IJE TRANSACTIONS A: Basics Vol. 26, No. 4, (April 2013) 383-392

International Journal of Engineering

Journal Homepage: www.ije.ir

Nusselt Number Estimation along a Wavy Wall in an Inclined Lid-driven Cavity using Adaptive Neuro-Fuzzy Inference System (ANFIS)

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PAPER INFO

Paper history:
Received 08 October 2012
Received in revised form 27 November 2012
Accepted 13 December 2012

Keywords:
ANFIS
Lattice Boltzmann Method (LBM)
Mixed Convection
Nusselt Number

ABSTRACT

In this study, an adaptive neuro-fuzzy inference system (ANFIS) was developed to determine the Nusselt number (Nu) along a wavy wall in a lid-driven cavity under mixed convection regime. Firstly, the main data set of input/output vectors for training, checking and testing of the ANFIS was prepared based on the numerical results of the lattice Boltzmann method (LBM). Then, the ANFIS was developed and validated using the randomly selected data series for network testing. The applied ANFIS model has four inputs including Reynolds number (Re), Richardson number (Ri), wavy wall amplitude (A) and inclination angle (θ). Nusselt number (Nu) was the unique output of the ANFIS model. To select the best ANFIS model, the average errors of various architectures for three different data series of training, checking and testing of the main data set are calculated. Results indicated that the developed ANFIS has acceptable performance to predict the Nu number for the cited convection problem. This method can reduce computing time and cost considering acceptable accuracy of results.

doi: *10.5829/idosi.ije.2013.26.04a.07*

NOMENCLATURE

c_s	Speed of sound in Lattice scale
c_p	Specific heat at constant pressure, $(kJ.kg^{-1}.K^{-1})$
e_α	Discrete lattice velocity in α direction
F_α	External force in direction of lattice velocity
f_α^{eq}	Equilibrium distribution
g_a	Acceleration due to gravity, $(m.s^{-2})$
Gr	Grashof number, $(g.\beta.(T_h - T_c).L^3/v^2)$
k	Thermal conductivity, $(W.m^{-1}.k^{-1})$
L	Characteristic Length, (m)
Nu_{ave}	Average Nusselt number
Nu	Local Nusselt number
Pr	Prandtl number, (v/α)
Re	Reynolds number, $(u_0.L/v)$
Ri	Richardson number, (Gr/Re^2)
A	wavy wall curve amplitude
T	Temperature, (k)
T_{ref}	Bulk temperature (k), $(T_{ref} = (T_h + T_c)/2)$
u_0	Velocity of lid wall, $(m.s^{-1})$
w_α	Weighting factor

Greek Symbols

β	Thermal expansion coefficient, $(1.k^{-1})$
Δt	Lattice time step
θ	Inclination angle
ρ	Density, $(kg.m^{-3})$
τ	Lattice relaxation time

Subscripts

ave	average
c	cold
f	fluid
h	hot
max	maximum

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1. INTRODUCTION

Convection heat transfer from wavy surfaces is observed in many engineering and scientific applications such as electronic devices, solar collectors, and wavy-plate condensers in refrigerators. In recent years many studies about convection heat transfer and fluid flow in wavy enclosures have been carried out using traditional CFD methods [1-6].

Al-Amiri et al. [1] have conducted a study on mixed convection heat transfer in a lid-driven cavity with a sinusoidal bottom wall. They have used a finite element approach based on the Galerkin method. They studied the effect of the Richardson number, amplitude of the surface, and the number of undulations on the fluid flow and heat transfer. Adjlout et al. [2] presented a numerical simulation of natural convection in an inclined cavity with wavy wall. They used partial differential equations in their work and illustrated that the Nusselt number decreases in comparison with the square cavity. Also, they investigated the effect of inclination on natural convection for different rotation angles.

The corrugated wall geometry is one of the several devices being used in industrial transport processes. The viscous flow in wavy channels was analytically studied by Bums and Parkes [7]. Also, Goldstein and Sparrow [8] used the naphthalene technique to measure local and average heat transfer coefficients in a corrugated wall channel.

The lattice Boltzmann method (LBM) is a numerical technique based on kinetic theory for simulating fluid flows and modeling the physics in fluids. In the last decade, LBM has been used as a powerful numerical technique to simulate heat transfer and fluid flow [9-17]. LBM has well-known advantages such as easy implementation, possibility of parallel coding and simulating of complex fluid dynamic problems (e.g. complex geometries [11], multiphase flow [12], porous media [13], fuel cell modeling [14], and Nanofluids [15]). The LBM utilizes two distribution functions, for the flow and temperature fields. It models the movement of fluid particles to define macroscopic parameters of fluid flow. Basically, LBM applies uniform Cartesian cells to the discrete problem domain. Each cell of the grid contains a constant number of distribution functions, which represent the number of fluid particle movement in these separated directions. The distribution functions are obtained by solving the lattice Boltzmann equation (LBE), which is a special form of the Kinetic Boltzmann Equation.

