IJE TRANSACTIONS A: Basics Vol. 25, No. 4, (October 2012) 399-410

International Journal of Engineering

Journal Homepage: www.ije.ir

Three Dimensional Laminar Convection Flow of Radiating Gas over a Backward Facing Step in a Duct

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PAPER INFO

Paper history:
Received 8 January 2012
Accepted in revised form 30 August 2012

Keywords:
Laminar Convection Flow
Backward Facing Step
Radiation Heat Transfer
DOM

ABSTRACT

In this study, three-dimensional simulations are presented for laminar forced convection flow of a radiating gas over a backward-facing step in rectangular duct. The fluid is treated as a gray, absorbing, emitting and scattering medium. The three-dimensional Cartesian coordinate system is used to solve the governing equations which are conservations of mass, momentum and energy. These equations are solved numerically using the CFD techniques to obtain the temperature and velocity fields. Discretized forms of these equations are obtained by the finite volume method and solved using the SIMPLE algorithm. Since the gas is considered as a radiating medium, all of the convection, conduction and radiation terms are presented in the energy equation. For computation of radiative term in energy equation, the radiative transfer equation (RTE) is solved numerically by the discrete ordinates method (DOM) to find the divergence of radiative heat flux distribution inside the radiating medium. The numerical results are presented graphically and the effects of the radiation-conduction parameter, optical thickness and albedo coefficient on heat transfer behavior of the system are investigated. Comparison of numerical results with the available data published in open literature shows a good consistency.

doi: *10.5829/idosi.ije.2012.25.04a.11*

NOMENCLATURE

A_x, A_y, A_z	Areas of control volume's faces normal to the x-, y- and z- directions, respectively, (m^2)	X, Y, Z	Dimensionless horizontal and vertical coordinate, respectively
AR	Aspect ratio	**Greek Symbols**	
Cp	Specific heat, ($J.kg^{-1}.K^{-1}$)	α	Thermal diffusivity, ($m^2.s^{-1}$)
ER	Expansion ratio	σ	Stefan Boatsman's constant = 5.67×10^{-8}, ($W.m^{-2}.k^{-4}$)
I	Radiation intensity, ($W.m^{-2}$)	ε	Wall emissivity
I*	Dimensionless radiation intensity	κ	Thermal conductivity
Nu_t	Total Nusselt number	μ	Dynamic viscosity, ($N.s.m^{-2}$)
Nu_r	Radiative Nusselt number	ν	Kinematic viscosity, ($m^2.s^{-1}$)
Nu_c	Convective Nusselt number	ρ	Density ($kg.\ m^{-3}$)
Pe	Peclet number	τ	Optical thickness
Pr	Prandtl number	Θ	Dimensionless temperature
q_t	Total heat flux, ($W.m^{-2}$)	Θ_b	Mean bulk temperature
q_r	Radiative heat flux, ($W.m^{-2}$)	Θ_M	Mean temperature
q_c	Convective heat flux, ($W.m^{-2}$)	θ_1, θ_2	Dimensionless temperature parameters
Re	Reynolds number	**Subscripts**	
RC	Radiation-conduction parameter	c	Convective
S	Radiation source function	in	Inlet section
S*	Dimensionless radiation source function	r	Radiative
U_o	Average velocity of the incoming flow at the inlet section, ($m.\ s^{-1}$)	t	Total
x, y, z	Horizontal and vertical distance, respectively, (m)	w	Wall

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1. INTRODUCTION

Forced convection flow in three-dimensional channels with abrupt contraction or expansion in flow geometry is widely encountered in engineering applications. Separation flows accompanied with heat transfer are frequently encountered in many systems, such as heat exchangers, gas turbine blades, combustion chamber and duct flows used in industrial applications. In some of the mentioned devices, specially, when soot particles exist in the combustion product, the radiation effect may be important. Besides, the trend toward increasing temperature in modern technological systems has promoted concerted effort to develop more comprehensive and accurate theoretical methods to treat radiation. Therefore, for having more accurate and reliable results in the analysis of these types of flow, the flowing gas must be considered as a radiating medium and all of the heat transfer mechanisms including convection, conduction and radiation, must be taken into account. The flow over backward-facing step (BFS) has the most features of separated flows. There are many studies in which, the BFS flows were analyzed from a fluid mechanics or a heat transfer perspective. Although the geometry of BFS flow is very simple, the heat transfer and fluid flow over this type of step contain most of complexities. Consequently, it has been used in the benchmark investigations. There are many studies about laminar convection flow over BFS in a two-dimensional duct by several investigators [1-4].

Velocity measurements were reported for three-dimensional laminar separated airflow adjacent to a backward-facing step using two-component laser Doppler velocimeter by Armaly et al. [5]. The results showed some interesting flow behaviors that could not be deduced from two-dimensional studies.

Iwai et al. [6] studied three-dimensional numerical simulation for flows over a backward-facing step at low Reynolds number in order to investigate the effects of the duct aspect ratio. In that study, distribution patterns of both Nusselt number and the skin friction coefficient on the bottom wall were considered. They reported that an aspect ratio of as large as AR=16 at least was needed to obtain a 2-D region at the mid-plane for Re= 250. Also, they reported that the maximum Nusselt number did not appear on the centerline but near the two side walls in every case. Three-dimensional numerical simulations were carried out for mixed convective flows over a backward-facing step in a rectangular duct by Iwai et al. [7]. Effect of the inclination angle was the main objective in that study where Reynolds number, expansion ratio and aspect ratio were constant.

Nie and Armaly [8] investigated the effects of step height on the flow and heat transfer characteristics of incompressible laminar forced convection flow adjacent to backward-facing step in a 3-D rectangular duct. They showed that the size of the primary recirculation region and the maximum Nusselt number on the heated wall increase as the step height increases. Distributions of wall temperature, Nusselt number and friction coefficient on all of the bounding walls of laminar three-dimensional forced convection flow adjacent to backward-facing step in a rectangular duct were reported by Nie and Armaly [9].

Beaudoin et al. [10] studied three-dimensional stationary structure of the flow over a backward-facing step, experimentally. That study revealed that actually three regions of two-dimensional flow was potentially unstable through the centrifugal instability.

Uruba et al. [11] experimentally investigated the control of narrow channel flow behind a backward-facing step by blowing and suction near the step foot. The intensity of flow control was characterized by the suction/blowing flow coefficient. Results indicated that both blowing and suction were able to reduce the length of the separation zone down to one third of its value without control. Besides, it was found that the existing three-dimensional vortex structures near the step were influenced by suction more than blowing.

