IJE TRANSACTIONS C: Aspects Vol. 27, No. 3, (March 2014) 385-394

International Journal of Engineering

Journal Homepage: www.ije.ir

Semi-analytical Approach for Free Vibration Analysis of Variable Cross-section Beams Resting on Elastic Foundation and under Axial Force

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PAPER INFO

Paper history:
Received 30 June 2013
Received in revised form 05 September 2013
Accepted 14 September 2013

Keywords:
Variable Cross-Section Beam
Elastic Foundation
Axial Force
Differential Transform Method

ABSTRACT

In this paper, free vibration of an Euler-Bernoulli beam with variable cross-section resting on elastic foundation and under axial tensile force is considered. Beam's constant height and exponentially varying width yields variable cross-section. The problem is handled for three different boundary conditions: clamped-clamped, simply supported-simply supported and clamp-free beams. First, the equation of motion that governs the free vibration is derived and then dimensionless frequencies are determined using differential transform method (DTM). DTM is a semi-analytical approach based on Taylor expansion series that is a powerful tool in solution ordinary and partial differential equations. The effects of axial force, elastic foundation coefficient and non-uniformity parameter on dimensionless frequencies are investigated. Wherever possible, comparisons are made with the studies in open literature. Results show, the DTM yields rapid convergence without any frequency missing although convergence rate depends on boundary conditions. Also, dimensionless frequencies are sensitive to axial force rather than other parameters.

doi: *10.5829/idosi.ije.2014.27.03c.05*

1. INTRODUCTION

Nowadays, non-uniform beams due to the wide applications in modern engineering such as turbine blade, helicopter blades, satellites structure, even robotic arms, and architectural applications are too considerable. It is clear that applications like above are because of achieving a better distribution of strength and weight in comparing with uniform beams. Therefore many researchers deal with analysis of dynamic behavior of non-uniform or variable cross-section beams. Cranch and Adler [1] presented the close form solutions for free vibration of non-uniform beams with four kinds of rectangular cross-section. Abrate [2] found that the equation of motion of a non-uniform beam may be transformed into that of a uniform beam. He presented simple formulas for predicting the fundamental frequency of non-uniform beams with various boundary conditions. Laura et al. [3] determined the natural frequencies of beams with constant width and bi-linearly varying thickness using different approaches and showed the optimized Rayligh-Ritz method has better accuracy than the differential quadrature method. Auciello [4] obtained lower and upper bounds on the free vibration frequencies of tapered slender beams, Rayligh-Ritz method was used for upper bounds and structure reduced to rigid bars with elastic hinges to lower bounds. Ece et al. [5] derived exact solution for free vibration of beams with constant height and exponentially varying width for three different types of boundary conditions associated with hinged, clamped and free ends. Firouz-Abadi et al. [6] utilized equation of motion of variable cross-section beam to obtain a singular differential equation in terms of the natural frequency of vibration and applied Wentzel, Kramers, Brillouin (WKB) expansion series to find solution. Nikkhah Bahrami et al. [7] used modified wave approach to calculate natural frequencies and mode shapes of arbitrary non-uniform beams.

Structural elements that are represented as a beam when resting on elastic foundation have wide application in numerous aspects of engineering. The well-known models for elastic foundations are Winkler, Pasternak and Vlasov. The Winkler model of elastic

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foundation is the most preliminary one in which the vertical displacement is assumed to be proportional to the contact pressure at an arbitrary point [8]. The Winkler model has been used for different problems of buckling and vibration of beams and plates that rest on elastic foundation [9-13].

The differential transform is a semi-analytic and powerful method for solving linear and nonlinear differential equations. This method was first used in the engineering domain by Zhou [14] to analyze electric circuits. This method was first used in structural dynamics by Malik and Dang [15], then different types of problem in structural dynamics were handled by differential transform method (DTM). For example, free vibration analysis of a centrifugally stiffened Timoshenko beam [16], vibration of an elastic beam supported on elastic soil [17], free vibration analysis of circular thin plates [18], vibration analysis of composite sandwich beams with viscoelastic core [19], vibration and modal stress analyses of circular plates made of two-directional functionally graded materials resting on elastic foundations [20], response of forced Euler-Bernoulli beams [21], buckling analysis of partially embedded pile in elastic soil [22], free vibration analysis of Timoshenko beams resting on two-parameter elastic foundation [23] and so on [24-28]. Beside applications in structural dynamics, DTM has wide applications in fluid flow and heat transfer problems [29] and nonlinear oscillators problems [30].