Although LBM is a powerful method to obtain the flow and temperature fields of convection problems in different shaped enclosures and channels, but achieving the adequate accuracy and stability for solving the complex convection heat transfer problems needs to pass a time consuming and expensive computation process.

To reduce the computational efforts, the soft programming techniques such as artificial neural network (ANN) and fuzzy-logic (FL) can be used in conjunction with LBM as powerful tools to determine the solution of flow, heat and mass transfer problems.

In this paper, firstly data set of Reynolds number (Re), Richardson number (Ri), wavy wall curve amplitude (A) and inclination angle (θ) assumed as inputs and Nusselt number (Nu) are derived from the LBM as output for the mixed convection problem. Then, an adaptive neuro-fuzzy inference system (ANFIS) is developed based on derived data from LBM to predict the effects of inclination phenomenon on convection heat transfer rate from a wavy wall in a lid driven cavity.

Novelty of this research is in using a combination of neural network and fuzzy logic (ANFIS model) to predict the Nusselt number which has not been worked before.

2. PROBLEM DESCRIPTION

In thermal convection problems, the Richardson number ($Ri=Gr/Re^2$) is a dimensionless parameter which plays a more effective role in natural convection compared with the forced convection. If the Richardson number becomes significantly less than unity, the buoyancy force term may be vanished and consequently the natural convection becomes negligible ($Ri<0.1$). On the other hand, when Ri grows, the natural convection becomes dominant and the forced convection is negligible ($Ri>10$). Furthermore, if Ri gets into the order of unity, the flow is likely to be buoyancy-driven and both forced and natural convection will be important ($0.1<Ri<10$).

Forced convection is independent from the gravity force, thus it is predicted that inclination has no effect on forced convection. On the other hand, natural convection completely depends on the buoyancy force that is in relation with the gravity force direction. Therefore, the inclination plays an important role in both fluid flow and temperature field when natural convection is dominant. To cover all conditions of convection heat transfer regimes including forced, natural and mixed convection, a wide range of input variables must be studied at the numerical solution. It can be very time consuming and more costly. Therefore, this manuscript tries to present an Adaptive Neuro-Fuzzy Inference System (ANFIS) to predict the needed output parameter in a wide range of input parameters. The modeled ANFIS architecture is based on the LBM results for special case studies at specific input data. The selected input data are covered necessary range of input variables to delineate the inclination effects on convection heat transfer from a wavy wall in a lid-driven cavity.

The geometry of the problem under study is shown in Figure 1. The vertical walls of the cavity are assumed to be insulated while the wavy bottom surface is maintained at a uniform temperature (T_h) higher than the top lid temperature (T_c). The lid wall has a constant velocity motion equal to 0.1 of sound speed in a left to right direction (see Figure 1).

The working fluid is assumed to be Newtonian with constant fluid properties, and the flow is considered to be laminar, incompressible, steady and two-dimensional. Viscous dissipation was neglected, because it has a negligible effect. The effect of different inclination angles ($0 \le \theta \le 180$) on both fluid flow and heat transfer is investigated for different Richardson numbers ($0.1 \le Ri \le 10$), amplitudes of wavy wall ($0.05 \le A \le 0.25$) while the Reynolds number is equal to 100, 150 and 200 and the Prandtl number is fixed to 0.71 for air flow. The wavy wall is defined as follows:

$$Y = A(1+\sin(2\pi X)) \tag{1}$$

where A is the curve amplitude. Also, the local Nusselt number along the wavy wall is defined as follows:

$$Nu = -L\frac{\partial T}{\partial n} \tag{2}$$

where n is the coordinate direction normal to the wavy wall and L is the characteristic length that is given by:

$$L = \int_0^1 \sqrt{1+(2A\pi\cos(2\pi X))^2}\, dX \tag{3}$$

By integrating the local Nusselt number along the wavy surface, the average Nusselt number is calculated as follows:

$$Nu_{ave} = \frac{1}{L}\int_0^L Nu\, ds \tag{4}$$

where s shows the integral calculated along the wavy line.

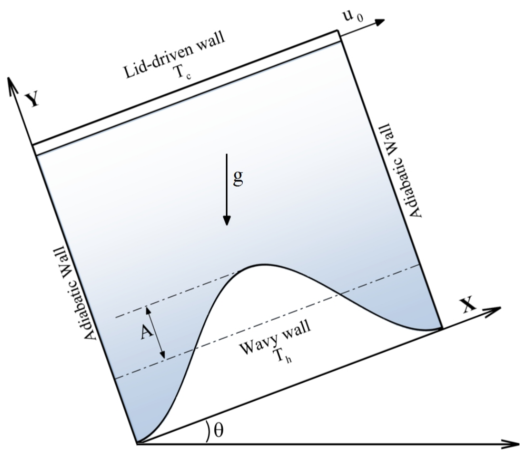

Figure 1. Geometry of the problem.

