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Accurate Model of Capacitance for MEMS Sensors using Corrugated Diaphragm with Residual Stress

B. Azizollah Ganji *, M. Taybi

Department of Electrical and Computer Engineering, Babol University of Technology, Babol, Iran

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ABSTRACT

In this paper, we present a new model for calculating the capacitance of MEMS sensor with corrugated diaphragm. In this work, the effect of residual stress is considered on deflection of diaphragm and capacitance of sensor. First, a new analytical analyzes have been carried out to derive mathematic expression for central deflection of corrugated diaphragm and its relationship with residual stress. Then, the capacitance and sensitivity of sensor using corrugated diaphragm with residual stress are calculated under bias voltage and pressure. The analytical results are compared with simulation using a Finite Element Method (FEM). The results show that the new analytical model is very close with simulation results.

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1. INTRODUCTION

Micro-electro-mechanical devices such as finger print sensor, acoustic sensor and tunable capacitors have been commercially available in the market in the past decade [1-4]. The capacitive sensors are interesting products, because they can sense motion, chemical composition, electric field, pressure, acceleration, fluid level and fluid composition. The sensors are fabricated using conductive electrodes with detection circuits that convert capacitance changes to voltage, pulse width variation or frequency [5]. Capacitor can be fabricated as a sensor or an actuator. Diaphragm with clamped edges is an important mechanical component of the MEMS sensors and actuators [6-9]. In general, thin-film diaphragms suffer from residual stresses, which can easily affect the mechanical behavior of structure [10]. High residual stress in film may give undesirable effects such as higher actuation voltage, film buckling and even diaphragm cracking. The sophisticated design requires precise control of the initial stresses. The residual stress can be controlled within certain limits by the parameters of the deposition process [11]. It would be better if the mechanical sensitivity of the diaphragm is not determined by deposition process. A possible method to solve the stress problem is the utilization of corrugation technique. The corrugations reduce the effects of the residual stress. Thus, they increase the mechanical sensitivity and linear range of the diaphragm [12].

The sensitivity of the capacitive sensors depends on capacitance changes. Therefore, the modeling of capacitance for calculating the accurate capacitance between the deformed diaphragm and fixed back plate is necessary. In previous works [13, 14], the capacitance of sensors with flat diaphragms are modeled without residual stress. In this paper, the capacitance of MEMS sensor using corrugated diaphragm with residual stress have been modeled and compared with simulation results using a Finite element method (FEM).

2. THE CAPACITANCE OF SENSORE WITH FLAT DIAPHRAGM

Figure 1 shows the MEMS capacitive structure with circular flat clamped diaphragm. For two parallel

*Corresponding Author Email: baganji@nit.ac.ir (B. Azizollah Ganji)

conductive plates with air gap, the capacitance between the plates can be calculated as [13]:

$$C=\varepsilon\frac{A}{d} \tag{1}$$

where, A is the area of the plates, d is the air gap and ε is the dielectric constant. It is assumed that the circumference of the plates is much bigger than the air gap. Thus, the fringe capacitance at the edges of the plates can be neglected. For clamped circular flat diaphragm, the capacitance is given by [13]:

$$C=\iint\frac{\varepsilon}{d-w(r,\theta)}rdrd\theta \tag{2}$$

where, w is the deflection of the diaphragm. The deflection of a flat circular diaphragm with clamped edges and high residual stress, using homogeneous pressure, P, for small deflection can be calculated by [15]:

$$w(r)=\frac{p}{\left(\frac{5.33}{(R^2-r^2)^2.(1-v^2)}t^3.E+\frac{4\sigma_0.t}{R^2-r^2}\right)} \tag{3}$$

where, E, v, R, σ_0 and t are Young's modulus, Poisson's ratio, radius, residual stress and thickness of diaphragm, respectively. By replacing Equation (3) in Equation (2), we have:

$$C=\int_0^{2\pi}\int_0^{R}\frac{\varepsilon.r.dr.d\theta}{d-\frac{p}{\left(\frac{5.33}{(R^2-r^2)^2.(1-v^2)}t^3.E+\frac{4\sigma_0.t}{R^2-r^2}\right)}} \tag{4}$$

The sensitivity of sensor can be calculated using equation $S = dC/dP$.