In all of the above works, the effect of radiative heat transfer in fluid flow was not studied, such that the gas energy equation only contained the convection and conduction terms. In a forced convection problem, when the flowing gas behaves as a participating medium, its complex absorption, emission and scattering introduce a considerable difficulty in the simulation of these flows. There are limited numbers of literatures about the radiative transfer problems with complex 2-D and 3-D geometries.

Azad and Modest [12] investigated the problem of combined radiation and turbulent forced convection in absorbing, emitting and linearly anisotropic scattering gas particulate flow through a circular tube. Bouali and Mezrhab [13] studied heat transfer by laminar forced convection and surface radiation in a divided vertical channel with isotherm side walls. They found that the surface radiation has important effect on the Nusselt number in convective flow with high Reynolds numbers. Two-dimensional forced convection laminar flow of radiating gas over a backward facing step in a duct was analyzed by Ansari and Gandjalikhan Nassab [14].

Effects of wall emissivity, Reynolds number and its interaction with the conduction-radiation parameter on heat transfer behavior of the system were investigated. Also, same authors [15-16] studied the laminar forced convection flow of a radiating gas adjacent to inclined backward and forward facing step in a duct. The two-dimensional Cartesian coordinate system was used to simulate flow over inclined surface by considering the Blocked-off region in regular grid. The fluid was treated as a gray, absorbing, emitting and scattering medium.

The effect of radiative properties on heat transfer behavior of fluid flow was investigated.

Mixed convection heat transfer in 3-D horizontal and inclined ducts with considering radiation effects has been numerically examined in detail by Chiu et al. [17-18]. Those works were primarily focused on the interaction of the thermal radiation with mixed convection for a gray fluid in rectangular ducts. The vorticity–velocity method was employed to solve the three-dimensional Navier–Stokes equations while the integro-differential radiative transfer equation was solved by the discrete ordinates method.

Results revealed that radiation effects have a considerable impact on the heat transfer and would reduce the thermal buoyancy effects. Besides, it was revealed that the development of temperature was accelerated by the radiation effects.

Although there are limited studies about combined heat transfer of radiation and forced convection flow of participating gases over a BFS, but based on the author's knowledge, 3-D laminar forced convection flow of a radiating gas over a backward-facing step in a duct is not still studied by discrete ordinates method (DOM). Therefore, the present research work deals with the three-dimensional simulations of incompressible laminar forced convection flow of a radiating gas over a backward-facing step in rectangular duct, while the well known DOM is employed to solve the radiation problem.

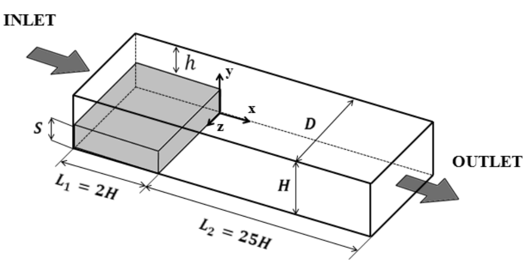


Figure 1. Schematic of computational domain

2. PROBLEM DESCRIPTION

Three-dimensional laminar forced convection of a radiating gas flow in a horizontal heated rectangular duct with a backward facing step is numerically simulated. Schematic of the computational domain is shown in Figure 1.

The upstream and downstream heights of the duct are h and H, respectively, such that this geometry provides the step height of s, with expansion ratio (ER=H/h) of 2. The width of the duct is D with an aspect ratio of AR=*D*/*h*, which is considered equal to 4 in the present computations. The upstream length of the duct is considered to be L_1=2H and the rest of the channel length is equal to L_2=25H. This is made to ensure that the flows at the inlet and outlet sections are not affected significantly by the sudden changes in the geometry and flow at the exit section becomes fully developed.

3. BASIC EQUATION

For incompressible, steady and three-dimensional laminar flow, the governing equations are the conservations of mass, momentum and energy that can be written as follows:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\frac{\mu}{\rho}\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2}\right) \quad (2)$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}+w\frac{\partial v}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\frac{\mu}{\rho}\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right) \quad (3)$$

$$u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}+w\frac{\partial w}{\partial z}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\frac{\mu}{\rho}\left(\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right) \quad (4)$$

$$\frac{\partial}{\partial x}(\rho u c_p T)+\frac{\partial}{\partial y}(\rho v c_p T)+\frac{\partial}{\partial z}(\rho w c_p T)=\kappa\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}+\frac{\partial^2 T}{\partial z^2}\right) -\nabla \vec{q}_r \quad (5)$$

In the above equations, u, v and w are the velocity components in x-, y- and z- directions, respectively, ρ the density, p the pressure, T the temperature, μ the dynamic viscosity, c_p specific heat, κ the thermal conductivity and $\vec{q}_r$ is the radiative flux vector.

The boundary conditions are treated as no slip condition at the solid walls (zero velocity) and constant temperature of T_w at the bottom, top, sides and step walls. At the inlet duct section, the flow has uniform velocity of $U_\circ$ with uniform temperature of T_{in}, which is assumed to be lower than T_w. At the outlet section, zero axial gradients for velocity components and gas temperature are employed.

3. 1. Gas Radiation Modeling

In the energy equation, besides the convective and conductive terms, the radiative term as the divergence of the radiative heat flux, i.e. $\nabla.\vec{q}_r$ is also presented. This radiative term can be computed as follow [19]:

$$\nabla.\vec{q}_r=\sigma_a\left(4\pi I_b(\vec{r})-\int_{4\pi} I(\vec{r},\vec{s})d\Omega\right) \quad (6)$$

In the above equation, $I(\vec{r},\vec{s})$ is the radiation intensity at the situation $\vec{r}$ and in the direction $\vec{s}$ and $I_b(\vec{r})=\frac{\sigma(T(\vec{r}))^4}{\pi}$ is the black body radiation intensity where

σ_a is the absorption coefficient. For calculation of $\nabla.\vec{q}_r$, the radiation intensity field is primary needed. To obtain this term, it is necessary to solve the radiative transfer equation (RTE).

