



Reliability Analysis of Three Elements in Series and Parallel Systems under Time-varying Fuzzy Failure Rate

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PAPER INFO

Paper history:

Received 02 June 2013

Received in revised form 02 September 2013

Accepted 14 September 2013

Keywords:

Reliability Analysis

Mean Time to Failure

Time-varying Fuzzy Failure Rates

Parallel System

Series System

ABSTRACT

Reliability is the most important performance issue in engineering design process but in the real world problems, there are limitations for using conventional reliability. Fuzzy logic has proved to be effective in expressing uncertainties in different fields, including engineering reliability. In this paper, for both series and parallel systems composed of three identical or different elements, the reliability has been analyzed using the fuzzy concepts and some characteristics such as the mean time to failure have been evaluated, taking in our account that all the operating units have increasable time varying failure rates with fuzzy triangular membership functions. This paper includes a numerical example to illustrate the model and validate the analytical results.

doi: 10.5829/idosi.ije.2014.27.04a.06

1. INTRODUCTION

System reliability occupies important and significant role in design of power plants, manufacturing systems, industrial systems, standby systems, etc. By the reliability analysis, a manager can decide the optimal strategy, by setting the range of Mean Time to Failure (MTTF) to reflect the desired failure rates and this in turn will minimize the total cost involved. The reliability of an item is the probability that the item will perform a required function during a given interval of time under specified conditions. The system reliability depends on the reliability of its components, configuration of the system, and the system failure criteria. During the study of the reliability of any system, the goal is predicting the suitable system reliability features based on the component failure data and the system design. Series or parallel systems or both of them can represent the large engineering system with similar or different components having constant [1-4], or time-varying failure rates. Many studies handled the problem of time-varying failure and repair rates [5, 6].

In the real world applications as sensor information, data are vaguely specified so fuzzy theory can handle such vagueness using membership function. The fuzzy set theory has a realistic and practical means to describe the real world. It has been introduced firstly by Lotfi zadah [7] and many researcher [8-10] who used the fuzzy sets in the reliability analysis to get some reliability characteristics. But the system parameters are often imprecise due to incomplete and accurate information, so failure and repair rates may be expressed with some uncertainty by experiences of experts. In fact, the fuzzy sets make descriptions of the objective world more realistic, practical, and accurate so they have been widely applied in decision-making and logic programming. Sharma et al. [11] used a system containing three elements established with constant and increasable fuzzy failure rates in real time situations and Jiang et al. [12] constructed an algorithm to get the fuzzy reliability. Wang [13] made the analysis with fuzzy random variables. Mishra et al. and Krishnan et al. [14, 15] presented the real time study of a k -out-of- n system having n identical elements with constant fuzzy failure rates. In this paper, both series and parallel system composite of three similar or different components are analyzed when the failure distribution of each operating

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component has an increasable failure rate. We can't expect the exact value of the failure rate of each unit due to the lack of sufficient data to demonstrate the uncertainty in the calculations. This parameter can be expressed using triangular fuzzy numbers which are estimated using chi-square distribution with $(1-\beta)100\%$ confidence interval. Then, we describe the reliability of the two systems in the fuzzy sense. At the end, we give numerical example and by comparing the driven results with the crisp model, fuzzy models indicate more realistic and practical results than the crisp one. Moreover, we can construct another un-repairable system and analyze it to get the reliability function and the mean time to failure under fuzzy assumptions.

2. NOTATIONS

- λ_i : Failure rate of the i^{th} element.
- L_i : Lower limit of the triangular fuzzy number related to the failure rate of the i^{th} element.
- M_i : Medium of the triangular fuzzy number related to the failure rate of the i^{th} element.
- U_i : Upper limit of the triangular fuzzy number related to the failure rate of the i^{th} element.
- $P_i(t)$: Probability that the system at the t moment to be in condition i .
- $Rs(t)$: Probability of the functionality of the series system.
- $Rp(t)$: Probability of the functionality of the parallel system.
- MTTF**: Mean time to failure.

