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Joint Pricing and Inventory Control for Seasonal and Substitutable Goods Mentioning the Symmetrical and Asymmetrical Substitution

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ABSTRACT

Nowadays many well-known firms may produce similar products at different prices in order to remain in the competitive environment. The price differences may cause substitution condition which motivates the customers to substitute similar cheaper product with an expensive one leading to an environment which is known as "customer-based price driven substitution". This research proposes a new mathematical model towards a joint dynamic pricing and inventory control for seasonal and substitutable goods in a competitive market over a finite time planning horizon. It is assumed that the two substitute goods belong to two different rival firms. The objective is to determine the optimal price, order quantity and the number of periods for one product in the presence of symmetrical and asymmetrical substitutions such that the total profit of the related firm is maximized. First it is showed that total profit is a concave function of price which leads us to a unique optimal solution. To provide the optimal solution a simple algorithm is developed. Finally, in order to evaluate the performance of proposed algorithm a numerical example is presented.

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1. INTRODUCTION

In recent years, pricing and inventory models for various types of products have been studied by many researchers [1-4]. Most of the researches refer to dynamic pricing of deteriorating items [5-7]. Dynamic pricing and inventory policies for seasonal goods have been studied first by [8]. He considered a two-period stochastic demand, where the price of second period depends on the price of first period. Later, [9]modeled the pricing of seasonal products by considering two classes of discount in the model: Inventory based and fixed discounting. The optimal solution of the classes is found by using Nash equilibrium as a game between the seller and the customers and optimization problem for the seller, respectively. [10] modeled dynamic pricing and order quantity for perishable seasonal goods with spot and forward purchase demand, which provide customers with both prompt delivery and scheduled delivery option. Demand functions are time and price dependent. [11] extended a dynamic model for consumer choice behavior in markets. In their research the products are seasonal with limited availability and the costumers are classified in two types: strategic customers and myopic customers. Strategic customers are willing to purchase later with higher stock-out risk but lower price and myopic customers are willing to purchase early in the season with higher price but lower stock-out risk.

Furthermore competition in the markets usually motivates the firms to produce similar products at different prices. Demand of the products depends on the customers' tendency whether to substitute one product for another or not. [12] classified the substitution in two types: manufacture driven substitution in which a manufacture substitute the higher value product for the lower value one when there is stock-out and customer

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driven substitution in which the customer substitutes one product for another. In the second type, if shortages lead to substitution, we call this inventory-driven substitution, and if the price differences cause the substitution, it is called price driven substitution. Both types of these substitutions lead to demand substitution. Also, [13] categorized substitution as symmetrical and asymmetrical. Based on their definition, the symmetrical substitution happens when all of the lost demands of one product are added to the demands of the substitute product whereas asymmetrical substitution happens when just a fraction of lost demands are added to the demands of the substitutable product. In this paper both symmetrical and asymmetrical price driven substitution are taken into account.

According to the above mentioned issues, some researchers were motivated to study the substitutable goods. Stochastic demand in the pricing model and inventory control for substitutable products was considered by [14]. The model is based on two essential assumptions: the demand function is price dependent and substitution is one-way. [15] proposed a stochastic pricing model for perishable goods with inventory driven substitution and demand correlation that maximizes the seller's cumulative revenues. The pricing model for services such as concerts or sporting events with dynamic substitution was proposed by [16]. The model aims to maximize the revenue in which the customers choose a product that maximizes their consumer surplus. [17] studied stochastic programming of pricing and inventory control for products with static and dynamic inventory driven substitution. [13] developed a model of pricing and production capacity decisions for symmetrical and asymmetrical price-driven substitution products with deterministic and stochastic demands. [18] established a model for substitutable products under operational postponement to determine optimal capacity. They studied the impact of operational delay and degree of substitution on the profit and optimal capacity. [19] studied the order quantity of the retailer for two substitutable products with stochastic inventory dependent demand. He proposed two heuristics solutions as procedure solutions.

