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TECHNICAL NOTE

Maintainability Policy for Deteriorating System with Inspection and Common Cause Failure

A. Mishra [a]*; M. Jain[b]

[a] *Department of Mathematics, I.B.S. Khandari , Dr. B. R. Ambedkar University, Agra,*
[b] *Department of Mathematics, I.I.T Roorkee,*

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ABSTRACT

A condition based on preventive and corrective maintenance policy is proposed for a continuously operating system. The condition of the system is assumed to deteriorate with time. The model incorporates both deterioration as well as random common cause failures. The deterioration stages are modeled as discrete state processes. The system is put to random inspection to know the condition. The mean times between inspections are exponentially distributed. If the observed condition at an inspection exceeds the threshold (N) deterioration level, the preventive maintenance (PM) is performed, else no action is taken and the system continues to run. The proposed model considers an accumulated deterioration based on increasing intensity for the random failures. The transient solutions using Laplace transform as well as steady state solutions using recursive technique are suggested to compute the state probabilities of the system. Various reliability measures of the system have been established and validated numerically by taking an illustration.

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1. INTRODUCTION

Preventive maintenance is a schedule of planned maintenance actions aimed at the prevention of breakdowns and failures. The primary goal of preventive maintenance is to prevent the failure of equipment before it actually occurs. It is designed to preserve and enhance the equipment reliability by replacing worn out components before they actually fail. Preventive maintenance activities include equipment checks, partial or complete overhauls at specified periods, oil changes, lubrication and so on. In addition, workers can record equipment deterioration so that they know when to replace or repair worn out parts before they cause system failure. Recent technological advances in tools for inspection and diagnosis have proved even more accurate and effective for equipment maintenance. The ideal preventive maintenance program prevents all equipment failure before it occurs.
The relative ease and cost of preventing failures (retaining an item in a specified condition) or correcting failures (restoring an item to a specified condition) can be justified through maintenance actions. Although other factors, such as highly trained people and a responsive supply system can help keeping downtime to an absolute minimum; it is the inherent maintainability that determines this minimum time. Improving training or support cannot effectively compensate for the effect on availability of a poorly designed (in terms of maintainability) product. Minimizing the cost to support a product and maximizing the availability of that product are best done by designing the product to be reliable and maintainable. Testability, an important subset of maintainability, is a design characteristic that allows the status (operable, inoperable or degraded) of an item to be determined, and faults within the item to be isolated in a timely and efficient manner. The ability to detect and isolate faults within a system, and to do so efficiently and cost effectively, is not only important in the field, but also during manufacturing. All products must be tested and verified prior to release to the customer. Trade-off must be made upfront on the use of built-in-test (BIT) versus other means of fault detection and isolation. The poor testability is main reason of

*Corresponding Author Email: *ashishddive@gmail.com* (A. Mishra)

higher manufacturing costs, higher support costs, and lower customer satisfaction.

The dangerous failure of many components may be unlikely if the devices are designed, installed, operated, inspected, and maintained according to the specifications. However, when the device specifications are violated, multiple simultaneous failures can occur. This is known as a common cause failure (CCF). Goel and Gupta [1] studied a two-engine aeroplane model with two types of failure and preventive maintenance. Sharma et al. [2] made the stochastic analysis of a parallel system with common cause failure, preventive maintenance and two types of repair. Reliability analysis of a system with preventive maintenance, inspection and two types of repair was considered by Goel et al. [3]. Mokaddis et al. [4] gave the probabilistic analysis of a two-unit system with a warm standby subject to preventive maintenance and a single service facility. Goel and Murari [5] investigated the two-unit cold-standby redundant system subject to random checking, corrective maintenance and system replacement with repairable and non-repairable types of failure. For a 1-out-of-n system with uptime, downtime and related costs, the concept of preventive maintenance was incorporated by Smith and Dekker [6]. A delay time multi-level on-condition preventive maintenance inspection model based on constant base interval risk when inspection which detects pending failure was developed by Williams and Hirani [7].

The new method to minimize the preventive maintenance cost of series-parallel systems was suggested by Bris et al. [8]. Zhao [9] considered a preventive maintenance policy of a critical reliability level for the system subject to degradation. Amari and McLaughlin [10] investigated the optimal design of a condition based maintenance model. The availability of a k-out-of-N system with limited spares and repair capacity under a condition based maintenance strategy was considered by Smidt-Destombes et al. [11]. A study of availability-centered preventive maintenance for multi-component systems was done by Tsai et al. [12]. The maintainability analysis of repairable machining system was analysed by Jain et al. [13]. Jain and Mishra obtained the steady state availability of multistage degraded machining system and made Bayesian estimation of unknown parameters for two unit system with common cause failures. Rao and Naikan [14] studied the conditions based preventive maintenance policy for Markov deteriorating systems. Ruiz et al. [15] considered scheduling and preventive maintenance in the flow-shop sequencing problem. Zequeira et al. [16] studied the optimal buffer inventory and opportunistic preventive maintenance under random production capacity availability. Kenne and Nkeungoue [17] suggested simultaneous control of production, preventive and corrective maintenance rates of a failure-prone manufacturing system. The method for effects evaluation of some forms of power transformers preventive maintenance was made by Mijailovic [18]. Darghouth et al. [19] considered a profit assessment model for equipment inspection and replacement under renewing free replacement warranty policy. The maintenance scheduling of a protection system subject to imperfect inspection and replacement was done by Berrade et al. [20]. Berrade [21] obtained the two-phase inspection policy with imperfect testing. Tang et al. [22] investigated the availability of a system subject to hidden failure inspected at constant intervals with non-negligible downtime due to inspection and downtime due to repair/replacement.

