IJE TRANSACTIONS A: Basics Vol. 27, No. 4, (April 2014) 599-608

International Journal of Engineering

Journal Homepage: www.ije.ir

Modeling and Availability Analysis of Internet Data Center with various Maintenance Policies

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PAPER INFO

Paper history:
Received 01 March 2013
Received in revised form 19 May 2013
Accepted 22 August 2013

Keywords:
Reliability Analysis
Internet Data Center (IDC)
Router Failure
Cost Analysis

ABSTRACT

In this paper, the authors have focused on the stochastic analysis of an internet data center (IDC), which consists of a database main server connected to a redundant server. Observing the different possibilities of functioning of the system, analysis has been done to evaluate the various reliability characteristics of the system. The system can completely fail due to failure of redundant server before repair of database server, router failure and switch failure. The system can also fail completely due to a cooling failure or some natural calamity like earthquake, fire; etc. All the failure rates are assumed constant while the repairs follow two types of distributions namely general and Gumbel-Hougaard family copula.

doi: *10.5829/idosi.ije.2014.27.04a.10*

1. INTRODUCTION

The system reliability has been extensively studied by various authors like Cui and Li [1], Govil [2], Gupta and Sharma [3] and many others. They have discussed the reliability characteristic of complex systems by taking various failures and one repair policy. Network analysis is an important approach to model real-world systems. Considering the present scenario with the complexity of advance technology and modern demands of the networking system, it is necessary to study the internet data center that has become an essential requirement of usual life. Aggrarwal et al. [4] proposed a concept that failure of a node implies the failure of arcs incident from it. Kui et al. [5] studied terminal reliability of a computer communication network. This paper deals with the study of functioning of internet data center (IDC) with a redundant mail server. The internet data center can have two types of failure namely partial failure and complete failure. The information technology enabled architecture of IDC to be handled by two switches L_2 and L_3. The L_3 switch is a six-port switch, connected to a server via L_2 switch. Whenever the main mail server fails, redundant server comes into functioning automatically by a switch over device. The switch over device is instantaneous and automatic. The system can fail due to some failure like

(i) Failure of redundant server before repair of main mail server.
(ii) Failure of switch.
(iii) Router failure.
(iv) Cooling of server failure.
(v) Failure due to a natural calamity like earthquake or fire etc.

The system will be in a degraded state, when the main mail server is in completely failure mode and redundant server is in the partial failure mode.

The authors [6-12] have considered reliability and MTTF of a system, with different types of failures and one type of repair. They discussed the reliability of systems with different failure and common cause failure under the preemptive resume policy using Gumbel-Hougaard family copula distribution. However, there are many situations in real life systems where more than one repair is possible between two transition states. When this possibility exists, reliability of the system can be analyzed with the help of the copula [13, 14]. Therefore, in reference to the earlier models, here the

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authors have considered an internet data center model in which they tried to address the problem where two different repair facilities are available between adjacent states i.e. the initial state and complete failed states. All failure rates are assumed constant. The repairs follow general and Gumbel-Hougaard family copula distributions. In the present paper, S_0 is a state where the system is in good working condition. S_1, S_3, S_5 are states where the system is in degraded mode. States S_2, S_4, S_7, S_8, S_9 and S_{10} are the states where the system is in the complete failure mode. When the redundant server is in degraded state and the repair facility is not available, then system has to wait for repair, which is represented in state S_6, and whenever repair facility is available, the system is repaired and is ready for further functioning. Whenever the system is in degraded mode, it is repaired by general repair and whenever the system is in the complete failure mode, the system is repaired with the help of Gumbel-Hougaard family copula. The system is analyzed by supplementary variable technique. The various measures of reliability have been discussed and some particular cases are taken to highlight the result. The transition diagram of the designed model has been shown in Figure 1.