In this study, free vibration of variable cross-section beams resting on elastic foundation and under axial tensile force with different types of boundary conditions is considered. Euler-Bernoulli beam theory has been used. Constant height and exponentially varying in width is considered as non-uniformity in cross-section. This type of non-uniform cross-section has been considered for free vibration of beams without elastic foundation and axial force [5, 31] and cantilever beams with tip mass [32]. After deriving equation of motion, a semi-analytical technique called differential transform method is used to determine dimensionless frequencies and investigation on effect of different parameters on behavior of such beams. Also, for the first time, effects of boundary conditions, elastic foundation and axial force on convergence rate of DTM is investigated.

2. THE MATHEMATICAL MODEL AND FORMULATION

Consider a beam with exponentially varying width and constant height that rests on elastic foundation undergoing transverse vibration under axial tensile force as shown in Figure 1. The width of beam (b) varies over length coordinates as follow:

$$b(x) = b_0 e^{-\sigma x}, \tag{1}$$

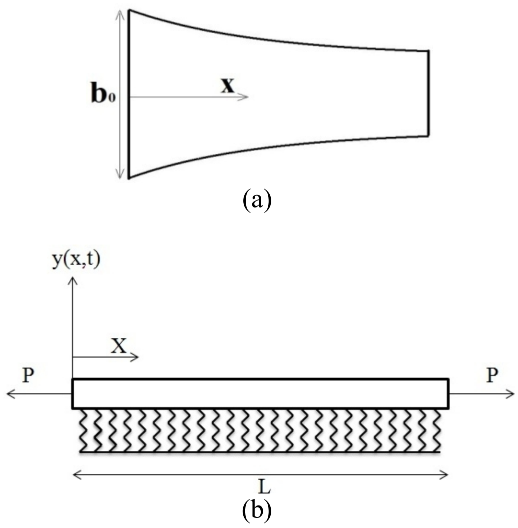

Figure 1. (a) Top view of beam with exponentially varying width and (b) lateral view of beam resting on elastic foundation and under axial tensile force with constant height and non-uniform width.

Here b_0 is width in the left end of beam and σ is the non-uniformity parameter. When width of beam varies over the beam length as Equation (1), then cross-section and moment of inertia varies as follow:

$$A(x) = A_0 e^{-\sigma x}, \tag{2}$$

$$I(x) = I_0 e^{-\sigma x}, \tag{3}$$

A_0 and I_0 are cross-section and moment of inertia in the left end of beam, respectively. Rao [33] governed the equation of motion for an Euler-Bernoulli beam with arbitrary cross-section resting on elastic foundation and under axial tensile force as follows:

$$\frac{\partial^2}{\partial x^2}(EI(x)\frac{\partial^2 y(x,t)}{\partial x^2}) - \frac{\partial}{\partial x}(p\frac{\partial y(x,t)}{\partial x}) + \rho A(x)\frac{\partial^2 y(x,t)}{\partial t^2} + ky(x,t) = 0, 0 \le x \le L \tag{4}$$

where $y(x,t)$ represents transverse displacement of the beam, E is modulus of elasticity, $I(x)$ is moment of inertia, p is the axial tensile force, $A(x)$ is cross-section of the beam, k is elastic coefficient of foundation, L is the length of the beam and x is coordinate along the longitudinal axis.

