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A Robust Model for a Dynamic Cellular Manufacturing System with Production Planning

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ABSTRACT

This paper develops a robust optimization approach for a dynamic cellular manufacturing system (DCMS) integrated with production planning under uncertainty of parts processing time. To deal with this uncertainty, a robust optimization as a tractable approach is adopted. The model includes cell formation, inter-cell layout and production planning concepts under a dynamic environment. The aim of the model is to minimize inter and intra-cell material handling, inventory holding, back order and reconfiguration costs. To verify the behavior of the presented model and the performance of the developed approach, a numerical example solved in finding an optimal solution.

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NOMENCLATURE

Decision variables

B_i^h	Backorder of part type i in period h ($B_i^0 = 0$)
I_i^h	Inventory of part type i at the end of period h ($I_i^0 = 0$)
$N_{m,c}^h$	$\begin{cases} 1 & \text{if machine type m is located in cell } c \text{ in period h;} \\ 0 & \text{otherwise} \end{cases}$
PQ_i^h	Production quantity of part type i to be produced in period h
PQB_i^h	$\begin{cases} 1 & \text{if } PQ_i^h > 0 \\ 0 & \text{otherwise} \end{cases}$
l, l'	Index for a candidate locations to be a cell ($l = 1,..., L$)
m	Index for machines ($m = 1,..., M$)
n	Index of different constraints
r	Index for routings required by part i in period h ($r = 1,..., R_i^h$)
t_n	Uncertain element of the n-th constraint ($1 \le n \le CN$) adopting values from truncated uncertainty interval
$\left\{U_{r,i}^{(1)}, U_{r,i}^{(2)}, U_{r,i}^{(3)}, ..., U_{r,i}^{(K_{r,i})}\right\}$	Machine index in routing r of part type i

Indices

c, c'	Index for machine cells ($c = 1,..., C$)
h	Index for production periods ($h = 1,..., H$)
i	Index for parts ($i = 1,..., I$)
j	Index of different decision variables
M	Number of machines
R_i^h	Number of available routings for part type i in period h
$t_{i,m}^h$	Processing time of part i on machine m in period h
$\tilde{t}_{i,m}^h$	Uncertain processing time of part i on machine m in period h
$\hat{t}_{i,m}^h$	Range of uncertain processing time of part i on machine m in period h
$t'_{h,m}$	Time-capacity of machine type m in period h

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Input parameters		**Indices**	
A_1	Inter-cell part trip unit cost	UP_c	Maximum number of machines should be located in cell c
A_2	Intra-cell part trip unit cost	α_i^h	Unit holding cost of part type i in period h
A_∞	A large positive quantity	β_i^h	Unit backorder cost of part type i in period h
$\tilde{a}_{n,j}$	Uncertain element of the n-th row and the j-th column in A	γ_m	Relocation cost of machine m between production periods
$a_{n,j}$	Estimated nominal value of $\tilde{a}_{n,j}$	Γ_n	Conservation level value for the n-th constraint ($1 \leq n \leq CN$)
$\hat{a}_{n,j}$	Estimated range of $\tilde{a}_{n,j}$	$\lvert J_n \rvert$	Number of elements of J_n
C	Number of machine cells should be constructed	$\lvert S_n \rvert$	Number of elements of S_n
CN	Total number of Constraints	**Matrices and Vectors**	
D_i^h	Demand value for part i in period h	A	$CN \times DN$ matrix of coefficients
$Dis_{l,l'}$	Distances between two candidate locations l and l'	X	DN-dimensional decision vector
DN	Total number of decision variables	**Sets**	
H	Number of production periods	J_n	Set of uncertain elements of the n-th constraint ($1 \leq n \leq CN$)
I	Number of parts	S_n	Set of uncertain elements of either objective function (n=0) or n-th constraint ($1 \leq n \leq CN$) adopting values from a respective uncertainty interval
$K_{r,i}^h$	Number of machines in routing r of part type i in period h	**Variables**	
L	Number of candidate locations to be a cell ($L \geq C$)	p, y, z	Continuous auxiliary robust modeling variables
Low_c	Minimum number of machines should be located in cell c	x_j	Element of the j-th row in X
$V_{r,i}^h$	$\begin{cases} 1 & \text{if routing } r \text{ of part type } i \text{ is selected as process plan in period } h; \\ 0; & \text{otherwise} \end{cases}$		
$X_{c,l}^h$	$\begin{cases} 1 & \text{if cell } c \text{ is to be constructed in location } l \text{ in period } h; \\ 0 & \text{otherwise} \end{cases}$		

1. INTRODUCTION

Today's industrial world witnesses an increasing global competition, where old technologies failed to overcome the new form of change in demand. The application of group technology to production systems has in industries led to the introduction of cellular manufacturing (CM) which tries to take advantage of the similarity between parts. Each CMS design is consisted of four important decisions; namely cell formation (CF), group layout (GL), group scheduling (GS) and resource allocation [1], in which most of studies have developed CF problems [2, 3]. Only a few studies have concentrated on integrating two or more CMS decisions. Kia et al. [4] proposed an integration of CF and GL models considering the multi-rows layout utilization to locate machines in the cells configured with flexible shapes and several design features (e.g., alternative process routings, operation sequence, processing time, production volume of parts, purchasing machine, duplicate machines, machine capacity, lot splitting, intra-cell layout, inter-cell layout, multi-rows layout of equal area facilities and flexible reconfiguration). Jolai et al. [5] considered the integration of CL and GL models and proposed an electromagnetism-like algorithm to solve the problem. Arkat et al. [6] proposed a model that integrates CF, GL and cellular scheduling to minimize the total movement and completion time of parts. Their results show that considering three CM decisions simultaneously can significantly improve the performance of CM systems.