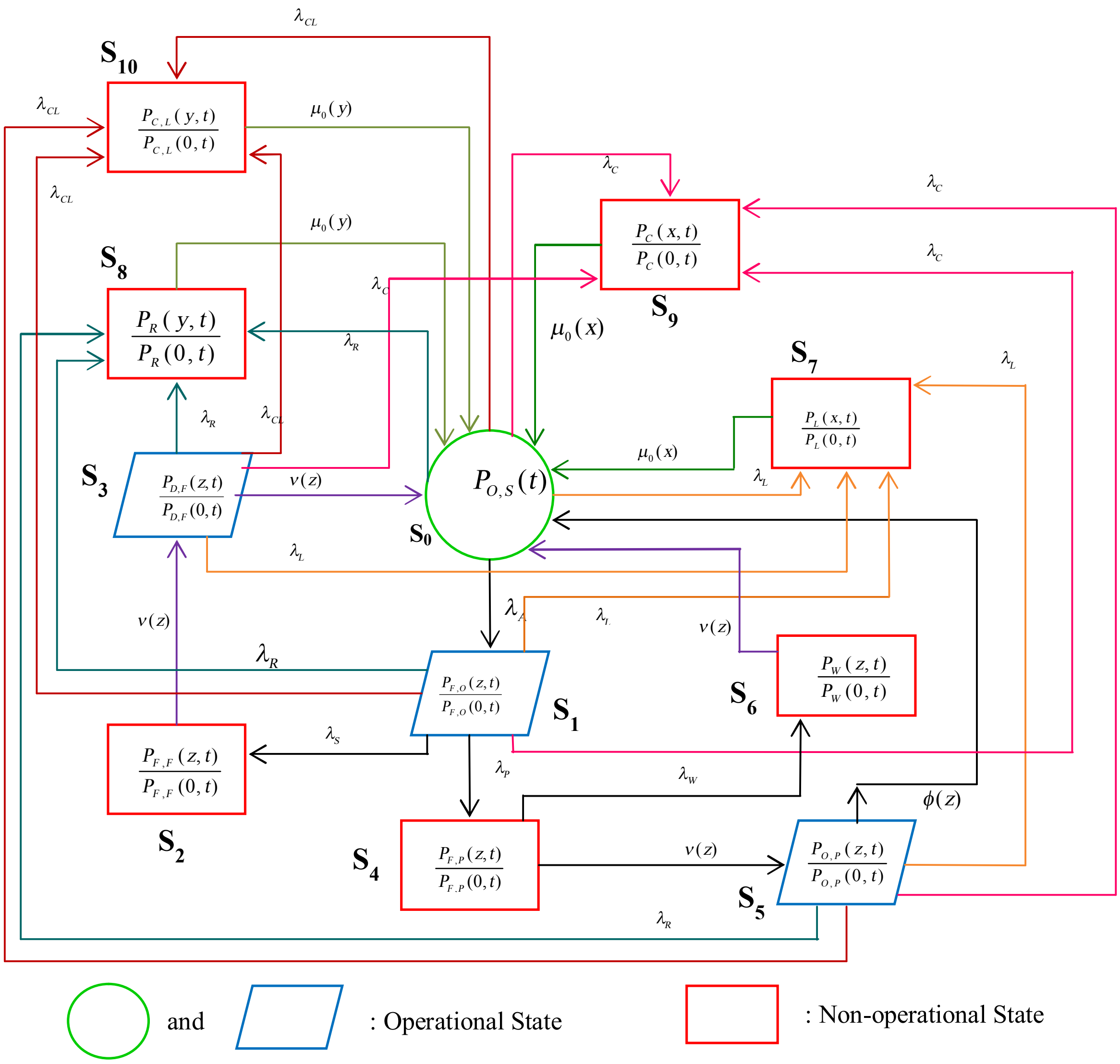

Figure 1. State Transition Diagram

2. ASSUMPTIONS

The following assumptions are taken throughout the discussion of the model:

(i) Initially the system is in S_0 state where both main as well as redundant server is in good condition.
(ii) When the main mail server fails, the redundant server starts working and repair is employed to the failed server.
(iii) The system waits for repair, if repair facility is not available; as soon as the repair facility is available, the repairing is employed to failed unit.
(iv) In repair, the preference is given to that unit which has failed first; i. e. main mail server.
(v) All failure rates are constant.
(vi) Switch failure /router failure/ cooling failure/ failure due to natural calamity need fast repairing i. e. Copula distribution is employed to repair (Gumbel-Hougaard family copula).
(vii) Repaired system works like a new and the repair does not damage anything.

3. NOTATIONS

The following notations are associated with the model:

$\lambda_A / \lambda_S / \lambda_P / \lambda_W / \lambda_L / \lambda_R / \lambda_C / \lambda_{CL}$	Failure rates of the main mail server/ standby redundant server/ partial failure rate of redundant server/ waiting rate of main mail server/switch failure rate/router failure/cooling failure/failure due to natural calamity.
$\phi(z) / \psi(z)$	Repair rates for state S_5 /S_2, S_3, S_4, S_6.
$\overline{P}(s)$	Laplace transformation of P(t).
$E_p(t)$	Expected profit during the interval [0, t).
K_1, K_2	Revenue per unit time and service cost per unit time, respectively.
C_θ $(u_1(p), u_2(p))$	The expression of joint probability (failed state to the initial state) according to Gumbel-Hougaard family is given as: $C_\theta(u_1(p), u_2(p)) = \mu_0(p) = \exp[p^\theta + \{\log\phi(p)\}^\theta]^{1/\theta}$ Where $u_1(p) = \phi(p)$, and $u_2(p) = e^p$; p=x, y.
$P_{o,s}(t)$	The probability that the system is in Operable State S_0.
$P_C(x,t)$	The probability that the system is in state S_9, the system is in complete failed state due to the failure of cooling system. The system is under repair and the elapse repair time is x, t.
$P_L(x,t)$	The probability that the system is in state S_7, the system is in complete failed state due to the switch failure. The system is under repair and the elapse repair time is x, t.
$P_{C,L}(y,t)$	The probability that the system is in state S_{10}, the system is in complete failed state due to the natural calamity. The system is under repair and the elapse repair time is y, t.
$P_R(y,t)$	The probability that the system is in state S_8, the system is in complete failed state due to the router failure. The system is under repair and the elapse repair time is y, t.
$P_{O,F}(z,t)$	The probability that the system is in state S_3, the system is in a degraded state and is in the operational state after repair of the main mail server. The redundant server is running under repair and the elapse repair time is z, t.
$P_{F,F}(z,t)$	The probability that the system is in state S_2, the system is in complete failed state due to the failure of redundant server before repair of the main mail server. The main mail server is running under repair and the elapse repair time is z, t.
$P_{F,O}(z,t)$	The probability that the system is in state S_1 after failure of the main mail server. The system is in degraded state but is in operational state. The system is under repair and the elapse repair time is z, t.
$P_{F,P}(z,t)$	The probability that the system is in state S_4 ,the system is in complete failed state due to failure of main mail server and partial failure in redundant server, the main mail server is running under repair and the elapse repair time is z, t.
$P_{O,P}(z,t)$	The probability that the system is in state S_5, the system is in the operational state as the main mail server has been repaired and is in operational state. The redundant server is running under repair and the elapse repair time is z, t.
$P_W(z,t)$	The probability that the system is in state S_6, the system is in failed state after a failure of the main mail server. The system is under repair and the elapse repair time is z, t.