The relevant boundary conditions associated with Equation (4) are:

Clamped-clamped (C-C):

$$y(0) = y(L) = \frac{\partial y}{\partial x}(0) = \frac{\partial y}{\partial x}(L) = 0, \tag{5}$$

Simply supported –simply supported (S-S):

$$y(0) = y(L) = \frac{\partial^2 y}{\partial x^2}(0) = \frac{\partial^2 y}{\partial x^2}(L) = 0, \tag{6}$$

Clamped-free (C-F):

$$y(0) = \frac{\partial y}{\partial x}(0) = \frac{\partial^2 y}{\partial x^2}(L) = \frac{\partial^3 y}{\partial x^3}(L) = 0, \tag{7}$$

Assuming that the motion is harmonic we consider $y(x,t)$ as follow:

$$y(x,t) = \bar{y}(x).e^{i\Omega t} \tag{8}$$

Substituting Equations (2), (3) and (8) into Equation (4) yields the following equation of motion:

$$EI_0 e^{-\sigma x}\frac{\partial^4 \bar{y}}{\partial x^4} - 2\sigma EI_0 e^{-\sigma x}\frac{\partial^3 \bar{y}}{\partial x^3} + \sigma^2 EI_0 e^{-\sigma x}\frac{\partial^2 \bar{y}}{\partial x^2} - p\frac{\partial^2 \bar{y}}{\partial x^2} - \rho A_0 e^{-\sigma x}\Omega^2 \bar{y} + k\bar{y} = 0 \tag{9}$$

Defining a non-dimensional coordinate $\xi = x/L$, the equation of motion is obtained in non-dimensional form as:

$$\frac{\partial^4 \tilde{y}}{\partial \xi^4} - 2\delta\frac{\partial^3 \tilde{y}}{\partial \xi^3} + (\delta^2 - Pe^{\delta\xi})\frac{\partial^2 \tilde{y}}{\partial \xi^2} + (Ke^{\delta\xi} - \omega^2)\tilde{y} = 0, 0 < \xi < 1 \tag{10}$$

where other parameters in Equation (10) are:

$$\tilde{y} = \frac{\bar{y}}{L}, \quad \delta = \sigma L, \quad P = \frac{pL^2}{EI_0}, \quad K = \frac{kL^4}{EI_0}, \quad \omega^2 = \frac{\rho A_0 \Omega^2 L^4}{EI_0}. \tag{11}$$

Additionally, dimensionless boundary conditions can be written as follows:
Clamped-clamped (C-C):

$$\tilde{y}(0) = \tilde{y}(1) = \frac{\partial \tilde{y}}{\partial \xi}(0) = \frac{\partial \tilde{y}}{\partial \xi}(1) = 0, \tag{12}$$

Simply supported –simply supported (S-S):

$$\tilde{y}(0) = \tilde{y}(1) = \frac{\partial^2 \tilde{y}}{\partial \xi^2}(0) = \frac{\partial^2 \tilde{y}}{\partial \xi^2}(1) = 0, \tag{13}$$

Clamped-free (C-F):

$$\tilde{y}(0) = \frac{\partial \tilde{y}}{\partial \xi}(0) = \frac{\partial^2 \tilde{y}}{\partial \xi^2}(1) = \frac{\partial^3 \tilde{y}}{\partial \xi^3}(1) = 0. \tag{14}$$

3. THE DIFFERENTIAL TRANSFORM METHOD (DTM)

Differential transform method is an efficient semi-analytic approach for solving ordinary and partial differential equation that use the form of polynomials as the approximations to the exact solutions that are sufficiently differentiable. It is different from high-order Taylor series expansions because Taylor series expansions requires symbolic computation of the necessary derivatives of the data functions and is expensive for large orders [14-30].

The conceptual feature of the DTM is to transform the governing differential equations and boundary conditions as well as continuity conditions into a set of algebraic equations using a transformation role. Solving the algebraic equations in the usual way leads to accurate results with fast convergence rate and small computational effort.

A function $x(t)$, analytical in domain D, can be represented by a power series around any arbitrary point in this domain. Differential transform of a function $x(t)$ is defined as follows:

$$X(k) = \frac{1}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=0}, \tag{15}$$

In Equation (15), $x(t)$ is the original function and $X(k)$ is the transformed function. Differential inverse transform of $X(k)$ is defined as:

$$x(t) = \sum_{k=0}^{\infty} X(k)t^k, \tag{16}$$

Combining Equation (15) and (16), we obtain the following equation:

$$x(t) = \sum_{k=0}^{\infty} \frac{t^k}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=0}, \tag{17}$$

In principal applications, the function $x(t)$ is shown by a finite numbers of terms and Equation (17) can be written as:

$$x(t) = \sum_{k=0}^{N} \frac{t^k}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=0}, \tag{18}$$

which implies that:

$$x(t) = \sum_{k=N+1}^{\infty} \frac{t^k}{k!}\left[\frac{d^k x(t)}{dt^k}\right]_{t=0}. \tag{19}$$

is negligibly small. In this study, the convergence of the natural frequencies determines the value of N. Basic transformation rules depending on the DTM for differential equations and boundary conditions are tabulated in Tables 1 and 2, respectively.

