IJE TRANSACTIONS A: Basics Vol. 25, No. 4, (October 2012) 301-307

International Journal of Engineering

Journal Homepage: www.ije.ir

Effects Analysis of Frozen Conditions for Spacecraft Relative Motion Dynamics

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PAPER INFO

Paper history:
Received 7 March 2012
Received in revised form 20 May 2012
Accepted 14 June 2012

Keywords:
Frozen Orbits
Relative Motion
Argument of Perigee
Dynamical Modeling
Eccentricity Rate
Propellant Saving
Simulation Results
Zonal Harmonic Perturbation

ABSTRACT

The purpose of this rersearch is to analyze the effective application of particular earth orbits in dynamical modeling of relative motion problem between two spacecraft. One challenge in implementing these motions is maintaining the relations as it experiences orbital perturbations (zonal harmonics J_2 and J_3), notably due to the Earth's oblateness. Certain aspects of the orbital geometry can remain virtually fixed over extended periods of time due to a natural phenomenon called a frozen orbit. Specifically, the elements of the orbital geometry that can remain fixed are the argument of perigee (ω) and eccentricity (e). Simulation results show that, using frozen orbits phenomenon results in considerable propellant saving and performance duration a relative orbital mission is preferable. In this regard, an method is developed that determines if the relative orbit conditions will be met given the initial differences in frozen orbit elements (argument of perigee and eccentricity) for each of two spacecraft. Using the frozen conditions in relative motion dynamics can be reduced the amount of required propellant for orbit correction manoeuvres due to the harmonic perturbations over the course of a year.

doi: *10.5829/idosi.ije.2012.25.04a.01*

1. INTRODUCTION

The first application where a frozen orbit was used is the TOPEX mission that treated with target of topography experiment in the 1986. In this mission, frozen orbits are characterized by almost no long-term change in eccentricity or argument of perigee [1]. The standard method of frozen orbit prediction is shown to be inadequate for TOPEX inclinations due to the effect of higher degree zonal harmonic gravity terms. This method is described from which long-term motion in mean eccentricity, argument of perigee, and inclination can be predicted without numerical integration and is used to locate frozen orbits. Frauenholz, Bhat, and Shapiro discuss the observed behavior over three years of the semi-major axis, inclination and the frozen eccentricity vector in this mission. The precision orbit determination used on this project produced a radial accuracy of about 4 cm. The osculating to mean element conversion technique was a method developed by Guinn [2]. Here, the mean orbital elements remove all central and third-body perturbations acting over ten days. The predicted decay of the semi-major axis was almost entirely due to drag [3].

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As estimation observed, other forces influencing the semi-major axis include solar radiation and thermal gradients. These body-fixed forces either increase or decrease the orbit, depending on the orientation of the satellite. As for the inclination, the observed variations in inclination corresponded very well to the predictions. The primary perturbing factors were the third-body forces of lunar and solar gravity. Next, Chao and et al. [4] describe a formation that uses a frozen orbit as the reference orbit of the centersat. This strategy also uses differential GPS measurements, MEMS and auto-feedback control. Station-keeping maneuvers are necessary to keep it in a frozen orbit. Formation-keeping maneuvers are required to have each subsat follow its reference orbit. The results of the analysis show only 10 to 30 m/s per year per satellite in ΔV are required to control a 1 km radius cluster for LEO orbit [5]. Basis researches show that the eccentricity frozen orbit was first described for SEASAT, but it has been studied for the Earth-orbiting missions of the atmospheric explorer, the heat capacity mapping mission, LANDSAT, GEOSAT, and TOPEX and for Martian and Lunar orbiters [6]. In another paper, Shapiro [7] states in his abstract perspective: "frozen orbits arise from bifurcations or singularities in the relevant system of differential equations obtained via the appropriate Hamiltonian or Lagrangian formulation". He gives equations for de/dt and dω/dt where all harmonics can be included.

Zhou et al. [8] also discuss frozen orbits. They say that the effects of solar radiation and atmospheric drag greatly affect the oscillation pattern, while the gravitational effects of the Sun and Moon only slightly affect this pattern and the forces correction maneuvers to maintain the frozen orbit [8].

Therefore, by considering the above-noted, the objective of this paper is description and application of this particular earth orbits in dynamical modeling of relative motion between two spacecraft. In simulation of spacecraft relative motion equations, using this frozen orbit phenomenon results in considerable propellant saving and spacecraft performance. Also, an algorithm is developed that determines if the relative orbit conditions will be met given the initial differences in particular orbit elements for two spacecraft.

