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TECHNICAL NOTE

Vibration Analysis of a Nonlinear System with a Nonlinear Absorber under the Primary and Super-harmonic Resonances

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A B S T R A C T

In vibratory systems, linear and nonlinear vibration absorbers can be used to suppress primary and super-harmonic resonance responses. In this paper, the behavior of a nonlinear system with a nonlinear absorber, under the primary and super-harmonic resonances is investigated. Comparison of the effects of attached nonlinear absorber on a nonlinear system with that of a linear one, under the resonance cases is performed. The stiffness of the main system and the absorber are considered to be cubically nonlinear, whereas the behavior of the dampers is supposed to be linear. Using multiple time scales (MTS) method, approximate solution of the nonlinear equations of motion is obtained. It is concluded that at primary resonance, a linear absorber can suppress the peak amplitude of the system better than a non-linear one, but under super-harmonic resonance, effective reduction in the vibration amplitude can be achieved using a nonlinear absorber.

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NOMENCLATURE

x	Absolute displacement
m	Mass
k	Linear stiffness
k'	Nonlinear stiffness
c	Linear damping constant
a	The domain of the main system
b	The domain of the absorber
Greek Symbols	
θ	The phase of the main system
γ	The phase of the absorber
Subscripts	
1	Main system
2	absorber

1. INTRODUCTION

Vibration amplitude in linear or nonlinear vibratory systems can be under control using absorbers. They can decrease the vibration amplitude of a system near its resonance frequency. A 1-DOF nonlinear system can be considered as a plausible model for investigation of nonlinear phenomena in many mechanical systems. However, application of a linear or nonlinear absorber to the mentioned 1-DOF system leads to a 2-DOF system. Harmonic resonances of a 1-DOF nonlinear system under parametric and external excitations with quadratic, cubic and quartic nonlinearities have been investigated by Eissa and El-Bassiony [1]. They studied effects of different parameters on the response and stability of the system. Pai et al. [2] controlled steady state vibrations of a cantilevered skew aluminum plate using saturation phenomena for 1:2, 1:3 and 1:4 internal resonant cases. A nonlinear parametric feedback control has been used by Leung et al. [3] to eliminate instability resonance responses in a forced 1-DOF nonlinear system. It is shown that the proposed nonlinear feedback is effective in the control of primary, super- and sub-harmonic resonance responses. Chatterjee [4] proposed a method to control linear resonant vibrations, low-frequency non-resonant vibrations, and primary and

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1/3 sub-harmonic resonances of a forced oscillator. Nahvi [5] investigated a 2-DOF quadratic nonlinear self-excited system subjected to a parametric excitation. The multiple time scales method was used to study 1:2 internal and parametric resonances of the system. Amer and El-Sayed [6] studied stability of a 2-DOF system under primary and super-harmonic resonances. The system consists of the main system and an attached absorber, both with quadratic and cubic nonlinear springs. They obtained frequency-response equations of the 2-DOF system and showed that by adding an absorber, the peak amplitude of the main system is reduced to 2.5% of its maximum value.

Dynamics of a 2-DOF nonlinear system was studied by Zhu et al. [7]. They showed that by properly selecting the stiffness and damping constants of the spring and damper the vibration amplitude can be reduced. Liu and Liu [8] studied dynamics of a 2-DOF system consisting of a mass-spring system and a dynamic vibration absorber. Using the Brock's approach, the optimum parameters of the absorber which is effective in a wide range of forcing amplitude were obtained. Viguie and Kerschen [9] addressed the problem of mitigating vibration of mechanical systems using nonlinear dynamic absorbers.

Ji and Zhang [10] used a dynamic vibration absorber to suppress the primary resonance vibration of a forced nonlinear oscillator. To indicate the performance of vibration absorber on reducing the response, attenuation and desensitization ratios has been introduced. They stated that an absorber with small attenuation ratio and large desensitization ratio correspond to better performance of the absorber. Using a linear absorber, suppression of super-harmonic resonance response was studied by Ji and Zhang [11]. Results show that for fixed absorber parameters, better performance can be achieved by using smaller absorber mass, larger spring stiffness and larger damping of the absorber. Sayed and Kamel [12] carried out a comprehensive investigation for suppressing the vibration of the nonlinear plant when subjected to external and parametric excitations. They applied an active vibration absorber (by quadratic and cubic order of control) in the presence of 1:2 and 1:3 internal resonances. Gahary and Ganaini [13] studied vibration suppression of a beam under multi-parametric excitation forces. A time delayed absorber was applied to suppress chaotic vibrations and the effects of different absorber parameters on the system behavior were obtained numerically.

The performance of a new type of nonlinear vibration absorber that is attached to a 1-DOF linear\nonlinear oscillator, subjected to a periodic external excitation has been studied by Febbo and Machado [14].

Hsu et al. [15] presented an experimental and analytical investigation of the physical behavior and effectiveness of a nonlinear dynamic vibration absorber. The absorber is attached by a nonlinear hardening spring to a cantilever linear beam excited by a shaker.