3. 1. Solution Procedure with DTM In the solution stage, the DTM is applied to the Equation (10) using the transformation rules given in Table 1 and the following recurrence relation is obtained:

$$\begin{aligned}&(n+4)!\,Y(n+4)-2\delta(n+3)!\,Y(n+3)\\&+\delta^2(n+2)!\,Y(n+2)-n!\,P\sum_{l=0}^{n}[\frac{(l+2)!}{l!}Y(l+2).\frac{\delta^{n-l}}{(n-l)!}]\\&+n!\,K\sum_{l=0}^{n}[Y(l).\frac{\delta^{n-l}}{(n-l)!}]-\omega^2 n!\,Y(n)=0\end{aligned} \quad (20)$$

In the follow, solution procedure is explained to clamped-clamped beam, similar procedure is followed for the other two boundary conditions; simply supported- simply supported and clamped –free beam.

Applying the DTM rules given in Table 2 to Equation (12), the left end boundary conditions of C-C beam is obtained as:

$$Y(0)=Y(1)=0 \quad (21)$$

In the left end of C-C beam, values of bending moment and shear force corresponding to $\frac{\partial^2\tilde{y}}{\partial\xi^2}(0)$ and $\frac{\partial^3\tilde{y}}{\partial\xi^3}(0)$, respectively, are unknown. Therefore, the transformation of these value express as follow:

$$Y(2)=a,Y(3)=b \quad (22)$$

where a and b are unknown parameters.

Equation (20) is a recursive equation, that from it for different values of n, $Y(n)$ can be determined in terms of a, b, δ, K, P and ω.

The right end of C-C beam corresponds to $\xi=1$, applying the DTM rules given in Table 2 yields:

$$\sum_{n=0}^{N}Y(n)=0, \quad (23)$$

$$\sum_{n=0}^{N}nY(n)=0\cdot \quad (24)$$

Substituting $Y(n)$ into Equations (23) and (24), following matrix expression is obtained:

$$\begin{bmatrix}Q_{11}(\omega) & Q_{12}(\omega)\\ Q_{21}(\omega) & Q_{22}(\omega)\end{bmatrix}.\begin{bmatrix}a\\ b\end{bmatrix}=\begin{bmatrix}0\\ 0\end{bmatrix}, \quad (25)$$

where a and b are constants and Q_{j1} and Q_{j2} ($j=1,2$) are ploynomials of ω that the coefficients of these polynomials are determined from values of δ, K and P. For the non-trivial solutions of Equation (25), it is necessary that the determinant of the coefficient matrix is equal to zero:

$$\begin{vmatrix}Q_{11}(\omega) & Q_{12}(\omega)\\ Q_{21}(\omega) & Q_{22}(\omega)\end{vmatrix}=0. \quad (26)$$

Solving Equation (26), the dimensionless frequencies are calculated. The jth estimated dimensionless frequency, $\omega_j^{(N)}$ corresponds to N and the value of N is determined by the following convergence criterion:

$$\left|\omega_j^{(N)}-\omega_j^{(N-1)}\right|<\varepsilon. \quad (27)$$

TABLE 1. Basic transformation rules of DTM for equations [30]

Original function	Transformed function
$f(x)=\alpha.g(x)$	$F(n)=\alpha.G(n)$
$f(x)=g(x)\pm h(x)$	$F(n)=G(n)\pm H(n)$
$f(x)=\frac{d^m g(x)}{dx^m}$	$F(n)=\frac{(n+m)!}{n!}G(n+m)$
$f(x)=g(x)h(x)$	$F(n)=\sum_{l=0}^{n}G(l)H(n-l)$
$f(x)=e^x$	$F(n)=\frac{1}{n!}$

TABLE 2. Basic transformation rules of DTM for boundary conditions [17]