3. MODELING OF CAPACITANCE USING CORRUGATED DIAPHRAGM

The corrugated diaphragms play important role in sensors to reduce the effect of residual stress. Figure 2 shows the corrugated diaphragm and Figure 3 shows a geometric model of a corrugated structure that consists of flat and corrugated zone. In this figure, R, S, L and h_c are radius, spatial period, arc length and depth of corrugations, respectively. The equilibrium stress of corrugated diaphragm, σ , can be calculated as:

$$\sigma=\eta\sigma_0 \qquad \eta<1 \tag{5}$$

where, η is the attenuation coefficient of residual stress. The attenuation coefficient of residual stress is given by [15]:

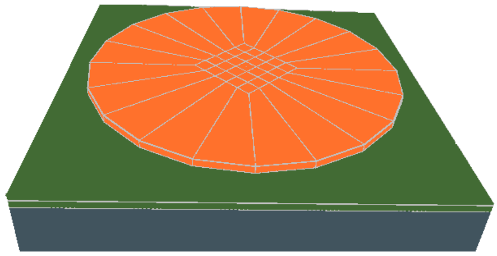
$$\eta=\frac{\sigma}{\sigma_0}=\frac{R.t^2}{R.t^2+6N_c.h_c^2.w_c.\sin\beta+8N_c.h_c^3.\sin^2\beta} \tag{6}$$

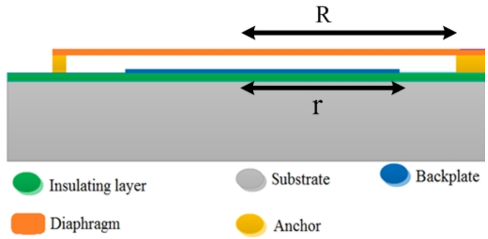
where, w_c and h_c are width and height of corrugations and N_c is the corrugations number. Since the diaphragm is clamped, the deflection at the central region of the diaphragm is higher than that at the diaphragm edges.

The central deflection of a corrugated circular diaphragm with clamped edges and residual stress is expressed as [16]:

$$P=\left[a_pE\frac{t^4}{R^4}+\frac{4t^2\sigma}{R^2}\right]\frac{w}{t}+b_p\frac{E}{(1-v^2)}.\frac{t^4}{R^4}.\frac{w^3}{t^3} \tag{7}$$

where, a_p is the dimensionless linear coefficient and b_p is the dimensionless non-linear coefficient [16].


(a) Top view



(b) Cross section view

Figure 1. MEMS capacitive sensor with circular clamped flat diaphragm

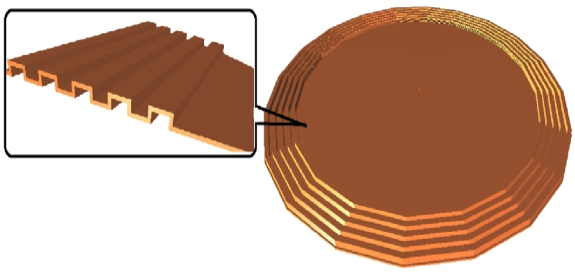


Figure 2. Corrugated diaphragm

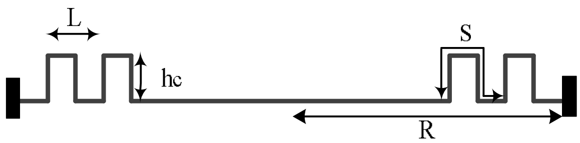


Figure 3. Cross section of corrugated diaphragm

$$a_p = \frac{2(q+1)(q+3)}{3(1+\frac{v^2}{q^2})} \tag{8}$$

$$b_p = 32\frac{1-v^2}{q^2-9}(\frac{1}{6}+\frac{3-v}{(q-v)(q+3)}) \tag{9}$$

where, q is corrugated profile factor which is given by the following equation [15]:

$$q^2 = \frac{S}{L}(1+1.5\frac{h_c^{\ 2}}{t^2}) \tag{10}$$

For small deflection [$(w/t)<<1$], the 3rd order term of Equation (7) can be neglected. Thus, the relation between central deflection and applied pressure is linear. From Equation (7), the central deflection of corrugated circular diaphragm for small deflections, can be written as:

$$W = \frac{p}{a_p.E\frac{t^3}{R^4}+\frac{4.t.\sigma}{R^2}} \tag{11}$$

According to Equation (11), the deflection of circular corrugated diaphragm can be defined as:

$$W(r) = \frac{p}{a_p.E\frac{t^3}{(R^2-r^2)^2}+\frac{4.t.\sigma}{R^2-r^2}} \tag{12}$$

Using Equations (2) and (12), the capacitance of MEMS sensor can be obtained as:

$$C = \int_0^b\int_0^{2\pi}\frac{\varepsilon.r.dr.d\theta}{d-\frac{p}{a_p.E.t^3(\frac{1}{(R^2-r^2)^2})+4.t.\sigma.(\frac{1}{(R^2-r^2)})}} \tag{13}$$

The sensitivity of sensor with corrugated diaphragm for small deflection can be calculated as Equation (14). The capacitance and sensitivity of sensor depends on diaphragm thickness, air gap, residual stress and diaphragm radius.

4. THE EFFECT OF BIAS VOLTAGE

An external voltage, V, is applied between tow plates of capacitor. The supply voltage providing a means for readout of the change in capacitance due to diaphragm deflection, the resulting electrostatic attraction force pulls the capacitor electrodes towards each other. Consequently, it causes the diaphragm to deflect, even in the absence of an external mechanical pressure. This electrostatic attraction force is nonlinear and increases with the decreasing gap width between the electrodes for a fixed voltage. The electrostatic attraction force F_E between the plates due to the charges on the plates can be found by differentiating the stored energy of the capacitor with respect to the position of the movable plate and is expressed as:

$$F_E = -\frac{d}{dx}(\frac{1}{2}CV^2) = \frac{\varepsilon_0.A.V^2}{2(d_0-x)^2} \tag{15}$$

where, x is the diaphragm deflection and d_0 is the initial distance between tow plates. The electrostatic pressure on diaphragm can be obtained as:

$$P_0 = \frac{F_E}{A} = \frac{\varepsilon_0.V^2}{2.(d-x)^2} \tag{16}$$

The capacitance of MEMS sensor with corrugated diaphragm under bias voltage is obtained by:

$$C = \int_0^b\int_0^{2\pi}\frac{\varepsilon.r.dr.d\theta}{d-\frac{p+p_0}{a_p.E.t^3(\frac{1}{(R^2-r^2)^2})+4.t.\sigma.(\frac{1}{(R^2-r^2)})}} \tag{17}$$

5. ESULTS AND DISCUSSION

In this section, the central deflection and capacitance of sensor with flat and corrugated diaphragm have been considered. We used MATLAB for mathematical analysis and IntelliSuite MEMS tool for simulation of sensor behaviours. In this example, two sensors with flat and corrugated diaphragms with Poisson's ratio of 0.22, Yung's modulus of 169 GPa, diaphragm radius of 0.270 µm, diaphragm thickness of 1µm and for corrugated diaphragm, h_c=2um, w_c=4um, b_c=4um, $\beta=90°$ have been assumed.