In this paper, the effects of using a nonlinear absorber on the amplitude of a nonlinear system are investigated. The springs of the main system and the absorber are considered to be cubically nonlinear, whereas the behavior of the dampers is supposed to be linear. Under the primary and super-harmonic resonances, the peak amplitude of the system with a nonlinear absorber is compared with the peak amplitude of the system with a linear one. Finally, using the attenuation and desensitization ratios, the best values of the nonlinear absorber parameters are determined. As far as we know, investigation on the primary and super-harmonic resonances in a nonlinear system with attached nonlinear absorber and comparison with the behavior of a linear one has not yet been reported.

2. MATHEMATICAL MODEL

Consider a 1-DOF nonlinear system which is attached to a nonlinear absorber (Figure 1). An external force excites the main mass (m_1). The nonlinear spring stiffness of the main oscillatory system and the absorber are k_1' and k_2', respectively, which are assumed to be weak enough.

The parameters of the absorber (m_2, k_2, k_2', c_2) and the main system (m_1, k_1, k_1', c_1) are listed in Table 1. The equations of motion for the 2-DOF nonlinear system, shown in Figure 1, are [16]:

$$\begin{aligned} &m_1\ddot{x}_1 + k_1 x_1 + k_1' x_1^3 + c_1\dot{x}_1 - k_2(x_2 - x_1) \\ &- k_2'(x_2 - x_1)^3 + c_2(\dot{x}_1 - \dot{x}_2) = \sum_{j=1}^{n} f_j \cos\Omega_j t \end{aligned} \tag{1a}$$

$$\begin{aligned} &m_2\ddot{x}_2 + k_2(x_2 - x_1) + k'_2(x_2 - x_1)^3 \\ &+ c_2(\dot{x}_2 - \dot{x}_1) = 0 \end{aligned} \tag{1b}$$

The behavior of this system will be studied for primary and super-harmonic resonant cases.

2. 1. Primary Resonance

For the primary resonance, it is assumed that $\Omega \cong \omega_1 + \varepsilon\sigma$, where Ω is the force frequency, ω_1 the natural frequency of the main system and σ the detuning parameter. Equation (1) may be written as:

$$\begin{aligned} &\ddot{x}_1 + \omega_1^2 x_1 + \varepsilon\alpha_1 x_1^3 + \varepsilon\xi_1\dot{x}_1 - \varepsilon\alpha_2 x_2 \\ &- \varepsilon\alpha_3(x_2 - x_1)^3 + \varepsilon\xi_2(\dot{x}_1 - \dot{x}_2) \\ &= \varepsilon f\cos\Omega t \end{aligned} \tag{2a}$$

$$\begin{aligned} &\ddot{x}_2 + \omega_2^2(x_2 - x_1) + \varepsilon\beta_1(x_2 - x_1)^3 \\ &+ \varepsilon\xi_3(\dot{x}_2 - \dot{x}_1) = 0 \end{aligned} \tag{2b}$$

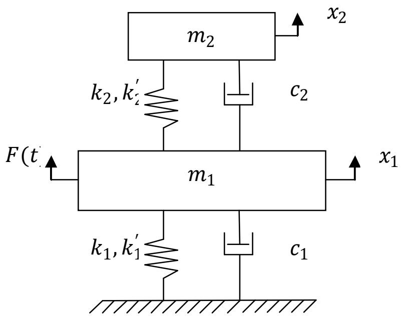


Figure 1. 2-DOF nonlinear system with nonlinear absorber

TABLE 1. Parameter values of the 2-DOF system

Symbol	Variable	Value	Unit
m_1	Mass of the nonlinear main system	10	kg
m_2	Absorber mass	0.8	kg
k_1	Linear stiffness of the main system	44	N/m
k_1'	Nonlinear stiffness of the main system	8	N/m^3
k_2	Linear absorber stiffness	2	N/m
k_2'	Nonlinear absorber stiffness	0.5	N/m^3
c_1	Linear damping constant of the main system	0.8	Ns/m
c_2	Linear absorber damping constant	0.08	Ns/m

where

$$\omega_1^2 = \frac{k_1 + k_2}{m_1}, \quad \omega_2^2 = \frac{k_2}{m_2}, \quad \alpha_1 = \frac{k_1'}{m_1},$$
$$\alpha_2 = \frac{k_2}{m_1}, \quad \alpha_3 = \frac{k_2'}{m_1}, \quad \xi_1 = \frac{c_1}{m_1},$$
$$\xi_2 = \frac{c_2}{m_1}, \quad \xi_3 = \frac{c_2}{m_2}, \quad \beta_1 = \frac{k_2'}{m_2},$$
$$f_0 = \sum_{j=1}^{n} f_j, \quad f = \frac{f_0}{m_1} \tag{3}$$

and ε is introduced to show weakness of a parameter. Applying multiple time scale (MTS) method, a set of four equations is obtained which can be used to determine the amplitude and phase of the steady state solutions. Considering two terms expansion of x, the solution of Equation (2) can be expressed as [17]:

$$x_1(t;\varepsilon) = x_{10}(T_0, T_1) + \varepsilon x_{11}(T_0, T_1) + O(\varepsilon^2) \tag{4a}$$

$$x_2(t;\varepsilon) = x_{20}(T_0, T_1) + \varepsilon x_{21}(T_0, T_1) + O(\varepsilon^2) \tag{4b}$$

The first and second derivatives of x with respect to time are given by:

$$\frac{dx}{dt} = D_0 + \varepsilon D_1 \tag{5a}$$

$$\frac{d^2 x}{dt^2} = D_0^2 + 2\varepsilon D_0 D_1 \tag{5b}$$

and the time scales are:

$T_0 = t,\ T_1 = \varepsilon t$.