Original boundary conditions	Transformed boundary conditions
$f(0)=0$	$F(0)=0$
$\frac{df(0)}{dx}=0$	$F(1)=0$
$\frac{d^2 f(0)}{dx^2}=0$	$F(2)=0$
$\frac{d^3 f(0)}{dx^3}=0$	$F(3)=0$
$f(1)=0$	$\sum_{n=0}^{N}F(n)=0$
$\frac{df(1)}{dx}=0$	$\sum_{n=0}^{N}nF(n)=0$
$\frac{d^2 f(1)}{dx^2}=0$	$\sum_{n=0}^{N}n(n-1)F(n)=0$
$\frac{d^3 f(1)}{dx^3}=0$	$\sum_{n=0}^{N}n(n-1)(n-2)F(n)=0$

where $\omega_j^{(N-1)}$ is the jth estimated dimensionless frequency corresponding to $N-1$ and ε is the tolerance parameter.

If Equation (27) is satisfied, the jth dimensionless frequency ($\omega_j^{(N)}$) is obtained. Required CPU time for calculation of dimensionless frequencies is directly dependent on the value of ε, when ε decreases, the required CPU time increases. Therefore, different values are considered for ε, for results presented in figures $\varepsilon = 0.002$ and for results presented in tables $\varepsilon = 0.00001$.

The explained procedure for C-C beam can be extended to the S-S and C-F beams. For S-S beam boundary conditions at the left end, using DTM rules is transformed as follows:

$$Y(0) = Y(2) = 0, \tag{28}$$

and for C-F beam is transformed as follows:

$$Y(0) = Y(1) = 0, \tag{29}$$

Also, applying DTM rules given in Table 2, the boundary conditions at the right end of S-S beam is transformed as follow:

$$\sum_{n=0}^{N} Y(n) = 0, \tag{30}$$

$$\sum_{n=0}^{N} n(n-1)Y(n) = 0, \tag{31}$$

and for C-F beam, boundary conditions at the right end is transformed as follow:

$$\sum_{n=0}^{N} n(n-1)Y(n) = 0, \tag{32}$$

$$\sum_{n=0}^{N} n(n-1)(n-2)Y(n) = 0 \cdot \tag{33}$$

4. RESULTS AND DISCUSSION

Procedure for determining dimensionless frequencies by DTM has been implemented in computer program using Matlab software. In order to verify accuracy of DTM, comparison between results obtained by DTM and exact solution [5] in the case of non-uniform beam without elastic foundation and axial force has been done in Table 3 for the first five dimensionless frequencies and for three different boundary conditions i.e. C-C, S-S and C-F. Comparison of results indicates the excellent accuracy of DTM.

In Figures 2-4, the convergence of first five dimensionless frequencies for $P = K = 0$ and $\delta = 1$ with respect to the number of terms considered is presented for C-C, S-S and C-F boundary conditions, respectively. It is observed from these figures that the DTM shows rapid convergence and stability in computation without any frequency missing. The accuracy of the method increases dramatically with the number of terms taken into consideration. Rate of convergence for C-F boundary condition is higher than two others kind of boundary conditions.

In Tables 4-9, effect of axial tensile force and elastic foundation coefficients on the first two dimensionless frequencies when δ is constant has been investigated. The results show, in constant value of the δ, dimensionless frequencies are more sensitive to axial force rather elastic foundation coefficient. First and second dimensionless frequencies in C-C boundary condition have same response to variation of elastic foundation coefficient, but in S-S and C-F boundary conditions first dimensionless frequency has significant sensitivity to elastic coefficient variation rather than second dimensionless frequency.

In Figures 5-7, effect of parameter, corresponding to non-uniformity in cross-section (δ) on the first three dimensionless frequencies has been investigated for C-C, S-S and C-F boundary conditions, respectively. Three special cases of elastic foundation and axial force values are considered in these figures. In the case A, $K = P = 0$, in the case B, $K = 500, P = 10$, and in the case C, $K = 10, P = 50$, are considered. It is observed from these figures that in the case B, beam with C-F boundary condition has different behaviors in the two first dimensionless frequencies, whereas two other boundary conditions have same behavior when δ is changed. It is clear, first dimensionless frequency of beam with C-F boundary condition has valuable sensitivity to δ when elastic foundation is greater than axial force.