4. FORMULATION AND SOLUTION OF MATHEMATICAL MODEL

By the probability of considerations and continuity arguments, we can obtain the following set of difference differential equations governing the present mathematical model.

$$\left[\frac{\partial}{\partial t}+\lambda_A+\lambda_L+\lambda_C+\lambda_{CL}+\lambda_R\right]P_{O,S}(t)$$

$$=\left[\int_0^\infty \nu(z)P_W(z,t)dz+\int_0^\infty \phi(z)P_{O,P}(z,t)dz\right.$$

$$+\int_0^\infty \mu_o(x)P_C(x,t)dx+\int_0^\infty \mu_0(y)P_{CL}(y,t)dy \tag{1}$$

$$+\int_0^\infty \nu(z)P_{O,F}(z,t)dz+\int_0^\infty \mu_o(x)P_L(x,t)dx$$

$$+\int_0^\infty \mu_o(y)P_R(y,t)dy$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_R+\lambda_C+\lambda_L+\lambda_{CL}+\lambda_S+\lambda_P\right]P_{F,O}(z,t)=0 \tag{2}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\nu(z)\right]P_{F,F}(z,t)=0 \tag{3}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\lambda_R+\lambda_C+\lambda_L+\lambda_{CL}+\nu(z)\right]P_{O,F}(z,t)=0 \tag{4}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\nu(z)+\lambda_W\right]P_{F,P}(z,t)=0 \tag{5}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\lambda_R+\lambda_C+\lambda_L+\lambda_{CL}+\phi(z)\right]P_{O,P}(z,t)=0 \tag{6}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\nu(z)\right]P_W(z,t)=0 \tag{7}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_0(x)\right]P_L(x,t)=0 \tag{8}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_0(x)\right]P_C(x,t)=0 \tag{9}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\mu_0(y)\right]P_R(y,t)=0 \tag{10}$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\mu_0(y)\right]P_{CL}(y,t)=0 \tag{11}$$

Boundary conditions

$$P_{F,O}(0,t)=\lambda_A P_{O,S}(t) \tag{12}$$

$$P_{F,F}(0,t)=\lambda_s\lambda_A P_{O,S}(t) \tag{13}$$

$$P_{O,F}(0,t)=\int_0^\infty \nu(z)P_{F,F}(z,t)dz \tag{14}$$

$$P_{F,P}(0,t)=\lambda_P\lambda_A P_{O,S}(t) \tag{15}$$

$$P_{O,P}(0,t)=\int_0^\infty \nu(z)P_{F,P}(z,t)dz \tag{16}$$

$$P_W(0,t)=\lambda_W\lambda_P\lambda_A P_{O,S}(t) \tag{17}$$

$$P_L(0,t)=\lambda_{CL}\left(P_{O,S}(t)+P_{O,P}(0,t)+P_{O,F}(0,t)+P_{F,O}(0,t)\right) \tag{18}$$

$$P_R(0,t)=\lambda_R\left(P_{O,S}(t)+P_{O,P}(0,t)+P_{O,F}(0,t)+P_{F,O}(0,t)\right) \tag{19}$$

$$P_C(0,t)=\lambda_C\left(P_{O,S}(t)+P_{O,P}(0,t)+P_{O,F}(0,t)+P_{F,O}(0,t)\right) \tag{20}$$

$$P_{CL}(0,t)=\lambda_{CL}\left(P_{O,S}(t)+P_{O,P}(0,t)+P_{O,F}(0,t)+P_{F,O}(0,t)\right) \tag{21}$$

Initials condition

$P_{O,S}(0)=1$ and other state probabilities at t=0 are zero. (22)