Substituting the approximate solution (4) into Equation (2) and equating the same coefficients of ε^0 and ε^1 to zero, yields:

$$\varepsilon^0 : (D_0^2 + \omega_1^2)x_{10} = 0 \tag{6a}$$

$$(D_0^2 + \omega_2^2)x_{20} = \omega_2^2 x_{10} \tag{6b}$$

$$\begin{aligned}\varepsilon^1 : (D_0^2 + \omega_1^2)x_{11} &= f\cos(\Omega T_0) - 2D_0 D_1 x_{10} \\ &- \xi_1 D_0 x_{10} - \alpha_1 x_{10}^3 + \alpha_2 x_{20} + \alpha_3 (x_{20} - x_{10})^3 \\ &- \xi_2 D_0 (x_{10} - x_{20})\end{aligned} \tag{6c}$$

$$\begin{aligned}(D_0^2 + \omega_2^2)x_{21} &= -2D_0 D_1 x_{20} + \omega_2^2 x_{11} \\ &- \beta_1 (x_{20} - x_{10})^3 - \xi_3 D_0 (x_{20} - x_{10})\end{aligned} \tag{6d}$$

Eliminating the secular terms in the solution of Equation (6), the following set of equations is obtained:

$$\begin{cases} a\eta' = (g_1 - \sigma)a + g_2 a^3 + g_3 a b^2 - e\cos(\eta) \\ a' = g_4 a - e\sin(\eta) \\ b\gamma' = n_1 b + n_2 b^3 + n_3 a^2 b \\ b' = n_4 b \end{cases} \tag{7}$$

where $\eta = \theta - \sigma T_1$, and a and θ are amplitude and phase of the main system, respectively, and b and γ amplitude and phase of the absorber. Moreover,

$$\begin{aligned}
&e = \frac{f}{2\omega_1}, \quad g_1 = -\frac{1}{2\omega_1}\alpha_2 \Gamma_1, \\
&g_2 = 3\frac{1}{8\omega_1}(\alpha_1 - \alpha_3(\Gamma_1 - 1)^3), \\
&g_3 = -3\frac{1}{4\omega_1}\alpha_3(\Gamma_1 - 1), \\
&g_4 = \frac{1}{2}(-\xi_1 + \xi_2(\Gamma_1 - 1)), \\
&\Gamma_1 = \frac{\omega_2^2}{\omega_2^2 - \omega_1^2}, \quad \Gamma_2 = \frac{1}{\omega_1^2 - \omega_2^2}, \\
&n_1 = -\frac{1}{2}\alpha_2 \Gamma_2 \omega_2, \quad n_2 = -3\alpha_3 \frac{1}{8}\omega_2 \Gamma_2 + 3\beta_1 \frac{1}{8\omega_2}, \\
&n_3 = -3\alpha_3 \frac{1}{4}(\Gamma_1 - 1)^2 \omega_2 \Gamma_2 + 3\beta_1 \frac{1}{4\omega_2}(\Gamma_1 - 1)^2 \\
&n_4 = \frac{1}{2}(\xi_2 \omega_2^2 \Gamma_2 - \xi_3)
\end{aligned} \tag{8}$$

2. 1. 1. Frequency-response Equation At the fixed points of Equation (7) that lead to the steady state solution of the main system, the phase and amplitude are constant, therefore,

$$a' = b' = 0, \quad \eta' = \gamma' = 0 \tag{9}$$

Then, from Equation (7) one can write:

$$b' = n_4 b \Rightarrow b = 0 \tag{10}$$

Among the three possible cases, $i) a \neq 0, b = 0$ $ii) a = 0, b \neq 0$ ideal case and $iii) a \neq 0, b \neq 0$ real case, only the first one can happen. Considering *b=0*, the frequency-response equation can be expressed as:

$$(g_1 - \sigma + g_2 a^2)^2 a^2 + g_4^2 a^2 = e^2 \tag{11}$$

2. 1. 2. Stability Analysis Knowing the eigenvalues of the Jacobin matrix of Equation (7), here, the stability of the system under resonances is studied using the method of Andronov and Vitt [17]. Consider the following relations:

$$\begin{cases} a = a_0 + a_1 \\ \eta = \eta_0 + \eta_1 \\ b = b_0 + b_1 \end{cases} \tag{12}$$

where a_0, η_0 and b_0 are steady state solutions of Equation (7) and a_1, η_1 and b_1 small perturbations. Substitution of Equation (12) into Equation (7) leads to the following matrix equation:

$$\begin{bmatrix} a_1' \\ \eta_1' \\ b_1' \end{bmatrix} = J \begin{bmatrix} a_1 \\ \eta_1 \\ b_1 \end{bmatrix} \tag{13}$$

where:

$$J = \begin{bmatrix} g_4 & -(g_1 - \sigma)a_0 - g_2 a_0^3 & 0 \\ \frac{(g_1 - \sigma)}{a_0} + 3 g_2 a_0 & g_4 & 0 \\ 0 & 0 & n_4 \end{bmatrix} \tag{14}$$

The eigenvalues of J are computed as:

$$\lambda_{1,2} = g_4 \pm \sqrt{-(3 g_2 a_0^2 + g_1 - \sigma)(g_1 - \sigma + g_2 a_0^2)}$$
$$\lambda_3 = n_4 \tag{15}$$

In the next section, the eigenvalues are evaluated by specifying the force excitation values.