To the best of our knowledge the effect of price driven substitution on the models of dynamic pricing and inventory control has not been mentioned previously, while in the real reaction, most of the seasonal goods can be substituted with similar products. So, the demand depends on the customers whether to substitute the products or not. The assumption is that two seasonal substitutable goods belong to two rival firms. In this paper symmetrical and asymmetrical customer-based price driven substitution is considered. For the seasonal goods with price-driven substitution, the demand function is dependent on time, product price and price differences between the substitutable goods. The major objective is to determine the optimal price, order quantity and the optimal number of price settings for one product, such that the total profit of the firm will be maximized. This model can be used by the firms which sell seasonal goods; an example for such situations can arise in air conditioner firms. Suppose that a firm sells the known brand of air conditioner. Undoubtedly, there are many rival firms with different brands which can be easily substituted by the customers. So, the products are seasonal and substitutable. Hence, the firm can use the following extended model to gain maximum profit according to the optimum prices, order quantity and the number of price settings.

Accordingly, in Section 2, the notations and assumptions of the paper are presented. In Section 3, the mathematical model based on the mentioned objective, and also related constraints is proposed. Then, it is proved that for any given number of price settings, the objective function is a strictly concave function of selling price. In Section 4, a solution procedure is introduced to find the optimal price, order quantity and number of periods for the product. In Section 5, a numerical example is solved to show the efficiency of the model and algorithm, and finally Section 6 refers to the conclusions and some future directions.

2. NOTATIONS AND ASSUMPTIONS

In this section, we introduce the products and the firms as *A* and *B*. *A* is used for the main product and the related firm for which we want to determine the optimal solution, and *B* for the substitute product and the related firm. The following notations and assumptions are used in this paper.

2.1. Notations:

Parameters

M	Length of sales season
T	Time space for each period
N_{max}	Maximum number of price settings
L	Substitution factor $L \geq 0$
Θ	The fraction of the lost demands of high priced product that are added to the demand of low priced substitutable product
β_a, α_a, g_a	Constant values of d_a
pb_j	Unit selling price of products *B*
h_a	Unit time inventory holding cost per unit product *A*
c_0	Unit price setting cost
SC	Price setting cost
c_s	Unit Delivery setup cost
DC_a	Delivery setup cost of product *A* (loading and unloading

Functions

d_a	Demand rate of product *A*
$R_a(j)$	Sales revenues of product *A* during period *j*
TR_a	Total sales revenues of product *A*
$IC_a(j)$	Inventory holding cost during period j for product*A*
TIC_a	Total inventory holding cost for product *A*

c_a	Unit purchasing cost of product *A*
PC_a	Purchasing cost of product *A*
$S_a(k)$	Cumulative sales amount up to end of period k for product *A*
$I_{a,j}(t)$	Inventory level at time *t* of period *j* for product *A*
$F_a(n, pa_j)$	Profit of the firm *A*

Decision variables

n	Number of price settings ($n \le N_{max}$)
pa_j	Unit selling price of products $A(pa_j > c_a$
q_a	Order quantity of product *A*

2. 2. Assumptions

- Two seasonal substitute items (*A*,*B*) are assumed, which belong to rival firms and are sold at different prices.
- The time horizon is limited to the sale season.
- Demand function is dependent on time, product price and price differences between substitutable goods.
- Symmetrical and asymmetrical customer based price-driven substitution is assumed.
- The products A and B are more or less similar. Hence, the number of periods for both of them can be assumed as the same.
- In the start of the season, the rival firm (B) determines pb_j before firm A. So, firm A determines pa_j according to the known pb_j. For the next periods, pb_j is affected by the pa_j in the previous period and also is determined earlier than pa_j (For example: for $\theta = \theta_1$, $n = n_1$ and $j = j_1 > 1$, pb_j is affected by pa_j for $\theta = \theta_1$, $n = n_1$ and $j = j_1 - 1$).
- For any given Θ we have different values of pb_j, but for simplicity we assumed that the average of the pb_j for different values of Θ has been given.
- pa_j is assumed to be smaller than pb_j ($pa_j < pb_j$).
- The items are not perishable.
- The ordering is just done at the beginning of the sale.
- The sales season (*M*) is divided into equal time spaces *T* which is given by $T=M/n$ (refer to [8]).
- Period *j* (j=1,2,…,n) indicates the time interval between the *j-th* and *j+1-th* price setting. The Prices are constant during the periods but are reset (decrease) at the beginning of the next period.
- Delivery setup cost per sales amount cannot be ignored (refer to [8]).
- Shortages are not allowed.

3. THE MODEL FORMULATION

In this paper an inventory system is considered where q_a units of items arrive to the inventory system at the beginning of the sales season and are sold during that season. Two seasonal and substitutable items with symmetrical and asymmetrical price-driven substitution are considered where two rival firms or manufactures offer the items at different prices.