The RTE for an absorbing, emitting and scattering gray medium can be expressed as [19]:

$$(\vec{s}.\nabla)I(\vec{r},\vec{s}) = -\beta I(\vec{r},\vec{s}) + \sigma_a I_b(\vec{r}) + \frac{\sigma_s}{4\pi}\int_{4\pi} I(\vec{r},\vec{s})\varphi(\vec{s},\vec{s}')d\Omega' \quad (7)$$

where σ_s is the scattering coefficient, $\beta = \sigma_a + \sigma_s$ is the extinction coefficient and $\phi(\vec{s},\vec{s}')$ is the scattering phase function for the radiation from incoming direction $\vec{s}'$ and confined within the solid angle $d\Omega'$ to scattered direction $\vec{s}$ confined within the solid angle $d\Omega$. In this study, the phase function is equal to unity because of the assumption of isotropic scattering medium. The boundary condition for a diffusely emitting and reflecting gray wall is:

$$I(\vec{r}_w,\vec{s}) = \varepsilon_w I_b(\vec{r}_w) + \frac{(1-\varepsilon_w)}{\pi}\int_{\vec{n}_w.\vec{s}'<0} I(\vec{r}_w,\vec{s})|\vec{n}_w.\vec{s}'|d\Omega' \quad \vec{n}_w.\vec{s} > 0 \quad (8)$$

in which ε_w is the wall emissivity, $I_b(\vec{r}_w)$ is the black body radiation intensity at the temperature of the boundary surface and $\vec{n}_w$ is the outward unit vector normal to the surface. Since, the RTE depends on the temperature fields through the emission term $I_b(\vec{r}_w)$, thus it must be solved simultaneously with overall energy equation. RTE is an integro-differential equation that can be solved with DOM.

In the DOM, Equation (7) is solved for a set of n different directions, $\vec{s}_i$, i=1,2,3,…,n and integrals over solid angle are replaced by the numerical quadrature, that is:

$$\int_{4\pi} f(\vec{s})d\Omega \cong \sum_{i=1}^{n} w_i f(\vec{s}_i) \quad (9)$$

where w_i is the quadrature weights associated with the direction $\vec{s}_i$. Thus, according to this method, Eq. (7) is approximated by a set of n equations, as follows:

$$(\vec{s}_i.\nabla)I(\vec{r},\vec{s}_i) = -\beta I(\vec{r},\vec{s}_i) + \sigma_a I_b(\vec{r}) + \frac{\sigma_s}{4\pi}\sum_{j=1}^{n} I(\vec{r},\vec{s}_j)\phi(\vec{s}_j,\vec{s}_i)w_j \qquad i = 1,2,3,...,n \quad (10)$$

subjected to the boundary conditions:

$$I(\vec{r}_w,\vec{s}_i) = \varepsilon_w I_b(\vec{r}_w) + \frac{(1-\varepsilon_w)}{\pi}\sum_{\vec{n}_w.\vec{s}_j<0} I(\vec{r}_w,\vec{s}_j)|\vec{n}_w.\vec{s}_j|w_j \quad (11)$$

$$\vec{n}_w.\vec{s}_i > 0$$

Also $\nabla.\vec{q}_r$ is represented as:

$$\nabla.\vec{q}_r = \sigma_a\left(4\pi I_b(\vec{r}) - \sum_{i=1}^{n} I(\vec{r},\vec{s}_i)w_i\right) \quad (12)$$

At any arbitrary surface, heat flux may also be determined from surface energy balance as:

$$\vec{q}.n(r_w) = \varepsilon_w(\pi I_b(r_w) - \sum_{\vec{n}_w.\vec{s}_j<0} I_i(r_w)|n_w.s_i|w_i) \quad (13)$$

In 3-D Cartesian coordinate system, Eq. (10) becomes as follows [19]:

$$\xi_i\frac{\partial I_i}{\partial x} + \eta_i\frac{\partial I_i}{\partial y} + \mu_i\frac{\partial I_i}{\partial z} + \beta I_i = \beta S_i \qquad i = 1,2,3,...,n \quad (14)$$

where

$$S_i = (1-\omega)I_b + \frac{\omega}{4\pi}\sum_{j=1}^{n} I(\vec{r},\vec{s}_j)\phi(\vec{s}_j,\vec{s}_i)w_j \qquad i = 1,2,3,...,n \quad (15)$$

In fact S_i is a shorthand for the radiative source function. In Eq. (15), ω is the albedo coefficient, defined as $\omega = \frac{\sigma_s}{\beta}$. The finite difference form of Eq. (14) gives the following form for radiant intensity [19]:

$$I_{p_i} = \frac{|\xi_i|A_x I_{xi}i/\gamma_x + |\eta_i|A_y I_{yi}i/\gamma_y + |\mu_i|A_z I_{zi}i/\gamma_z + \beta\forall S_{pi}}{\beta\forall + |\xi_i|A_x/\gamma_x + |\eta_i|A_y/\gamma_y + |\mu_i|A_z/\gamma_z} \quad (16)$$

in which ξ_i, η_i and μ_i are the direction cosines for the direction $\vec{s}_i$ and $\forall$ is the element cell volume.

The details of the numerical solution of RTE by DOM were also described in the previous work in which the thermal characteristics of porous radiant burners were investigated [20].

For the radiative boundary conditions, the walls are assumed to emit and reflect diffusely with constant wall emissivity, $\varepsilon_w = 0.8$. In addition, the inlet and outlet sections are considered for radiative transfer as black walls at the fluid temperature in inlet and outlet sections, respectively.

3. 2. Non-Dimensional Forms of the Governing Equations

In numerical solution of the set of governing equations including the continuity, momentum and energy, the following dimensionless parameters are used to obtain the non dimensional forms of these equations:

$$(X,Y,Z) = (\frac{x}{H},\frac{y}{H},\frac{z}{H}), \qquad (U,V,W) = (\frac{u}{U_\circ},\frac{v}{U_\circ},\frac{w}{U_\circ}),$$

$$P = \frac{p}{\rho U_\circ^2} \quad \Theta = \frac{T-T_{in}}{T_w-T_{in}}, \quad \theta_1 = \frac{T_{in}}{T_w-T_{in}}, \quad \theta_2 = \frac{T_w}{T_{in}},$$

$$I^* = \frac{I}{\sigma T_w^4}, \quad S^* = \frac{S}{\sigma T_w^4} \quad \tau = \beta.H, \quad (1-\omega) = \frac{\sigma_a}{\beta}, \quad (17)$$

$$Pr = \frac{\nu}{\alpha}, \qquad Re = \frac{\rho U_0 H}{\mu}, \qquad Pe = Re.Pr$$