Taking Laplace transformation of equations (1)-(21) and using equation (22), we obtain

$$\left[s+\lambda_A+\lambda_L+\lambda_C+\lambda_{CL}+\lambda_R\right]\overline{P}_{O,S}(s) \tag{23}$$

$$=1+\int_0^\infty \nu(z)\overline{P}_W(z,s)dz+\int_0^\infty \phi(z)\overline{P}_{O,P}(z,s)dz$$

$$+\int_0^\infty \mu_o(x)\overline{P}_C(x,s)dx+\int_0^\infty \mu_0(y)\overline{P}_{CL}(y,s)dy$$

$$+\int_0^\infty \nu(z)\overline{P}_{O,F}(z,s)dz+\int_0^\infty \mu_o(x)\overline{P}_L(x,s)dx$$

$$+\int_0^\infty \mu_o(y)\overline{P}_R(y,s)dy$$

$$\left[s+\frac{\partial}{\partial z}+\lambda_S+\lambda_{CL}+\lambda_C+\lambda_L+\lambda_R+\lambda_P\right]\overline{P}_{F,O}(z,s)=0 \tag{24}$$

$$\left[s+\frac{\partial}{\partial z}+\nu(z)\right]\overline{P}_{F,F}(z,s)=0 \tag{25}$$

$$\left[s+\frac{\partial}{\partial z}+\lambda_C+\lambda_{CL}+\lambda_R+\lambda_L+\nu(z)\right]\overline{P}_{O,F}(z,s)=0 \tag{26}$$

$$\left[s+\frac{\partial}{\partial z}+\nu(z)+\lambda_W\right]\overline{P}_{F,P}(z,s)=0 \tag{27}$$

$$\left[s+\frac{\partial}{\partial \nu}+\lambda_L+\lambda_C+\lambda_{CL}+\lambda_R+\phi(z)\right]\overline{P}_{O,P}(z,s)=0 \tag{28}$$

$$\left[s+\frac{\partial}{\partial z}+\nu(z)\right]\overline{P}_w(z,s)=0 \tag{29}$$

$$\left[s+\frac{\partial}{\partial x}+\mu_0(x)\right]\overline{P}_L(x,s)=0 \tag{30}$$

$$\left[s+\frac{\partial}{\partial x}+\mu_0(x)\right]\overline{P}_C(x,s)=0 \tag{31}$$

$$\left[s+\frac{\partial}{\partial y}+\mu_0(y)\right]\overline{P}_R(y,s)=0 \quad (32)$$

$$\left[s+\frac{\partial}{\partial y}+\mu_0(y)\right]\overline{P}_{CL}(y,s)=0 \quad (33)$$

$$\overline{P}_{F,O}(0,s)=\lambda_A\overline{P}_{O,S}(s) \quad (34)$$

$$\overline{P}_{F,F}(0,s)=\lambda_s\lambda_A\overline{P}_{O,S}(s) \quad (35)$$

$$\overline{P}_{O,F}(0,s)=\int_0^\infty v(z)\overline{P}_{F,F}(z,s)dz \quad (36)$$

$$\overline{P}_{O,P}(0,s)=\int_0^\infty v(z)\overline{P}_{F,P}(z,s)dz \quad (37)$$

$$\overline{P}_{F,P}(0,s)=\lambda_p\lambda_A\overline{P}_{O,S}(s) \quad (38)$$

$$\overline{P}_L(0,s)=\lambda_L\left(\overline{P}_{O,S}(s)+\overline{P}_{F,O}(0,s)+\overline{P}_{O,P}(0,s)+\overline{P}_{O,F}(0,s)\right) \quad (39)$$

$$\overline{P}_R(0,s)=\lambda_R\left(\overline{P}_{O,S}(s)+\overline{P}_{F,O}(0,s)+\overline{P}_{O,P}(0,s)+\overline{P}_{O,F}(0,s)\right) \quad (40)$$

$$\overline{P}_W(0,s)=\lambda_W\lambda_p\lambda_A\overline{P}_{O,S}(s) \quad (41)$$

$$\overline{P}_C(0,s)=\lambda_C\left(\overline{P}_{O,S}(s)+\overline{P}_{F,O}(0,s)+\overline{P}_{O,P}(0,s)+\overline{P}_{O,F}(0,s)\right) \quad (42)$$

$$\overline{P}_{CL}(0,s)=\lambda_{CL}\left(\overline{P}_{O,S}(s)+\overline{P}_{F,O}(0,s)+\overline{P}_{O,P}(0,s)+\overline{P}_{O,F}(0,s)\right) \quad (43)$$