In this paper, we study the preventive maintenance issues for equipment whose condition is subject to deterioration with time. The rest of the paper is organized as follows. The assumptions and notations of the proposed model are given in section 2. Section 3 deals with the analysis of transient equations and their solution. The steady state balance equations are constructed and recursive solutions are obtained in section 4. The reliability measures have been obtained in section 5. In section 6, the numerical results have been displayed via graphs and tables. In section 7, conclusions are drawn by highlighting the significance and future scope of the work done.

2. MODEL DESCRIPTIONS

Consider a system which is subjected to deterioration. There may be common cause failure at different stages of deterioration. The state i $(1 \le i \le N)$ of the system is defined in increasing order of deterioration; the common cause failure may occur and no maintenance is performed. In state i, $N+1 \le i \le k$, the preventive maintenance is performed. The preventive maintenance (PM) does not help in case of the failure of a device. The life time distribution of the device has an increasing failure rate. We assume that the component time to successive failure in i^{th} state of the system is exponential distributed with rate λ_i where $1 \le i \le k$. The inspection time of the system is also assumed to be exponentially distributed with rate α_i in i^{th} $1 \le i \le N$ deterioration stage. During an inspection, the system neither operates nor deteriorates. The replacement of the components takes place if the identified deterioration stage of the system at an inspection exceeds a threshold N; $1 \le N \le k$, which is a preventive replacement threshold. The system has a deterioration failure immediately following the completion of k stages of deterioration. Following a deterioration failure, a corrective maintenance is performed which makes the system as good as new condition (Figure 1). We also assume that the time to repair is exponentially distributed.

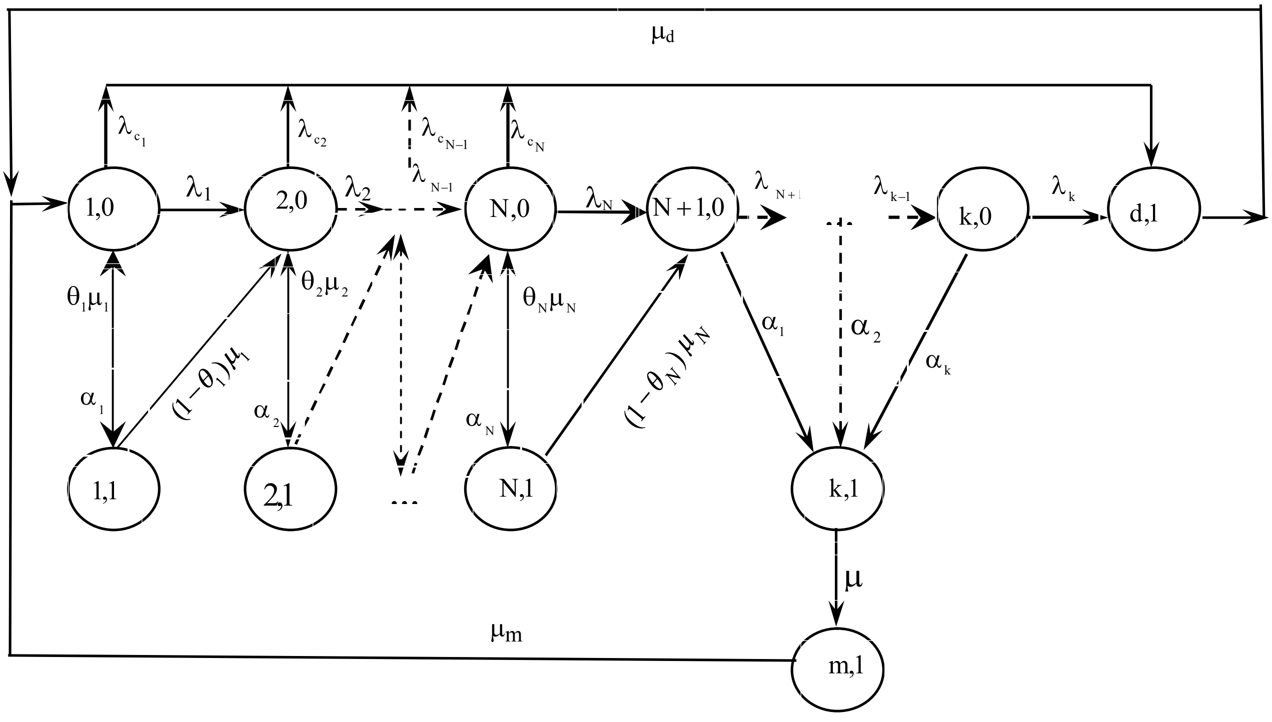

Figure 1. State transition flow diagram for the condition based preventive maintenance (CBPM) model

Further assume that an inspection is triggered after a mean duration $1/\lambda_i$ and takes an average time of $1/\mu_i$. Let θ_i be the probability that after inspection at i^{th} stage, the system remains in the same stage; $1 \le i \le N$; $(1-\theta_i)$ is the probability which the system continues to deteriorate and at the end of inspection reaches to next $(i+1)^{th}$ stage. When an inspection is completed, no action is taken if the device is found to be in the first stage of its life time. λ_{c_i} is common cause failure in i^{th} deterioration state $1 \le i \le N$; α_i is the successive inspection rate of the system, $1 \le i \le k$.