$$RC = \frac{\sigma T_w^3 H}{k}, \qquad q_r^* = \frac{q_r}{\sigma T_w^4}$$

The non-dimensional forms of the governing equations are as follows:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}+\frac{\partial W}{\partial Z}=0 \tag{18}$$

$$\frac{\partial}{\partial X}\left(U^2-\frac{1}{Re}\frac{\partial U}{\partial X}\right)+\frac{\partial}{\partial Y}\left(UV-\frac{1}{Re}\frac{\partial U}{\partial Y}\right)+\frac{\partial}{\partial Z}\left(UW-\frac{1}{Re}\frac{\partial U}{\partial Z}\right)=-\frac{\partial P}{\partial X} \tag{19}$$

$$\frac{\partial}{\partial X}\left(UV-\frac{1}{Re}\frac{\partial V}{\partial X}\right)+\frac{\partial}{\partial Y}\left(V^2-\frac{1}{Re}\frac{\partial V}{\partial Y}\right)+\frac{\partial}{\partial Z}\left(VW-\frac{1}{Re}\frac{\partial V}{\partial Z}\right)=-\frac{\partial P}{\partial Y} \tag{20}$$

$$\frac{\partial}{\partial X}\left(UW-\frac{1}{Re}\frac{\partial W}{\partial X}\right)+\frac{\partial}{\partial Y}\left(VW-\frac{1}{Re}\frac{\partial W}{\partial Y}\right)+\frac{\partial}{\partial Z}\left(W^2-\frac{1}{Re}\frac{\partial W}{\partial Z}\right)=-\frac{\partial P}{\partial Z} \tag{21}$$

$$\frac{\partial}{\partial X}\left(U\Theta-\frac{1}{Pe}\frac{\partial \Theta}{\partial X}\right)+\frac{\partial}{\partial Y}\left(V\Theta-\frac{1}{Pe}\frac{\partial \Theta}{\partial Y}\right)+\frac{\partial}{\partial Z}\left(W\Theta-\frac{1}{Pe}\frac{\partial \Theta}{\partial Z}\right)+\frac{\tau(1-\omega)RC\,\theta_1\theta_2}{Pe}\left[\frac{4}{\theta_2^4}\left(\frac{\Theta}{\theta_1}+1\right)^4-\sum_{i=1}^{n}I_i^* w_i\right]=0 \tag{22}$$

3. 3. The Main Physical Quantities The main physical quantities of interest in heat transfer study are the mean bulk temperature and Nusselt number.

The mean bulk temperature along the channel was calculated using the following equation:

$$\Theta_b=\frac{\int_0^2\int_0^1 \Theta\, UdYdZ}{\int_0^2\int_0^1 UdYdZ} \tag{23}$$

In the combined convection-radiation heat transfer, the energy transport from the duct wall to the gas flow depends on two related factors:

1. Fluid temperature gradient on the wall
2. Rate of radiative heat exchange on boundary surface

Therefore, total heat flux on the wall is the sum of convective and radiative heat fluxes such that $q_t = q_c + q_r = -k(\frac{\partial T}{\partial y}) + q_r$. Therefore, the function of total Nusselt number ($Nu_t = \frac{q_t H}{k(T_W - T_M)}$) is the sum of local convective Nusselt number (Nu_c), and local radiative Nusselt number (Nu_r).

Total Nusselt numbers is given as follows [16]:

$$Nu_t = Nu_c + Nu_r = \frac{-1}{\Theta_w - \Theta_M}\left.\frac{\partial \Theta}{\partial Y}\right|_{Y=0} + \frac{RC.\theta_1.\theta_2}{\Theta_w - \Theta_M} q_r^* \tag{24}$$

where

$$\Theta_M=\frac{\int_0^1 \Theta\, UdY}{\int_0^1 UdY} \tag{25}$$

4. NUMERICAL PROCEDURE

Finite difference forms of the partial differential Eqs. (18) to (22) were obtained by integrating over an elemental cell volume with staggered control volumes for the x-, y- and z- velocity components. Other variables of interest were computed at the grid nodes. The discretized forms of the governing equations were numerically solved by the SIMPLE algorithm of Patankar and Spalding [21].

Numerical solutions were obtained iteratively by the line-by-line method. Numerical calculations were performed by writing a computer program in FORTRAN. Based on the result of grid tests for obtaining the grid-independent solutions, six different meshes were used in the grid independence study.

The corresponding maximum values of convective and total Nusselt numbers along the bottom wall in the mid-plane of the duct are calculated and tabulated in Table 1. As it is seen, a grid size of 460×36×36 can be chosen for obtaining the grid independent solution, such that the subsequent numerical calculations are made based on this grid size. It should be mentioned that near the top, bottom and step walls, clustering is employed in the x- and y- directions for obtaining more accuracy in the numerical calculations.

Also, for computation the divergence of radiative heat flux, which is needed for the numerical solution of the energy equation by DOM, S_4 approximation has been used in this study. Numerical solutions are obtained iteratively such that iterations are terminated when sum of the absolute residuals is less than 10^{-4} for momentum and energy equations. But in the numerical solution of RTE, the maximum difference between the radiative intensities computed during two consecutive iteration levels did not exceed 10^{-6} at each nodal point for the converged solution. By this numerical strategy, the velocity, temperature and radiation intensity distributions inside the flow domain can be obtained.

TABLE 1. Grid independence study, Re=250, RC=100, ω=0.5, τ=0.005

Grid size	Value of the maximum convective Nusselt number	Value of the maximum total Nusselt number
140×14×14	-	-
240×20×20	3.94	4.66
330×25×25	4.23	4.98
400×30×30	4.39	5.16
460×36×36	4.50	5.32
500×40×40	4.52	5.35

5. CODE VALIDATION

To verify the accuracy of computations in obtaining the heat transfer and flow characteristics of pure forced convection flow over BFS, the numerical implementation is validated by reproducing the results of two investigators.

First, the reattachment length on the bottom wall for a three-dimensional fluid flow over a BFS is compared with the experimental data, obtained by Li [22], in Figure 2. The maximum reattachment length occurs at the sidewall and not at the mid-plane, as one would expect. However, a good consistency is seen between experimental and present numerical results.