$$\begin{aligned} S_S = \frac{\partial C}{\partial p} = \frac{-t.\sigma}{P^2}.\left(\ln\left[d.t^3.ap.E+4.d.t.\sigma.(R^2-r^2)-p.(R^2-r^2)^2\right]+\frac{p.(R^2-r^2)^2}{d.t^3.ap.E+4.d.t.\sigma.(R^2-r^2)-p.(R^2-r^2)^2}\right)\frac{\tanh^{-1}\left(\frac{-4.d.t.\sigma+2.p.(R^2-r^2)}{2\sqrt{p.d.t^3.ap.E+4.d^2.t^2.\sigma^2}}\right)}{\sqrt{p.d.t^3.ap.E+4.d^2.t^2.\sigma^2}} \\ \left[\frac{ap^2.E^2.t^4+8.d.t^3.\sigma^2.ap.E}{4.(p.t.ap.E+4.d.\sigma^2)}+4.d.t^2.\sigma^2\right]-\left(\frac{ap.E.t^3}{2}+\frac{4.d.t^2.\sigma^2}{p}\right)\frac{\frac{1}{\sqrt{p.d.t^3.ap.E+4.d^2.t^2.\sigma^2}}}{\sqrt{p.d.t^3.ap.E+4.d^2.t^2.\sigma^2}}.\frac{\left[(R^2-r^2)-\frac{(-4.d.t.\sigma+2.p.R^2-2.p.r^2).d.t^3.ap.E}{4.(p.d.t^3.ap.E+4.d^2.t^2.\sigma^2)}\right]}{\left(1-\frac{(-4.d.t.\sigma+2.p.R^2-2.p.r^2).d.t^3.ap.E}{4.(p.d.t^3.ap.E+4.d^2.t^2.\sigma^2)}\right)} \end{aligned} \tag{14}$$

Figure 4 shows the MEMS capacitive sensor with corrugated circular diaphragm.

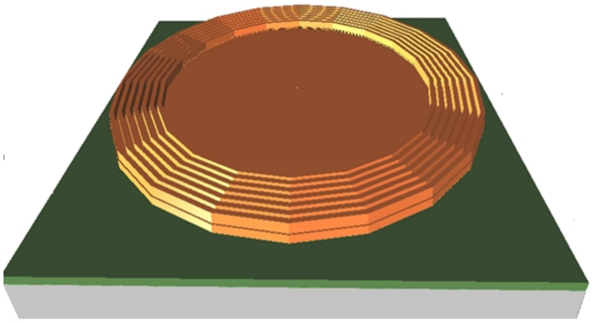
Figuer 5 shows the capacitance of sesnor versus homogeneous pressure. It can be seen that, the sensor with corrugated diaphragm has higher capacitance than the sensor with flat one. Figure 6 shows the sensitivity of sensor with flat and corrugated diaphragm versus pressure. The results show that the sensor with corrugated diaphragm has higher sensitivity and also the sensitivity of sensor increase with pressure. It means that the corrugations reduce the residual stress effect of diaphragm and increase the sensor sensitivity.

Figure 7 shows the sensitivity of sensor with flat and corrugated diaphragm versus voltage. It can be seen that the sensitivity of sensor increase by an increase in the voltage.

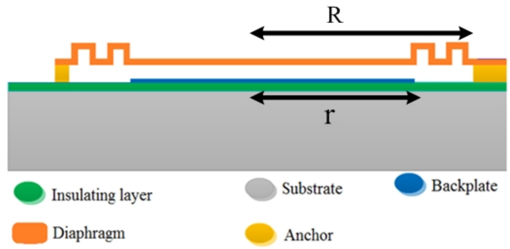
All results show that the new analytical model of capacitance for MEMS sensors using corrugated diaphragm with residual stress is very close with simulation result. Therefore, the new model is verified to design and analyze of MEMS capacitive sensors.