3. LATTICE BOLTZMANN METHOD

The basic form of the Lattice Boltzmann Equation which is a special form of the kinetic Boltzmann equation with an external force by introducing BGK approximations can be written as follows for both flow and the temperature fields [9]:

$$f_\alpha(x + e_\alpha \Delta t, t + \Delta t) = f_\alpha(x,t) + \frac{\Delta t}{\tau_m}[f_\alpha^{eq}(x,t) - f_\alpha(x,t)] + \Delta t e_\alpha F_\alpha \tag{5}$$

$$g_\alpha(x + e_\alpha \Delta t, t + \Delta t) = g_\alpha(x,t) + \frac{\Delta t}{\tau_t}[g_\alpha^{eq}(x,t) - g_\alpha(x,t)] \tag{6}$$

where $f_\alpha(x,t)$, e_α and F_α are the distribution function on the mesoscopic level, the discrete lattice velocity and F_α ithe external force term in α direction, respectively. f_α^{eq} and g_α^{eq} are equilibrium distribution functions that are calculated f as follows:

$$f_\alpha^{eq} = w_\alpha \rho[1 + \frac{e_\alpha u}{c_s^2} + \frac{1}{2}\frac{(e_\alpha u)^2}{c_s^4} - \frac{1}{2}\frac{u^2}{c_s^2}] \tag{7}$$

$$g_\alpha^{eq} = \begin{cases} -\frac{3}{2} w_0 \rho R T \vec{u}^2 & \alpha=0 \\ \frac{3}{2} w_\alpha \rho R T[1+(e_\alpha.\vec{u})+\frac{3}{2}(e_\alpha.\vec{u})^2-\vec{u}^2] & \alpha=1\text{-}4 \\ w_\alpha \rho R T[3+6(e_\alpha.\vec{u})+\frac{9}{2}(e_\alpha.\vec{u})^2-\frac{3}{2}\vec{u}^2] & \alpha=5\text{-}8 \end{cases} \tag{8}$$

where ρ and w_α are the lattice fluid density and weighting factor, respectively. The latter has these values: $w_0 = 4/9$ for $|c_0| = 0$, $w_{1\text{-}4} = 1/9$ for $|c_{1\text{-}4}| = 1$ and $w_{5\text{-}9} = 1/36$ for $|c_{5\text{-}9}| = \sqrt{2}$ in the D2Q9 model [9]. To model buoyancy force, the force term in "Equal 5" needs to be assumed as below in the required direction:

$$F_\alpha = 3 w_\alpha g_\alpha \beta(T - T_{ref}) \tag{9}$$

where T_{ref} is assumed as average of T_h and T_c. Macroscopic variables can be calculated as follows:

$$\rho = \sum_\alpha f_\alpha,\ \ \rho u_i = \sum_\alpha f_\alpha c_{i\alpha},\ \ T = \sum_\alpha g_\alpha \tag{10}$$

The boundary fitting method (BFM) accuracy at velocity and temperature fields is used to simulate the curved boundary in the LBM. The problem is investigated for different Richardson numbers ($0.1 \le Ri \le 10$), curve amplitudes ($0.05 \le A \le 0.25$) and inclination angles ($0 \le \theta \le 180$) when the Reynolds number is equal to 100, 150 and 200. The numerical code of LBM is validated with the results presented by Al-Amiri et al. [1] for mixed convection in a lid-driven cavity with a wavy sinusoidal bottom surface. Table 1 shows the comparison of the values of average Nusselt number of this study with those presented by Al-amiri et al. [1] exhibited good agreement between the present results and other published data.

TABLE 1. Average Nusselt number along the wavy wall at Gr = 104, Pr = 1, A = 0.05 and θ = 0.

Ri Value	Present study	Al-Amiri et al. [1]
Ri = 0.1	7.242	7.412
Ri = 1	3.200	3.192
Ri = 10	2.829	2.694

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4. ADAPTIVE NEURO-FUZZY INFERENCE SYSTEM (ANFIS)

An ANFIS gives the mapping relation between the input and output data using hybrid learning method to determine the optimal distribution of membership functions [18]. Both artificial neural network and fuzzy logic are used in ANFIS architecture. Basically, five layers are used to construct this inference system. Each ANFIS layer consists of several nodes described by the node function. The inputs for the layers are obtained from the nodes in the previous layers. Figure 2 shows the ANFIS structure for a system with m inputs ($X_1 \ldots X_m$), each with n membership functions (MFs), a fuzzy rule base of R rules and one output (Y). The network consisting of five layers is used for training Sugeno-type fuzzy interface system (FIS) through learning and adaptation. Number of nodes (N) in layer 1 is the product of numbers of inputs (m) and MFs (n) for each input, i.e., N=mn. Number of nodes in layers 2-4 depends on the number of rules (R) in the fuzzy rule base. Five Layers of ANFIS model are described in the following.