Solving (23) -(33) with the help of (34) -(43), one may get

$$\overline{P}_{O,S}(s)=\frac{1}{D(s)} \quad (44)$$

$$\overline{P}_{F,O}(s)=\frac{\lambda_A}{D(s)(s+\lambda_P+\lambda_S+\lambda_C+\lambda_{CL}+\lambda_R+\lambda_L)} \quad (45)$$

$$\overline{P}_{O,F}(s)=\frac{\lambda_A\lambda_S}{D(s)}S_v(s+\lambda_C+\lambda_{CL}+\lambda_R+\lambda_L) \quad (46)$$

$$\overline{P}_{F,F}(s)=\frac{\lambda_A\lambda_S}{D(s)}\frac{(1-S_v(s))}{s} \quad (47)$$

$$\overline{P}_{F,P}(s)=\frac{\lambda_A\lambda_P}{D(s)}\frac{(1-S_v(s+\lambda_W))}{s+\lambda_W} \quad (48)$$

$$\overline{P}_{O,P}(s)=\frac{\lambda_A\lambda_P}{D(s)}S_v(s+\lambda_W)\frac{(1-S_\phi(s+\lambda_C+\lambda_{CL}+\lambda_L+\lambda_R))}{s+\lambda_C+\lambda_{CL}+\lambda_L+\lambda_R} \quad (49)$$

$$\overline{P}_W(s)=\frac{\lambda_W\lambda_P\lambda_A}{D(s)}\frac{(1-S_v(s))}{s} \quad (50)$$

$$\overline{P}_L(s)=\frac{\lambda_L\left(1+\lambda_A+\lambda_A\lambda_P S_v(s+\lambda_W)S_v(s)+\lambda_A\lambda_S S_v(s)\right)}{D(s)}\frac{(1-S_{\mu_0}(s))}{s} \quad (51)$$

$$\overline{P}_C(s)=\frac{\lambda_C\left(1+\lambda_A+\lambda_A\lambda_P S_v(s+\lambda_W)S_v(s)+\lambda_A\lambda_S S_v(s)\right)}{D(s)}\frac{(1-S_{\mu_0}(s))}{s} \quad (52)$$

$$\overline{P}_{CL}(s)=\frac{\lambda_{CL}\left(1+\lambda_A+\lambda_A\lambda_P S_v(s+\lambda_W)S_v(s)+\lambda_A\lambda_S S_v(s)\right)}{D(s)}\frac{(1-S_{\mu_0}(s))}{s} \quad (53)$$

$$\overline{P}_R(s)=\frac{\lambda_R\left(1+\lambda_A+\lambda_A\lambda_P S_v(s+\lambda_W)S_v(s)+\lambda_A\lambda_S S_v(s)\right)}{D(s)}\frac{(1-S_{\mu_0}(s))}{s} \quad (54)$$

Where

$$D(s)=\begin{pmatrix} s+\lambda_A+\lambda_L+\lambda_C+\lambda_{CL}+\lambda_R-(\lambda_W\lambda_P\lambda_A S_v(s)+\lambda_P\lambda_A S_v(s+\lambda_W)S_\phi(s+\lambda_C+\lambda_L+\lambda_R+\lambda_{CL}) \\ +\lambda_C(1+\lambda_A+\lambda_A\lambda_P S_v(s+\lambda_W)S_v(s)+\lambda_A\lambda_S S_v(s))S_{\mu_0}(s)+\lambda_{CL}((1+\lambda_A+\lambda_A\lambda_P S_v(s+\lambda_W)S_v(s) \\ +\lambda_A\lambda_S S_v(s))S_{\mu_0}(s)+\lambda_S\lambda_A S_v(s+\lambda_L+\lambda_C+\lambda_{CL}+\lambda_R)S_{\mu_0}(s)+\lambda_L((1+\lambda_A+\lambda_A\lambda_P S_v(s+\lambda_W)S_v(s) \\ +\lambda_A\lambda_S S_v(s)))S_{\mu_0}(s)+\lambda_R((1+\lambda_A+\lambda_A\lambda_P S_v(s+\lambda_W)S_v(s)+\lambda_A\lambda_S S_v(s)S_{\mu_0}(s)) \end{pmatrix}$$

The Laplace transformations of the probabilities that the system is in up (i.e. either good or degraded state) and failed state at any time are as follows:

$$\overline{P}_{up}(s)=\overline{P}_{O,S}(s)+\overline{P}_{F,O}(s)+\overline{P}_{O,F}(s)+\overline{P}_{O,P}(s) \quad (55)$$

$$\overline{P}_{failed}(s)=\overline{P}_{F,F}(s)+\overline{P}_w(s)+\overline{P}_L(s)+\overline{P}_C(s)+\overline{P}_{CL}(s)+\overline{P}_R(s)+\overline{P}_{F,P}(s) \quad (56)$$

5. PARTICULAR CASES

5. 1. Availability When repair follows an exponential distribution, setting

$$S_{\mu_0}(s)=\overline{S}_{\exp[x^\theta+\{\log\phi(x)\}^\theta]^{1/\theta}}(s)=\frac{\exp[x^\theta+\{\log\phi(x)\}^\theta]^{1/\theta}}{s+\exp[x^\theta+\{\log\phi(x)\}^\theta]^{1/\theta}}$$