2. 1. 3. Saddle-node Bifurcation At the saddle-nodes, the tangency of the frequency-response curve is vertical; therefore, these points can be determined by differentiating the frequency-response equation (11) with respect to a^2 and considering $d\sigma/da^2 = 0$. The saddle-node points occur at points with the detuning parameters:

$$\sigma_{\pm} = g_1 + 2 g_2 a^2 \pm \sqrt{g_2^2 a^4 - g_4^2} \tag{16}$$

2. 1. 4. Attenuation and Desensitization Ratios To find the best values of the absorber parameters, two ratios namely, attenuation and desensitization ratios are introduced by [10, 11].

By definition, attenuation ratio is given by:

$$R = \frac{a_p}{a_{0p}} \tag{17}$$

where a_p and a_{0p} are the peak amplitudes of the main system with and without the absorber, respectively. To find the peak amplitude of the system without absorber, we should obtain the frequency-response of the system without absorber by eliminating parameters related to the nonlinear terms of the absorber in Equation (11). Desensitization ratio is expressed as:

$$E = \frac{e_{crit}}{e_{0crit}} \tag{18}$$

where e_{crit} and e_{0crit} are the critical excitation amplitudes of the system for the occurrence of saddle-node bifurcation with and without absorber, respectively, that may be found from Equation (16). By eliminating the nonlinear terms related to the absorber from Equation (16), e_{0crit} will be obtained. Noting these definitions, it is clear that for an oscillatory system, an absorber with a small *R* and a large *E* works better. The variations of *R* and *E* with k_2' are discussed in the next section and the best values of k_2' are found.

2. 2. Super-harmonic Resonance For the super-harmonic resonance, i.e., $3\Omega \cong \omega_1 + \varepsilon\sigma$, a hard force excitation is assumed. Using MTS method, eliminating the secular terms and with some manipulations, one obtains the frequency-response equation as:

$$h_1^2 a^2 + (h_2 - \sigma + h_3 a^2) a^2 = q^2 \tag{19}$$

where

$$q = -\frac{F^3}{\omega_1}(-\alpha_1 + \alpha_3(\Gamma_3 - 1)^3) \qquad F = \frac{f}{2(\omega_1^2 - \Omega^2)}$$
$$h_1 = -\frac{1}{2}\xi_1 + \frac{1}{2}\xi_2(\Gamma_1 - 1)$$
$$h_2 = \frac{3}{\omega_1}\alpha_1 F^2 - \frac{1}{2\omega_1}\alpha_2 \Gamma_1 - \frac{3}{\omega_1}\alpha_3 F^2((\Gamma_3 - 1)^2(\Gamma_1 - 1)) \tag{20}$$
$$h_3 = \frac{3}{8\omega_1}(\alpha_1 - \alpha_3(\Gamma_1 - 1)^3)$$
$$\Gamma_1 = \frac{\omega_2^2}{\omega_2^2 - \omega_1^2}, \qquad \Gamma_3 = \frac{\omega_2^2}{\omega_2^2 - \Omega^2}$$

By applying the method of Andronov and Vitt, the eigenvalues for the super-harmonic resonance can be obtained as:

$$\lambda_{1,2} = h_1 \pm\sqrt{-(h_3a^2 + h_2 - \sigma)(3h_3a^2 + h_2 - \sigma)}$$
$$\lambda_3 = \frac{1}{2}\omega_2^2\Gamma_3\xi_2 - \frac{1}{2}\xi_3 \qquad (21)$$

The saddle-node points occur at:

$$\sigma_{\pm} = h_2 + 2h_3a^2 \pm \sqrt{h_3^2a^4 - h_1^2} \qquad (22)$$

3. NUMERICAL EXAMPLE

For the primary resonant case, in order to plot the frequency-response curve, Equation (11) can be used to obtain σ in terms of *a* as:

$$\sigma_{\pm} = g_1 + g_2a^2 \pm \sqrt{\frac{e^2}{a^2} - g_4^2} \qquad (23)$$

To achieve convergent responses, the suitable amplitude of the force excitation should be determined [1]. The detuning parameter related to the peak amplitude, a_p, can be obtained from Equation (23) as $\sigma = g_1 + g_2a_p^2$. Knowing that jump phenomenon does not occur at the detuning parameter related to this point therefore, the detuning parameter should be equal to the detuning parameter obtained from Equation (16), i.e., $\sigma = g_1 + 2g_2a^2$, to get the suitable force amplitude.

For this system with the nonlinear absorber under the primary resonance, the applied forces for convergent response is found to be f_0= 0.2106(N) and f_0=0.0537 (N), respectively for with and without the absorber.