The demand function for seasonal goods is time and price dependent and has the form of: $dt(p) = e^{-\alpha t} - \beta p$ (where p is sales price and α, β, a>0) [8]. Moreover, the demand function for a substitutable item with a price-driven substitution is a function of product price and price differences of substitute goods as follows: $d_a = A_a - B_a - L\theta(pa - pb))$, where $pa < pb$, A_a and B_a>0 are constant and known values, $L \ge 0$ is the substitution factor, $\theta \ge 0$ the fraction of asymmetrical substitution and *pa*, *pb* the price of substitutable products A and B respectively [11]. According to these functions, the effect of substitution is not considered in the first function, while in the real world, most of the similar products can be substituted. Moreover, in the second function the demand is constant over time so it is not efficient for the seasonal products. Therefore, we introduce a demand function for seasonal and substitute items as follows:

$$d_a(t, pa_j) = \alpha_a e^{-g_a t} - \beta_a pa_j - \theta L(pa_j - pb_j)$$
$$(j-1)T \le t \le jT, 1 \le j \le n \tag{1}$$

It is clear from Equation (1)that the demand function is dependent on time, price and price difference of the products. As mentioned in the assumptions, for both symmetrical and asymmetrical substitution, the selling price during each period is constant while it decreases in the start of the next period. According to (1), by decreasing the selling price, the demand rate increases. Thus, the demand decreases exponentially during the time in each period and increases in the start of the next period because of the price reduction as shown in Figure 1. If $pa_j < pb_j$, in the symmetrical substitution (θ =1) all of the lost demand of product *B* are added to d_a, while in the asymmetrical substitution just a fraction of lost demands of product *B* are added to d_a. So, in the symmetrical case, d_a is expected to be more than the case of asymmetrical with 0<θ <1 as shown in Figure 1.

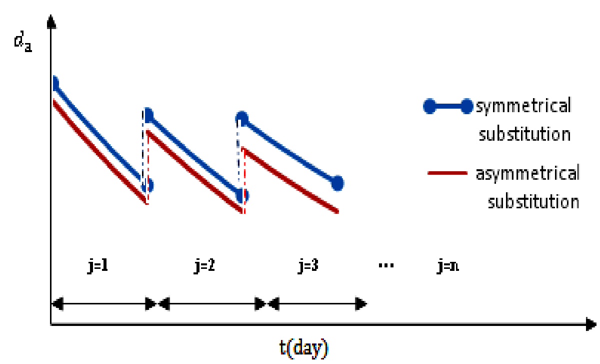

Figure 1. Representation of d_a during the time

As mentioned before, shortages are not allowed. So, the cumulative sales amount from the start of the sales season up to the end of period k is given by:

$$S_a(k)=\sum_{j=1}^{k} S_a(j)=\sum_{j=1}^{k}\int_{(j-1)T}^{jT} d_a\left(t,\mathrm{pa}_j\right)dt=$$
$$\frac{e^{-kTg_a}(-1+e^{kTg_a})\alpha_a}{g_a}+\sum_{j=1}^{k}(L\theta Tpb_j-Tpa_j(L\theta+\beta_a)) \tag{2}$$

The differential equation governing the system at time t of period $j$is given by:

$$\frac{dI_{a,j}(t)}{dt}=-d_a((j-1)T+t,pa_j)$$
$$0<t<T,1<\mathrm{j}<\mathrm{n} \tag{3}$$

It is clear that the inventory level of product A at the start of period j is the difference value between order quantity and the cumulative sales amount up to period j-1; so we have: $I_{a,j}(0)=\mathrm{q}_a-S_a(j-1)$. Using this condition, the Equation (3) is:

$$I_{a,j}(t)=\mathrm{q}_a-S_a(j-1)-\int_{(j-1)T}^{(j-1)T+t} d_a\left(t,\mathrm{pa}_j\right)dt=-L\theta tpb_j$$
$$+q_a-\frac{\alpha_a}{g_a}+\frac{e^{-(t+(j-1)T)g_a}\alpha_a}{g_a}+tpa_j(L\theta+\beta_a)- \tag{4}$$
$$\sum_{j=1}^{j-1}(L\theta Tpb_j-Tpa_j(L\theta+\beta_a))$$