In Table 10, for three special cases of elastic foundation and axial force values, required number of terms in DTM to reach $\varepsilon = 0.001$ in convergence criterion are tabulated. Since, effects of parameters on required number of terms for calculation of first and second frequencies is negligible, required number of terms for calculation of third and fourth frequencies are presented. Results show C-C boundary condition needs more number of terms rather than S-S and C-F boundary conditions. Also, when axial force increases, required number of terms in DTM increases to reach desired ε.

TABLE 3. Comparison of DTM with exact solution in the case of beams without elastic foundation and axial force

δ		C-C		S-S		C-F	
		Present	**[5]**	**Present**	**[5]**	**Present**	**[5]**
1	ω_1	22.51168	22.51167	9.77291	9.77291	4.73490	4.72298
	ω_2	61.85969	61.85968	39.57036	39.57036	24.20181	24.20168
	ω_3	121.10798	121.10799	88.97052	88.97052	63.86449	63.86448
	ω_4	200.07412	200.07411	158.08418	158.08418	123.09790	123.09790
	ω_5	298.77615	298.77661	246.92634	246.92650	202.06833	202.06876
2	ω_1	22.93772	22.93771	9.48725	9.48725	6.26264	6.25877
	ω_2	62.42273	62.42272	39.85231	39.85231	26.58359	26.58350
	ω_3	121.72273	121.72272	89.40520	89.40520	66.37449	66.37449
	ω_4	200.71859	200.71860	158.59689	158.59689	125.68471	125.68471
	ω_5	299.43096	299.44012	247.48601	247.48629	204.69504	204.69531

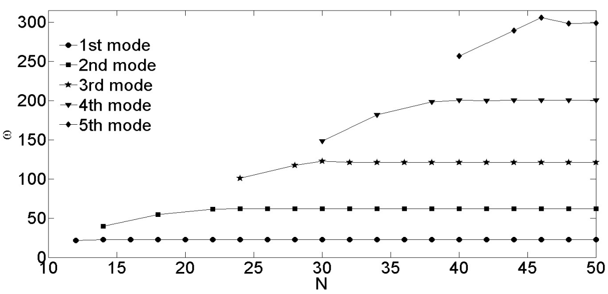

Figure 2. Convergence of dimensionless frequencies for C-C boundary condition ($P = K = 0, \delta = 1$)

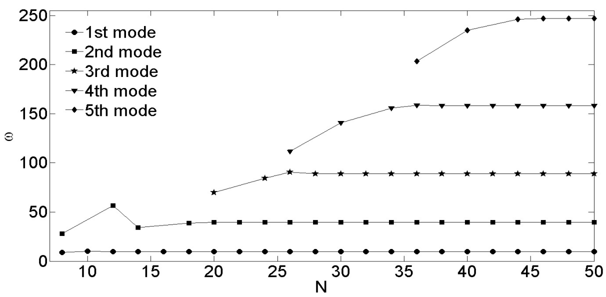

Figure 3. Convergence of dimensionless frequencies for S-S boundary condition ($P = K = 0, \delta = 1$)

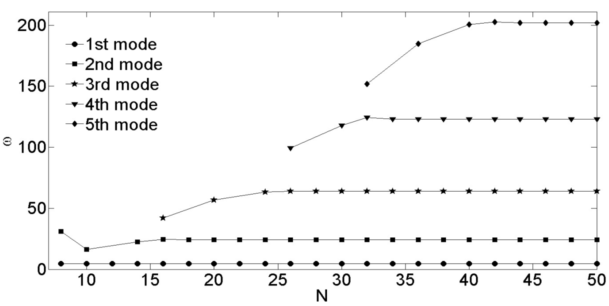

Figure 4. Convergence of dimensionless frequencies for C-F boundary condition ($P = K = 0, \delta = 1$)

TABLE 4. Effects of elastic foundation and axial force on first dimensionless frequency of C-C beam ($\delta = 0.5$)

P	K=0	K=50	K=100
0	22.4077111	23.8007532	25.1163792
20	28.5432877	29.6478479	30.7125262
40	33.4626373	34.4086608	35.3292263
60	37.673419	38.515488	39.3394305
80	41.4085593	42.1756351	42.9289204
100	44.7975795	45.5072019	46.2058559