In Equation (16), if $g_2^2a^4 - g_4^2 > 0$, there is an interval $\sigma_- < \sigma < \sigma_+$ that three responses exist for the amplitude of the system. So, by assuming f_0= 0.0537(N), if the maximum amplitude is $a_{max}>0.2497m$, an interval exists for three responses. When the system is with the absorber, $a_{max}=0.0909m$ that occurs at $\sigma = 0.0574\,rad/s$ whereas, without the absorber $a_{max}=0.2560m$ which is larger than $0.2497m$. Therefore, by applying f_0=0.0537(N) without the absorber, at the interval $0.00938 < \sigma < 0.0108$ the system has two stable and one unstable responses but with the absorber, the response of the system is stable.

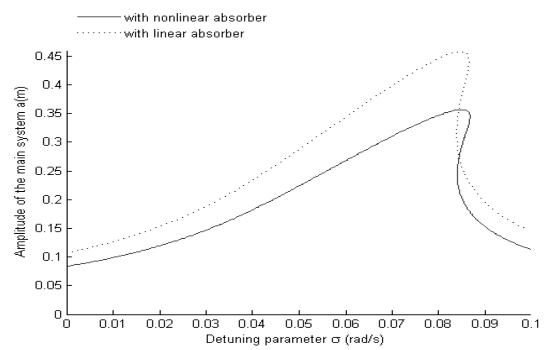
The eigenvalues of the system without the absorber under the force f_0=0.0537(N) are listed in Table 2. It can be seen that at a typical point with $\sigma = 0.0101\,rad/s$ which is located in the unstable region, three eigenvalues exist; the first and the third one are related to the stable response and the sign of the real part of the second one which is unstable changes.

TABLE 2. Eigenvalues of the system under primary resonance without the absorber

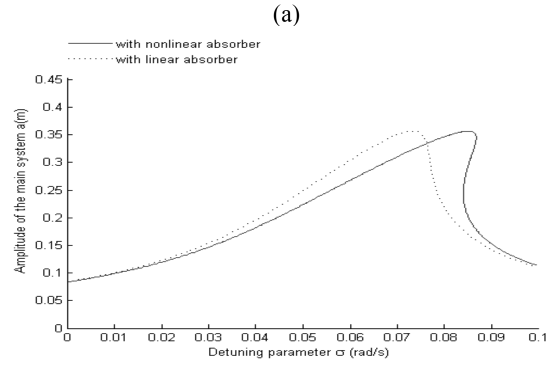
σ(rad / s)	λ_1	λ_2
0	-0.0050+ 0.0083i	-0.0050- 0.0083i
0.005	-0.0050+ 0.0066i	-0.0050+ 0.0066i
0.0101	$-0.0001 + 10^{-10}$ i	$-0.001 - 10^{-10}$ i
	$0.0001 - 10^{-10}$ i	$-0.0101 + 10^{-10}$ i
	-0.0050-0.0014i	-0.0050+0.0014i
0.02	-0.0050 - 0.0188i	-0.0050 + 0.0188i

TABLE 3. Eigenvalues of the system under primary resonance with the absorber

σ(rad / s)	λ_1	λ_2	λ_3
0	-0.0137 + 0.0557i	-0.0137 - 0.0557i	-0.0452
0.005	-0.0137 + 0.0507i	-0.0137 - 0.0507i	-0.0452
0.01	-0.0137 + 0.0458i	-0.0137 - 0.0458i	-0.0452
0.03	-0.0137 + 0.0263i	-0.0137 - 0.0263i	-0.0452
0.05	-0.0137 + 0.0084i	-0.0137 - 0.0084i	-0.0452
0.07	-0.0137+ 0.0125i	-0.0137- 0.0125i	-0.0452
0.09	-0.0137-0.0339i	-0.0137+0.0339i	-0.0452



(a)



(b)

Figure 3. Amplitude of the main system with the linear and nonlinear absorbers in the primary resonant case, (a) with different applied forces, (b) with the same force.

For the same force and detuning parameter values, the eigenvalues of the system with the absorber are shown in Table 3. It can be seen that by adding the absorber, all the eigenvalues of the system have negative real parts, hence; they are related to the stable responses. For the super-harmonic resonance, from Equation (19) one obtains:

$$\sigma_{\pm} = h_2 + h_3 a^2 \pm \sqrt{\frac{q^2}{a^2} - h_1^2} \tag{24}$$

The suitable force for this case is found as f_0=11.8596(N) without the absorber, and f_0=19.3824(N) when the absorber is attached to the system.

In Figure (2a, b), the frequency-response curve is plotted using Equation (24) with f_0=11.8596(N). In the interval $0.0093 < \sigma < 0.0108$, without the absorber, the responses are unstable, whereas, with the absorber the responses are stable. Figure (2a, b) shows the behavior of the oscillatory system before and after adding the nonlinear absorber for the primary and super-harmonic resonances. The amplitudes of the applied force are f_0=0.0537 (N) and f_0=11.8596(N) for the primary and super-harmonic resonances, respectively.