Since the objective function is to maximize the profit, it includes some revenue and cost terms. The revenue is related to the sales revenue and the costs consist of inventory holding cost, purchasing cost, delivery setup cost and price setting cost. The inventory holding cost during period j and total inventory holding cost are respectively given by:

$$IC_a(j)=\int_{(j-1)T}^{jT} I_{a,j}(t)h_a dt=\frac{e^{-2jTg_a}h_a}{2g_a{}^2}(2e^{Tg_a}(e^{Tg_a}-1)*$$
$$\alpha_a-e^{2jTg_a}g_aT\alpha_a+e^{2jTg_a}g^2{}_aT((1-2j)L\theta Tpb_j+ \tag{5}$$
$$(2j-1)*Tpa_j(L+\beta_a)+2(q_a-\sum_{j=1}^{j-1}(L\theta Tpb_j-Tpa_j*$$
$$(\mathrm{L}\theta+\beta_a))))$$

$$TIC_a=\sum_{j=1}^{n} IC_a(\mathrm{j})=\frac{(1-e^{-2nTg_a})h_a\alpha_a}{(1+e^{-Tg_a})g_a{}^2}-\frac{nTh_a\alpha_a}{g_a}+$$
$$0.5T^2h_a\beta_a*\sum_{j=1}^{n}(2j-1)pa_j+T^2h_a\beta_a\sum_{j=1}^{n-1}(n-j)pa_j$$
$$-0.5L\theta T^2h_a\sum_{j=1}^{n}(1-2j)(pa_j-pb_j)-L\theta T^2h_a* \tag{6}$$
$$\sum_{j=1}^{n-1}(n-j)(pa_j-pb_j)+nTh_a(\frac{e^{-nTg_a}(e^{nTg_a}-1)\alpha_a}{g_a}+$$
$$\sum_{j=1}^{n}(L\theta Tpb_j-Tpa_j(L\theta+\beta_a)))$$

Also, the cost terms including purchasing cost, delivery setup cost and the price setting cost are respectively given by following equations:

$$PC_a=q_ac_a \tag{7}$$
$$DC_a=S_a(n)c_s \tag{8}$$
$$SC=nc_0 \tag{9}$$

As mentioned, $S_a(j)$ is the sales amount of period j. So, the sales revenue during period j and total sales revenue are respectively given by:

$$R_a(j)=S_a(j)pa_j \tag{10}$$

$$TR_a=\sum_{j=1}^{n}R_a(j)=\sum_{j=1}^{n}pa_j(L\theta Tpb_j+\frac{e^{-jTg_a}(e^{Tg_a}-1)\alpha_a}{g_a}$$
$$Tpa_j(L\theta+\beta_a)) \tag{11}$$

When there is no inventory shortage, the order quantity is given by the total sales amount. If we set k=n into (2), the total amount of sales can be calculated as follows:

$$q_a=S_a(n)=\frac{e^{-nTg_a}(e^{nTg_a}-1)\alpha_a}{g_a}+\sum_{j=1}^{n}(LT\theta\, pb_j-Tpa_j*$$
$$(L\theta+\beta_a)) \tag{12}$$

As mentioned before, the objective function is the difference between revenues and the costs. Thus, our problem is formulated as follows:

$$F(n,pa_j)=TR_a-TIC_a-q_ac_a-nc_0-S_a(n)cs=$$
$$-nc_0-\frac{\left(1-e^{-2nTg_a}\right)h_a\alpha_a}{\left(1+e^{-Tg_a}\right)g_a^2}+\frac{nTh_a\alpha_a}{g_a}-\frac{1}{2}T^2h_a\beta_a$$
$$*\sum_{j=1}^{n}(-1+2j)pa_j-T^2h_a\beta_a\sum_{j=1}^{-1+n}(-j+n)pa_j+$$
$$0.5L*T^2h_a\sum_{j=1}^{n}(1-2j)\left(pa_j-pb_j\right)+T^2h_a*$$
$$\sum_{j=1}^{-1+n}(-j+n)\left(pa_j-pb_j\right)-c_a*$$
$$\left(\frac{\left(1-e^{-nTg_a}\right)\alpha_a}{g_a}+\sum_{j=1}^{n}(\left(L\theta Tpb_j-Tpa_j\left(L\theta+\beta_a\right)\right)\right)- \tag{13}$$
$$c_s\left(\frac{\left(1-e^{-nTg_a}\right)\alpha_a}{g_a}+\sum_{j=1}^{n}\left(L\theta Tpb_j-Tpa_j\left(L\theta+\beta_a\right)\right)\right)-$$
$$nTh_a\left(\frac{\left(1-e^{-nTg_a}\right)\alpha_a}{g_a}+\sum_{j=1}^{n}\left(L\theta Tpb_j-Tpa_j\left(L\theta+\beta_a\right)\right)\right)+$$
$$\sum_{j=1}^{n}pa_j\left(L\theta Tpb_j+\frac{e^{-jTg_a}\left(-1+e^{Tg_a}\right)\alpha_a}{g_a}-Tpa_j\left(L\theta+\beta_a\right)\right)$$