2. EQUATIONS OF MOTION ANALYSIS

The following subsections will discuss the linearized relative motion and frozen orbit equations for using this set of equations in development of dynamical modeling and next, obtaining simulation results.

2. 1. Relative Motion (Hill's) Equations One technology that makes satellite relative motion feasible is Clohessy-Wiltshire-Hill's orbits equations which have historically been used in Rendezvous and Docking missions. The full development of these equations can be found in multiple texts including Vallado and McClain, as followed here. The derivation begins with the orbital two-body equation of motion of the reference satellite [9]:

$$\ddot{\vec{r}}_{ref} = -\frac{\mu \vec{r}_{ref}}{r_{ref}^{3}} \tag{1}$$

Here, $\vec{r}_{ref}$ is the position vector from centre of the earth to the reference satellite, r_{ref} is the magnitude of this vector and μ is the gravitational coefficient. The orbital two-body equation of motion of the chaser satellite is similar, but with the addition of a force for Rendezvous problem, F_{thrust}:

$$\ddot{\vec{r}}_{ch} = -\frac{\mu \vec{r}_{ch}}{r_{ch}^{3}} + \vec{F}_{thurst} \tag{2}$$

So, $\vec{r}_{ch}$ is the position vector from centre of the earth to the chaser satellite, r_{ch} is the magnitude of this vector. Next, define the relative vector as the difference between the chaser and the reference satellite [9].

$$\vec{r}_{rel} = \vec{r}_{ch} - \vec{r}_{ref} \tag{3}$$

Differentiate this equation twice, so Equations (1) and (2) can be substituted in. The equations are rearranged and linearized about the reference orbit. The Hill's equations are then expressed in terms of Cartesian, respectively:

$$\begin{cases} \ddot{x} - 2n\dot{y} - 3n^2 x = f_x \\ \ddot{y} + 2n\dot{x} = f_y \\ \ddot{z} + n^2 z = f_z \end{cases} \tag{4}$$

Since the orbits are circular, n is the mean motion and is given by:

$$n = \sqrt{\mu / r_{ref}^{3}} \tag{5}$$

To solve Hill's equations for coasting or unforced motion, given the initial relative difference in position between the two satellites (x_0, y_0, z_0), the initial relative difference in velocity $(\dot{x}_0, \dot{y}_0, \dot{z}_0)$, the mean motion of the reference satellite (n), and the time interval of interest, these equations give the relative motion described as following [10]:

$$\begin{cases} x(t) = \frac{\dot{x}_0}{n}\sin(nt) - (3x_0 + \frac{2\dot{y}_0}{n})\cos(nt) + (4x_0 + \frac{2\dot{y}_0}{n}) \\ y(t) = (6x_0 + \frac{4\dot{y}_0}{n})\sin(nt) + \frac{2\dot{x}_0}{n}\cos(nt) + (y_0 - \frac{2\dot{x}_0}{n}) \\ \quad - (6nx_0 + 3\dot{y}_0)t \\ z(t) = z_0\cos(nt) + \frac{\dot{z}_0}{n}\sin(nt) \end{cases} \tag{6}$$

The corresponding rates are then,

$$\begin{cases} \dot{x}(t) = \dot{x}_0\cos(nt) + (3nx_0 + 2\dot{y}_0)\sin(nt) \\ \dot{y}(t) = (6nx_0 + 4\dot{y}_0)\cos(nt) - 2\dot{x}_0\sin(nt) - (6nx_0 + 3\dot{y}_0) \\ \dot{z}(t) = -nz_0\sin(nt) + \dot{z}_0\cos(nt) \end{cases} \tag{7}$$