Krishnan et al. [7] developed a CF model to integrate group technology with a facility layout problem. They included three basic steps; (1) presenting a mathematical model for grouping the machines in order to minimize inter-cell part trips, (2) proposing a new measure (i.e., bonding efficiency) to balance the inter-cell flow and (3) implementing a genetic algorithm to determine the best facility layout. Moreover, some recent studies try to integrate CMS with other concepts (e.g., production planning), which conduct the model to be more real, since in real world the product demand is not equal to production quantity. Ah Kioon et al. [8] developed a model that integrates the cell formation, system reconfiguration along with the consideration of multiple process routings, production planning (PP), machine capacities and availabilities. Solimanpur et al. [9] developed a fuzzy goal programming-based approach for solving a multi-objective CF and PP in dynamic virtual cellular manufacturing systems considering worker flexibility. In most studies related to CMS under a dynamic environment, input parameters are considered deterministic and certain. While in reality, a number of parameters (e.g., processing time, part demand, product mix, inter-arrival time and available machine capacity) are uncertain. Mahdavi et al. [10] developed the multi-period cell formation and production planning in a DCMS considering worker assignment. The objective of the model is to minimize machine, reconfiguration, inter-cell material handling, inventory holding, backorder, worker hiring, firing and salary costs.

Some studies of considering uncertainty are as follows. Szwarc et al. [11] considered uncertainty in demand and machines capacity in CMS problem, and is resolved by fuzzy approach. Tavakkoli-Moghaddam et al. [12] proposed a multi-objective model for a cell formation problem under fuzzy and dynamic conditions, the main goal of the proposed model was to select a process plan with the minimum cost and also to identify the most appropriate production volume with respect to fuzzy demands and capacities. Asgharpour and Javadian [13] presented a nonlinear integer CMS model in dynamic and stochastic states solved by a genetic algorithm (GA) and considered a dynamic production, a stochastic demand, routing flexibility and machine flexibility.

Tavakkoli-Moghaddam et al. [14] developed a model for facility layout problem in CMS with stochastic demands, where the aim of objective function is to minimize inter-cell and intra-cell costs. Ghezavati and Saidi-Mehrabad [15] proposed a CM model and assumed that processing and arrival times for parts are stochastic. After formulating the problem with queuing theory, it was solved with new combination of the GA and simulated annealing (SA) algorithm. In addition, Ghezavati and Saidi-Mehrabad [16] applied a scenario-based stochastic programming technique to solve the CF problem integrated with GS decision. Rabbani et al. [17] proposed a bi-objective cell formation problem with stochastic demand quantities and solved with a two-phase fuzzy linear programming approach. Studies considering uncertainty can be categorized to four approaches: stochastic programming approach, fuzzy programming approach, stochastic dynamic programming approach, and robust optimization approach [18, 19]. Fuzzy optimization (FO) is an alternative method to cope with uncertainty that represents uncertainty through fuzzy numbers. Its aim is to find the best decision alternative under a membership to a given set that is inexact. On the other hand, stochastic programming (SP) is a methodology for solving optimization problems under uncertainty, which is usually characterized by a probability distribution on some parameters. In other words, a scenario generation approach is used to produce some scenarios from a probability distribution representing realizations of random variables associated with uncertain sources.

In real-world applications of linear programming (LP), there is the possibility that uncertainty in the input parameters may make the usual optimal solution no longer optimal or even infeasible. Therefore, the need to use approaches, which are immune to data uncertainty, increases. A recent methodology for optimization under uncertainty is robust optimization that models data uncertainties through a set of deterministic and bounded intervals [20]. The robust optimization approach solves a deterministic version of the original uncertain problem to obtain an optimal solution that is immunized against data uncertainties [21].

It is proved that the RO method outperforms other FO and SO methods. The main advantages of this method can be described as follows:

- Many FO methods increase the solving complexity and are typically difficult to be solved in a reasonable computational time, especially in comparison with the proposed RO method that is less sophisticated.
- Standard approaches (e.g., RO), which utilizes real-valued quantities, are less difficult to understand than fuzzy optimization using fuzzy numbers.
- In scenario-based SP methods, a number of scenarios may be huge and can increase the model complexity strictly. However, the RO method remains computationally tractable irrespectiveness of its number of uncertain parameters [22, 23].

In this paper, a robust optimization approach is proposed for the integrated mathematical model of cell formation, inter-cellular layout and production planning with alternative process routing under a dynamic environment to minimize the presented model against the product processing time uncertainty. The aim of the objective function is to minimize inter-cell, intra-cell, inventory holding, back order and machine reconfiguration costs.

This paper is organized as follows. In section 2, the mathematical programming model is presented. Section 3 presents an example with computational results to demonstrate the behavior of the presented model and verify the performance of the developed approach. The paper ends with conclusion.

2. PROPOSED FORMULATION

In this section, the nonlinear mathematical model is first presented in a deterministic form integrated of cell formation, inter-cell layout design and production planning with the aim of minimizing five main costs: inter-cell and intra-cell part trip, inventory holding, backorder and machine relocation. Different routings for each part are also considered; where for one part, one of its routings with the lowest cost is chosen among other routings. Then, the presented nonlinear model is linearized and afterwards a robust optimization approach is applied throughout the model as a tractable optimization technique to cope with the product processing time uncertainty.

The problem is formulated according to the following assumptions:

- The demand for each part type in each period is known and deterministic.
- Parts have different processing routings where each routing has different sequence of machines.
- Inter-cell movement cost is dependent on the distance traveled, while intra-cell movement cost is regardless of the distance.
- The inventory holding and back orders are considered.
- The time capacity of each machine type is known.
- The upper and lower bound of cell size is known.