We define transient and steady state probabilities as follows:

$P_{i,0}(t)$: prob. {system is functioning in i^{th} deterioration stage at time t, $1 \le i \le k$ }

$P_{i,1}(t)$: prob. {system is under inspection in i^{th} deterioration stage at time t, $1 \le i \le k$ }

$P_{d,1}(t)$: prob. {system has failed due to deterioration and is under corrective maintenance at time t}

$P_{m,1}(t)$: prob. {system is under preventive maintenance at time t}

For steady state, we denote

$$P_{i,0} = \lim_{t\to\infty} P_{i,0}(t)\,;\; P_{i,1} = \lim_{t\to\infty} P_{i,1}(t)$$

$$P_{d,1} = \lim_{t\to\infty} P_{d,1}(t)\,;\; P_{m,1} = \lim_{t\to\infty} P_{m,1}(t)$$

Some other notations used for model formulation are

k	number of stages of deterioration before deterioration failure
μ	inspection rate after random failure in i^{th} $(N+1 \le i \le k)$ stage.
μ_m	preventive maintenance rate
μ_d	corrective maintenance rate of the component following a deterioration failure.

3. THE ANALYSIS

The differential difference equations for different stages are constructed as follows:

$$\frac{d}{dt}P_{1,0}(t) = -(\lambda_1 + \alpha_1 + \lambda_{c_1})\, P_{1,0}(t) + \mu_d\, P_{d,1}(t) + \mu_m\, P_{m,1}(t) + (\theta_1\mu_1)\, P_{1,1}(t) \quad (1)$$

$$\frac{d}{dt}P_{i,0}(t) = -(\lambda_i + \alpha_i + \lambda_{c_i})\, P_{i,0}(t) + (\theta_i\mu_i)\, P_{i,1}(t) + \{(1-\theta_{i-1})\mu_{i-1}\}\, P_{i-1,1}(t) + \lambda_{i-1}\, P_{i-1,1}(t), \quad i = 2,3,\dots,N \quad (2)$$

$$\frac{d}{dt}P_{N+1,0}(t)=-(\lambda_{N+1}+\alpha_1)\,P_{N+1,0}(t)+\lambda_N P_{N,0}(t)+\{(1-\theta_N)\mu_N\}P_{N,1}(t) \tag{3}$$

$$\frac{d}{dt}P_{i,0}(t)=-(\lambda_i+\alpha_{i-N})\,P_{i,0}(t)+\lambda_{i-1}P_{i-1,0}(t),\quad i=N+2,N+3,\dots,k \tag{4}$$

$$\frac{d}{dt}P_{1,1}(t)=-\{(1-\theta_1)\mu_1+\theta_1\mu_1\}\,P_{1,1}(t)+\alpha_1 P_{1,0}(t) \tag{5}$$

$$\frac{d}{dt}P_{i,1}(t)=-\{(1-\theta_i)\mu_i+\theta_i\mu_i\}\,P_{i,1}(t)+\alpha_i P_{i,0}(t),\quad i=1,2,\dots,N \tag{6}$$

$$\frac{d}{dt}P_{k,1}(t)=-\mu\,P_{k,1}(t)+\left\{\sum_{i=N+1}^{k}\alpha_{i-N}\right\}P_{i,0}(t) \tag{7}$$

$$\frac{d}{dt}P_{m,1}(t)=-\mu_m P_{m,1}(t)+\mu\,P_{k,1}(t) \tag{8}$$

$$\frac{d}{dt}P_{d,1}(t)=-\mu_d P_{d,1}(t)+\left\{\sum_{i=1}^{N}\lambda_{c_i}\right\}P_{i,0}(t)+\lambda_k P_{k,0}(t) \tag{9}$$

The transient solution is obtained by solving the differential difference Equations (1)-(9) as follows. Using Laplace transform of the set of Equations (1)-(9) with initial conditions $P_{1,0}(0)=1$, we obtain

$$1=(s+\lambda_1+\alpha_1+\lambda_{c_1})\,\tilde{P}_{1,0}(s)-\mu_d\,\tilde{P}_{d,1}(s)-\mu_m\,\tilde{P}_{m,1}(s)-\theta_1\mu_1\,\tilde{P}_{1,1}(s) \tag{10}$$

$$0=(s+\lambda_i+\alpha_i+\lambda_{c_i})\,\tilde{P}_{i,0}(s)-\{\theta_i\mu_i\}\tilde{P}_{i,1}(s)-\{(1-\theta_{i-1})\mu_{i-1}\}\,\tilde{P}_{i-1,1}(s)+\lambda_{i-1}\,\tilde{P}_{i-1,0}(s),\quad i=2,3,\dots,N \tag{11}$$

$$0=(s+\lambda_{N+1}+\alpha_1)\,\tilde{P}_{N+1,0}(s)-\lambda_N\tilde{P}_{N,0}(s)-\{(1-\theta_N)\mu_N\}\tilde{P}_{N,1}(s) \tag{12}$$

$$0=(s+\lambda_i+\lambda_{i-N})\,\tilde{P}_{i,0}(s)-\lambda_{i-1}\tilde{P}_{i-1,0}(s),\quad i=N+2,\dots,k \tag{13}$$

$$0=(s+\{1-\theta_1\}\mu_1+\theta_1\mu_1)\,\tilde{P}_{1,1}(s)-\alpha_1\tilde{P}_{1,0}(s) \tag{14}$$