, $\bar{S}_v(s)=\dfrac{v}{s+v}$, $\bar{S}_\phi(s)=\dfrac{\phi}{s+\phi}$ and the values of different parameters are $\lambda_A=0.01$, $\lambda_S=0.015$, $\lambda_P=0.02$, $\lambda_W=0.025$, $\lambda_L=0.03$, $\lambda_C=0.035$,$\lambda_{CL}=0.022$, $\lambda_R=0.04$, $\phi(v)=1, v(z)=1$, $\phi=1$,$\theta=1$ in (55)

5. 1. 1. Expression for availability, when repair follows general and Gumbel-Hougaard family copula time distribution**.**

$$P_{up}(t)=\begin{pmatrix} -0.06177729500e^{(-0.162000t)} \\ +0.04513535841e^{(-2.846875602\,t)} \\ +0.001402599367e^{(-1.125568762t)} \\ -0.0011622793492e^{(-1.025325698\,t)} \\ +0.35990465e^{(-1.001537909\,t)} \\ +1.016398532e^{(-0.007992029066\,t)} \end{pmatrix} \quad (57\text{ a})$$

5. 1. 2. Expression for availability, when repair follows the general time distribution.

$$P_{up}(t)=\begin{pmatrix} -0.05605364342e^{(-0.162000t)} \\ +0.1118791750e^{(-1.129144601\,t)}\cos(0.01495793895\,t) \\ -0.02613259173\,e^{(-1.129144601\,t)}\sin(0.01495793895t) \\ +0.002237256039\,e^{(-1.011595711\,t)}\cos(0.007838586453\,t) \\ -0.002048265310\,e^{(-1.011595711\,t)}\sin(0.007838586453\,t) \\ +0.9419372123\,e^{(-0.0075193731\,t)} \end{pmatrix} \quad (57\text{ b})$$

For, t= 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 and so on, one may get different values of $P_{up}(t)$ as shown in Table 1.

5. 2. Mean Time to Failure (MTTF) Taking all repairs to zero in the Equation (55). Taking the limit as *s* tends to zero, one can obtain the MTTF as:

$$MTTF=\frac{1}{(\lambda_A+\lambda_C+\lambda_L+\lambda_{CL}+\lambda_R)}\left(1+\frac{\lambda_A}{(\lambda_P+\lambda_C+\lambda_L+\lambda_{CL}+\lambda_R+\lambda_S)}\right) \quad (58)$$

Setting $\lambda_A=0.01, \lambda_L=0.03$ $\lambda_C=0.035$, $\lambda_{CL}=0.022$, $\lambda_R=0.04$, $\lambda_P=0.02$ $\lambda_S=0.015$ and varying λ_A, λ_L, λ_C, λ_{CL},λ_R,λ_P,λ_S as 0.01, 0.02, 0.03, 0.04, 0.05, 0.06, 0.07, 0.08, 0.09 one by one in (58). One may obtain the Table 2, which demonstrates the variation of MTTF with respect to failure rates.

5. 3. Cost Analysis Let the service facility be always available, then expected profit during the interval [0, t) is

$$E_p(t)=K_1\int_0^t P_{up}(t)dt-K_2t \quad (59)$$

Using (57 a) in (59), the expected profit of the system is given by

$$E_p(t)=\begin{pmatrix} K_1(-0.005516e^{(-2.7607t)}\,0.00090376e^{(-1.2227t)} \\ -0.000021733e^{(-1.0563t)}-639.21e^{(0.0015579t)} \\ +639.21)-K_2t \end{pmatrix} \quad (60)$$

Setting K_1= 1and K_2= 0.5, 0.4, 0.3, 0.2, 0.1, 0.05, 0.01, respectively and varying t =0, 1, 2, 3, 4, 5, 6, 7, 8, 9 and so on units of time in (60), one get Table 3 .

TABLE 1. Time vs. Availability

Time(t)	$P_{up}(t)$	
	Case 5. 1. 1	**Case 5. 1. 2**
0	1.0000	1.0000
1	0.9584	0.9241
2	0.9558	0.8992
3	0.9543	0.8930
4	0.9521	0.8859
5	0.9491	0.8826
6	0.9454	0.8756
7	0.9412	0.8716
8	0.9365	0.8672
9	0.9315	0.8626