4. 1. Fuzzification Layer

It transforms the crisp inputs X_i to linguistic labels (A_{ij}, like small, medium, large, etc.) with a degree of membership. The output of node *ij* is expressed as follows:

$$O^1_{ij} = \mu_{ij}(X_i),\quad i = 1\ldots m,\ \ j = 1\ldots n \tag{11}$$

where μ_{ij} is the *j*th membership function for the input X_i. Several types of MFs can be used, including triangular curve, generalized bell function, trapezoidal curve, Gaussian function and the sigmoidal function that are used in this study.

The triangular curve is a function of a vector, *x*, and depends on three scalar parameters *a*, *b*, and *c*, as follows:

$$f(x;a,b,c) = \begin{Bmatrix} 0 & x \le a \\ x - a/b - a & a \le x \le b \\ c - x/c - b & b \le x \le c \\ 0 & c \le x \end{Bmatrix} \tag{12}$$

where a and c locate the "feet" of the triangle and the parameter b locates the peak. The generalized bell function depends on three parameters a, b, and c as follows:

$$f(x;a,b,c) = \frac{1}{1 + \left|\frac{x-c}{a}\right|^{2b}} \tag{13}$$

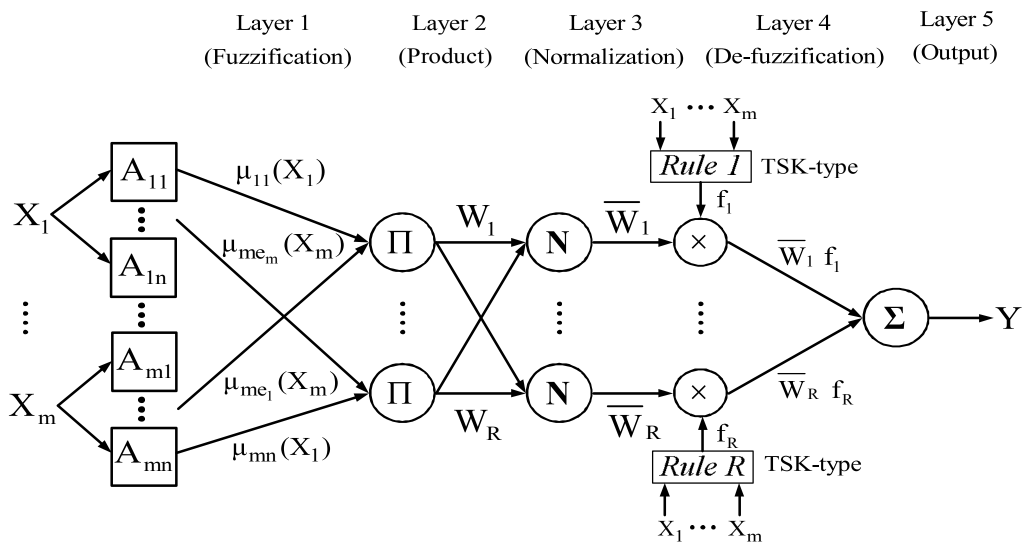

Figure 2. Schematic of the ANFIS structure.

where a and b vary the width of the curve and the parameter c locates the center of the curve, where b is usually positive. The trapezoidal curve is a function of a vector, x, and depends on four scalar parameters a, b, c, and d, as given by the following:

$$f(x;a,b,c,d)=\begin{Bmatrix} 0 & x\le a \\ (x\text{-}a)/(b\text{-}a) & a\le x\le b \\ 1 & b\le x\le c \\ (d\text{-}x)/(d-c) & c\le x\le d \\ 0 & d\le x \end{Bmatrix} \quad (14)$$

where the parameters a and d locate the "feet" of the trapezoid and the parameters b and c locate the "shoulders". The Gaussian function depends on two parameters σ and c as given by the following:

$$f(x;\sigma,c)=e^{\frac{-(x-c)^2}{2\sigma^2}} \quad (15)$$

where the parameter c locates the peak.

The sigmoidal membership function is a mapping on a vector x, and depends on two parameters a and c as given in the following equation:

$$f(x;\sigma,c)=\frac{1}{1+e^{-a(x-c)}} \quad (16)$$

Depending on the sign of the parameter a, the sigmoidal membership function is inherently open to the right or to the left.