$$0=(s+\{1-\theta_i\}\mu_i+\theta_i\mu_i)\,\tilde{P}_{i,1}(s)-\alpha_i\tilde{P}_{i,0}(s),\quad i=1,2,\dots,N \tag{15}$$

$$0=(s+\mu)\,\tilde{P}_{k,1}(s)-\left\{\sum_{i=N+1}^{k}\alpha_{i-N}\right\}\tilde{P}_{i-N}(s) \tag{16}$$

$$0=(s+\mu_m)\,\tilde{P}_{m,1}(t)-\mu\tilde{P}_{k,1}(t) \tag{17}$$

$$0=(s+\mu_d)\,\tilde{P}_{d,1}(s)-\left\{\sum_{i=1}^{N}\lambda_{c_i}\right\}\tilde{P}_{i,0}(s)+\lambda_k\tilde{P}_{k,0}(s) \tag{18}$$

The Equations (10)-(18) can be written in the matrix form as

$$B(s)\tilde{P}(s)=P(0) \tag{19}$$

where,

$$B(s)=\begin{bmatrix}A_1 & 0 & A_6 & A_{10}\\ A_5 & A_2 & 0 & A_{11}\\ A_7 & 0 & A_3 & A_9\\ A_8 & 0 & 0 & A_4\end{bmatrix}_{(2k+1)\times(2k+1)}$$

Here

$$A_1=\begin{bmatrix}-(s+\lambda_1+\lambda_{c_1}+\alpha_1) & \lambda_1 & 0 & \cdots & 0\\ 0 & -(s+\lambda_2+\lambda_{c_2}+\alpha_2) & \lambda_2 & \cdots & 0\\ 0 & 0 & -(s+\lambda_3+\lambda_{c_3}+\alpha_3) & \cdots & 0\\ 0 & 0 & 0 & \ddots & \vdots\\ 0 & 0 & 0 & \cdots & \lambda_N\\ 0 & 0 & 0 & & \end{bmatrix}_{N+1\times N}$$

$$A_2=\begin{bmatrix}-(s+\lambda_6+\alpha_1) & \lambda_6 & 0 & \cdots & 0\\ 0 & -(s+\lambda_7+\alpha_2) & \lambda_7 & \cdots & 0\\ 0 & 0 & -(s+\lambda_8+\alpha_3) & \cdots & 0\\ 0 & 0 & 0 & \ddots & \lambda_{k-1}\\ 0 & 0 & 0 & \cdots & -(s+\lambda_k+\alpha_N)\end{bmatrix}_{N\times N}$$

$$A_3=\begin{bmatrix}-(s+\mu_1) & 0 & 0 & \cdots & 0\\ 0 & -(s+\mu_2) & 0 & \cdots & 0\\ 0 & 0 & -(s+\mu_3) & \cdots & 0\\ 0 & 0 & 0 & \ddots & \vdots\\ 0 & 0 & 0 & \cdots & -(s+\mu_N)\end{bmatrix}_{N\times N}$$

$$A_4=\begin{bmatrix}-(s+\mu) & \mu & 0\\ 0 & -(s+\mu_m) & 0\\ 0 & 0 & -(s+\mu_d)\end{bmatrix}_{3\times 3}$$

$$A_5=\begin{bmatrix}\theta_1\mu_1 & (1-\theta_1)\mu_1 & 0 & \dots & 0\\ 0 & \theta_2\mu_2 & (1-\theta_2)\mu_2 & \dots & 0\\ 0 & 0 & \theta_3\mu_3 & \dots & 0\\ 0 & 0 & 0 & \ddots & \vdots\\ 0 & 0 & 0 & \dots & (1-\theta_N)\mu_N\end{bmatrix}_{N+1\times N}$$

$A_6=\alpha_N I$

$A_7=[\mu_d]_{1\times 1}$

$A_8=[\mu_m]_{1\times 1}$

$A_9=[\lambda_k]_{1\times 1}$

$A_{10}=\left[\lambda_{c_1}\quad \lambda_{c_2}\quad \dots\quad \lambda_{c_N}\right]^T_{1\times N}$

$A_{11}=\left[\alpha_1\quad \alpha_2\quad \dots\quad \alpha_N\right]^T_{1\times N}$

Also denote

$$\tilde{P}(s)=\left[\tilde{P}_{1,0}(s),\tilde{P}_{2,0}(s),\dots\tilde{P}_{k,0}(s),\tilde{P}_{1,1}(s),\tilde{P}_{2,1}(s),\dots\tilde{P}_{N,1}(s),\tilde{P}_{k,1}(s),\tilde{P}_{m,1}(s),\tilde{P}_{d,1}(s)\right]^T$$

To compute $\tilde{P}_{i,j}(s)\,(i=1,2,\dots,k;\ j=0,1),\tilde{P}_{d,1}(s)$ and $\tilde{P}_{m,1}(s)$,

we apply Cramer's rule on matrix B(s) and obtain

$$\widetilde{P}_{i,j}(s)=\frac{\Delta B'(s)}{\Delta B(s)} \tag{20}$$

where, $\Delta B(s)$ is the determinant of matrix $B(s)$, and $B'(s)$ is the determinant of matrix which has been obtained by replacing the respective column vectors of $B(s)$ with the initial vector $P(0)$. Now, we calculate characteristic roots of tridiagonal matrix B(s). It is clear that s >= 0 is a root of $\Delta B(s)=0$. Now substituting s = (-d), we obtain

$$B(-d) = (B-dI) \tag{21}$$

Now Equation (19) becomes

$$B(-d)\widetilde{P}(s)=(B-dI)\widetilde{P}(s)=P(0) \tag{22}$$

Other (L-1) roots in which x are real and y are complex roots in pair are denoted by $d_1, d_2, ..., d_x$ and $(d_{x+1}, \overline{d}_{x+1}), (d_{x+2}, \overline{d}_{x+2}), ..., (d_{x+y}, \overline{d}_{x+y})$, respectively. Thus, we have

$$\Delta B(s)=s\left[\prod_{u=1}^{x}(s+d_u)\right]\left[\prod_{u=1}^{y}(s+d_{x+u})(s+\overline{d}_{x+u})\right] \tag{23}$$