3. SYSTEM RELIABILITY

3. 1. System Reliability in Crisp Case In this section, we discuss the reliability of the general unrepairable series system and parallel system in the crisp case, Figure 1 represents the configuration of the series and the parallel systems, with components P_1, P_2, \dots, P_n , assuming their reliabilities are R_1, R_2, \dots, R_n ($0 \leq R_i \leq 1; i = 1, 2, \dots, n$), respectively. For each component, the failure rate is not constant but linearly changes with time and equals $(\lambda_i t)$, then:

$$R_i(t) = e^{-\int_0^t \lambda_i t dt} = e^{-\frac{1}{2} \lambda_i t^2} \tag{1}$$

Then for the series system of three oponents we can get:

$$Rs(t) = \prod_{i=1}^3 R_i(t) = e^{-\frac{1}{2} (\lambda_1 + \lambda_2 + \lambda_3) t^2} \tag{2}$$

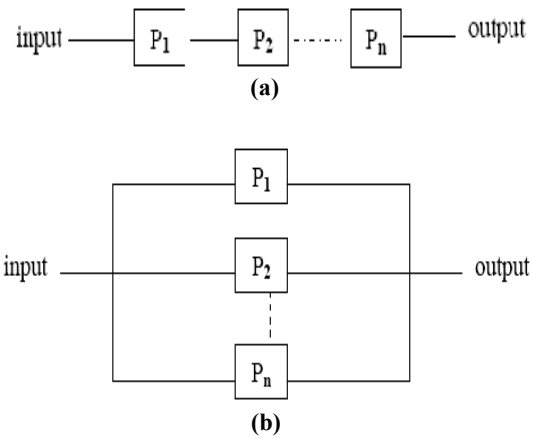


Figure 1. (a) Series system configuration and (b) parallel system configuration

$$MTTFs = \int_0^\infty Rs(t) dt = \sqrt{\frac{\pi}{2(\lambda_1 + \lambda_2 + \lambda_3)}} \tag{3}$$

and for the parallel system

$$Rp(t) = 1 - (1 - e^{-\frac{1}{2} \lambda_1 t^2})(1 - e^{-\frac{1}{2} \lambda_2 t^2})(1 - e^{-\frac{1}{2} \lambda_3 t^2}) \tag{4}$$

$$MTTFp = \int_0^\infty Rp(t) dt = \sqrt{\frac{\pi}{2 \lambda_1}} + \sqrt{\frac{\pi}{2 \lambda_2}} + \sqrt{\frac{\pi}{2 \lambda_3}} - \sqrt{\frac{\pi}{2(\lambda_1 + \lambda_2)}} - \sqrt{\frac{\pi}{2(\lambda_1 + \lambda_3)}} - \sqrt{\frac{\pi}{2(\lambda_2 + \lambda_3)}} + \sqrt{\frac{\pi}{2(\lambda_1 + \lambda_2 + \lambda_3)}} \tag{5}$$

3. 2. System Reliability under Fuzzy Failure Rates

Most important consideration is that the values of the failure rates $\lambda_i; i = 1, 2, 3$ are not fixed, since they are driven from collected data or the opinions of the experts, uncertainty of these values is an undeniable fact. Most of the times, these failure rates are considered as known values or have known distribution function. In this paper, we assume that failure rates, $\lambda_i; i = 1, 2, 3$ in the form of triangular fuzzy numbers are as follows:

$$\bar{\lambda}_i = (L_i / M_i / U_i); i = 1, 2, 3 \tag{6}$$

in which M_i is the point estimation and L_i, U_i are the lower and the upper limits of $(1-\beta)100\%$ confidence interval of $\bar{\lambda}_i$ where the α -cuts of these failure rates can be calculated as follows:

$$\bar{\lambda}_i[\alpha] = [\lambda_{iL}(\alpha), \lambda_{iU}(\alpha)] = [L_i + \alpha(M_i - L_i), U_i - \alpha(U_i - M_i)] ; i = 1, 2, 3 \tag{7}$$