4. 2. Product Layer For each node *k* in this layer, the output represents weighting factor (firing strength) of the rule *k.* The output W_k is the product of all its inputs as follows:

$$O_k^2=W_k=\mu_{1e_1}(X_1)\,\mu_{2e_2}(X_2)\,...\,\mu_{me_m}(X_m)$$
$$k=1...R,\quad e_1,e_2,...,e_m=1...n \quad (17)$$

4. 3. Normalized Layer The output of each node k in this layer represents the normalized weighting factor $\overline{W}_k$ of the k^{th} rule as follows:

$$O_k^3=\overline{W}_k=\frac{W_k}{W_1+W_2+...+W_R} \quad (18)$$

4. 4. De-Fuzzification Layer Each node of this layer gives a weighted output of the first order TSK-type fuzzy if-then rule as follows:

$$O_k^4=\overline{W}_k\ f_k \quad (19)$$

where f_k represents the output of k^{th} TSK-type fuzzy rules as follows:

$$\text{If } (X_1 \text{ is } A_{1e_1}) \text{ and } (X_2 \text{ is } A_{2e_2}) \text{ and ...}$$
$$\text{and } (X_m \text{ is } A_{me_m}) \text{ Then } f_k=\sum_{i=1}^{m}p_{ie_i}X_i+r_k \quad (20)$$
$$(e_1,\ e_2\dots\ e_m=1\dots n),\ (k=1...R)$$

where p_{ie_i} and r_k are called consequent parameters.

4. 5. Output Layer This single-node layer represents the overall output (Y) of the network as the sum of all weighted outputs of the rules:

$$O^5=Y=\sum_{k=1}^{n}\overline{W_k}\ f_k \quad (21)$$

5. RESULTS AND DISSCUTION

The procedure of the ANFIS development for this study consists of three main steps include: data set preparation, data pre-processing and developing the ANFIS network. All these steps are described in the following sections.

5. 1. Data Set Preparation The main data set comprising of 189 pairs of four inputs (Reynolds number (Re), Richardson number (Ri), wavy wall curve amplitude (A) or inclination angle (θ)) and one output (Nusselt number (Nu)) is prepared based on the already mentioned LBM. A Sample of this database is shown in Table 2.

TABLE 2. Part of data set obtained via LBM.

No.	Inputs				Output	Train data	Check data	Test data
	Re	Ri	A	θ	Nu			
1	100	0.1	0.05	0	2.125	*	–	–
2	100	0.1	0.05	30	2.046	*	–	–
3	100	0.1	0.05	60	1.960	–	*	–
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
.	.	.	.	.	.	.	.	.
186	200	10	0.25	90	3.412	*	–	–
187	200	10	0.25	120	2.477	*	–	–
188	200	10	0.25	150	1.623	–	–	*

For each input data, such Reynolds and Richardson numbers and curve wall amplitude, three values, is selected according to the needed range to describe the physics of the problem truly when inclination angels vary from 0 to 180^o at 30^o steps. The selected values of Ri must cover all convection heat transfer regimes. Thus 0.1, 1 and 10 were selected. When Ri is equal to 0.1 it means that forced convection is dominant. On the other hand, when the Ri is equal to 10, natural convection is dominant. Considering the values of Ri, the Re values are assumed to be equal to 100, 150 and 200 to have a laminar flow field.

5. 2. Data Pre-processing It is important to process the data set into patterns before the ANFIS can be trained and the mapping learnt. Training/ checking/ testing pattern vectors are formed. Each pattern is formed with an input condition vector and the corresponding target vector [19, 20]. The scale of the input and output data is an important matter to consider, especially when the operating ranges of process parameters are different. The scaling or normalizing ensures that the ANFIS will be trained effectively, without any particular variable skewing the results significantly. As a result, all of the input parameters are equally important in the training of the network. The scaling is performed by mapping each term to a value between "0" and "1" using the following equation:

$$V_{norm} = \frac{V_i - V_{min}}{V_{max} - V_{min}} \quad (22)$$

where V_{norm} is the normalized value, V_i is the value of a certain variable (Re, Ri, A or θ), and V_{max} and V_{min} are the maximum and minimum values of the of the independent variable, respectively.

The input pattern vectors are then formed; comprising of 137, 22 and 30 pairs of input/output ones respectively for training, checking and testing the network on the basis of the main data set.

5. 3. Development of ANFIS Model This section is composed of three stages: determining the suitable topology of the ANFIS model by trial and error, training the ANFIS along with checking to prevent overfitting of the network and validating the developed ANFIS by using the test data series. After data preprocessing, 137, 22 and 30 pairs of input/output ones are randomly selected on the basis of the main data set as training, checking and testing data series, respectively (e.g. data marked by " * " in Table 2). The testing data series is used in ANFIS learning process and the checking data series is used for testing the ANFIS network along with the training to prevent overfitting of the network. Also, the testing data series is presented to the trained network as new application data for verifying or testing the predictive accuracy of the network model. Thus, the network is evaluated using data that have not been used for training and checking.