Equations (20) and (23) yield to

$$\widetilde{P}_{i,j}(s)=\frac{\Delta B'(s)}{s\left[\prod_{u=1}^{x}(s+d_u)\right]\left[\prod_{u=1}^{y}(s+d_{x+u})(s+\overline{d}_{x+u})\right]} \tag{24}$$

On expanding by partial fractions, we get

$$\widetilde{P}_{i,j}(s)=\frac{a_0}{s}+\frac{a_1}{(s+d_1)}+...+\frac{a_x}{(s+d_x)}+\frac{b_1 s+c_1}{(s+d_{x+1})(s+\overline{d}_{x+1})}+...+\frac{b_y s+c_y}{(s+d_{x+y})(s+\overline{d}_{x+y})} \tag{25}$$

Here, a_0 and $a_l\,(l=1,2,...,x)$ are real numbers calculated as follows

$$a_0=\frac{\Delta B'(0)}{\left(\prod_{u=1}^{x}d_u\right)\left(\prod_{u=1}^{y}d_{x+u}\overline{d}_{x+u}\right)} \tag{26 a}$$

$$a_l=\frac{\Delta B'(-d_l)}{(-d_l)\left(\prod_{\substack{u=1\\u\neq l}}^{x}(d_u-d_l)\right)\left(\prod_{\substack{u=1\\u\neq l}}^{y}(-d_l+\overline{d}_{x+u})(-d_l+\overline{d}_{x+u})\right)} \tag{26 b}$$

Let complex characteristic root d_{x+l} be a combination of real part v_l and imaginary part w_l. Then

$$b_l(-d_{x+l})+c_l=\frac{\Delta B'(-d_{x+l})}{(-d_{x+l})\left[\prod_{\substack{u=1\\u\neq l}}^{x}(d_u-d_{x+u})\right]\left[\prod_{\substack{u=1\\u\neq l}}^{y}(-d_{x+l}+d_{x+u})(-d_{x+l}+\overline{d}_{x+u})\right]} \tag{27}$$

$l=1,2,...,y$

On taking inverse Laplace transform of Equation (25), we get

$$P_{i,j}(t)=a_0+\sum_{l=1}^{x}a_l\exp(-d_l t)+\sum_{l=1}^{y}[b_l\exp(-v_l t)\cos(w_l t)+\frac{c_l-b_l v_l}{w_l}\exp(-v_l t)\sin(w_l t)] \tag{28}$$

where, $a_0, a_l, b_l, c_l, v_l, w_l$ are all real numbers.

We obtain system availability using transient probabilities in previous section as follows:

System availability is obtained as:

$$A(t)=\sum_{i=1}^{k}P_{i,0}(t) \tag{29}$$

4. STEADY STATE SOLUTION

By taking limit $t\to\infty$ in Equations (1)-(9), the steady state balance equations governing the model are as follows:

$$(\lambda_1+\alpha_1+\lambda_{c_1})P_{1,0}=\mu_d\,P_{d,1}+\mu_m P_{m,1}+(\theta_1\mu_1)P_{1,1} \tag{30}$$

$$(\lambda_i+\alpha_i+\lambda_{ci})P_{i,0}=\{\theta_i\mu_i\}P_{i,1}+\{(1-\theta_{i-1})\mu_{i-1}\}P_{i-1,1}+\lambda_{i-1}P_{i-1,0}, i=2,3,...,N \tag{31}$$

$$(\lambda_N+\alpha_1)\,P_{N+1,0}=\lambda_N P_{N,0}+\{(1-\theta_N)\mu_N\}\,P_{N,1} \tag{32}$$

$$(\lambda_i+\lambda_{i-n})P_{i,0}=\lambda_{i-1}P_{i-1,0},\ i=N+2,...,k \tag{33}$$

$$\{(1-\theta_i)\mu_i+\theta_i\mu_i\}P_{i,1}=\alpha_i P_{i,0},\ i=1,2,...,N \tag{34}$$

$$\mu\ P_{k,1}=\left\{\sum_{i=N+1}^{k}\alpha_{i-N}\right\}P_{i,0} \tag{35}$$

$$\mu_m\ P_{m,1}=\mu\ P_{k,1} \tag{36}$$

$$\mu_d\ P_{d,1}=\left\{\sum_{i=1}^{N}\lambda_{ci}\right\}P_{i,0}+\lambda_k P_{k,0} \tag{37}$$