By using the extension principle, the reliability function and MTTF can be calculated in each case as follows:

$$\begin{aligned} \overline{RS}[t, \alpha] &= [RS_L(t, \alpha), RS_U(t, \alpha)] \\ RS_L(t, \alpha) &= e^{-\frac{1}{2} (U_1 + U_2 + U_3 - \alpha (U_1 + U_2 + U_3 - M_1 - M_2 - M_3)) t^2} \end{aligned} \tag{8.1}$$

$$RS_U(t, \alpha) = e^{-\frac{1}{2}(L_1+L_2+L_3 + \alpha(M_1+M_2+M_3-L_1-L_2-L_3))t^2} \tag{8.2}$$

$$\overline{MTTFs}[\alpha] = [MTTFs_L(\alpha), MTTFs_U(t, \alpha)]$$

$$MTTFs_L(\alpha) = \int_0^\infty RS_L(t) dt = \sqrt{\frac{\pi}{2(U_1+U_2+U_3-\alpha(U_1+U_2+U_3-M_1-M_2-M_3))}} \tag{9.1}$$

$$MTTFs_U(\alpha) = \int_0^\infty RS_U(t) dt = \sqrt{\frac{\pi}{2(L_1+L_2+L_3+\alpha(M_1+M_2+M_3-L_1-L_2-L_3))}} \tag{9.2}$$

$$\overline{Rp}[t, \alpha] = [Rp_L(t, \alpha), Rp_U(t, \alpha)]$$

$$Rp_L(t, \alpha) = 1 - (1 - e^{-\frac{1}{2}(U_1-\alpha(U_1-M_1))t^2})(1 - e^{-\frac{1}{2}(U_2-\alpha(U_2-M_2))t^2})(1 - e^{-\frac{1}{2}(U_3-\alpha(U_3-M_3))t^2}) \tag{10.1}$$

$$Rp_U(t, \alpha) = 1 - (1 - e^{-\frac{1}{2}(L_1+\alpha(M_1-L_1))t^2})(1 - e^{-\frac{1}{2}(L_2+\alpha(M_2-L_2))t^2})(1 - e^{-\frac{1}{2}(L_3+\alpha(M_3-L_3))t^2}) \tag{10.2}$$

$$\overline{MTTFp}[\alpha] = [MTTFp_L(\alpha), MTTFp_U(t, \alpha)]$$

$$MTTFp_L(\alpha) = \int_0^\infty Rpl(t)dt = \sqrt{\frac{\pi}{2(U_1-\alpha(U_1-M_1))}} + \sqrt{\frac{\pi}{2(U_2-\alpha(U_2-M_2))}} + \sqrt{\frac{\pi}{2(U_3-\alpha(U_3-M_3))}} - \sqrt{\frac{\pi}{2(L_1+L_2+\alpha(M_1+M_2-L_1-L_2))}} - \sqrt{\frac{\pi}{2(L_1+L_3+\alpha(M_1+M_3-L_1-L_3))}} - \sqrt{\frac{\pi}{2(L_2+L_3+\alpha(M_2+M_3-L_2-L_3))}} + \sqrt{\frac{\pi}{2(U_1+U_2+U_3-\alpha(U_1+U_2+U_3-M_1-M_2-M_3))}} \tag{11.1}$$

$$MTTFp_U(\alpha) = \int_0^\infty Rpu(t)dt = \sqrt{\frac{\pi}{2(L_1+\alpha(M_1-L_1))}} + \sqrt{\frac{\pi}{2(L_2+\alpha(M_2-L_2))}} + \sqrt{\frac{\pi}{2(L_3+\alpha(M_3-L_3))}} - \sqrt{\frac{\pi}{2(U_1+U_2-\alpha(U_1+U_2-M_1-M_2))}} - \sqrt{\frac{\pi}{2(U_1+U_3-\alpha(U_1+U_3-M_1-M_3))}} - \sqrt{\frac{\pi}{2(U_2+U_3-\alpha(U_2+U_3-M_2-M_3))}} + \sqrt{\frac{\pi}{2(L_1+L_2+L_3+\alpha(M_1+M_2+M_3-L_1-L_2-L_3))}} \tag{11.2}$$