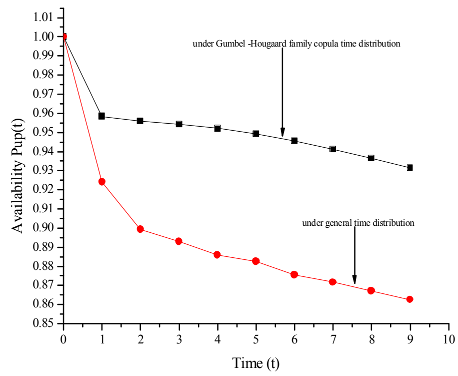

Figure 2. Time vs. Availability

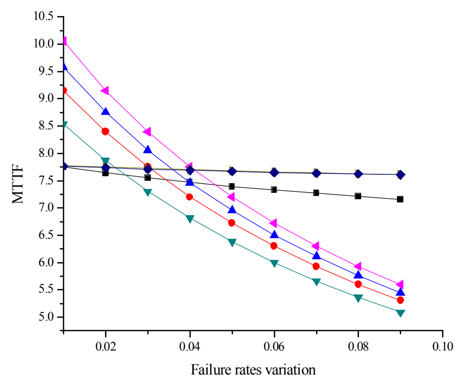

Figure 3. MTTF as function of Failure rate

TABLE 2. Failure rates vs. MTTF

Failure rate	MTTF with respect to						
	λ_A	λ_L	λ_C	λ_{CL}	λ_R	λ_P	λ_S
0.01	7.7498	9.1489	9.5802	8.5333	10.0538	7.7794	7.7642
0.02	7.6425	8.3920	8.7543	7.8704	9.1489	7.7498	7.7363
0.03	7.5489	7.7498	8.0583	7.3022	8.3920	7.7236	7.7116
0.04	7.4665	7.1982	7.4639	6.8100	7.7498	7.7003	7.6896
0.05	7.3934	6.7193	6.9564	6.3795	7.1982	7.6794	7.6697
0.06	7.3281	6.2999	6.5029	6.0000	6.7193	7.6606	7.6519
0.07	7.2695	5.9292	6.1091	5.6628	6.2999	7.6435	7.6356
0.08	7.2165	5.5998	5.7599	5.3613	5.9294	7.6280	7.6208
0.09	7.1584	5.3047	5.4483	5.0901	5.5998	7.6138	7.6072

TABLE 3. Time vs. Expected profit

Time(t)	Ep(t)						
	K_2=0.5	K_2=0.4	K_2=0.3	K_2=0.2	K_2=0.1	K_2=0.05	K_2=0.01
0	0	0	0	0	0	0	0
1	0.4703	0.5703	0.6703	0.7703	0.8703	0.9203	0.9603
2	0.9269	1.1269	1.3269	1.5269	1.7269	1.8269	1.9069
3	1.3820	1.6820	1.9820	2.2820	2.5820	2.7330	2.8520
4	1.8353	2.2353	2.6353	3.0353	3.4353	3.6353	3.7953
5	2.2660	2.7860	3.2860	3.7860	4.2860	4.5360	4.7360
6	2.7333	3.3333	3.9333	4.5333	5.1333	5.4333	5.6733
7	3.1767	3.8767	4.5767	5.2767	5.9767	6.3267	6.6067
8	3.6156	4.4156	5.2156	6.0156	6.8156	7.2156	7.5356
9	4.0496	4.9496	5.8496	6.7496	7.6496	8.0996	8.4896

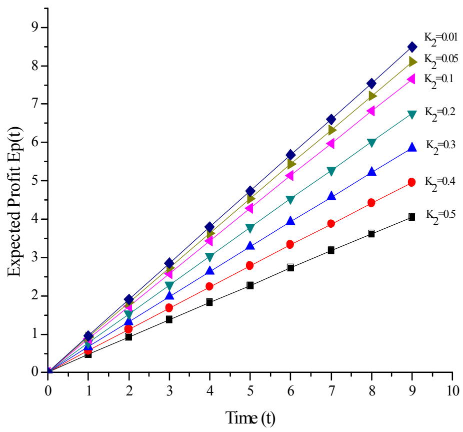

Figure 4. Time vs. Expected profit

6. DISCUSSION AND CONCLUSION

Table 1 and Figure 2 show information how availability of the complex repairable system changes with respect to the time when failure rates are fixed at different values. When failure rates are fixed at lower values $\lambda_A = 0.01$, $\lambda_S = 0.015$, $\lambda_P = 0.02$, $\lambda_W = 0.025$, $\lambda_L = 0.03$, $\lambda_C = 0.035$, $\lambda_{CL} = 0.022$, $\lambda_R = 0.04$ the availability of the system decreases. Probability of failure also increases, with the passage of time and ultimately becomes steady to the value zero after a sufficient long interval of time. Hence, one can safely predict the future behavior of a complex system at any time for any given set of parametric values, as is evident by the graphical consideration of the model.

Furthermore, availability of the system can be obtained with the help of copula distribution and general distribution in repair as per Equations (57 a) and (57 b), respectively and found that copula distribution improve availability of the system.

Table 2 and corresponding Figure 2 yields the mean-time-to-failure (MTTF) of the system with respect to variation in λ_A, λ_C,λ_{CL}, λ_R, λ_P,λ_S and λ_L respectively when other parameters have been taken as constant.

When revenue cost per unit time K_1 fixed at 1, service cost K_2 = 0.5, 0.4, 0.3, 0.2, 0.1, 0.05, 0.01, profit has been calculated. Results are also demonstrated by the graphs. It can be observed from Table 3 and corresponding Figure 4 that as service cost decreases, profit increases.

Hence, this modeling is very useful in engineering problems and copula applications in reliability theory.