Following recursive approach for the solving Equations (30)-(37), the steady state probabilities of different states are obtained as:

$$P_{i,1}=\left(\frac{\alpha_i}{\mu_i}\right)P_{i,0};\ i=1,2,...,N \tag{38}$$

$$P_{i,0}=(\phi_1)P_{1,0};\ i=2,3,...,N \tag{39}$$

$$P_{N+1,0}=(\phi_2)P_{1,0} \tag{40}$$

$$P_{i,0}=(\phi_3)P_{1,0};\ \ i=N+2,...,k \tag{41}$$

$$P_{k,1}=(\phi_4)P_{1,0} \tag{42}$$

$$P_{m,1}=(\phi_5)P_{1,0} \tag{43}$$

$$P_{d,1} = (\phi_6)P_{1,0} \quad (44)$$

where,

$$\phi_1 = \prod_{i=1}^{j-1}\left[\frac{(\lambda_i + (1-\theta_i)\alpha_i)}{(\lambda_{i+1} + \lambda_{c_{i+1}} + (1-\theta_{i+1})\alpha_{i+1})}\right]$$

$$\phi_2 = \left[\frac{(\lambda_N + (1-\theta_N)\alpha_N)}{(\lambda_N + \alpha_1)}\prod_{i=1}^{N-1}\frac{(\lambda_i + (1-\theta_i)\alpha_i)}{(\lambda_{i+1} + \lambda_{c_{i+1}} + (1-\theta_{i+1})\alpha_{i+1})}\right]$$

$$\phi_3 = \left[(\lambda_N + (1-\theta_N)\alpha_N)\prod_{i=1}^{N-1}\frac{(\lambda_i + (1-\theta_i)\alpha_i)}{(\lambda_{i+1} + \lambda_{c_{i+1}} + (1-\theta_{i+1})\alpha_{i+1})}\prod_{i=N+1}^{i-1}\frac{\lambda_i}{\lambda_i + \alpha_{i-n}}\right]$$

$$\phi_4 = \left[\frac{1}{\mu}\left(\sum_{i=N+1}^{k}\alpha_{i-N}\{(\phi_2) + (\phi_3)\}\right)\right]$$

$$\phi_5 = \left[\frac{1}{\mu_m}\left(\alpha_1(\phi_1) + \sum_{i=N+2}^{k}\alpha_{i-N}(\phi_2)\right)\right]$$

$$\phi_6 = \left[\frac{1}{\mu_d}\left(\left\{\sum_{i=1}^{N}\lambda_{c_i}(\phi_1) + \lambda_k(\phi_2)\right\}\right)\right]$$

Using normalizing condition i.e. the sum of probabilities of all states is equal to unity, we get

$$P_{1,0} = [1 + \phi_1 + \phi_2 + \phi_3 + \phi_4 + \phi_5 + \phi_6]^{-1} \quad (45)$$

5. PERFORMANCES MEASURES

We derive various reliability indices using steady state probabilities obtained in previous section as follows:

- Steady state availability is given by

$$A_v = \sum_{i=1}^{k} P_{i,0} \quad (46)$$

- Mean time between corrective maintenance of the system is given by

$$MTBCM = \frac{1}{\mu_d P_{d,1}} \quad (47)$$

- Mean time between preventive maintenance is obtained as

$$MTBPM = \frac{1}{\mu_m P_{m,1}} \quad (48)$$

- Mean time between inspection of the system is given by

$$MTBI = \frac{1}{A_v \alpha} \quad (49)$$

6. NUMERICAL RESULTS

In this section, we validate the analytical results established in previous section by taking the numerical illustration. For various performance indices namely system availability A(t), steady state availability Av, mean time between corrective maintenance (MTBCM), mean time between preventive maintenance (MTBPM), mean time between inspection (MTBI), the numerical results are displayed by varying different parameters in tabular and graphical forms. The default parameters chosen for computation purpose are k = 15, $\lambda = 0.01$, $\lambda_c = 0.001$, $\alpha = 0.1$, $\theta = 1$, $\mu_m = 0.5$, $\mu = 5$ $\mu_d = 0.3$.

Tables 1-4 exhibit the results for Av, MTBCM, MTBPM, and MTBI for increasing values of λ, λ_c,α and θ, respectively. From Table 1, it is noticed that as λ increases, Av starts decreasing reasonably whereas a sharp decrement is seen in MTBCM. Opposite to these trends MTBI increases by increasing λ, we notice that as λ increases, MTBPM first decreases and then increases. Table 2 shows the effect of λ_c on the performance indices. It is found that availability decreases slightly with the increasing values of λ_c whereas MTBI and MTBPM increase significantly. When the effect of λ_c is examined for MTBCM, it is observed that MTBCM decreases initially for lower values of λ_c but it starts increasing for higher values of λ_c. In Table 3, availability, mean time between preventive maintenance and mean time between inspection display decreasing patterns with the rising values of α. Availability decreases moderately whereas a sharp decrement can be seen in the case of MTBCM and MTBI. In case of MTBCM, it is noticed that it firstly comes down slightly and then rises remarkably. From Table 4, it is clear that availability and MTBCM increase but MTBPM and MTBI decrease with the increase in θ.

Tables 5-8 summarize the optimal values of threshold parameter (N^*) at which maximum availability is achieved. These threshold values and corresponding performance measures namely Av, MTBCM, MTBPM and MTBI against varying values of λ, λc, α and θ are given in Tables 5-8, respectively.

Figures 2-5 display the trend of A(t) against time (t) for different values of parameters $\lambda, \lambda_c, \alpha, \theta$. From Figures 2-4 it is clear that availability shows decreasing pattern with time whereas in Figure 5, it firstly increases sharply and after reaching certain height starts coming down before attaining almost constant value. Figure 2 reveals that availability first drops sharply and then gradually by increasing λ but as the time grows, near about t = 15, it becomes almost constant and coincides almost for each value of λ. From Figure 3, it is clear that availability decreases with the increasing values of λ_c. In Figure 4, it is seen that as α increases, the availability is initially constant (upto = 5) but later it decreases. Figure 5 shows a distinct pattern of availability with respect to t as compared to previous cases. However, in this case, once again it decreases as θ increases but diminishes with time t.