2. 2. Frozen Orbits Equations Chobotov in his text book describes a frozen orbit as one for which the mean elements chosen to produce constant values of eccentricity (e) and argument of perigee (ω) with time [11]. This property stops the rotation of perigee. The eccentricity of the frozen orbit will remain constant for years if the solar radiation pressure and the atmospheric drag are not too influential. The variation rate equations for eccentricity and argument of perigee that incorporate only zonal harmonics J_2 and J_3 are as follows:

$$\frac{de}{dt} = -\frac{3J_3 n}{2(1-e^2)^2}\left(\frac{R_E}{a}\right)^3 \sin(i)\left(1 - \frac{5}{4}\sin^2(i)\right)\cos(\omega) \tag{8}$$

$$\frac{d\omega}{dt} = \frac{3J_2 n}{(1-e^2)^2}\left(\frac{R_E}{a}\right)^2\left(1 - \frac{5}{4}\sin^2(i)\right) \times \left[1 + \frac{J_3}{2J_2(1-e^2)} \times \left(\frac{R_E}{a}\right)\frac{\sin(i)\sin(\omega)}{e}\right] \tag{9}$$

Here, R_E is the Earth's mean equatorial radius, a is Semi-major axis and i is orbit inclination. At the critical inclination of i=63.4° or 116.6°, the term becomes zero in both Equations (8) and (9), resulting in solutions with nearly constant values of eccentricity and argument of perigee. The Russian Molniya orbits are near the critical inclination of 63.4°and had this characteristic [11].

The eccentricity rate de/dt is zero if i=0 or the critical inclination, or if ω=90 or 270 degrees. Since most missions do not fly in the equatorial plane or at critical inclination, ω must be one of above-noted values for this condition to occur. The argument of perigee rate expression $d\omega/dt$ is zero at the critical inclination or when the square bracketed term in Equation (9) is equal to zero. By setting this term equal to zero, for a given value of a and i, the mean "frozen eccentricity" can thus be found. For ω=90°, the frozen eccentricity is [11]:

$$e_f \approx -\frac{J_3}{2J_2}\left(\frac{R_E}{a}\right)\sin(i) \tag{10}$$

This eccentricity is of the order of 10^{-3} because J_3 is three orders of magnitude less than J_2. Chobotov also states that, "For initial conditions that are near, but not at, the frozen point, e and ω will move counter clockwise in closed contours. For inclinations less than 63.435 *deg* or greater than 116.565 *deg*, the motion is clockwise". For conditions farther from the frozen point, the contours do not close [11]. Figure 1 shows an example of the closed contours about the frozen conditions for the TOPEX mission [1].

2.3. Development of Difference Equations In order to directly find the time history of the differences in eccentricity and argument of perigee between the satellites about discussion of relative motion conditions, expressions for $d(\Delta e)/dt$ and $d(\Delta\omega)/dt$ are needed that $\Delta\omega$ is the difference in argument of perigee and Δe is the difference in eccentricity between the satellites.

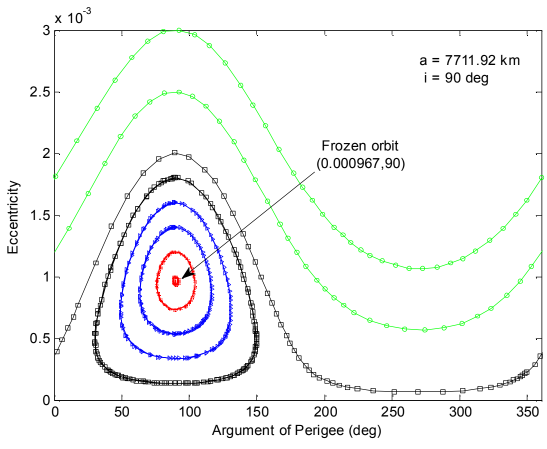


Figure 1. Closed contours about the frozen orbit conditions.

As developed previously, the equations for the change in eccentricity and argument of perigee with time are defined [11]. Using the delta functions above with Equations (8) and (9), it follows that:

$$\frac{d(e_1+\Delta e)}{dt} = -\frac{3J_3 n}{2(1-(e_1+\Delta e)^2)^2}\left(\frac{R_E}{a}\right)^3 \sin(i)\times \left(1-\frac{5}{4}\sin^2(i)\right)\cos(\omega_1+\Delta\omega) \tag{11}$$

$$\frac{d(\omega_1+\Delta\omega)}{dt} = \frac{3J_2 n}{(1-(e_1+\Delta e)^2)^2}\left(\frac{R_E}{a}\right)^2\left(1-\frac{5}{4}\sin^2(i)\right)\times \left[1+\frac{J_3}{2J_2(1-(e_1+\Delta e)^2)}\times\left(\frac{R_E}{a}\right)\frac{\sin(i)\sin(\omega_1+\Delta\omega)}{(e_1+\Delta e)}\right] \tag{12}$$