Because the fuzzy rate of each component linearly changes with time ($\bar{\lambda}_i, t$) and has exponential distribution with unknown fuzzy parameter ($\bar{\lambda}_i$) which can be calculated, see [17], using the point estimation and $(1-\beta)100\%$ confidence intervals (two ways) of this parameter as:

$$M_i = \frac{n_i}{\sum T_i} \tag{12.1}$$

$$L_i = \frac{x_{2n_i(1-\beta/2)}^2}{2\sum T_i} \tag{12.2}$$

$$U_i = \frac{x_{2n_i(\beta/2)}^2}{2\sum T_i} \tag{12.3}$$

where the data is synthesized for each component i ; $i=1, 2, 3$ in two statistics: a total test time is denoted as $\sum T_i$ and a sample size is denoted as n_i .

4. NUMERICAL AND GRAPHICAL EXAMPLE

The Maple software is used to analysis the reliability and MTTF of the series and parallel systems.

4. 1. For Three Different Components Assume a system consisting of three major units. These are continuously monitored by a failure detection device. For efficiency, the manager wants to analysis the system and get fuzzy reliability and MTTF. Due to uncertainty or the imprecision of data, the failure rates of the three operating units can be represented by triangular fuzzy numbers.

Assume each unit is tested with a sample size 25, 35, 30 and total failure test times are 920, 700, 1200. Based on that statistical data with a confidence level 95%, the values of the medium, lower and upper limits of these three triangular fuzzy numbers related to failure rates $\bar{\lambda}_1, \bar{\lambda}_2, \bar{\lambda}_3$ are calculated according to Equation (12) as follows:

$$\bar{\lambda}_1 = (0.0176/ 0.0272/ 0.03882)$$

$$\bar{\lambda}_2 = (0.0231/ 0.05/ 0.051)$$

$$\bar{\lambda}_3 = (0.01348/ 0.025/ 0.02975)$$

It is easy to find that the fuzzy reliability function for both the series and parallel systems using Equations (8) and (10) and represent them graphically at different values of α as shown in Figure 2 and according to Equations (9) and (11), we can get the crisp intervals of the failuer rates of the three operating units and the mean-time to failure of both series and parallel systems at different possibility α levels as shown in Table 1 and their overall shapes are represented in Figure 3.

4. 2. For Three Identical Components If each component has the same fuzzy failure rate $\bar{\lambda}_1 = \bar{\lambda}_2 = \bar{\lambda}_3 = \bar{\lambda}$, we test with a sample size of 25 and a total test time of 920. Based on that statistical data with a confidence level 95%, the triangular fuzzy number related to that failure rate is $\bar{\lambda} = (0.0176/ 0.0272/ 0.03882)$. The fuzzy reliability function for both the series and parallel systems can be represented graphically at different values of α as shown in Figure 4 and the crisp intervals of that failuer rate and the mean-

time to failure of both series and parallel systems at different α -cuts are shown in Table 2 and their overall shapes are represented in Figure 5.