TABLE 5. Effects of elastic foundation and axial force on second dimensionless frequency of C-C beam ($\delta = 0.5$)

P	K=0	K=50	K=100
0	61.7195079	62.2408934	62.7579278
20	70.6532331	71.1090426	71.56199
40	78.4967698	78.9070515	79.3152374
60	85.5488499	85.9247423	86.2990022
80	91.932558	92.2797888	92.6256715
100	97.5280812	97.8469308	98.1645879

TABLE 6. Effects of elastic foundation and axial force on first dimensionless frequency of S-S beam ($\delta = 0.5$)

P	K=0	K=50	K=100
0	9.84536167	12.6907833	15.0045044
20	18.7035776	20.3412689	21.8559106
40	24.5383684	25.8067144	27.015169
60	29.226896	30.2987457	31.3336709
80	33.259269	34.2044849	35.1240654
100	36.8521768	37.7071084	38.542919

TABLE 7. Effects of elastic foundation and axial force on second dimensionless frequency of S-S beam ($\delta = 0.5$)

P	K=0	K=50	K=100
0	39.5013091	40.3139375	41.1109098
20	50.7902139	51.4243339	52.0509159
40	59.9667104	60.5041402	61.0369469
60	67.9000716	68.374686	68.8461006
80	74.9905425	75.4201931	75.8474622
100	81.4633345	81.858839	82.2524848

TABLE 8. Effects of elastic foundation and axial force on first dimensionless frequency of C-F beam ($\delta = 0.5$)

P	K=0	K=50	K=100
0	4.08932796	9.56002449	12.8816721
20	1.95734337	8.98634193	12.5548989
40	1.06982322	8.89674306	12.5352566
60	0.629893134	8.89110061	12.5574896
80	0.390184444	8.90173763	12.5824923
100	0.251046312	8.91426918	12.6038883

TABLE 9. Effects of elastic foundation and axial force on second dimensionless frequency of C-F beam ($\delta = 0.5$)

P	K=0	K=50	K=100
0	39.5013091	40.3139375	41.1109098
20	50.7902139	51.4243339	52.0509159
40	59.9667104	60.5041402	61.0369469
60	67.9000716	68.374686	68.8461006
80	74.9905425	75.4201931	75.8474622
100	81.4633345	81.858839	82.2524848

TABLE 10. Required number of terms in DTM to reach $\varepsilon = 0.001$

δ	P	K	Number of modes	C-C	S-S	C-F
0.2	0	0	Third	38	35	32
			fourth	46	43	43
0.2	10	100	Third	40	35	33
			fourth	48	43	41
0.2	100	10	Third	47	41	44
			fourth	53	47	51

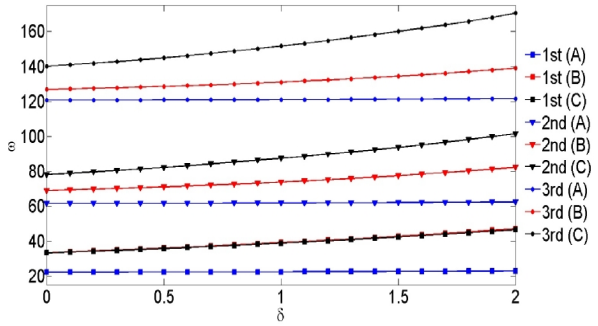

Figure 5. Effect of δ on first three dimensionless frequencies of C-C beam. $A: K = P = 0$, $B: K = 500, P = 10$ and $C: K = 10, P = 50$

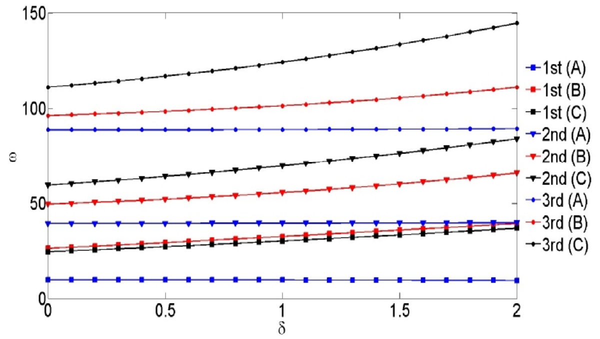

Figure 6. Effect of δ on first three dimensionless frequencies of S-S beam. $A: K = P = 0, B: K = 500, P = 10$ and $C: K = 10, P = 50$