In another test case, the present numerical implementation was validated by reproducing the results of Iwai et al. [6] in which a forced convection gas flow over a BFS in a three-dimensional duct was studied. Variation of Nusselt number along the centerline on the bottom wall is compared with that obtained by Iwai et al. [6] in the cases of ER=2 and AR=4. The results are presented graphically in Figure 3. In this test case, at the upstream boundary, inlet flow is assumed to be hydrodynamically fully developed, while the fluid is assumed to have a uniform temperature at this section. The side walls are treated adiabatic and the other walls are isotherm with uniform temperature T_w which is greater than the fluid inlet temperature. It is seen from Figure 3 that the value of Nusselt number increases downstream the step corner after which the Nu_{max} occurs near the reattachment point, and then approaches to a constant value far from the step location. However, Figure 3 shows a good agreement between the present numerical results with those reported by Iwai et al. [6].

It should be mentioned that as the radiating effect of the gas flow is neglected in the study by Iwai et al. [6], therefore, the optical thickness of the media is set equal to zero with neglecting the surface radiation in the computations of Figure 3.

Validation of the numerical solution of RTE by DOM was also done in the previous works, in which the radiative heat transfer problems with complex 2-D and 3-D geometries were under study [16, 23].

6. RESULTS AND DISCUSSION

The present numerical results are about forced convective of a radiating gas flow over a BFS in three - dimensional duct with an expansion ratio of ER=2 and aspect ratio of AR=4 at different conditions. In numerical calculations, the Reynolds number is equal to 250, while the Prandtl number is kept constant at 0.71 to guarantee the constant fluid physical properties.

6. 1. The Effect of Radiation Heat Transfer in Forced Convection Flow of Radiating Gas

In order to illustrate the radiation effect in thermal behavior of a convection flow, contours of total Nusselt number along the bottom wall with and without considering the radiation term in energy equation are presented in Figure 4. This figure presents that the minimum value of total Nusselt number occurs on the bottom wall at the backward step corner, where the fluid is at rest. Also, it is seen that downstream the step location, the Nu_t increases sharply and approaches to a maximum value, after which Nu_t decreases and reaches to a constant value as the distance continues to increase in the stream-wise direction. The effect of radiation in Nu distribution can be found if one compares Figures 4(a) and (b) with each other. Figures 4(a) and (b) illustrate similar pattern for distribution of convection coefficient, but, it can be seen that the radiative effect increases the value of total Nusselt number specially near the duct walls, because of the surface radiation. This behavior will be explained in the explanation of next figures.

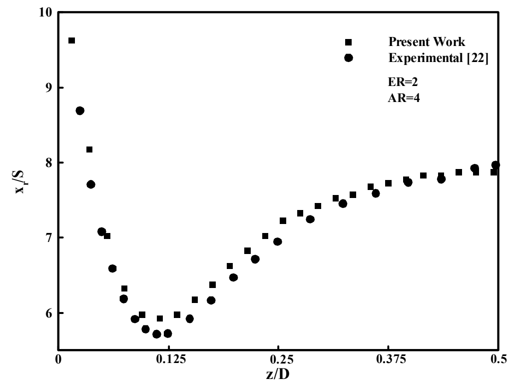


Figure 2. Comparison of the computed reattachment point with experiment

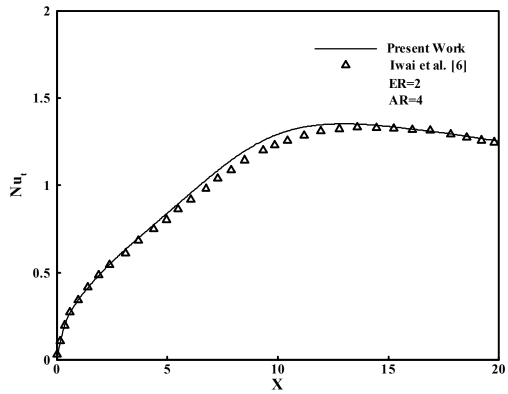
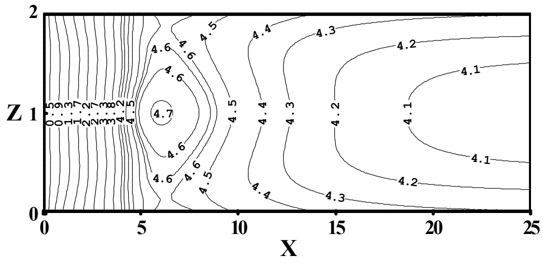
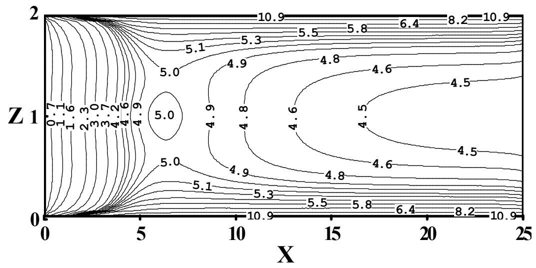


Figure 3. Comparison of numerical result for distribution of total Nusselt number along the centerline on the bottom wall in pure convection flow

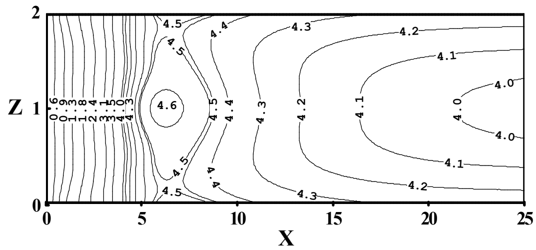


(a) without radiation effect

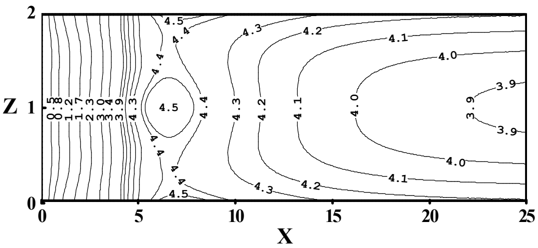


(b) with radiation effect, RC=50, ω =0.5, τ =0.005

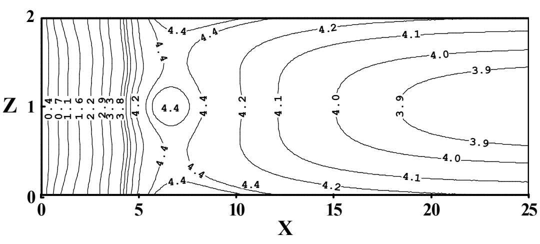
Figure 4. Distribution of total Nusselt number contours along the bottom wall



(a) RC=50



(b) RC=100



(c) RC=150

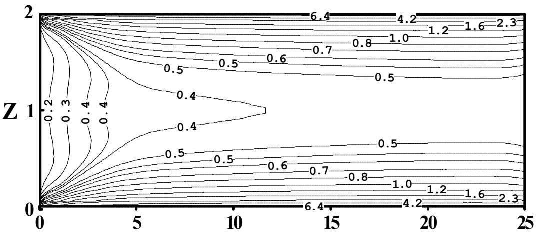
Figure 5. Distribution of convective Nusselt number contours along the bottom wall at different values of RC, ω =0.5, τ =0.005

In the convection flow of radiating fluid, the radiation-conduction parameter (RC), albedo coefficient (ω) and optical thickness (τ) are the main parameters. In the next sections, an attempt is made to study the effect of these parameters on thermal behavior of the combined convection-radiation system.