2. 1. Mathematical Model

$$\text{Min} \sum_{h=1}^{H}\sum_{i=1}^{I}\sum_{r=1}^{R_i^h}\sum_{m=1}^{K_{r,i-1}^h}\sum_{c=1}^{C}\sum_{c'=1}^{C}\sum_{l=1}^{L}\sum_{l'=1}^{L} A_1 . Dis_{l,l'} . V_{r,i}^h . PQ_i^h . N_{U_{r,i}^m,c}^h . X_{c,l}^h . N_{U_{r,i}^{m+1},c'}^h . X_{c',l'}^h \tag{1a}$$

$$+\sum_{h=1}^{H}\sum_{i=1}^{I}\sum_{r=1}^{R_i^h}\sum_{i=1}^{K_{r,i-1}}\sum_{c=1}^{C} A_2 . V_{r,i}^h . PQ_i^h . N_{U_{r,i}^m,c}^h . N_{U_{r,i}^{m+1},c}^h \tag{1b}$$

$$+\sum_{h=1}^{H}\sum_{i=1}^{I}\alpha_i^h . I_i^h \tag{1c}$$

$$+\sum_{h=1}^{H}\sum_{i=1}^{I}\beta_i^h . B_i^h + \tag{1d}$$

$$\sum_{h=1}^{H-1}\sum_{m=1}^{M}\sum_{c=1}^{C}\sum_{c'=1}^{C}\sum_{l=1}^{L}\sum_{l'=1}^{L}\gamma_m . N_{m,c}^h . N_{m,c'}^{h+1} . X_{c,l}^h . X_{c',l'}^{h+1} . Dis_{l,l'} \tag{1e}$$

s.t.

$$\sum_{m=1}^{M} N_{m,c}^h \geq Low_c \qquad \forall h,c \tag{2}$$

$$\sum_{m=1}^{M} N_{m,c}^h \leq UP_c \qquad \forall h,c \tag{3}$$

$$\sum_{c=1}^{C} N_{m,c}^h = 1 \qquad \forall h,m \tag{4}$$

$$\sum_{r=1}^{R_i^h} V_{r,i}^h = PQB_i^h \qquad \forall h,i \tag{5}$$

$$\sum_{l=1}^{L} X_{c,l}^h = 1 \qquad \forall h,c \tag{6}$$

$$\sum_{c=1}^{C} X_{c,l}^h \leq 1 \qquad \forall h,l \tag{7}$$

$$\sum_{i=1}^{I}\sum_{r=1}^{R_i^h} V_{r,i}^h . PQ_i^h . t_{i,U_{r,i}^m}^h \leq t'_{h,m} \qquad \forall h,m \tag{8}$$

$$PQ_i^h = D_i^h - I_i^{h-1} + B_i^{h-1} + I_i^h - B_i^h \qquad \forall h,i \tag{9}$$

$$PQ_i^h \leq A_\infty . PQB_i^h \qquad \forall h,i \tag{10}$$

$$N_{m,c}^h \in (0,1) \qquad \forall h,m,c \tag{11}$$

$$X_{c,l}^h \in (0,1) \qquad \forall h,c,l \tag{12}$$

$$PQB_i^h \in (0,1) \qquad \forall h,i \tag{13}$$

$$V_{r,i}^h \in (0,1) \qquad \forall h,r,i \tag{14}$$

$$PQ_i^h, I_i^h, B_i^h \geq 0 \quad \text{and int.} \qquad \forall h,i \tag{15}$$

The objective function (OF) of the presented model consists of five terms as follows. Equation (1a) represents the inter-cell material handling cost where, this cost happens when parts need to be processed in more than one cell. In Equation (1b), the intra-cell material part trip occurs only when two consecutive operations in one routing are allocated to the same cell but to different machines. Equation (1c) represents the inventory holding cost which happens due to keeping inventories for all parts. In Equation (1d), the backorder cost occurs when the manufacturing system is unable to fill an order and must complete it later. Equation (1e) represents the relocation cost of machines between periods where the distance between cells for these reconfigurations are considered. Equations (2) and (3) ensure that the number of machines for one cell is not exceeded lower and upper bound of cell size. The lower bound is used to prevent all machines from being assigned to a single cell.

Equation (4) is for ensuring that each machine is only being assigned to one cell. Equation (5) indicates that just one process routing will be chosen for each part and this routing will be selected only if that part type is to be produced in corresponding period. Equations (6) and (7) ensure that each cell is assigned to only one location and each location is assigned to one cell, respectively. Equation (8) is for forcing machine workload not to exceed its capacity. Equation (9) makes the inventory and/or backorders balanced with those from the previous period, production quantity, and the demand quantity. Equation (10) is a logical equation which guarantees that quantity of a part type produced in a particular period can be a positive quantity only when its corresponding binary viable is equal to 1. At last, Equations (11) to (15) are to define the decision variables types.

2. 2. Linearization In this section, the linearization of the nonlinear model is developed based on linearization methods [4, 24]. The nonlinearity of the model is due to Equations (1a), (1b) and (1e) and (8). Therefore, to linearize the model, some new variables should be defined as follows:

$$VPQ_{r,i}^{h} = PQ_i^h \cdot V_{r,i}^{h}$$

$$\psi_{U_{r,i}^{m},U_{r,i}^{m+1},c}^{h} = VPQ_{r,i}^{h} \cdot N_{U_{r,i}^{m},c}^{h} \cdot N_{U_{r,i}^{m+1},c}^{h}$$

$$\varphi_{U_{r,i}^{m},c,l,U_{r,i}^{m+1},c',l'}^{h} = VPQ_{r,i}^{h} \cdot N_{U_{r,i}^{m},c}^{h} \cdot N_{U_{r,i}^{m+1},c'}^{h} \cdot X_{c,l}^{h} \cdot X_{c',l'}^{h}$$

$$\eta_{m,c,c',l,l'}^{h} = N_{m,c}^{h} \cdot N_{m,c'}^{h+1} \cdot X_{c,l}^{h} \cdot X_{c',l'}^{h+1}$$