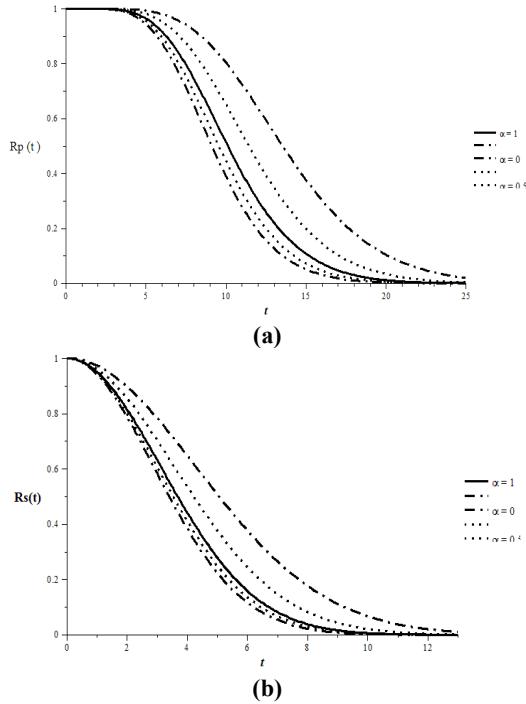


Figure 2. (a) Fuzzy reliability of parallel system and (b) Fuzzy reliability of series system with different three units

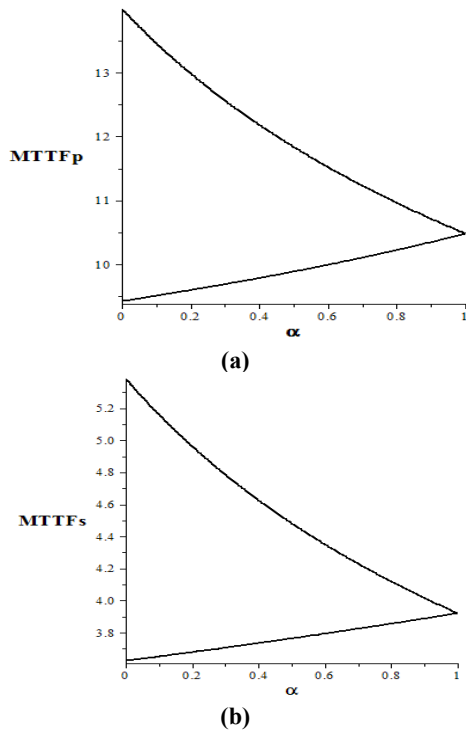


Figure 3. (a) Fuzzy MTTF of parallel system and (b) Fuzzy MTTF of series system with different three units

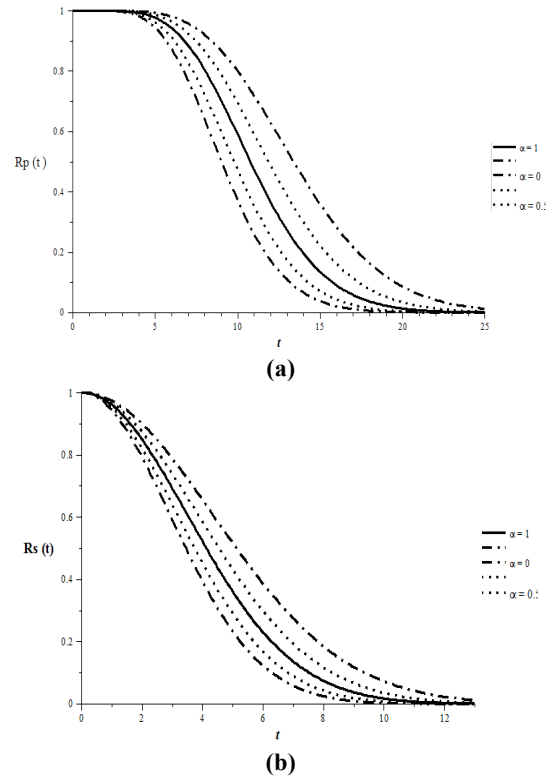


Figure 4. (a) Fuzzy reliability of parallel system and (b) Fuzzy reliability of series system with similar units