Remember that:

$$\frac{de_2}{dt} = \frac{d(e_1+\Delta e)}{dt} \tag{13}$$

$$\frac{d\omega_2}{dt} = \frac{d(\omega_1+\Delta\omega)}{dt} \tag{14}$$

In the above equations, e_i is indexed the eccentricity and ω_i is argument of perigee of satellite and n is mean angular rate of circular reference orbit. Then, as follows:

$$\frac{d(\Delta e)}{dt} = \frac{d(e_1+\Delta e)}{dt} - \frac{de_1}{dt} \tag{15}$$

$$\frac{d(\Delta\omega)}{dt} = \frac{d(\omega_1+\Delta\omega)}{dt} - \frac{d\omega_1}{dt} \tag{16}$$

In order to simplify Equations (15) and (16), one might want to make a few assumptions using small angle approximations.

2. 4. Application of Developed Equations to Hill's Problem In the analysis discussed so far, the orbital elements have been used to describe the orbits motion of either satellite. For understanding the general behavior of these orbits, Hill's equations are used. To study of relative motion under these particular orbits, a translation between the orbital elements and the Hill's initial conditions is desired. Therefore, an algorithm is developed for this translation that derived by Vallado and McClain [9]. The resulting radius and velocity vectors are in the PQW coordinate system. These coordinates are transformed to IJK coordinates, then, these IJK coordinates are transformed to RSW coordinates. The relative RSW coordinates are used to determine the Hill's initial conditions [9]. This algorithm is defined as follows:

$$r_{PQW} = \begin{bmatrix} \dfrac{p\cos(v)}{1+e\cos(v)} \\ \dfrac{p\sin(v)}{1+e\cos(v)} \\ 0 \end{bmatrix}, \quad V_{PQW} = \begin{bmatrix} -\sqrt{\dfrac{\mu}{p}}\sin(v) \\ \sqrt{\dfrac{\mu}{p}}(e+\cos(v)) \\ 0 \end{bmatrix} \tag{17}$$

$$\vec{r}_{IJK} = \left[\frac{IJK}{PQW}\right]\vec{r}_{PQW} \quad , \vec{V}_{IJK} = \left[\frac{IJK}{PQW}\right]\vec{V}_{PQW} \tag{18}$$

Here, p is the semi-parameter and v is the true anomaly. The transfer matrixes of described systems are available in some articles [9]. Next, the relative distance between the chaser and reference satellites in the IJK system can now be found:

$$\vec{r}_{rel,IJK} = \vec{r}_{ch,IJK} - \vec{r}_{ref,IJK} \tag{19}$$

These IJK vectors must be converted to the RSW system. The relative radius vector in the RSW coordinate system is as follows:

$$\vec{r}_{rel,RSW} = \left[\frac{RSW}{IJK}\right]\vec{r}_{rel,IJK} \Rightarrow \begin{cases} r_{rel,RSW} = (x_0, y_0, z_0)_{Hill} \\ V_{rel,RSW} = (\dot{x}_0, \dot{y}_0, \dot{z}_0)_{Hill} \end{cases} \tag{20}$$

2. 5. Orbit Correction Manoeuvres Since satellite relative motions are utilized smaller satellites with little or no propellant capability, the amount of propellant required to maintain a relation is of primary concern. An analysis was performed in order to understand the annual propellant requirements for orbits at frozen conditions. This initial study quantifies the amount of propellant needed to correct the orbital perturbations over the course of a year. As discussed previously, the orbital perturbations of interest are quantified by the equations that describe the change in eccentricity and argument of perigee difference with time [3]. These equations for $d(\Delta e)/dt$ and $d(\Delta\omega)/dt$ are Equations (15) and (16), respectively. These values are then multiplied by the time interval of one year to get an approximation for the yearly deviation in eccentricity and argument of perigee. The resulting values will be referred to as δe and $\delta\omega$, the deviations in the eccentricity and argument of perigee difference. The amount of propellant (ΔV) calculated is the amount needed to correct these δe and $\delta\omega$ values. This is only a snapshot in time of a cyclic motion, but for comparison the same snapshot in time is made for a range of initial settings in eccentricity and argument of perigee near by the frozen condition [5]. A purely tangential thrust is assumed, since this type of thrust is the most efficient in altering the eccentricity. The amount of propellant needed can be obtained as following approximate expression [12]:

$$\Delta V \cong \frac{1}{2} V_C \sqrt{(\delta e)^2 + e^2 (\delta\omega)^2} \tag{21}$$