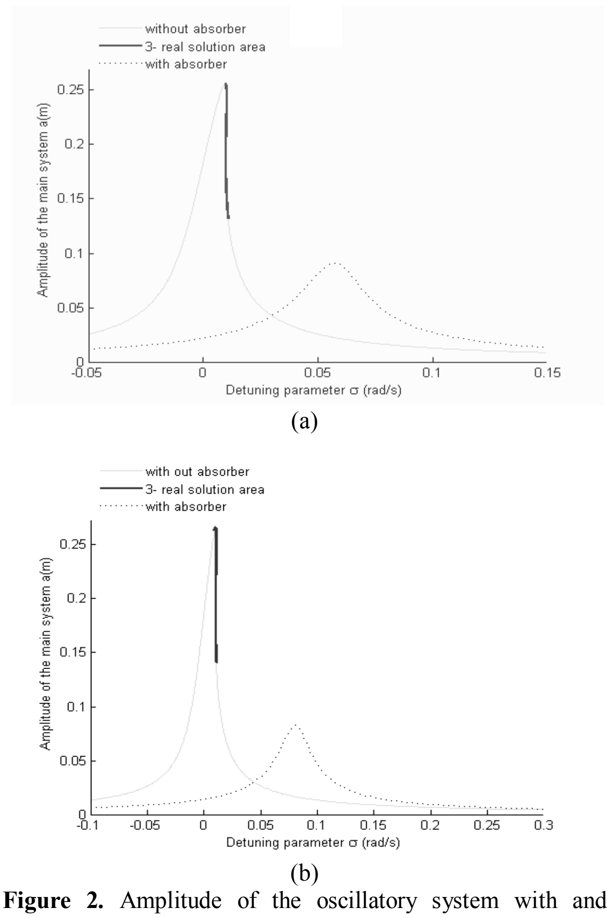


Figure 2. Amplitude of the oscillatory system with and without the absorber, (a) primary resonance, (b) super-harmonic resonance.

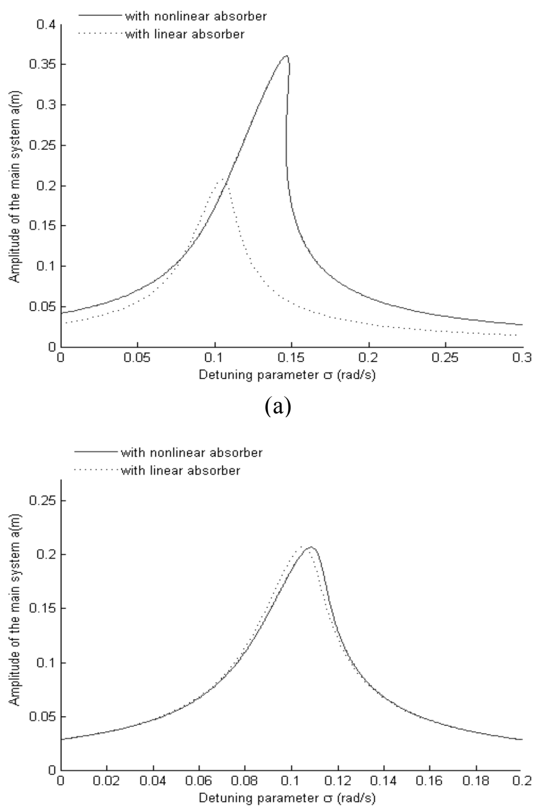
The thicker parts in the figure show the region with three responses. As can be seen, by adding the absorber, the peak amplitude of the oscillatory system reduces significantly near the resonance and will shift to a larger detuning parameter. For the primary resonance, behavior of the system with the linear and nonlinear absorbers is shown in Figure (3). It can be seen that by eliminating the nonlinear term of the absorber, the amplitude of the applied force can be larger, therefore, the peak amplitude of the main system increases, as well. In Figure (3a), the dotted line curve shows the frequency-amplitude response of the system with the linear absorber under f_0=0.2699 (N) and the solid line curve shows the response with the nonlinear absorber under f_0=0.2106 (N). For both linear and nonlinear absorbers, the system has an area with unstable response. The frequency-amplitude responses of the system with the linear and nonlinear absorbers under the same force f_0=0.2106(N) are shown in Figure (3b). The peak amplitudes are obtained as $a_{\max}$=0.3566 m where with the linear absorber it occurs at $\sigma = 0.0733\ rad/s$ and the response is stable in the interval of the detuning parameter $0 < \sigma < 0.1$. However, with the nonlinear absorber the response has an unstable region and the peak amplitude occurs at $= 0.0850\ rad/s$. Hence, it can be concluded that a linear absorber works better for the primary resonant case. Figure (4) shows the frequency-amplitude response in the super-harmonic resonant case with the linear and nonlinear absorbers. In Figure (4a), the dotted and the solid line curves are plotted for f0=16.1170 (N) and f0=19.3824 (N), for the linear and nonlinear absorbers, respectively. By eliminating the nonlinear term of the absorber, unlike the primary resonance case, the force that produces the peak amplitude decreases, and as a result, the peak amplitude of the oscillatory system reduces. In Figure (4b), the same force f0=16.1170 (N) is applied to the system with both the linear and nonlinear absorbers. The peak amplitude with the linear absorber is $a_{\max}$=0.2075m, which occurred at $\sigma = 0.1049\ rad/s$, and with the nonlinear absorber, $a_{\max}$=0.2073m at $\sigma = 0.1084 rad/s$ Moreover, with the linear absorber, the behavior of the system is like a linear system. To find the best value of the nonlinear parameter of the absorber, the attenuation and desensitization ratios will be used. By computing attenuation and desensitization ratios, it is found that for both primary and super-harmonic resonances, the attenuation ratio is independent of the nonlinear term of the absorber (k_2'). With the nonlinear absorber, the critical amplitude excitation is obtained from Equations (22 and 16), respectively, as:

$$e_{crit}\text{(for super-harmonic resonance)} = q_{crit} = \sqrt{2 h_1^3 / h_3}$$
$$e_{crit}\text{(for primary resonance)} = e_{crit} = \sqrt{2 g_4^3 / g_2} \tag{25}$$