Defining fuzzy membership functions and corresponding values can be considered as an important stage in the modeling. In order to determine the suitable architecture of the ANFIS by trial and error, different membership functions with [3 3 3 3] structure, 81 fuzzy "If–then rules" to train the model and hybrid optimization method were tested on three different combination of training, checking and testing data sets. The average Root Mean Square Error (RMSE) was calculated (Table 3) according to Equation (23). The advantage of hybrid method is that it uses back propagation for parameter associated with input membership function and least square estimation for parameters associated with output membership. As shown in Table 3, the Triangular, Gaussian and Sigmoidal membership functions (the zero-order Sugeno fuzzy rules) have demonstrated the best performances respectively (with least resultant testing RMSE). Finally, the triangular membership functions (see Figures 3a–d), the simplest MF formed from straight lines, has been chosen because of its valid results (see Figure 4 and Table 4). The structure of the developed ANFIS model is shown in Figure 4.

TABLE 3. Average root mean squared error (RMSE) of different membership functions.

MF type	Average Error (All Data sets)					
	Average training Error		Average checking Error		Average testing Error	
	Cons.	Lin.	Cons.	Lin.	Cons.	Lin.
Triangular	***0.108***	0.001	***0.275***	0.470	***0.313***	0.542
Bell- shaped	0.101	0.001	0.381	0.390	0.350	0.502
Trapezoidal	0.264	0.031	0.372	1.279	0.414	1.263
Gaussian	***0.108***	0.001	***0.285***	0.456	***0.334***	0.648
Sigmoidal	***0.104***	0.001	***0.292***	0.525	***0.337***	0.688

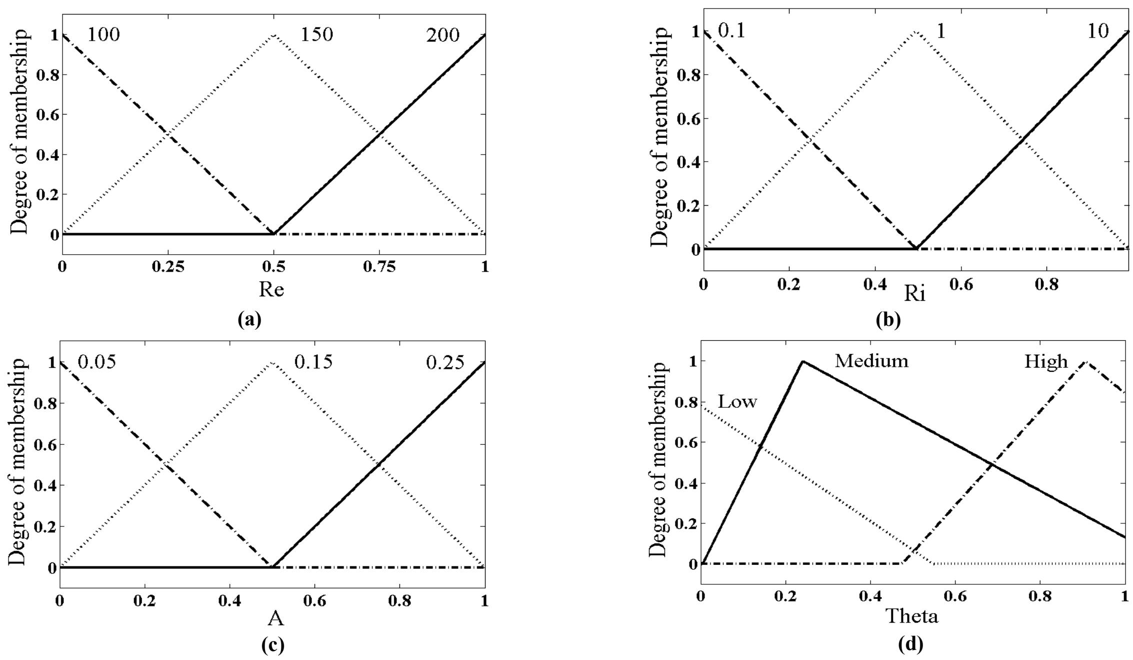

Figure 3. Membership function plot for inputs: (a) “Reynolds number (Re)”, (b) “Richardson number (Ri)”, (c) “wavy wall curve amplitude (A)”, (d) “inclination angle (θ)”.

TABLE 4. Obtained (LBM) and estimated (ANFIS) Nusselt number values from neuro-fuzzy model together with percentage error.