The following equations respect to new variables must be added to the original model:

$$VPQ_{r,i}^{h} \geq PQ_i^h - A_\infty \cdot \left(1 - V_{r,i}^{h}\right) \qquad \forall h,r,i \tag{16}$$

$$VPQ_{r,i}^{h} \geq 0 \quad \text{and int.} \qquad \forall h,r,i \tag{17}$$

$$\psi_{U_{r,i}^{m},U_{r,i}^{m+1},c}^{h} \geq VPQ_{r,i}^{h} - A_\infty \cdot \left(2 - N_{U_{r,i}^{m},c}^{h} - N_{U_{r,i}^{m+1},c}^{h}\right) \qquad \forall h,r,i,m,c \tag{18}$$

$$\psi_{U_{r,i}^{m},U_{r,i}^{m+1},c}^{h} \geq 0 \quad \text{and int.} \quad \forall h,r,i,m,c \tag{19}$$

$$\varphi_{U_{r,i}^{m},c,l,U_{r,i}^{m+1},c',l'}^{h} \geq VPQ_{r,i}^{h} - A_\infty \cdot \left(4 - N_{U_{r,i}^{m},c}^{h} - N_{U_{r,i}^{m+1},c'}^{h} - X_{c,l}^{h} - X_{c',l'}^{h}\right) \qquad \forall h,r,i,m,c,c',l,l' \tag{20}$$

$$\varphi_{U_{r,i}^{m},c,l,U_{r,i}^{m+1},c',l'}^{h} \geq 0 \quad \text{and int.} \quad \forall h,r,i,m,c,c',l,l' \tag{21}$$

$$4\eta_{m,c,c',l,l'}^{h} \leq N_{m,c}^{h} + N_{m,c'}^{h+1} + X_{c,l}^{h} + X_{c',l'}^{h+1} \qquad \forall h = 1,\dots,H-1,\ m,c,c',l,l' \tag{22}$$

$$\eta_{m,c,c',l,l'}^{h} \geq N_{m,c}^{h} + N_{m,c'}^{h+1} + X_{c,l}^{h} + X_{c',l'}^{h+1} - 3 \qquad \forall h = 1,\dots,H-1,\ m,c,c',l,l' \tag{23}$$

$$\eta_{m,c,c',l,l'}^{h} \ \text{is bin.} \quad \forall h,m,c,c',l,l' \tag{24}$$

By substituting new variables in the model, the linear form of the model is as follows:

$$\begin{aligned}\text{Min} & \sum_{h=1}^{H}\sum_{i=1}^{I}\sum_{r=1}^{R_i^h}\sum_{m=1}^{K_{r,i-1}^h}\sum_{c=1}^{C}\sum_{c'=1}^{C}\sum_{l=1}^{L}\sum_{l'=1}^{L} A_1 . Dis_{l,l'} . \varphi_{U_{r,i}^{m},c,l,U_{r,i}^{m+1},c',l'}^{h} \\ & + \sum_{h=1}^{H}\sum_{i=1}^{I}\sum_{r=1}^{R_i^h}\sum_{m=1}^{K_{r,i-1}^h}\sum_{c=1}^{C} A_2 . \psi_{U_{r,i}^{m},U_{r,i}^{m+1},c}^{h} \\ & + \sum_{h=1}^{H-1}\sum_{m=1}^{M}\sum_{c=1}^{C}\sum_{c'=1}^{C}\sum_{l=1}^{L}\sum_{l'=1}^{L} \gamma_m . \eta_{m,c,c',l,l'}^{h} . Dis_{l,l'} \\ & + \sum_{h=1}^{H}\sum_{i=1}^{I} \alpha_i^h . I_i^h + \sum_{h=1}^{H}\sum_{i=1}^{I} \beta_i^h . B_i^h \end{aligned} \tag{25}$$

s.t.

Equations (2) to (7), (9) to (15), (16) to (24)

$$\sum_{i=1}^{I}\sum_{r=1}^{R_i^h} VPQ_{r,i}^{h} . t_{i,U_{r,i}^{m}}^{h} \leq t'_{h,m} \qquad \forall h,m \tag{26}$$

2. 2. Robust Optimization Approach In recent years, dealing with uncertain data is a major challenge in optimization. One approach to address data uncertainty is developed under the name of robust optimization, which means finding a solution that can cope best with all possible realizations of the uncertain data. Various approaches of robust optimization are developed. Idea behind robust optimization is to consider the worst case scenario without a specific distribution assumption. The roots of robust optimization can be found in the field of robust control and in the work of Soyster [25] considered a deterministic linear optimization model, which is feasible for all data lying in a convex set. However, the model is very conservative and is protected against the worst-case scenario. Subsequently, a number of important robust formulations are developed by Ben-Tal and Nemirovski [26-28], El Ghaoui et al. [29] and Bertsimas et al. [30].

In this section, we present a robust approach developed by Bertsimas et al. [31] for discrete optimization problems with uncertain parameters based on a polyhedral uncertainty set. Aforementioned studies are based on ellipsoidal uncertainty or box uncertainty sets. The approach proposed by Bertsimas et al. [31] is adopted according to the following justifications:

- The robust optimization (RO) model proposed by Bertsimas [31] is more tractable than RO frameworks with ellipsoidal uncertainty sets. A robust counterpart of a model considering uncertain linear programming (LP) with ellipsoidal uncertainty sets is solved in form of second-order conic programming (SOCP). However, the robust counterpart of this model with polyhedral uncertainty sets remains in form of linear programming. Besides, obtaining a solution from SOCP as a non-linear model is more difficult than LP. Then, it is not particularly attractive for solving robust discrete optimization problems[31, 32].
- Although, RO's tractability in the model based on both polyhedral and box uncertainty sets are equal (i.e., the robust counterpart of an uncertain LP model is in form of LP), RO with a polyhedral uncertainty set handles uncertainties more

flexiblely than RO with a box uncertainty set (e.g., Soyster's RO [25]). Since RO with polyhedral uncertainty offers can assign each conservation level to each uncertainty, RO with a box uncertainty assigns only one conservation level to all uncertainties of each equation [31].