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APPENDIX 1

If system is in state S_0 then the state transition probability of system to remains in this state will be based on the fact that it should not move in other state. As the failure rate to move in other states are given as $\lambda_R, \lambda_L, \lambda_A, \lambda_{CL}, \lambda_C$ then rate to be in the state S_0 will be as $(1-\lambda_R\Delta t), (1-\lambda_L\Delta t), (1-\lambda_A\Delta t), (1-\lambda_{CL}\Delta t), (1-\lambda_C\Delta t)$. The differential equation for system to be in state S_0 during time (t, t+Δt) will be as

$$P_{O,S}(t+\Delta t) = (1-\lambda_R\Delta t)\;(1-\lambda_L\Delta t)\,(1-\lambda_A\Delta t)(1-\lambda_{CL}\Delta t)(1-\lambda_C\Delta t)P_{O,S}(t)$$
$$+\int_0^\infty \mu_0(x)P_L(x,t)dx\,\Delta t+\int_0^\infty \mu_0(x)P_C(x,t)dx\,\Delta t+\int_0^\infty \mu_0(y)P_{CL}(y,t)dy\,\Delta t+\int_0^\infty \mu_0(y)P_R(y,t)dy\,\Delta t$$
$$+\int_0^\infty v(z)P_{O,F}(z,t)dz\,\Delta t+\int_0^\infty v(z)P_w(z,t)dz\,\Delta t$$

$$\Rightarrow Lt_{\;\Delta t\to 0}\frac{P_{O,S}(t+\Delta t)-P_{O,S}(t)}{\Delta t}+(\lambda_R+\lambda_L+\lambda_A+\lambda_{CL}+\lambda_C)P_{O,S}(t)=\int_0^\infty \mu_0(x)P_L(x,t)dx+\int_0^\infty \mu_0(x)P_C(x,t)dx$$
$$+\int_0^\infty \mu_0(y)P_{CL}(y,t)dy++\int_0^\infty \mu_0(y)P_R(y,t)dy+\int_0^\infty v(z)P_{O,F}(z,t)dz+\int_0^\infty v(z)P_w(z,t)dz$$

$$\Rightarrow\left(\frac{\partial}{\partial t}+\lambda_R+\lambda_L+\lambda_A+\lambda_{CL}+\lambda_C\right)P_{O,S}(t)=\int_0^\infty \mu_0(x)P_L(x,t)dx+\int_0^\infty \mu_0(x)P_C(x,t)dx+\int_0^\infty \mu_0(y)P_{CL}(y,t)dy$$
$$+\int_0^\infty \mu_0(y)P_R(y,t)dy+\int_0^\infty v(z)P_{O,F}(z,t)dz+\int_0^\infty v(z)P_w(z,t)dz$$

Similarly remaining Equations (2)-(11) for other states can be obtained.

For boundary and initial conditions as:

If at any time the system is in state S_{i+1}, after failure from the state S_i, *x* is repair variable; then if repair variable not assigned; then at x=0 the state transition probability of state S_{i+1},= failure rate × state transition probability of its previous state.

Therefore,

$$P_{F,O}(0,t)=\lambda_A P_{O,S}(t),\;P_{F,F}(0,t)=\lambda_S\lambda_A P_{O,S}(t),\;P_{O,F}(0,t)=\int_0^\infty v(z)P_{F,F}(z,t)dz$$

and so on.

APPENDIX 2

Availability of the system can be obtained with the help of copula distribution and general distribution in repair as per Equations (57 a) and (57 b), respectively.

In a long run, when the repair is not assigned i.e. treating all repairs to be zero in the expression of $P_{up}(s)$ and taking limit ass tends to be zero, the given expression for MTTF can be obtain. Now fixing the failure rates at different values and varying one of them, the MTTF can obtain as given in Table 2 and in Figure 3. Expected profit, in interval [0, t) can be obtain if service facility is continuously available as per Equation (59).

Modeling and Availability Analysis of Internet Data Center with various Maintenance Policies

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PAPER INFO

Paper history:
Received 01 March 2013
Received in revised form 19 May 2013
Accepted 22 August 2013

Keywords:
Reliability Analysis
Internet Data Center (IDC)
Router Failure
Cost Analysis

چکیده

در این مقاله، نویسندگان روی تحلیل احتمالی یک مرکز اطلاعات اینترنت (IDC) که شامل سرور اصلی پایگاه داده ها متصل به سرور اصلی می باشد، متمرکز شده اند. با مشاهده احتمال های مختلف در مورد عملکرد سیستم، تحلیل برای تخمین واقعیت مختلف خصوصیات سیستم انجام شده است. سیستم می تواند به علت خرابی سرور اضافی قبل از تعمیر سرور پایگاه داده ها، خرابی مسیریاب و خرابی سویچ به طور کامل با شکست مواجه شود. همچنین، سیستم می تواند به علت خرابی خنک کننده یا بعضی از بلاهای طبیعی مانند زمین لرزه، آتش سوزی و ... به طور کامل با شکست مواجه شود. تمام سرعت های شکست، هنگامی که تعمیرات از دو نوع توزیع با نام های رابط خانواده عمومی و گومبل- هوگارد پیروی می کند، ثابت فرض می شوند.

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doi: *10.5829/idosi.ije.2014.27.04a.10*