S.t

$$d_a(t,pa_j)\geq 0 \longrightarrow pa_j<\frac{\alpha_a e^{-g_a jT}+\theta Lpb_j}{\beta_a+L\theta} \tag{14}$$
$$n\leq N_{\max} \tag{15}$$

The main goal of this model is to find the optimal pa_j, q_a and *n* which maximize the profit of firm *A*. The optimal order quantity for product *A* is given by (12) and pa_j, n can be obtained according to (13)-(15). pa_j is a real continuous variable, while *n* is a integer variable. For simplicity, we solve the problem for any given *n* and denote this problem as G_n. Then, the global maximum of the main problem $F^*(n, pa_j)$ is the maximum of G_n where $n \le N_{max}$.

First, we show that the profit function is concave. So, let discuss the following theorem.

Theorem. *For any given* $n, F(n, pa_j)$ *is a concave function of* pa_j

Proof. The second derivation of $F(n, pa_j)$ with respect to pa_j is taken as follows:

$$\frac{\delta F}{\delta pa_j} = 0.5L\theta T^2 h_a - 2jL\theta T^2 h_a + 2L\theta n T^2 h_a - 2L\theta T pa_j + L\theta T pb_j + \frac{e^{(-j+1)Tg_a}\alpha_a}{g_a} - \frac{e^{-jTg_a}\alpha_a}{g_a} + 0.5T^2 h_a \beta_a - 2T pa_j \beta_a + Tc_a(L\theta + \beta_a) + Tc_s(L\theta + \beta_a) \tag{16}$$

$$\frac{\partial^2 F}{\partial pa_j^2} = -2T(\beta_a + L\theta) \tag{17}$$

It is clear that $\frac{\partial^2 F}{\partial pa_j^2}$<0, so the objective function is strictly concave and the proof is completed. Θ The root of (16) gives pa_j as follows:

$$pa_j(1) = (\frac{1}{2}L\theta T^2 h_a + 2jL\theta T^2 h_a - 2L\theta n T^2 h_a - L\theta T pb_j - \frac{e^{-(-1+j)Tg_a}\alpha_a}{g_a} + \frac{e^{-jTg_a}\alpha_a}{g_a} - \frac{1}{2}T^2 h_a \beta_a - Tc_a(L\theta + \beta_a) - Tc_s(L\theta + \beta_a)) / 2T(L\theta + \beta_a) \tag{18}$$

4. ALGORITHM

To obtain the optimal values of pa_j and *n* the following Algorithm is developed. Obviously, the algorithm starts with *n*=2. This is true because *n*=1 is used for the static pricing strategy while here the problem encountered with dynamic pricing strategy.

1. Start with *j*=1, *n*=2, *N*=2, F^*=0.
2. While $n \le N_{max}$ do steps 3-8, else go to step 9.
3. While *j*≤*n* do steps 4-5, else go to 6.
4. Calculate $pa_j(1)$ according to (18) and calculate the upper bound of pa_j according to (14) named *Up*.
   4.1. If $pa_j(1)$ <*Up*, set $pa_j^* = pa_j(1)$, otherwise set $pa_j^* = Up$
5. Set *j*=*j*+1 and go to step 3.
6. Calculate $q_a^*, F(n, pa_j^*)$ respectively by (12), (13)
7. If $F(n, pa_j^*) > F^*$, then let *N*=*n*, $pa_j = pa_j^*$, $q_a = q_a^*$, $F^* = F(n, pa_j^*)$.
8. Set *n*=*n*+1 and go back to step 2.
9. Stop.

5. NUMERICAL EXAMPLE

To illustrate the algorithm, the following numerical example is presented. The results are based on the results applied by Mathematica 8.0.1.