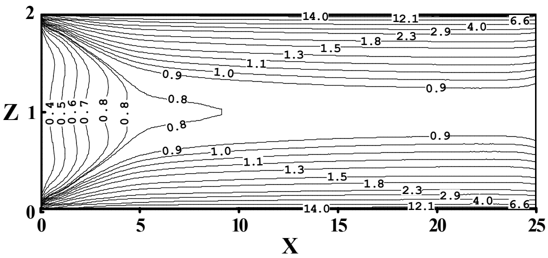
6. 2. The Effect of Radiation-Conduction Parameter Radiation-conduction parameter (RC) is one of the main parameters in the combined radiation-conduction systems, which shows the relative importance of the radiation mechanism compared with its conduction counterpart. High value of RC parameter shows the radiation dominance in a thermal system.

For the convective flow with radiating heat transfer in a 3-D channel including a backward facing step, as shown in Figure 1, the contours of convective, radiative and total Nusselt numbers along the bottom wall at different values of the RC are presented in Figures 5, 6 and 7, respectively.

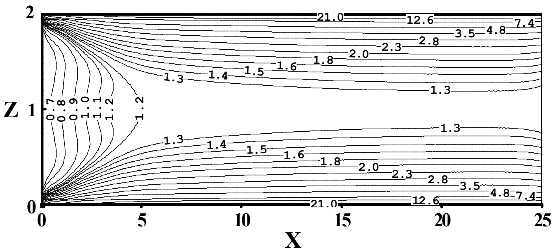
Figure 5 shows that the convective Nusselt number decreases by increasing in RC parameter. This behavior is due to this fact that under the effective presence of radiation mechanism at high values of *RC*, the temperature field inside the flow domain becomes more uniform; consequently, the amount of temperature gradient inside the flow domain decreases that causes a decrease in the value of local convective Nusselt number. But a different trend is seen from Figure 6 for the variation of radiative Nusselt number with RC parameter. This figure shows that Nu_r starts from a minimum value at the backward step corner where the minimum radiative heat flux leaves the bottom wall. As the distance increases from the step corner, the amount of radiative heat flux and consequently radiative Nusselt number increases sharply, which is due to a decrease in bottom wall incident radiative heat flux incoming from the step surface. After this sharp slope, Nu_r approaches a constant value as the distance continues to increase in the stream-wise direction. Furthermore, Figure 6 shows that the radiative Nusselt number increases by increasing in RC parameter, which is due to the increase in bottom wall's outgoing radiative heat flux in radiation dominance condition. It is seen that RC parameter has a greater influence on the radiative Nusselt number than the convective one such that total Nusselt number increases by increasing in RC, as shown in Figure 7. In order to show more about the influence of RC on the Nusselt number in the combined convection-radiation system, the distributions of convective, radiative and total Nusselt numbers in the mid-plane of the duct are plotted in Figures 8(a), (b) and (c), respectively. These figures show the same trend for variations of all types of Nusselt number as it was seen before in Figures 5, 6 and 7.



(a) RC=50

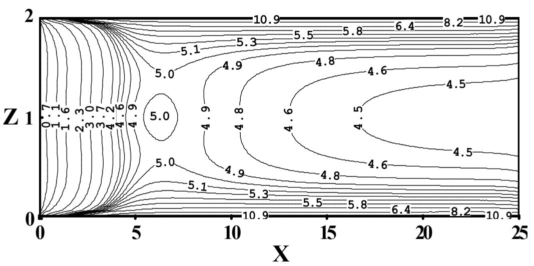


(b) RC=100

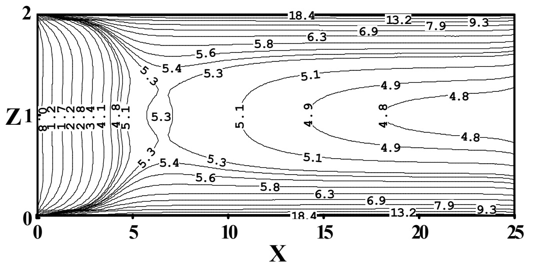


(c) RC=150

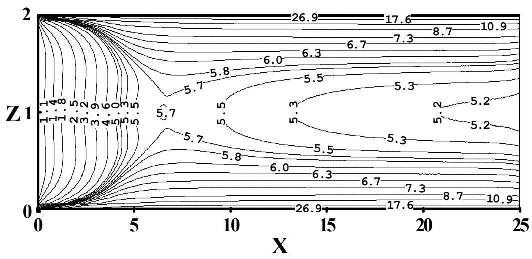
Figure 6. Distribution of radiative Nusselt number contours along the bottom wall at different values of RC, ω =0.5, τ =0.005



(a) RC=50



(b) RC=100



(c) RC=150

Figure 7. Distribution of total Nusselt number contours along the bottom wall at different values of RC, ω =0.5, τ =0.005

6. 3. The Effect of Optical Thickness

A well-known radiation property and one of the important parameters in participating medium is the optical thickness that affects the temperature distribution inside the participating medium. High optical thickness, means that the medium has great ability to absorb and emit radiant energy.

The effect of optical thickness on contours of total Nusselt number along the bottom wall is shown in Figure 9. Four different values for the optical thickness of the participating media are used in the computations of this figure. It is seen that as the medium's ability to absorb and emit thermal radiation becomes greater at high values of the optical thickness, such systems have high values for the total Nusselt number along the heated wall.