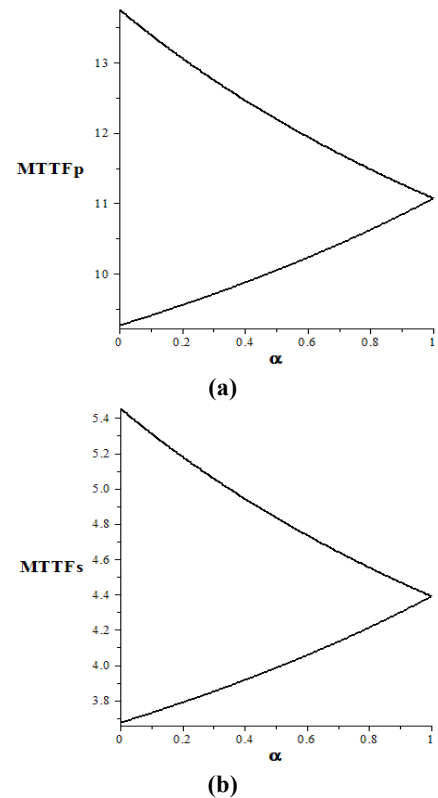


Figure 5. (a) Fuzzy MTTF of parallel system and (b) Fuzzy MTTF of series system with similar units

TABLE 1. α -cuts of the failure rates, the fuzzy mean-time to failure of the series and parallel system with different three units

α	$\lambda_{1L}(\alpha)$	$\lambda_{1U}(\alpha)$	$\lambda_{2L}(\alpha)$	$\lambda_{2U}(\alpha)$	$\lambda_{3L}(\alpha)$	$\lambda_{3U}(\alpha)$	$MTTF_{sL}(\alpha)$	$MTTF_{sU}(\alpha)$	$MTTF_{pL}(\alpha)$	$MTTF_{pU}(\alpha)$
0.0	0.0176	0.03882	0.0231	0.051	0.01348	0.02976	3.6244	5.3843	9.42249	13.997
0.1	0.0185	0.0377	0.0258	0.051	0.0146	0.02928	3.651	5.1606	9.50717	13.461
0.2	0.0195	0.0365	0.0285	0.0508	0.01578	0.0288	3.678	4.9626	9.5955	12.9877
0.3	0.0205	0.0353	0.0312	0.0507	0.01694	0.0283	3.706	4.7859	9.6879	12.565
0.4	0.0214	0.0342	0.03386	0.0506	0.0181	0.02785	3.7347	4.6267	9.875	12.1843
0.5	0.0224	0.03299	0.0365	0.0505	0.0192	0.02738	3.7639	4.4825	9.886	11.8384
0.6	0.0233	0.0318	0.0392	0.0504	0.0204	0.0269	3.7938	4.351	9.993	11.522
0.7	0.0243	0.03067	0.04193	0.0503	0.0215	0.0264	3.8244	4.23	10.1047	11.2313
0.8	0.02526	0.0295	0.04462	0.0502	0.0227	0.02595	3.8558	4.1192	10.223	10.9625
0.9	0.0262	0.02834	0.0473	0.0501	0.02385	0.0255	3.888	4.0164	10.348	10.7131
1.0	0.0272	0.0272	0.05	0.05	0.025	0.025	3.9209	3.9209	10.4806	10.4806

TABLE 2. α -cuts of the failure rate, the fuzzy mean-time to failure of the series and parallel system with similar three units

α	$\lambda_L(\alpha)$	$\lambda_U(\alpha)$	$MTTF_{sL}(\alpha)$	$MTTF_{sU}(\alpha)$	$MTTF_{pL}(\alpha)$	$MTTF_{pU}(\alpha)$
0.0	0.0176	0.03882	3.6726	5.454	9.26195	13.7554
0.1	0.0185	0.0377	3.7289	5.3118	9.4041	13.39589
0.2	0.0195	0.0365	3.78798	5.17985	9.553	13.06316
0.3	0.0205	0.0353	3.8499	5.057	9.7092	12.754
0.4	0.0214	0.0342	3.91499	4.94	9.873	12.4659
0.5	0.0224	0.03299	3.983	4.8362	10.046	12.1964
0.6	0.0233	0.0318	4.0557	4.73596	10.228	11.9437
0.7	0.0243	0.03067	4.132	4.642	10.421	11.7061
0.8	0.02526	0.0295	4.2127	4.553	10.624	11.4821
0.9	0.0262	0.02834	4.298	4.469	10.8403	11.2705
1.0	0.0272	0.0272	4.3896	4.3896	11.07	11.07