Figures 6 (a-b) show the steady state availability and mean time between inspection, respectively for different threshold values of N. From these figures it is observed that steady state availability (mean time between inspection) is maximum (minimum) at N = 6 as such optimal value N^* is achieved at N = 6.

From these numerical results, we can conclude that availability of the system starts decreasing when it works for a long time. Similarly when the system becomes more prone to failures, the availability again comes down. It is also noticed that by doing frequent inspections the system can be checked at proper time, which aids in the enhancement of system availability.

TABLE 1. Performance indices by varying λ

λ	A_v	MTBCM	MTBPM	MTBI
0.001	0.9740	78047	410.9	205.32
0.002	0.9549	31699	397.4	209.44
0.003	0.9316	18316	391.3	214.60
0.004	0.9058	12427	390.3	220.79
0.005	0.8784	9264	392.6	227.67
0.006	0.8504	7347	397.5	235.16
0.007	0.8225	6086	404.3	243.15
0.008	0.7950	5206	412.5	251.56
0.009	0.7686	4563	421.8	260.32
0.01	0.7424	4077	432.0	269.38

TABLE 2. Performance indices by varying λ_c

λ_c	A_v	MTBCM	MTBPM	MTBI
0.0001	0.9640	22822.46	1425.34	490.01
0.0002	0.9621	21263.01	1527.38	700.05
0.0003	0.9600	21482.02	1584.01	909.52
0.0004	0.9579	22201.57	1627.18	1050.26
0.0005	0.9558	23169.94	1664.31	1160.65
0.0006	0.9538	24306.67	1698.98	1246.25
0.0007	0.9526	25580.43	1730.84	1305.85
0.0008	0.9518	26978.55	1762.54	1356.65
0.0009	0.9502	28496.80	1793.45	1400.54
0.001	0.9496	30135.37	1825.21	1432.58

TABLE 3. Performance indices by different α

α	A_v	MTBCM	MTBPM	MTBI
0.01	0.9344	32391.84	312.03	107.01
0.02	0.8942	32279.30	128.31	55.91
0.03	0.8561	35389.44	77.81	38.93
0.04	0.8190	39572.19	55.46	30.52
0.05	0.7834	44412.74	43.21	25.52
0.06	0.7495	49775.27	35.61	22.23
0.07	0.7176	55602.21	30.50	19.90
0.08	0.6876	61865.01	26.85	18.17
0.09	0.6595	68547.85	24.13	16.84
0.1	0.6332	75641.32	22.03	15.79

TABLE 4. Performance indices by varying θ

θ	A_v	MTBCM	MTBPM	MTBI
0.1	0.9612	27063.31	5711.10	208.70
0.2	0.9693	34914.38	5020.03	206.14
0.3	0.9754	44480.10	4637.98	205.30
0.4	0.9801	56257.14	4410.09	204.58
0.5	0.9837	70916.33	4271.81	203.04
0.6	0.9866	89375.31	4191.70	202.09
0.7	0.9889	11290.70	4152.82	202.34
0.8	0.9908	14330.10	4145.50	201.52
0.9	0.9923	18312.10	4134.01	201.42
1	0.9935	23611.50	4105.08	201.21

TABLE 5. Optimal values of threshold (N^*) for varying λ

λ	0.01	0.03	0.05	0.07	0.09
N*	7	9	8	7	3
A_v	0.7020	0.6305	0.7273	0.7092	0.6569
MTBCM	6086.54	4563.25	5206.64	6086.81	18316.42
MTBPM	404.35	421.80	412.50	404.30	391.30
MTBI	243.15	260.32	251.56	243.15	214.60

TABLE 6. Optimal values of threshold (N^*) for varying λ_c

λ_c	0.001	0.003	0.005	0.007	0.009
N*	3	2	5	6	4
A_v	0.8989	0.9221	0.9584	0.9602	0.9601
MTBCM	25580.43	21482.02	26978.55	25580.43	24896.8
MTBPM	1755.84	1584.01	1762.54	1730.84	1793.45
MTBI	1043.42	1052.27	1041.83	1043.42	1039.91

TABLE 7. Optimal values of threshold (N^*) for varying α

α	0.001	0.003	0.005	0.007	0.009
N*	5	3	6	7	9
A_v	0.9047	0.7723	0.6283	0.5456	0.4893
MTBCM	5299.65	1416.96	1623.25	2063.12	1982.84
MTBPM	155.86	50.84	37.50	30.59	27.50
MTBI	110.52	45.66	31.82	26.18	22.70

TABLE 8. Optimal values of threshold (N^*) for varying θ

θ	0.1	0.3	0.5	0.7	0.9
N*	4	3	5	6	8
A_v	0.9276	0.9433	0.9472	0.9492	0.9545
MTBCM	348.67	304.44	398.77	491.75	489.75
MTBPM	128.49	113.47	115.80	115.87	119.77
MTBI	101.19	105.93	105.02	104.94	104.86