By substituting the terms (h_1, h_3) and (g_4, g_2), it can be find that

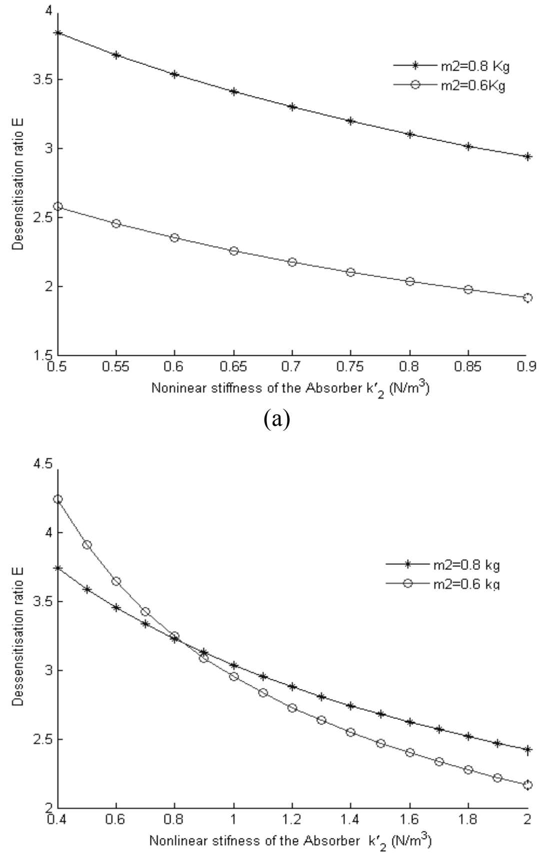
$$q_{crit} = e_{crit} = \sqrt{\left| \frac{(-\xi_1 + \xi_2(\Gamma_1 - 1))^3}{\frac{3}{8\omega_1}(\alpha_1 - \alpha_3(\Gamma_1 - 1)^3)} \right|} \tag{26}$$

According to Equation (26), the critical excitation amplitude of the system and subsequently, the system desensitization ratio can be affected by the nonlinear parameters of the 2-DOF system, k_1' and k_2'. Variations of the desensitization ratio with the nonlinear stiffness of the absorber for the primary and super-harmonic resonances are shown in Figure (5). It can be seen that for fixed damping constant and mass of the absorber, increase in the value of absorber nonlinear stiffness k_2' leads to a decrease in the desensitization ratio for both primary and super-harmonic resonances. It should be noted that to study the variations of R and E, the values of m_1, m_2, k_1 and k_2 should be taken such that the internal resonance may not occur; i.e., $\omega_1 \neq 3\omega_2, \frac{\omega_2}{3}, \omega_2$.



(b)

Figure 4. Amplitude of the main system with the linear and nonlinear absorbers in the super-harmonic resonant case: (a) with different applied forces, and (b) with the same applied force



(b)

Figure 5. Variations of the desensitization ratio with the nonlinear stiffness of the absorber for different absorber masses, (a) primary resonance, (b) super-harmonic resonance

4. CONCLUSION

By adding a linear or nonlinear absorber to a 1-DOF By adding a linear or nonlinear absorber to a 1-DOF nonlinear oscillatory system, the resonance amplitudes can be reduced. For a 2-DOF system consisting of the main system and the absorber under primary and super-harmonic resonances, the frequency-response equation at steady state was obtained for $a \neq 0, b = 0$. The results show that:

a) In comparison to the system without absorber, adding the nonlinear absorber reduces the amplitude of the system. Moreover, the peak amplitude occurs at a larger detuning parameter and the response of the system with the absorber is stable. By setting the nonlinear term in Equations (11, 19) to zero, the same results as in Refs. [10] and [11] shall be reached.
b) In the primary resonant case, if the absorber is nonlinear, the smaller applied force leads to the system to have saddle-nodes at a smaller detuning

parameter and the peak amplitude is smaller than that with a linear absorber. It means that by using a linear absorber we need a larger exciting force for the system to reach the instability region where the saddle- nodes occur.

c) Under the primary resonance, by applying the same force, the system with the linear absorber has linear behavior and the peak amplitude takes place in a smaller detuning parameter compare with the case with the nonlinear absorber. This is in good agreement with Ref. [15] for a linear main system.
d) In the super-harmonic resonant case, the system with a linear absorber should be excited by a smaller force, so the peak amplitude is smaller. However, in order to have saddle-nodes and the 3-real solution area for the nonlinear absorber, the system should be excited by a larger force.
e) Under the super-harmonic resonance, by applying the same force, the peak amplitude of the oscillatory system with nonlinear absorber is slightly smaller than that with the linear absorber and it takes place at a larger detuning parameter which is in good agreement with the results of Ref. [15] for a linear main system.
f) For both primary and super-harmonic resonances, the attenuation ratio is independent of the nonlinear term of the absorber, k_2'.
g) The desensitization ratio is influenced by the nonlinear term of the absorber, k_2'. By increasing k_2', the desensitization ratio reduces in both primary and super-harmonic resonant cases.