No.	Re	Ri	A	θ	Obtained Nu	Predicted Nu	Error (%)
1	100	0.1	0.05	90	1.895	1.903	0.466
2	100	0.1	0.15	60	2.123	2.111	0.559
3	100	0.1	0.25	0	2.229	2.252	1.056
4	100	1	0.05	30	1.846	1.516	17.898
5	100	1	0.05	180	1.396	1.190	14.750
6	100	1	0.15	90	1.488	1.485	0.160
7	100	1	0.25	90	1.995	2.017	1.119
8	100	10	0.05	60	3.746	3.525	5.887
9	100	10	0.05	180	0.751	0.612	18.570
10	100	10	0.15	150	1.153	1.403	21.672
11	100	10	0.25	120	1.724	1.890	9.581
12	150	0	0.05	90	2.212	2.191	0.985
13	150	0	0.15	90	2.483	2.500	0.675
14	150	0	0.25	180	2.611	2.594	0.657
15	150	1	0.05	180	1.437	1.443	0.419
16	150	1	0.15	150	1.572	1.733	10.241
17	150	1	0.25	180	2.391	2.227	6.833
18	150	10	0.05	150	0.984	1.441	46.483
19	150	10	0.15	90	3.382	3.256	3.715
20	150	10	0.25	60	3.341	3.143	5.932
21	200	0	0.05	90	2.479	2.499	0.796
22	200	0	0.15	90	2.793	2.794	0.034
23	200	0	0.25	120	2.891	2.901	0.339
24	200	1	0.05	90	2.143	1.701	20.624
25	200	1	0.15	30	2.987	2.710	9.264
26	200	1	0.25	30	2.999	2.870	4.319
27	200	10	0.05	30	5.178	5.501	6.233
28	200	10	0.15	30	4.462	4.977	11.535
29	200	10	0.25	30	3.803	4.500	18.344
30	200	10	0.25	150	1.623	1.941	19.607

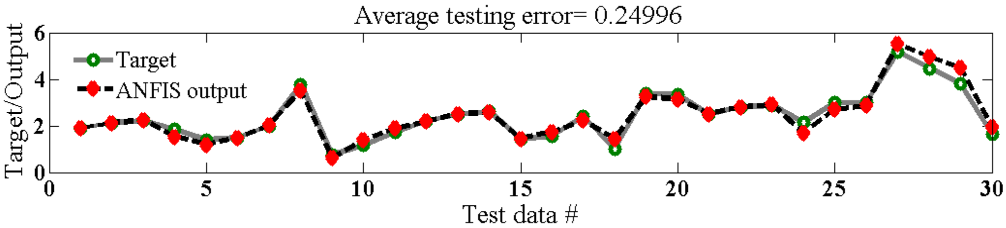

Figure 4. Testing (c) of the [3 3 3 3] ANFIS with Triangular membership function.

The training performance of the ANFIS model can be checked by the Root Mean Square Error (RMSE), Mean Absolute Percentage Error (MAPE) and Variance Accounted For (VAF) as follow:

$$\mathrm{RMSE}=\sqrt{\frac{1}{N}\sum_{i=1}^{N}(\mathrm{target}_i\text{-}\mathrm{output}_i)^2} \quad (23)$$

$$\mathrm{MAPE}=\frac{100}{N}\times\sum_{i=1}^{N}\left|\frac{\mathrm{target}_i-\mathrm{output}_i}{\mathrm{target}_i}\right| \quad (24)$$

$$\mathrm{VAF}=(1-\frac{\mathrm{var}(\mathrm{target}-\mathrm{outpt})}{\mathrm{var}(\mathrm{output})})\times 100 \quad (25)$$

$$\mathrm{var}(y)=\text{variance in set }(y)=\frac{1}{N}\sum_{i=1}^{N}(y_i-\bar{y})^2 \quad (26)$$

where N is the total number of testing patterns (30 patterns), *target*/*target*$_i$ is the target value, *output*$_i$ is the ANFIS output value, and $\bar{y}$ is the average of the set y. Subscript *i* indicates ith data in the set.

The performance indices RMSE, MAPE and VAF were calculated as 0.250, 8.625 and 94.698, respectively. Theoretically, a prediction model is accepted as excellent when RMSE and MAPE are equal to zero and VAF is 100%. Performance indices RMSE, MAPE and VAF indicate that assessed result had an acceptable accuracy in the specified range.

Also, the regression line of the targets/outputs is shown in Figures 5 and 6.

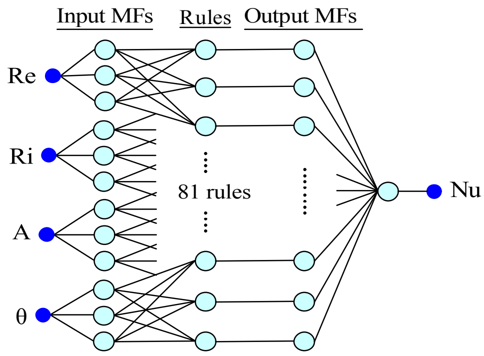

Figure 5. Schematic structure of the developed ANFIS model.