Bertsimas approach permits to control the conservatism level of the solution [31]. Solutions obtained from robust optimization approach guarantees more situations even worst ones. The important concern of the robust methodology in this paper is to present an optimal planning that is robust with regard to data uncertainties in product processing time. The deterministic compact form of the model can be rewritten as follows:

$$\begin{aligned} & Min \quad c^T.x \\ & \text{s.t.} \\ & \quad A\,x \le b \\ & \quad lb \le x \le ub \end{aligned} \tag{27}$$

Suppose that only matrix $A=(a_{n,j})$ elements are subjected to data uncertainty, the RO methodology models data uncertainties through bounded intervals designated as uncertainty set. Therefore, the matrix A's uncertain elements can be defined using the mean value and range of each uncertain element as follows:

$$\tilde{a}_{n,j} = \left[a_{n,j} - \hat{a}_{n,j},\ a_{n,j} + \hat{a}_{n,j} \right] \qquad \tilde{a}_{n,j} \in A \tag{28}$$

A number named conversion level (CL), symbolized by Γ_n (n= 0,1,…,CN) is introduced in [31] for robustness intentions and adjusting the robustness level which adopts different values in the interval $\left[0, \left|J_n\right|\right]$, where J_n is a set comprises uncertain elements of the n-th equation $J_n = \{ j \mid \hat{a}_{n,j} > 0 \}$. Therefore, the robust counterpart of Equation (27) which is nonlinear can be written as follows:

$$\begin{aligned} & \text{Min} \quad c^T.x \\ & \text{s.t.} \\ & \sum_j a_{n,j}.x_j \\ & + \max_{\{S_n \cup \{t_n\} \mid S_n \subseteq J_n, |S_n| = \lfloor \Gamma_n \rfloor, t_n \in J_n \setminus S_n\}} \left\{ \sum_{j \in S_n} \hat{a}_{n,j}.\left|x_j\right| + \left(\Gamma_n - \lfloor \Gamma_n \rfloor\right).\hat{a}_{n,t_n}.\left|x_{t_n}\right| \right\} \\ & \le b_n \qquad \forall n \\ & \quad lb \le x \le ub \end{aligned} \tag{29}$$

By applying the linearization technique for nonlinear Equation (28), we have:

$$Min \quad c^T.x$$

s.t.

$$\sum_j a_{n,j}.x_j + z_n.\Gamma_n + \sum_{j \in J_n} p_{n,j} \le b_n \qquad \forall n \tag{30}$$

$$z_n + p_{n,j} \ge \hat{a}_{n,j}.y_j \qquad \forall n, j \in J_n$$

$$-y_j \le x_j \le y_j \qquad \forall j$$

$$lb_j \le x_j \le ub_j \qquad \forall j$$

$$z_n \ge 0 \qquad \forall n$$

$$p_{n,j} \ge 0 \qquad \forall n, j \in J_n$$

$$y_j \ge 0 \qquad \forall j$$

Now, a robust DCMS model can be developed by introducing a set of symmetric bounded intervals which represent the uncertainty of the product processing time as follows:

$$\tilde{t}^h_{i,m} \in \left[t^h_{i,m} - \hat{t}^h_{i,m},\ t^h_{i,m} + \hat{t}^h_{i,m} \right] \qquad \forall h, i, m \tag{31}$$

Therefore, the robust counterpart of the proposed model is as follows:

$$\begin{aligned} \text{Min OF} = & \sum_{h=1}^{H} \sum_{i=1}^{I} \sum_{r=1}^{R_i^h} \sum_{m=1}^{K_{r,i-1}^h} \sum_{c=1}^{C} \sum_{c'=1}^{C} \sum_{l=1}^{L} \sum_{l'=1}^{L} A_1.Dis_{l,l'}.\varphi^h_{U^m_{r,i},c,l,U^{m+1}_{r,i},c',l'} \\ & + \sum_{h=1}^{H} \sum_{i=1}^{I} \sum_{r=1}^{R_i^h} \sum_{m=1}^{K_{r,i-1}^h} \sum_{c=1}^{C} A_2.\psi^h_{U^m_{r,i},U^{m+1}_{r,i},c} \\ & + \sum_{h=1}^{H-1} \sum_{m=1}^{M} \sum_{c=1}^{C} \sum_{c'=1}^{C} \sum_{l=1}^{L} \sum_{l'=1}^{L} \gamma_m.\eta^h_{m,c,c',l,l'}.Dis_{l,l'} \\ & + \sum_{h=1}^{H} \sum_{i=1}^{I} \alpha_i^h.I_i^h \\ & + \sum_{h=1}^{H} \sum_{i=1}^{I} \beta_i^h.B_i^h \end{aligned} \tag{32}$$

s.t.

