Loci of eigenvalues for the curved feet model at long period-one gait cycle (γ=0.0165 rad).

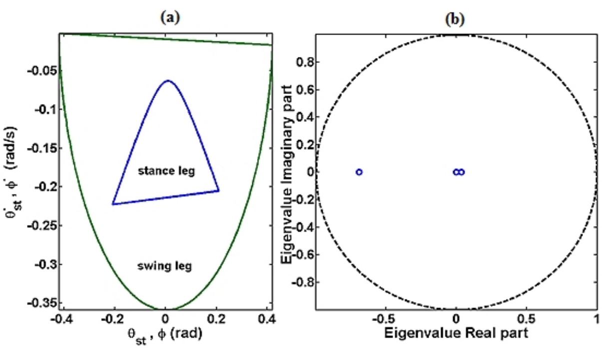

Figure 15. (a) A long-period limit cycle (b) loci of eigenvalues for the curved feet model ($\bar{r}$ =0.05, γ=0.009 rad).

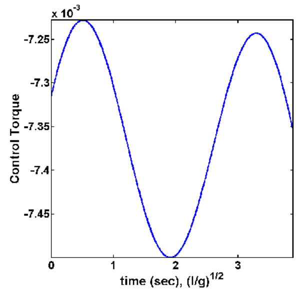

Figure 16. Control torque for stabilization the long period-one gait cycle of the curved feet model at ($\bar{r}$ =0.05, γ=0.0165 rad).

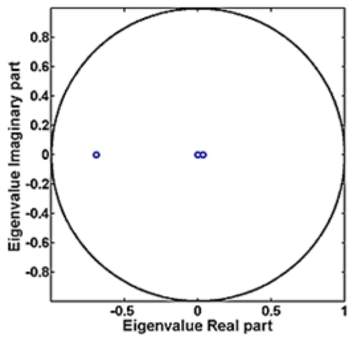

Figure 17. Loci of eigenvalues for the controlled curved feet model at long period-one gait cycle ($\bar{r}$ =0.05, γ=0.0165 rad).

6. CONCLUSION AND FUTURE WORKS

In this paper, we have studied the behavior of period-one gait limit cycles of a simplest passive walker with point and curved feet. Stability analysis shows that the short gaits of both models are unstable, while the long gaits are stable for a small range of slope angles. In order to stabilize the long period-one limit cycle on steeper slopes, we proposed a computed torque control method. Our control strategy is based on tracking stable desire walking on a gentle slope. This method can drive the unstable gait into a stable periodic gait with increasing the slope angle. Then, the passive robot can walk stably. Although, this approach has insured the stability and some energy efficiency, it cannot be able to stabilize the unstable short period-one limit cycles. Therefore, there is a need for a precise research on a new control method as a future work to achieve the goal of stabilization of these gaits.

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Stabilization and Walking Control for a Simple Passive Walker Using Computed Torque Method

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چکیده

راه‌رونده غیرفعال دینامیکی بدون وجود کنترل خارجی یا انرژی ورودی بر روی یک شیب ملایم راه می‌رود. با این وجود، پایداری گام پریود مرتبه اول تنها برای محدوده کمی از شیب‌ها امکان‌پذیر است. از آنجا که گام‌های غیرفعال به شدت به زاویه شیب سطح حساس هستند، استفاده از یک راهبرد کنترلی به منظور دست‌یابی به طیف گسترده‌ای از راه رفتن پایدار حایز اهمیت است. در پژوهش حاضر، روش گشتاور محاسبه شده به ایجاد سیکل‌های گام پریود مرتبه اول پایدار برای شیب‌های مختلف می‌پردازد. در این روش، پایدارسازی گام ناپایدار به کمک الگوی گام پریود مرتبه اول پایدار بر یک شیب ملایم معین انجام می‌شود. دستاورد پیشنهادی به وسیله ساده‌ترین راه‌رونده‌های غیرفعال با پاهای نقطه‌ای و منحنی به نمایش گذاشته شده‌است. نتایج شبیه‌سازی کارایی عملکرد روش کنترلی را در بهبود مشخصه‌های پایداری مدل‌های مورد بررسی نشان می‌دهد.

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