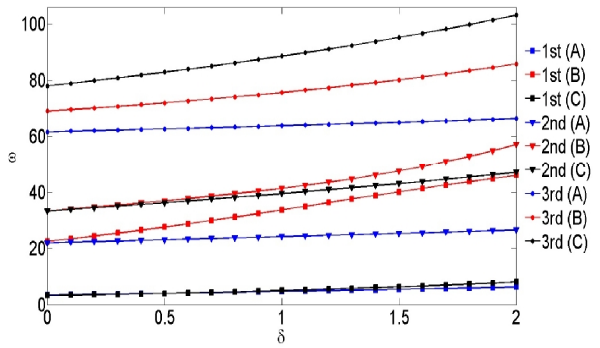

Figure 7. Effect of δ on first three dimensionless frequencies of C-F beam. $A: K = P = 0, B: K = 500, P = 10$ and $C: K = 10, P = 50$

5. CONCLUSION

In this study, free vibration of variable cross-section Euler-Bernoulli beam resting on elastic foundation and

under axial tensile force was studied by semi-analytical technique, namely differential transform method. The method possesses several benefits such as moderately simple procedure, stability in computations and high accuracy. Exponentially varying width of beam yields variable cross-section. Three boundary conditions are concerned: clamped-clamped, simply supported-simply supported and clamp-free beams. For some special cases comparison has made with exact solution in open literature. Effect of different parameters on dimensionless frequencies has been investigated.
The main conclusions are as follows:

1) Although DTM yields rapid convergence but convergence rate depends on boundary conditions.
2) For beams with exponentially varying in width, clamped –free boundary conditions have different behaviors from other boundary conditions when non-uniformity parameter is changed.
3) Dimensionless frequencies are more sensitive to axial force rather elastic foundation coefficients.
4) When axial forces increase, required number of terms in DTM increases to achieve the specified accuracy.

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Semi-analytical Approach for Free Vibration Analysis of Variable Cross-section Beams Resting on Elastic Foundation and under Axial Force

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PAPER INFO

Paper history:
Received 30 June 2013
Received in revised form 05 September 2013
Accepted 14 September 2013

Keywords:
Variable Cross-Section Beam
Elastic Foundation
Axial Force
Differential Transform Method

چکیده

در این مقاله ارتعاشات آزاد تیر اویلر–برنولی دارای سطح مقطع متغیر مستقر بر بستر الاستیک و تحت اثر بار کششی محوری مورد بررسی قرار می گیرد. ارتفاع تیر ثابت در نظر گرفته شده و عرض آن به صورت نمایی تغییر می کند. تحلیل برای سه حالات مختلف شرایط مرزی شامل دو سر گیردار، دو سر مفصل و یک سر گیردار–یک سر آزاد انجام شده است. بعد از آنکه معادله ارتعاشات آزاد تیر مورد بررسی استخراج شد، فرکانس های بدون بعد با استفاده از روش تبدیل دیفرانسیلی به دست می آیند. روش تبدیل دیفرانسیلی یک روش نیمه تحلیلی مبتنی بر بسط سری تیلور می باشد که به راحتی می تواند برای حل انواع معادلات دیفرانسیل معمولی و مشتقات جزئی به کار رود. تاثیر نیروی محوری، ضریب بستر الاستیک و پارامتر متناظر با تغییرات سطح مقطع بر فرکانس های بدون بعد بررسی شده است و در مواردی که در سایر منابع تحلیل مشابهی صورت گرفته است، مقایسه بین نتایج انجام شده است. نتایج نشان می دهد روش تبدیل دیفرانسیلی دارای همگرایی سریع بوده و قادر به محاسبه تمام فرکانس ها می باشد اگرچه سرعت همگرایی آن از شرایط تکیه گاهی اثر می پذیرد. همچنین فرکانس های بدون بعد حساسیت بیشتری نسبت به نیروی محوری از خود نشان می دهند.

doi: *10.5829/idosi.ije.2014.27.03c.05*