Example. In order to evaluate the performance of proposed model as well as the developed heuristic algorithm, the following numerical example is solved. This example is run on the Mathematica 8.01 optimization package.

Consider a firm which purchases seasonal goods named *A* at c_a =\$3per unit at the start of the sales season and sells them over that season. The assumption is that there is a similar product named *B* which is substitutable with product *A*. The example is based on the parameters as follows:

M=1200(units of time), h_a =0.003\$/per unit time, c_0 =100\$, c_s =1\$, N_{max}=4, *L*= 1(units of demand/per price differences of products (\$)).

The model is solved for θ =0, 0.1, 0.4, 0.7 and 1.

Demand function for the product *A* is assumed to be:

$$d_a(t, pa_j) = 10e^{-0.001t} - 0.7\, pa_j - \theta L(pa_j - pb_j)$$

These data are mostly taken from the study conducted by [10].

Table 1 shows the different values of pb_j. As mentioned in the assumptions, for any given θ we have different values of pb_j but for simplicity we assumed that the average of the pb_j for different values of θ has been given. By implementing the proposed algorithm, the optimal values of pa_j, q_a and $F(n, pa_j)$ are obtained as shown in Table 2.

In this research, the model was solved for N_{max}=6. In all cases, the optimal solution was obtained for n<4, so we ignored the results of n=5 and n=6. Hence, N_{max}=4 is considered. The results can be analyzed as follows:

TABLE 1. Data for different values of pb_j for $2 \le n \le 4$ and $1 \le j \le n$

n	**2**		**3**			**4**			
j	**1**	**2**	**1**	**2**	**3**	**1**	**2**	**3**	**4**
pb$_j$($)	7.9	5.3	8.8	6.8	5.3	9	8.1	6.8	5.2

TABLE 2. Computational results for the case of no substitution (θ =0)

$\theta = 0$	**n**	**2**		**3**			**4**			
	j	**1**	**2**	**1**	**2**	**3**	**1**	**2**	**3**	**4**
	pa$_j$($)	7.82	4.3	8.19	6.25	4.3	8.4	6.8	5.61	4.3
	F(n,**pa$_j$**)($)	2501.88		**2683.12**			2604.74			
	q_a	1898		1740			1715			

TABLE 3. Computational results for the case of asymmetrical substitution (0 <θ <1)

$\theta = 0.1$	**n**	**2**		**3**			**4**			
	j	**1**	**2**	**1**	**2**	**3**	**1**	**2**	**3**	**4**
	pa$_j$($)	7.85	4.43	8.3	6.33	4.43	8.52	6.96	5.73	4.41
	F(n,**pa$_j$**)($)	2587.84		**2679.33**			2536.41			
	q_a	1886		1725			1713			
$\theta = 0.4$	**n**	**2**		**3**			**4**			
	3	**1**	**2**	**1**	**2**	**3**	**1**	**2**	**3**	**4**
	4.67	8.77	7.26	5.94	4.63	4.67	8.77	7.26	5.94	4.63
	F(n,**pa$_j$**)($)	**2744.04**		2673.18			2415.8			
	q_a	1886		1678			1702			
$\theta = 0.7$	**n**	**2**		**3**			**4**			
	3	**1**	**2**	**1**	**2**	**3**	**1**	**2**	**3**	**4**
	4.8	8.91	7.44	6.7	4.75	4.8	8.91	7.44	6.7	4.75
	F(n,**pa$_j$**)($)	**2828.03**		2673.29			2356.16			
	q_a	1890		1635			1687.66			

TABLE 4. Computational results for the case of symmetrical substitution (θ **=1)**

$\theta = 1$	**n**	**2**		**3**			**4**			
	j	**1**	**2**	**1**	**2**	**3**	**1**	**2**	**3**	**4**
	pa$_j$($)	7.88	4.89	8.72	6.63	4.89	9	7.55	6.15	4.79
	F(n,**pa$_j$**)($)	**2888.41**		2678.76			2326.79			
	q_a	1883		1585			1699			

5. 1. Case of no Substitution ($\theta = 0$) In this case, as shown in Table 2, the prices of products *A* and *B* are independent from each other. It is clear from the table that the maximum profit is obtained as $2683.12 with q_a=1740 units for n=3. The number of price settings for n=3 is 2 times, at the time points 400 and 800. The optimal price of product *A* during time interval [0,400] is $8.19, during time interval [401,800] is $6.25 and during time interval [801, 1200] is $4.3.