5. RESULTS AND DISCUSSIONS

From the numerical example that introduced for analyzing a model for three items un-repairable system, as shown in Table 1, if the items have different triangular shaped fuzzy parameters, it is impossible for the value of the fuzzy mean-time to failure to fall below 3.6244 or exceed 5.3843 in the series connection and its most possible value is 3.9209. In the parallel connection, it can't fall below 9.42249 or exceed 13.997 and the most possible value is 10.4806. As shown in Table 2, if we have similar triangular shaped fuzzy parameters, the value of the fuzzy mean-time to failure in the series connection falls in the interval [3.6726, 5.454] and the most possible value is 4.3896. In the parallel connection, it falls in [9.26195, 13.7554] with the most possible value 11.07. After drawing the results using MAPLE software program, we find that the

$MTTF$ in all cases can be approximated to be a triangular shaped fuzzy number too and the fuzzy reliability for each value of α has a two reliability curves which represent the lower and the upper bounded curves. These are around the main reliability curve in the crisp case evaluated at $\alpha = 1$ and all those curves are bounded by the lower and upper limit curves at $\alpha = 0$.

6. CONCLUSION

Different reliability models can represent most electronic and electrical systems. In many practical situations, we can deal with them as series or parallel systems consisting of number of operating units with parameters driven from gathering data or the opinions of the experts. Due to uncertainty or imprecision of data in calculations of these parameters, they are estimated

through triangular fuzzy numbers. In this paper, we establish a model for a system consisting of three independent units connected in parallel or in series. We introduce a reliability analysis for the series and the parallel systems in the presence of fuzziness and model the uncertainty of their parameters values. For the three operating units having time-varying failure rates, we use triangular fuzzy numbers which are estimated using random samples to represent their parameters. In addition, we introduce a procedure to estimate the fuzzy reliability and the mean-time to failure. This procedure can be applied to similar reliability models. Finally, we introduce a numerical example for illustration and we use MAPLE program to show the results graphically. Further researches can focus on other un-repairable systems as parallel-series systems, series-parallel systems, k -out of n systems or standby systems to evaluate the reliability and mean time of failure.

7. ACKNOWLEDGMENTS

The authors are grateful to the editor and the referees for their valuable comments and helpful suggestions that have much improved the presentation.

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PAPER INFO

چکیده

Paper history:

Received 02 June 2013

Received in revised form 02 September 2013

Accepted 14 September 2013

Keywords:

Reliability Analysis

Mean Time to Failure

Time-varying Fuzzy Failure Rates

Parallel System

Series System

قابلیت اطمینان مهم ترین مسئله عملکرد در فرآیند طراحی و مهندسی است، اما در مسائل و مشکلات دنیای واقعی، محدودیت برای استفاده از قابلیت اطمینان متعارف وجود دارد. منطق فازی ثابت شده است که در بیان عدم قطعیت در زمینه های مختلف، از جمله مهندسی قابلیت اطمینان موثر است. در این مقاله، برای هر دو سری و سیستم های موازی متشکل از سه عنصر یکسان یا متفاوت، قابلیت اطمینان با استفاده از مفاهیم فازی و برخی از ویژگی های از قبیل زمان متوسط برای شکست ارزیابی شده است تجزیه و تحلیل شده، مصرف شده در حساب ما که همه عامل واحد زمان افزایش نرخ مختلف شکست با توابع عضویت مثلثی فازی. این مقاله شامل یک مثال عددی برای نشان دادن مدل و اعتبار نتایج تحلیلی است.

doi: 10.5829/idosi.ije.2014.27.04a.06