That, V_C is the velocity of a circular orbit that is obtained the following respect to:

$$V_C = \sqrt{\mu / r} \tag{22}$$

Figures 2 and 3 show the simple of simulation results of this survey. As obtained results show, the lowest ΔV value is at the frozen condition, as expected. The argument of perigee value requiring for the least propellant is ω=90 *deg*, Since the *cos(ω)* term in the Equation (8) is zero. Resulting in a *de/dt* value of zero, this means that the eccentricity value remains fixed even if the eccentricity is not the "frozen" value. Simulation results are interesting to note that the amount of propellant required is not linearly symmetric about the frozen condition. So, with eccentricity values increase, the amount of propellant needed for orbit corrections also increases. Moreover, obtained results show with inclination changes, the general shape of the resulting contours remains the same.

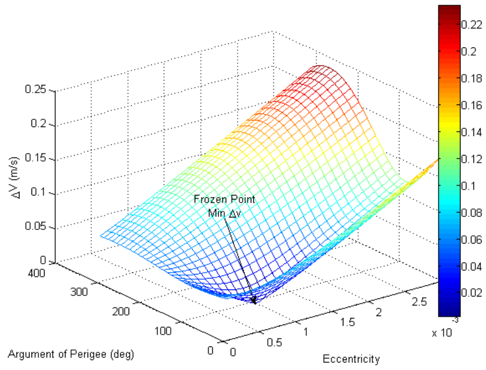


Figure 2a. ΔV values on eccentricity and argument of perigee (i= 90°).

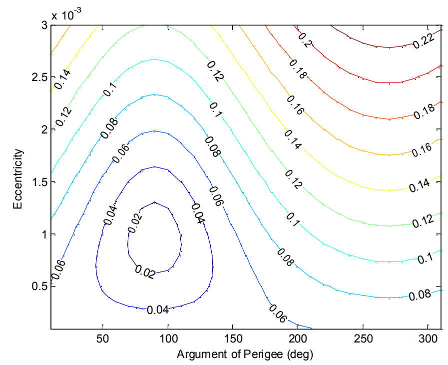


Figure 2b. ΔV contours on e and ω (i= 90°).

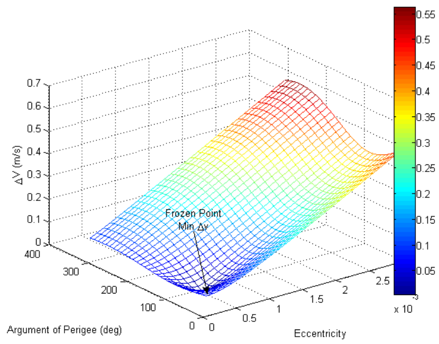


Figure 3a. ΔV values on eccentricity and argument of perigee (i= 25°)

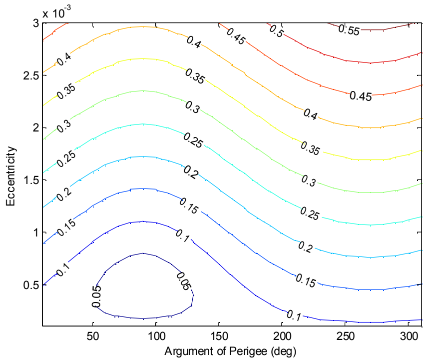


Figure 3b. ΔV contours on e and ω (i= 25°).

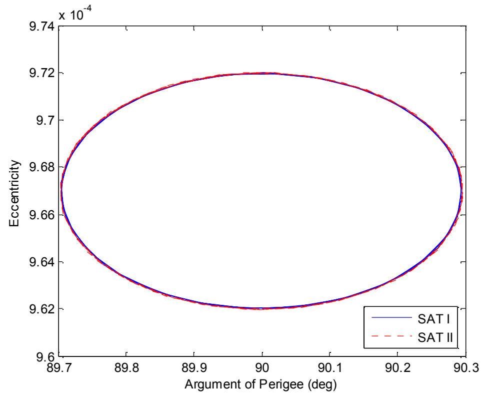
TABLE 1. Consideration and Simulation Initial Conditions.