5. 2. Case of Asymmetrical Substitution ($\theta = 0.1, 0.4, 0.7$) It can be concluded from Table 3 that for $\theta = 0.1$ the maximum profit is obtained as $2679.33 for n=3 and for $\theta = 0.4, 0.7$ the maximum profits are obtained as $2744.04 and $2828.03 respectively for n=2. The table shows that the optimal profit increases from $2679.33 to $2828.03 when θ increases. This is reasonable because by increasing θ more lost demands of product *B* are added to the demands of product *A*. So, the profit increases. Also, the dynamic prices for fixed *n* and fixed *j* increases when θ increases; for example for n=3, the dynamic prices for $\theta = 0.1$ are \$8.3, \$6.33 and \$4.43 for *j*=1-3; the dynamic prices for $\theta = 0.4$ are \$8.52, \$6.48 and \$4.67 for *j*=1-3 and the dynamic prices for $\theta = 0.7$ are \$8.64, \$6.57 and \$4.8 for *j*=1-3. It means that the firm *A* will increase the dynamic prices when θ increases.

5. 3. Case of Symmetrical Substitution ($\theta = 1$) In this case, all of the lost demands of product *B* are added to the demand of product *A*. In this case, as shown in Table 4**,** the maximum profit is obtained for n=2 with the value of \$2888.41**.** The profit is higher than both cases with no substitution ($\theta = 0$) and asymmetrical substitution (0<θ **<1**). As mentioned before, it is assumed that $pa_j < pb_j$. Hence, the customers will be motivated to substitute product *A* for *B*. So, the demand and the profit of firm *A* increases. It means that, in the condition of $pa_j < pb_j$, increases in θ have a positive effect on the profit of firm *A*.

6. CONCLUSION

This paper deals with a dynamic pricing and inventory models for symmetrical and asymmetrical substitution with a demand function which is dependent on time, price and price differences of the products. Most of the pricing and inventory papers assume that the products are either seasonal or substitutable. However, in real world, the competition in the market forces the firms to produce similar products at different prices. By considering these factors, a mathematical model has been developed to overcome the lack of previous works. We show that for any fixed number of price settings, the objective function is a concave function of the product price, so the optimal solution is attainable. Then, we proposed a solution procedure to find the optimal dynamic prices, order quantity and the number of price settings. Finally, a numerical example is provided to illustrate the efficiency of the algorithm.

The results show that the optimal profit increases when θ increases. So, increasing θ has a positive effect on the optimal profit.

Also, it can be concluded from the results, when θ is increasing the firm will increase the dynamic prices to obtain the optimal profit. Our model can be used by a firm with seasonal products, which have a substitution effect on similar products, to optimize the dynamic sales prices, inventory control variables and number of price settings. For possible future research, the model can be extended for situations in which the products are perishable and/or the substitutable product prices are not well-known.

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Joint Pricing and Inventory Control for Seasonal and Substitutable Goods Mentioning the Symmetrical and Asymmetrical Substitution

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چکیده

امروزه بسیاری از بنگاه‌های شناخته شده محصولات مشابه را با قیمت‌های مختلف تولید می‌کنند. گاهی اختلاف قیمتی سبب می‌شود تا مشتریان کالای مشابه ارزان‌تر را جایگزین کالای گران‌تر کنند که این جایگزینی بر مبنای مشتری و بر اساس تفاوت قیمتی نام دارد. در این تحقیق مدل ریاضی قیمت‌گذاری پویا و کنترل موجودی برای کالاهای فصلی و جایگزین در بازار رقابتی توسعه داده شده است. فرض بر این است که دو کالای جایگزین مربوط به دو بنگاه رقیب هستند.مدل ریاضی به دنبال تعیین همزمان قیمت دورهای، تعداد بهینه دوره و میزان سفارش یک بنگاه با هدف افزایش سود آن بنگاه است. جایگزینی در این تحقیق بر مبنای مشتری و بر اساس تفاوت قیمتی در نظر گرفته شده و در دو حالت متقارن و نامتقارن بررسی می‌شود.بعد از ارائه مدل اثبات می‌شود که تابع هدف، تابعی مقعرنسبت به قیمت است، سپس الگوریتم حل مساله ارائه شده و در نهایت مثال عددی به منظور بررسی کارایی مدل و الگوریتم بحث می‌شود.

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