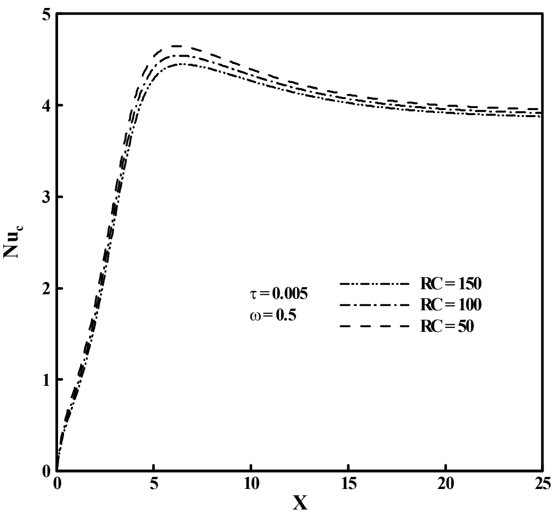
For more study about the effect of optical thickness on the thermal behavior of convection-radiation system, the distribution of mean bulk temperature along the duct at different values of the optical thickness is plotted in Figure 10. This figure shows that the mean bulk temperature increases along the duct because of both convection and radiation mechanisms.

As it is seen from Figure 10, increase of optical thickness and consequently increase of radiation heat transfer mechanism causes an increase in the amount of the gas mean bulk temperature. It is worth mentioning that in the case of no radiation (τ =0), in which the minimum mean bulk temperature exists along the duct, the radiative mechanism has not any role in the heat transfer process.

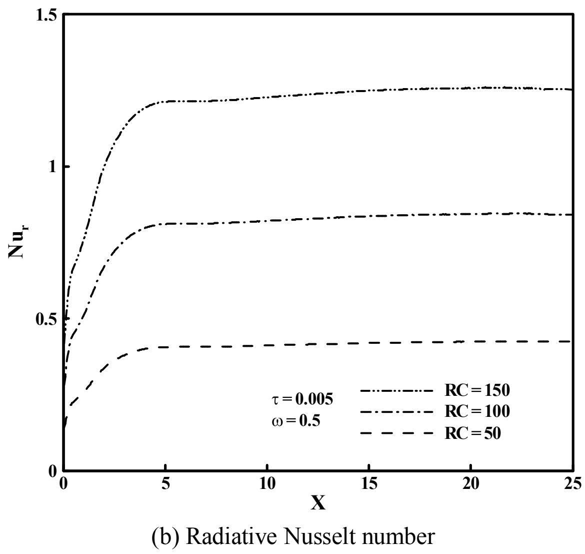
6. 4. The Effect of Scattering

The scattering albedo is another important parameter in radiating systems that shows the ability of participating medium to scatter thermal radiation. As it was mentioned before, scattering albedo, ω, is defined as $\omega = \sigma_s / \beta$. The extreme values of scattering albedo, i.e., ω=1.0 and ω=0.0 correspond to pure scattering and non-scattering cases, respectively. Therefore, the medium changes from pure absorption to pure scattering by increasing ω from 0 to 1.

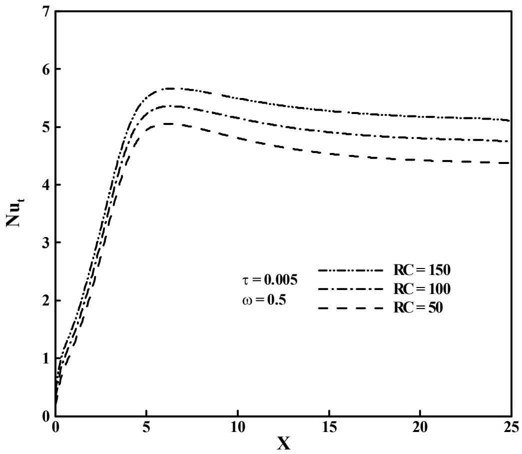
The effects of scattering albedo coefficient on the total Nusselt number and mean bulk temperature are presented in Figures 11 and 12, respectively. These figures show that the total Nusselt number and mean bulk temperature decrease by increasing in scattering albedo coefficient. Because, less radiative heat flux is converted to gas thermal energy in a pure scattering case compared to a pure absorption one. Beside, it can be found from Figures 11 and 12 that when the radiation term is omitted from the energy equation in the case of no-radiation problems, the convective system has the same trend and behavior as pure scattering case with ω=1.0.



(a) Convective Nusselt number

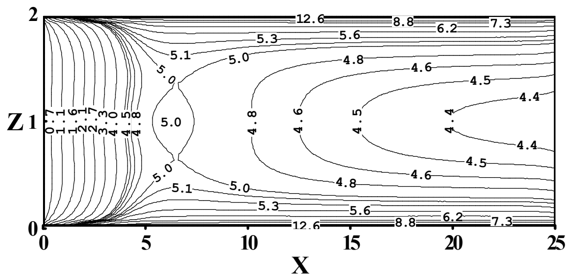


(b) Radiative Nusselt number

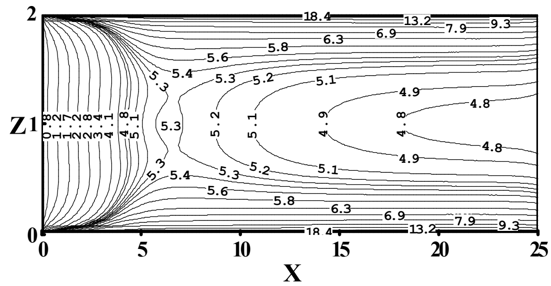


(c) Total Nusselt number

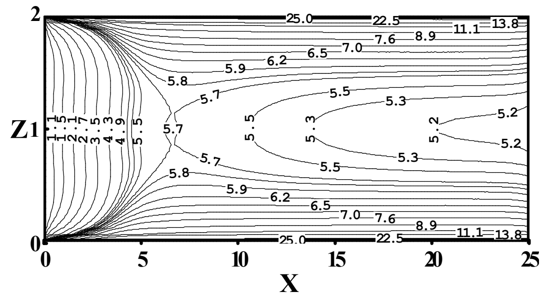
Figure 8. Effect of radiation-conduction parameter, RC, on the Nusselt number distribution along the bottom wall at the mid-plane of the duct