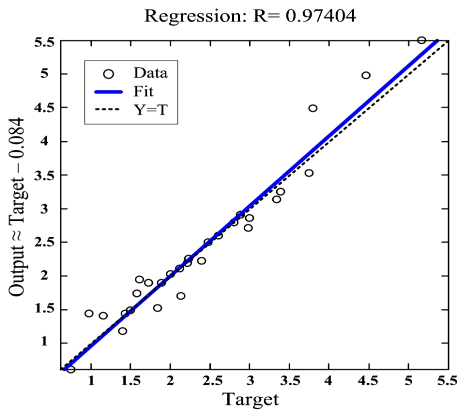

Figure 6. Cross-correlation between estimated and obtained Nusselt number values.

6. CONCLUSIONS

An Adaptive Neuro-Fuzzy Inference System (ANFIS) was developed to combine with the Lattice Boltzmann Method (LBM) in order to study mixed convection rate on a wavy wall in a 2D cavity. This combination reduced the calculation time and its cost. The problem was investigated for different Richardson numbers ($0.1 \le \mathrm{Ri} \le 10$), curve amplitudes ($0.05 \le \mathrm{A} \le 0.25$) and inclination angles ($0 \le \theta \le 180$) when the Reynolds number was equal to 100, 150 and 200. The prediction has been done based on LBM data for the cited problem with normalized inputs and output. The triangular membership function has been considered with [3 3 3 3] structure and hybrid optimization Method.

The ANFIS had four inputs including Reynolds number (Re), Richardson number (Ri), wavy wall curve amplitude (A), inclination angle (θ) and Nusselt number (Nu) as the one output. The input pattern vectors were formed comprising 137, 22 and 30 pairs of input/output ones respectively for training, checking and testing the network on the basis of the main data set. Results obtained from LBM were compared to the ANFIS results for the same case and it was seen that the developed ANFIS has a capability to properly estimate Nu number in the cited problem. The performance indices VAF, RMSE and MAPE, were calculated as 94.698, 0.250 and 8.625, respectively. This indicate that assessed result had an acceptable accuracy in the specified range.

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Nusselt Number Estimation along a Wavy Wall in an Inclined Lid-driven Cavity using Adaptive Neuro-Fuzzy Inference System (ANFIS)

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PAPER INFO

Paper history:
Received 08 October 2012
Received in revised form 27 November 2012
Accepted 13 December 2012

Keywords:
ANFIS
Lattice Boltzmann Method (LBM)
Mixed Convection
Nusselt Number

چکیده

در پژوهش حاضر، یک شبکه‌ی عصبی- فازی تطبیقی برای تعیین عدد ناسلت (Nu) بر روی دیواره‌ی مواج در یک محفظه‌ی درپوش‌دار با به‌کارگیری نتایج حاصل از روش شبکه‌ی بولتزمن ارائه شده‌است. در ابتدا مجموعه‌ی داده‌ها برای آموزش، ارزیابی و آزمایش شبکه‌ی عصبی- فازی تطبیقی، بر پایه‌ی نتایج به‌دست آمده از حل مسئله به روش شبکه‌ی بولتزمن، تهیه می‌شود. سپس شبکه‌ی عصبی- فازی تطبیقی توسعه و آموزش داده شده و در نهایت با نتایج به‌دست آمده از شبکه‌ی بولتزمن اعتبارسنجی می‌گردد. عدد رینولدز (Re)، عدد ریچاردسون (Ri)، دامنه‌ی دیواره‌ی مواج (A) و زاویه‌ی دوران (θ) به عنوان چهار ورودی و عدد ناسلت به عنوان تنها خروجی شبکه‌ی عصبی- فازی تطبیقی مدنظر قرار گرفته‌است. میانگین مجموع مربعات خطا برای سه آرایش گوناگون داده‌های آموزش، ارزیابی و آزمایش به ازای معماری‌های مختلف شبکه مورد مقایسه قرار گرفته و بر این اساس بهترین معماری شبکه انتخاب می‌گردد. پژوهش حاضر می‌کوشد تا با استفاده از شبکه‌ی عصبی- فازی تطبیقی آموزش داده‌شده برای مقادیر خاص ورودی، عدد ناسلت در هندسه‌ی بیان شده را برای سایر مقادیر ورودی در محدوده‌ی تعریف‌شده پیش‌بینی کند. نتایج نشان می‌دهند که استفاده از شبکه‌ی عصبی- فازی تطبیقی ضمن ارائه‌ی دقت قابل قبول می‌تواند موجب کاهش زمان و هزینه‌ی صرف‌شده در مقایسه با حل دقیق عددی شود.

doi: *10.5829/idosi.ije.2013.26.04a.07*