Equations (2-7), (9-15), (16-24)

$$\sum_{i=1}^{I} \sum_{r=1}^{R_i^h} VPQ^h_{r,i}.t^h_{i,U^m_{r,i}} + z_{h,m}.\Gamma_{h,m} + \sum_{i=1}^{I} p^h_{i,m} \le t'_{h,m} \quad \forall h, m \tag{33}$$

$$z_{h,m} + p^h_{i,m} \ge \sum_{r=1}^{R_i^h} \hat{t}^h_{i,U^m_{r,i}}.y_{t^h_{i,U^m_{r,i}}} \qquad \forall h, i, m \tag{34}$$

$$-y_{t^h_{i,U^m_{r,i}}} \le VPQ^h_{r,i} \le y_{t^h_{i,U^m_{r,i}}} \quad y_{t^h_{i,U^m_{r,i}}} \ge 0 \quad \forall h, r, i, m \tag{35}$$

$$z_{h,m} \ge 0 \qquad \forall h, m \tag{36}$$

$$p^h_{i,m} \ge 0 \qquad \forall h, i, m \tag{37}$$

However, the dimension of the processing time is equal to $H \times I \times M \times L$. However, note that the processing time for machine m in part i and period h is similar in all routings. Therefore, the total number of all uncertain variables of the set is equal to $H \times I \times M$. Besides, by considering Equations (28) and (31), it is revealed that $n=h \times m$ and $\Gamma_1, \Gamma_2, ..., \Gamma_{h \times m}$. Hence, in Equation (33), for each m and h, the number of uncertain set's elements are equal to the number of parts that machine m produces in period h (i.e. $J_n = \left[0, \left|J_n\right|\right] = \left[0, I_{h,m}\right]$), where, $I_{h,m}$ is the number of parts are produced by machine m in period h.

3. NUMERICAL RESULTS

A numerical example is considered to validate the performance of the presented model, in which eight machine types allocated to two possible cells to process 12 different part types, each having maximum three different routings and with the planning decisions over two time periods. This example is solved using the CPLEX 12.5 solver within GAMS software package on the personal computer 2.3 GHz Intel Core i5 with 4GB of RAM. The information related to the machine capacity, distance between candidate locations and part types are given in Tables 1 to 3, respectively. Table 3 contains the information related to the part sequences, routings, processing time and demand. For instance, part type one with the demand quantity of 95 has two different routings. The first routing consists of three operation sequences that each needs machines 7, 5 and 1, respectively. Furthermore, the minimum and maximum sizes of each cell in terms of machine numbers are assumed to be 2 and 4, respectively.

Based on definition in section 2.3, $\Gamma_1, \Gamma_2, ..., \Gamma_{16}$ ($n=h\times m=2\times 8=16$) can vary according to the Table 4. Note that when Γ_n =0, the Equations will be equivalent to the deterministic form. By changing $\Gamma_n \in \left[0, \left|J_n\right|\right]$ there will be also the flexibility of modifying the robustness of the method pertaining to the conservatism level of the solution. In the mentioned example, we adjust the processing time uncertainty $\hat{t}^{h}_{i,U^m_{r,i}}$ to equal 50% of the nominal processing time and $\Gamma_n = 0.5\times\left|J_n\right|$.

The obtained optimal solution of the proposed integrated model is presented in Tables 5 to 8. The cell configurations for two periods corresponding to the optimal solution of Table 5 are shown in Figure 1.

The optimal inter-cell layout is shown in Table 5. As an example in period one, cells 1 and 2 are constructed in locations 1 and 2, respectively, where they are 1 unit apart. Cell configurations for two periods are also presented in Table 5 and Figure 1. For instance, in period one, machine types M4, M5, M6 and M7 are assigned to cell 1. The efficient routing for each part type is shown in Table 6. By considering Tables 3, 5 and 6, it can be figured out how inter and intra-cell part trips happen. For instance, in period 2, part type 10 needs machines M3, M7 and M6 successively. On the other hand, the machines are assigned to C2, C1 and C1, respectively. Therefore, part type I2 after being processed on M3 in cell C2 would experience inter-cell part trip to cell C1 to be processed on machine M7, at last the part would experience intra-cell trip to M6.

TABLE 1. Distance matrix between cell locations

From \ To	L1	L2	L3
L1	0	1	2
L2	1	0	1
L3	2	1	0

TABLE 2. Capacity of machines

	M1	M2	M3	M4	M5	M6	M7	M8
Period 1	1800	1650	1950	2550	1500	1800	2025	1800
Period 2	1500	1350	1350	2400	1800	1950	2025	1500