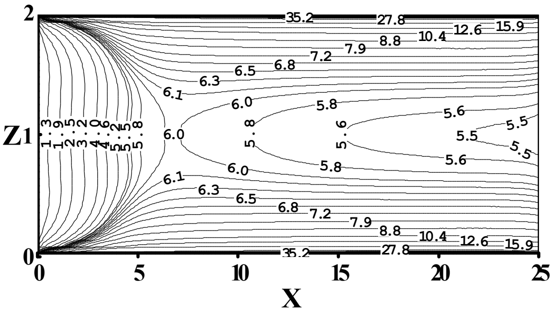
(a) τ =0.0025



(b) τ =0.005



(c) τ =0.0075



(d) τ =0.01

Figure 9. Distribution of total Nusselt number contours along the bottom wall at different values of τ , RC=100, ω =0.5

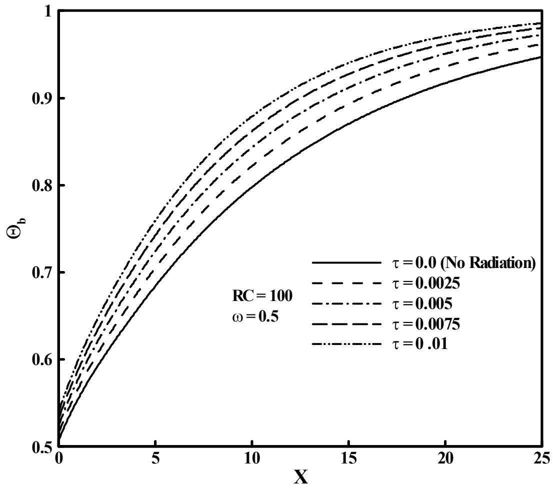


Figure 10. Effect of τ on the mean bulk temperature distribution along the duct

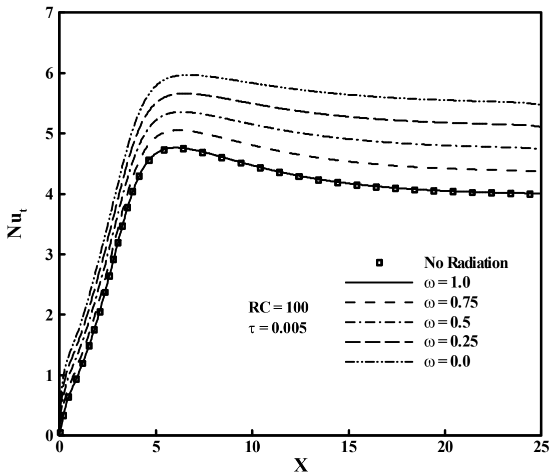


Figure 11. Effect of ω on the total Nusselt number distribution along the bottom wall in the duct mid-plane

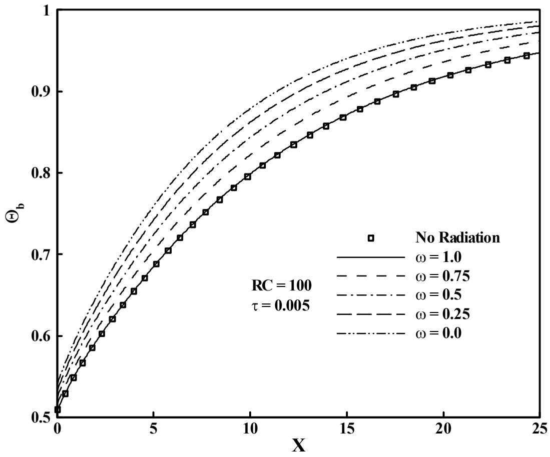


Figure 12. Effect of ω on the mean bulk temperature distribution along the duct

7. CONCLUSION

The present research work deals with the analysis of forced convection laminar flow of a radiating gas over a backward facing step in a 3-D horizontal duct. The set of governing equations consisting mass, momentum and energy is solved numerically by the CFD techniques in the Cartesian coordinate system. For calculating the radiative term in the energy equation, the RTE is solved by the DOM to obtain the distribution of radiant intensity inside the radiating medium. The effects of radiation-conduction parameter, the optical thickness and albedo coefficient on thermal behavior of the convection-radiation system are thoroughly explored by plotting the variations of Nusselt number (total, radiative and convective), and mean bulk temperature under different conditions. It was revealed that these parameters have great effect on thermal behavior of systems with combined convection-radiation heat transfer.

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Three Dimensional Laminar Convection Flow of Radiating Gas over a Backward Facing Step in a Duct

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P A P E R I N F O

Paper history:
Received 8 January 2012
Accepted in revised form 30 August 2012

Keywords:
Laminar Convection Flow
Backward Facing Step
Radiation Heat Transfer
DOM

چکیده

در این مطالعه، شبیه سازی سه بعدی جریان جابه‌جایی آرام و اجباری سیال بر روی یک پله پسرونده، در داخل یک کانال با مقطع مستطیلی و با در نظر گرفتن اثرات تابشی گاز مورد بررسی قرار گرفته است. سیال عامل همانند یک محیط خاکستری در جذب، صدور و پخش تشعشع شرکت می‌کند. برای حل معادلات حاکم که شامل معادلات بقای جرم، مومنتوم و انرژی هستند، دستگاه مختصات سه بعدی کارتزین مورد استفاده قرار گرفته است. جهت بدست آوردن میدان‌های سرعت و دما، این معادلات به صورت عددی و با استفاده از روشهای دینامیک سیالات محاسباتی حل می‌گردند. فرم جداسازی شده‌ی معادلات حاکم، توسط روش حجم محدود بدست آمده و با به کار بردن الگوریتم سیمپل حل می‌شوند. از آنجایی که گاز به عنوان یک محیط شرکت کننده در انتقال حرارت تشعشعی نقش دارد، تمام مکانیزم‌های انتقال حرارت که شامل جابه‌جایی، هدایت و تشعشع بوده به طور همزمان در معادله انرژی درنظر گرفته می‌شوند. برای محاسبه جمله تشعشع در معادله انرژی، معادله انتقال حرارت تابشی به طور عددی و با به کاربردن روش طول‌های مجزا حل شده و توزیع شار تشعشعی داخل جریان گاز محاسبه می‌شود. نتایج حل عددی به صورت رسم نمودارهایی برای بررسی اثرات ضخامت نوری، ضریب البدو ، عدد تشعشع-هدایت بر روی رفتار انتقال حرارت جریان گاز ارائه شده است. همچنین سازگاری خوبی بین نتایج عددی بدست آمده از مطالعه حاضر با نتایج مطالعات قبلی برقرار است.

doi: *10.5829/idosi.ije.2012.25.04a.11*