6. REFERENCES

1. El-Bassiouny, A. and Eissa, M., "Dynamics of a single-degree-of-freedom structure with quadratic, cubic and quartic non-linearities to a harmonic resonance", ***Applied Mathematics and Computation***, Vol. 139, No. 1, (2003), 1-21.
2. Pai, P., Rommel, B. and Schulz, M. J., "Non-linear vibration absorbers using higher order internal resonances", ***Journal of Sound and Vibration***, Vol. 234, No. 5, (2000), 799-817.
3. Leung, A. Y., Ji, J. C. and Chen, G., "Resonance control for a forced single-degree-of-freedom nonlinear system", ***International Journal of Bifurcation and Chaos***, Vol. 14, No. 04, (2004), 1423-1429.
4. Chatterjee, S., "Vibration control by recursive time-delayed acceleration feedback", ***Journal of Sound and Vibration***, Vol. 317, No. 1, (2008), 67-90.
5. Nahvi, H., "The response of two-degree-of-freedom self-sustained systems with quadratic nonlinearities to a parametric excitation", ***International Journal of Engineering***, Vol. 14, No. 3, (2001), 255-262.
6. Amer, Y. and El-Sayed, A., "Vibration suppression of non-linear system via non-linear absorber", ***Communications in Nonlinear Science and Numerical Simulation***, Vol. 13, No. 9, (2008), 1948-1963.
7. Zhu, S., Zheng, Y. and Fu, Y., "Analysis of non-linear dynamics of a two-degree-of-freedom vibration system with non-linear damping and non-linear spring", ***Journal of Sound and Vibration***, Vol. 271, No. 1, (2004), 15-24.
8. Liu, K. and Liu, J., "The damped dynamic vibration absorbers: Revisited and new result", ***Journal of Sound and Vibration***, Vol. 284, No. 3, (2005), 1181-1189.
9. Viguié, R. and Kerschen, G., "Nonlinear vibration absorber coupled to a nonlinear primary system: A tuning methodology", ***Journal of Sound and Vibration***, Vol. 326, No. 3, (2009), 780-793.
10. Ji, J. and Zhang, N., "Suppression of the primary resonance vibrations of a forced nonlinear system using a dynamic vibration absorber", ***Journal of Sound and Vibration***, Vol. 329, No. 11, (2010), 2044-2056.
11. Jinchen, J. and Nong, Z., "Suppression of super-harmonic resonance response using a linear vibration absorber", (2011).
12. Sayed, M. and Kamel, M., "1: 2 and 1: 3 internal resonance active absorber for non-linear vibrating system", ***Applied Mathematical Modelling***, Vol. 36, No. 1, (2012), 310-332.
13. H.A., E.-G. and El-Ganaini, W. A. A., "Vibration suppression of a dynamical system to multi- parametric excitation via time-delay absorber", ***Applied Mathematical Modeling***, Vol. 36, (2012), 35-45.
14. Febbo, M. and Machado, S., "Nonlinear dynamic vibration absorbers with a saturation", ***Journal of Sound and Vibration***, (2012).
15. Hsu, Y.-S., Ferguson, N. S. and Brennan, M. J., The experimental performance of a nonlinear dynamic vibration absorber, in Topics in nonlinear dynamics, Springer, Vol. 1. (2013), 247-257.
16. Rao, S. S., "Mechanical vibration", 4th ed ed, New Jersey, Person Prentice Hall, (2001).
17. Nayfeh, A. and Mook, D., "Nonlinear oscillations. ", ***John Willey and Sons, New York***, (1979).

Vibration Analysis of a Nonlinear System with a Nonlinear Absorber under the Primary and Super-harmonic Resonances

TECHNICAL NOTE

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چکیده

در سیستم‌های ارتعاشی، به منظور جلوگیری از تشدیدهای اولیه و سوپرهارمونیک، می‌توان ازجاذب‌های دینامیکی خطی یا غیرخطی استفاده کرد. در این مقاله، رفتار یک سیستم غیرخطی به همراه یک جاذب غیرخطی، تحت تشدیدهای اولیه و سوپرهارمونیک، بررسی شده است. مقایسه‌ی بین اثرات یک جاذب خطی و یک جاذب غیرخطی بر روی رفتار یک سیستم غیرخطی تحت تشدیدهای فوق انجام گرفته است. سختی سیستم اصلی و جاذب غیرخطی از نوع درجه‌ی سه است و میراکننده‌ها خطی هستند. برای به دست آوردن حل تقریبی معالات غیرخطی حرکت، از روش مقیاس زمانی متعدد - استفاده شده است. نتایج نشان می‌دهند که در حالت تشدید اولیه، در کاهش ماکزیمم دامنه‌ی سیستم، یک جاذب خطی عملکرد بهتری نسبت به جاذب غیرخطی دارد. اما تحت تشدید سوپرهارمونیک، با اضافه کردن یک جاذب غیرخطی به سیستم ارتعاشی، دامنه‌ی ارتعاش نسبت به حالتی که جاذب خطی است، کاهش بیشتری خواهد داشت.

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