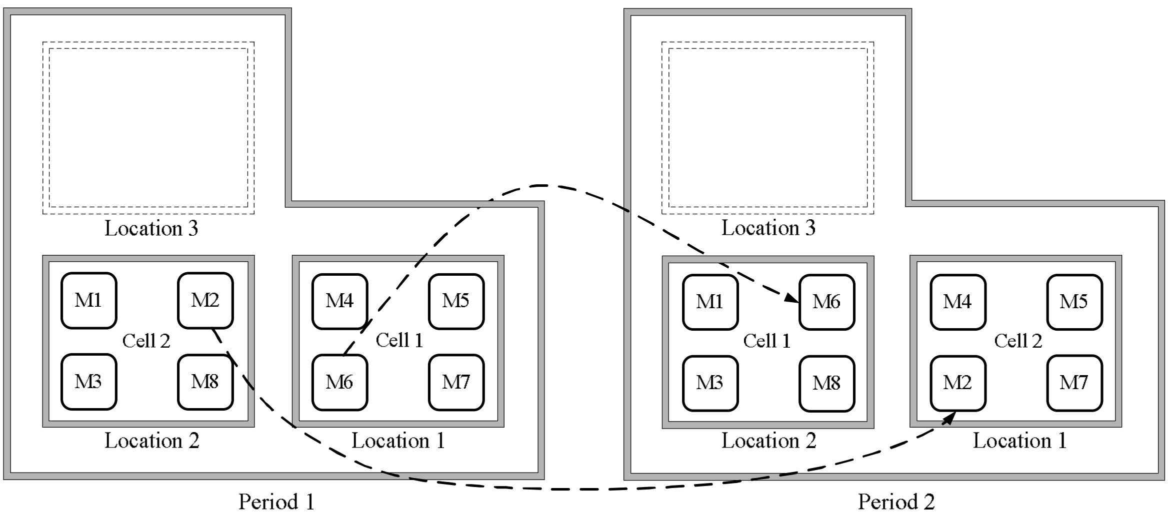

Figure 1. Optimal cell configuration

TABLE 3. Input data of part processing

Part	Period 1			Period 2		
	Process Sequence	Processing time (min)	Demand	Process Sequence	Processing time (min)	Demand
1	M7-M5-M1 M8-M7	2.4-3.2-1.6 4-2.4	95	M8-M5 M8-M7-M1	2.4-3.2 2.4-1.6-0.8	75
2	M2-M5 M3-M4	0.8-3.2 1.6-2.4	50	M2-M4-M5	0.8-4-2.4	0
3	M2-M5 M4-M6	2.4-4.8 3.2-4	65	M6-M7 M4-M6 M2-M5-M1	4-1.6 1.6-4 0.8-2.4-1.6	70
4	M1-M7-M8 M7-M4-M8 M1-M4-M3	2.4-4.8-0.8 4.8-3.2-0.8 2.4-3.2-1.6	60	M1-M4-M8 M4-M7-M1	2.4-3.2-1.6 4.2-2.4-2.4	50
5	M6-M3-M7 M2-M7-M4	3.2-1.6-0.8 2.4-0.8-1.6	50	M3-M6-M7 M2-M6-M7	3.2-4-0.8 2.4-4-0.8	45
6	M1-M4-M6 M1-M2-M4	0.8-2.4-1.6 0.8-1.6-2.4	80	M5-M4-M2 M6-M7-M3	1.6-1.6-2.4 0.8-1.6-3.2	60
7	M1-M4-M6 M3-M5-M2	1.6-0.8-2.4 3.2-4-0.8	40	M4-M5-M6 M1-M4-M5	0.8-2.4-3.2 1.6-0.8-2.4	45
8	M3-M7 M5-M8	2.4-4 3.2-4.8	50	M3-M7 M4-M6	1.6-3.2 1.6-4.8	60
9	M8-M6-M3 M2-M4-M3	1.6-4.8-6.4 3.2-2.4-6.4	0	M4-M8-M2	1.6-5.6-5.6	90
10	M3-M7-M6 M6-M8	0.8-2.4-1.6 1.6-3.2	50	M3-M7-M6	5.6-4-0.8	45
11	M8-M6-M7 M2-M4-M3	1.6-4-6.4 3.2-2.4-6.4	100	M6-M8-M7	1.6-5.6-5.6	85
12	M1-M7-M4 M4-M8 M1-M5-M4	1.6-3.2-2.4 2.4-4 1.6-2.4-2.4	70	M8-M5-M4 M3-M7-M4 M1-M5-M8	5.7-4-1.6 4.8-4.8-1.6 1.6-4-5.6	75

TABLE 4. Range of variation for conservation levels

Machine	Period 1	Period 2
M1	[0,5]	[0,5]
M2	[0,7]	[0,5]
M3	[0,8]	[0,5]
M4	[0,9]	[0,8]
M5	[0,6]	[0,6]
M6	[0,7]	[0,7]
M7	[0,7]	[0,9]
M8	[0,7]	[0,5]

TABLE 5. Optimal inter-cell layout and machine grouping

Cell	Period 1		Period 2	
	Cell location	Machines	Cell location	Machines
C1	L1	M4,M5, M6,M7	L2	M1,M8, M3,M6
C2	L2	M3,M8, M1,M2	L1	M2,M5, M7,M4

TABLE 6. Optimal part routings

Part	Period 1	Period 2
1	R2	R2
2	R1	R1
3	R1	R1
4	R1	R1
5	R1	R2
6	R2	R2
7	R2	R1
8	R2	R2
9	R2	R1
10	R2	R1
11	R2	R1
12	R2	R3

TABLE 7. Objective function value (OFV)

OFV	Inventory cost	Backorder cost	Relocation cost	Inter-cell cost	Intra-cell cost
3096	294	168	27	1580	1027

TABLE 8. Optimal production plan

Part	Period 1				Period 2			
	Inventory	Backorder	Production	Demand	Inventory	Backorder	Production	Demand
1	3		98	95			72	75
2			50	50			0	0
3			65	65			70	70
4	1		61	60			49	50
5			50	50			45	45
6			80	80			60	60
7	45		85	40			0	45
8			50	50			60	60
9	39		39	0			51	90
10			50	50			45	45
11			100	100		7	78	85
12	30		100	70			45	75

TABLE 9. Objective function value (OFV) for $\alpha_7^1 = 6$

OFV	Inventory cost	Backorder cost	Relocation cost	Inter-cell cost	Intra-cell cost
3828	404	168	86	1150	2020

TABLE 10. Computational results from different-sized problems

No.	*I×M×C×H*	No. variables	No. constraints	Objective function	Computational time (sec)	Gap (%)
1	12×8×2×2	2801	7551	3096	106	0
2	18×12×2×2	4973	12467	6453	291	0
3	24×16×2×2	8529	20767	11082	360	0
4	30×20×3×2	13469	41451	31078	914	0
5	36×24×3×2	17793	65519	44736	1159	0
6	42×28×3×2	26858	83587	60981	1453	0.01
7	48×32×4×2	51470	139547	94355	1898	0.01
8	54×36×4×2	97937	259859	135380	2642	0.08
9	60×42×4×2	129911	370775	176901	5648	10.3