6. CONCLUSION

In this paper, we presented a new model for calculating the capacitance of MEMS sensor using corrugated diaphragm with residual stress. The model includes the effect of different parameters on capacitance of sensor such as voltage and pressure. The sensitivity of sensor was calculated and the effect of bias voltage and presser was investigated on sensor sensitivity. The results show that the new model of capacitance for MEMS sensors using corrugated diaphragm with residual stress is very close with simulation result using fine element method. Thus, the new model can help designers to analyze and design of MEMS capacitive sensors. The results also show that the sensor with corrugated diaphragm has higher sensitivity than the flat one, because the corrugations reduce the effect of residual stress of diaphragm.



(a) Top view



(b) Cross section view

Figure 4. MEMS capacitive sensor with corrugated diaphragm

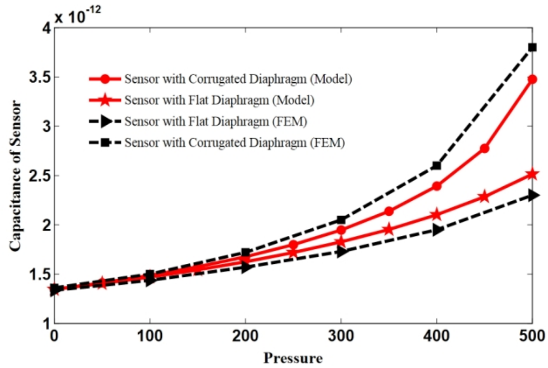


Figure 5. Capacitance of sensor vs. pressure

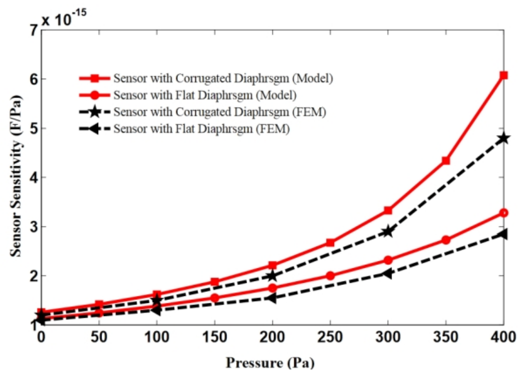


Figure 6. Sensitivity of sensor vs. pressure

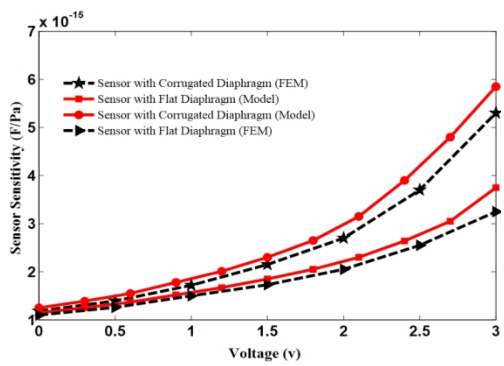


Figure 7. Sensitivity of sensor vs. voltage

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Accurate Model of Capacitance for MEMS Sensors using Corrugated Diaphragm with Residual Stress

B. Azizollah Ganji, M. Taybi

Department of Electrical and Computer Engineering, Babol University of Technology, Babol, Iran

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چکیده

در این مقاله روش جدیدی برای محاسبه ظرفیت خازنی سنسور میکروالکترومکانیکی با دیافراگم موج دار ارائه می‌شود. همچنین اثر تنش پسماند بر روی جابجایی دیافراگم و ظرفیت خازنی سنسور در نظر گرفته می‌شود. در ابتدا آنالیز تحلیلی برای بدست آوردن مقدار جابجایی مرکزی دیافراگم موج دار و رابطه‌ی آن با تنش پس ماند صورت می‌گیرد. سپس مقدار ظرفیت خازنی و حساسیت سنسور با دیافراگم موج دار دارای تنش پسماند تحت ولتاژ بایاس و فشار بدست آورده می‌شود. نتایج تحلیلی با شبیه سازی که با استفاده از روش اجزاء محدود انجام شده است، مقایسه می‌شود. مقایسه نشان می‌دهد روش تحلیلی جدید به خوبی توانسته است نتایج شبیه سازی را دنبال کند.

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