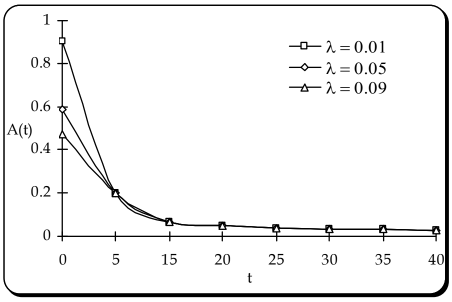

Figure 2. Availability vs t by varying λ

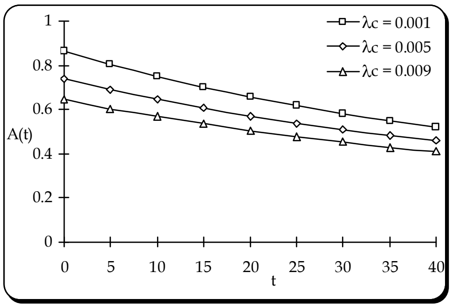

Figure 3. Availability vs t by varying λ_c

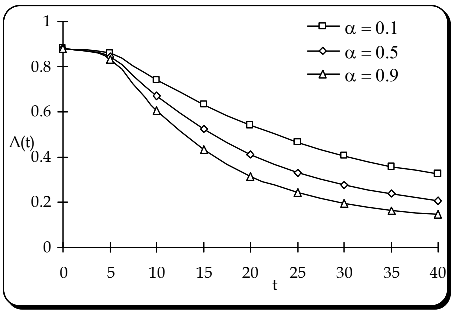

Figure 4. Availability vs t for different α

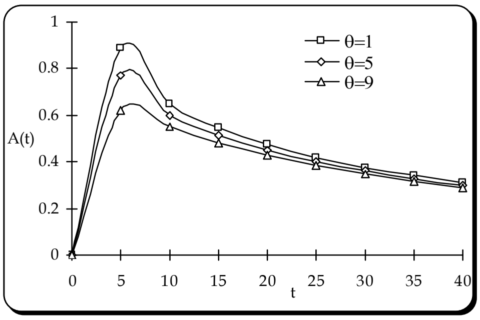

Figure 5. Availability vs t for different θ

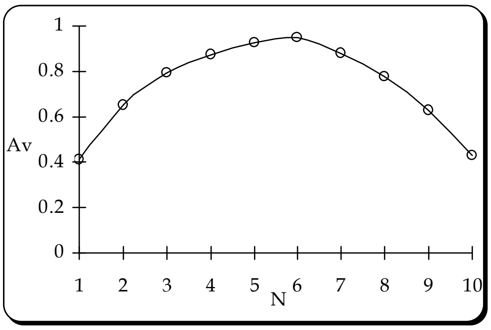

Figure 6(a). Steady state availability vs N

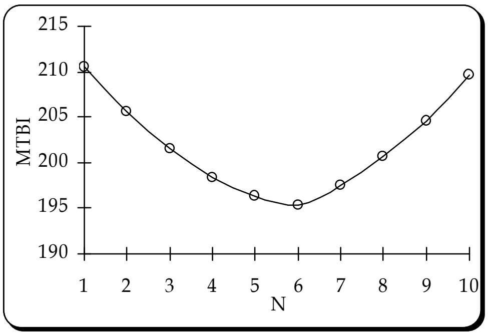

Figure 6(b). Mean time between inspection vs N

7. CONCLUSION

A decision model for determining the mean time between inspections of an object with sequential discrete states is presented. This includes the evaluation of the probabilities of various stages of a deteriorating system having preventive or corrective maintenance action at each inspection. Finally, the case study also facilitates the optimal threshold policy for preventive maintenance. The deterioration model developed as a Markov model allows the decision makers to properly propagate the uncertainty of the component's condition over time.

The model provides easy approach for solving the maintainability issues of many real time systems. The suitable examples can be found in defense wherein all weapons most of the time are in standby state and hence must be checked at periodic time.

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Maintainability Policy for Deteriorating System with Inspection and Common Cause Failure

TECHNICAL NOTE

A. Mishra[a]; M. Jain[b]

[a] *Department of Mathematics, I.B.S. Khandari , Dr. B. R. Ambedkar University, Agra,*
[b] *Department of Mathematics, I.I.T Roorkee,*

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چکیده

شرایطی بر پایه سیاست حفاظتی جلوگیری کننده و تصحیح کننده برای سیستم عملکردی پیوسته پیشنهاد شده است. فرض شده است که شرایط سیستم با زمان رو به زوال می رود. مدل هم زوال و هم شکستهای مشترک تصادفی را در بر می گیرد. مراحل زوال به صورت فرایندهای حالت مجزا مدل شده است. سیستم در بازرسی تصادفی قرار داده شد تا شرایط دانسته شود. زمان های متوسط بین بازرسی ها به صورت تصادفی توزیع شد. اگر شرط مشاهده شده در بازرسی از آستانه سطح زوال تجاوز کند، حفاظت جلوگیری کننده عمل می کند غیر از این هیچ عملی انجام نمی شود و سیستم تا اجرا ادامه می یابد. مدل پیشنهاد شده به صورت زوال تجمع یافته بر پایه افزایش شکست های تصادفی در نظر گرفته می شود. راه حل های ناپایدار با استفاده از تبدیل لاپلاس همانند راه حل های حالت پایدار با استفاده از روش بازگشتی برای محاسبه احتمال های حالت سیستم پیشنهاد شده است. آزمایشات مختلف قابل اعتماد بودن سیستم انجام شد و به صورت عددی با نمایش دادن معتبر شناخته شد.

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