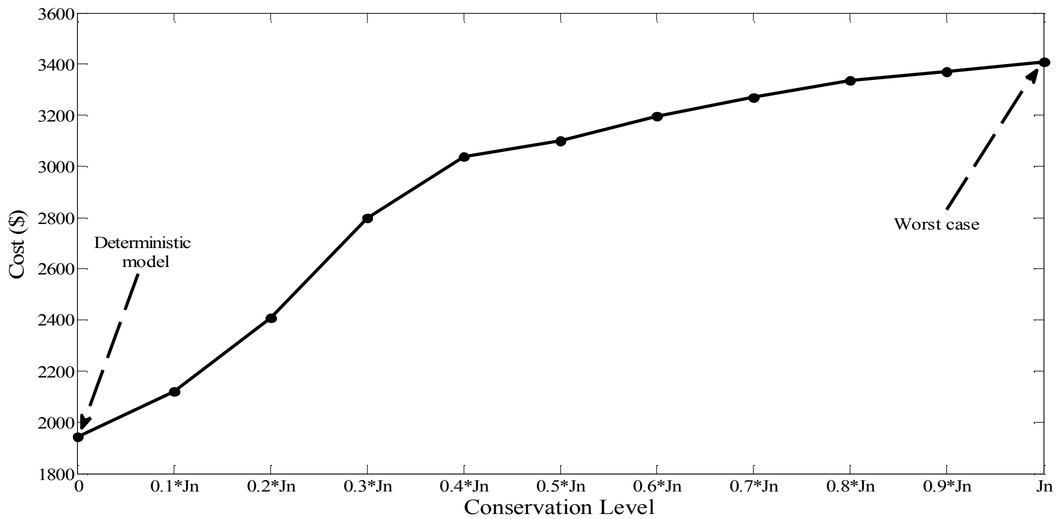

Figure 2. Effect of conservation level on objective function (cost)

The objective function value (OFV) is presented in Table 7. According to mentioned tables, for part type 1 in period 2, its routing R2 is chosen. This part should be processed on machines M8, M7 and M1 allocated to cells C2, C1 and C2, respectively. Hence, this part has two inter-cell trips. Based on the first term of the objective function, its first inter-cell part trip cost, is computed by:

$$A_1 . Dis_{2,1} . V^2_{2,1} . PQ^2_1 . N^2_{U^1_{2,1},2} . X^2_{2,2} . N^2_{U^2_{2,1},1} . X^2_{1,1}$$

$0.85\times1\times1\times72\times1\times1 = 30.6$

Note that: $U^1_{2,1}$ indicates to machine M8, which is the first machine in the second routing of part type 1. Table 8 demonstrates optimal production planning, demand quantity for each part type can be satisfied through production, backorder and/or inventory. For instance in period 1, the demand of part type 7 is 40 while 85 units are produced. Therefore, 45 units will be kept for next period as inventory and this amount can satisfy the next period's demand. Furthermore, type part 11 has been produced 78 units while the demand quantity is 85. Therefore, the demand cannot be satisfied completely and 7 units are considered as backorder. To verify the costing effect of inventories, assume that the unit inventory cost of part type 7 is increased from 3 to 6 per unit. Table 9 demonstrates the results in the objective function values. This increase makes the part type 7 demands to be satisfied in each pertaining period, in other words production quantity for part type 7 after this change is 40 and 45 in periods 1 and 2, respectively.

The robust optimization approach by solving the worst case problem presents an optimal solution immunized against all data uncertainties. To verify the behavior of the model, Figure 2 is plotted presenting the effect of the conservation level on the objective function value. According to this figure, the OFV is the function of conservation level. By increasing conservation level, the OFV increases. With robust optimization approach the desire to stay on the safe side can be achieved by enlarging uncertainty set. In the non-presence of conservation level (i.e., deterministic model), the optimal value is 1943. On the other hand, with maximum conservatism (i.e., worst case) the optimal value is increased by 75% to 3409. As far as we increase the conservatism, the presented model becomes more immune against processing time uncertainty. To better illustrate the ability of the robust DCMS model, the computational results from different-sized problems are presented in Table 10 illustrating the computational time, objective function, relative optimality criterion (Gap) and the number of variables and constraints for each problem. It is obvious that by increasing the problem size in terms of a number of variables, the computational time increases. The relative optimality criterion for an MIP problem is as follows:

$$\left(|BP-BF|\right)/\left(1.0e-10+|BF|\right)$$

where, BF is the objective function value of the current best integer solution, while BP is the best possible integer solution [33].

4. CONCLUSION

In this paper, a robust optimization approach has been developed for a new presented mathematical model integration of cell formation, inter-cell design, and production planning under dynamic environment, in order to cope with parts processing time uncertainty. This model has minimized inter and intra-cell material handling costs, relocation costs and production planning costs (e.g., inventory and backorder costs). This model has been able to determine the optimal cell configuration and production plan for each part type at each period over the planning horizon. The important advantages of this study are as follows:

- Applying a robust optimization approach to the DCMS.
- Distance-based relocations of machine types.
- Considering different routings for each part type.
- Incorporating inter-cell layout of machines with cell formation to exactly calculate inter-cell material handling cost.
- Considering uncertainty for parts processing time.

For future studies, providing frameworks that which considers more options of uncertainty can be interesting fields.

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A Robust Model for a Dynamic Cellular Manufacturing System with Production Planning

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چکیده

این مقاله یک روش بهینه سازی استوار برای طراحی یک سیستم پویای تولید سلولی ترکیب شده با برنامه ریزی تولید، تحت عدم قطعیت در زمان پردازش قطعات توسعه می‌دهد. برای مقابله با این عدم اطمینان، یک روش بهینه‌سازی استوار به عنوان یک رویکرد کنترل پذیر به کار گرفته شده است. این مدل شامل مفاهیم تشکیل سلول، طراحی درون سلولی و برنامه‌ریزی تولید در یک محیط پویا می‌باشد. هدف این مدل، کمینه‌سازی هزینه‌های حرکت درون و برون سلولی قطعات، نگهداری موجودی، سفارش تاخیر یافته و پیکربندی مجدد می‌باشد. در خاتمه یک مثال عددی برای نشان دادن رفتار مدل پیشنهادی و بررسی عملکرد روش توسعه یافته برای یافتن راه حل بهینه حل می‌شود.

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