Problem No.	Eccentricity SAT I	Eccentricity SAT II	A/Perigee (ω), SAT I	A/Perigee (ω), SAT II
(a)	9.61e-4	9.71e-4	90	90
(b)	1.0e-4	1.1e-4	90	90
(c)	1.0e-4	1.1e-4	60	120

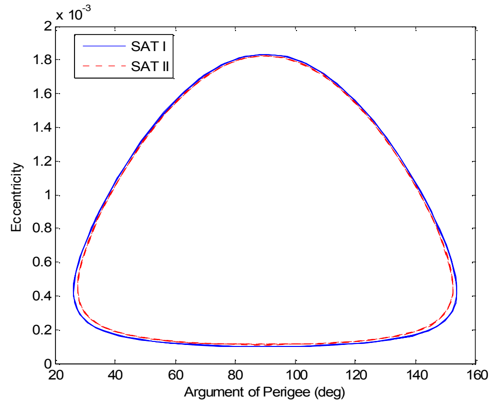
3. SIMULATION RESULTS AND DISCUSSION

For effects evaluation and analysis of these particular orbits in the relative motion dynamics, first is required to define the initial conditions for presented problem. Simulation results are obtained to use the following inputs (Table 1).

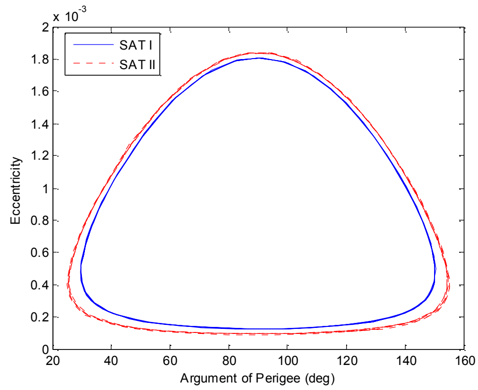
The first problem has one satellite at the frozen eccentricity value and other satellite with an eccentricity 10^{-5} greater than this value. In the second problem have two satellites in frozen conditions, with the different eccentricity. Similar to previous conditions, the third problem has the same eccentricity values, but with different argument of perigee. The value of the semi major axis (a) is assumed 7711.92 km that is obtained of TOPEX mission. The inclination is set at 90 degrees, since a polar orbit is being used. The gravitational parameter and perturbations are given in Ref [9]. When trying to simulation for finding Δe and Δω, e and ω cannot remain constant, since these values are changing with time. Due to the very small numbers used in this analysis, subtracting or adding combinations of independent states resulted in numerical errors. The results are obtained for a two-year time duration. Figures 4 show the closed contours that occur when the eccentricity is plotted against the argument of perigee near to the frozen condition. The results for first problem are closer to the frozen conditions and circular-type contours, while other are farther from the frozen conditions, and pear-shaped contours.



Problem (a)



Problem (b)

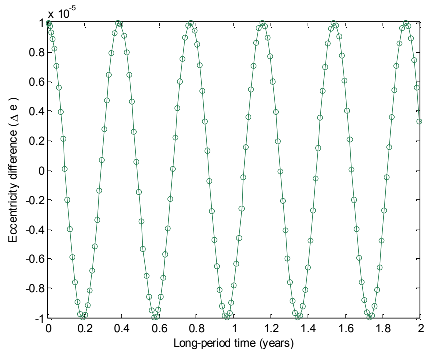


Problem (c)

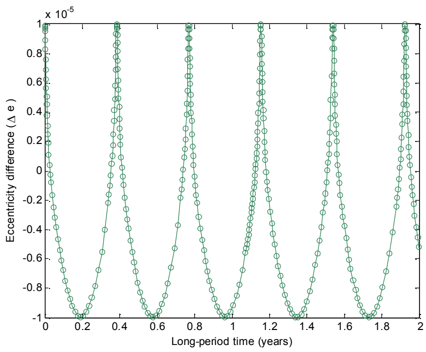
Figure 4. Variations of eccentricity vs. argument of perigee

The differences in eccentricity between Satellites I and II are shown in following plots (Figures 5). Similar to previous conditions, the plots repeat periodically to the same magnitudes. The magnitudes for problem (a) and (b) are varied between ± Δe_0, the initial eccentricity difference. However, for problem (c), the magnitude of this variation is an order of magnitude larger than the initial difference in eccentricity. Also, in this problem the difference in eccentricity rises sharply from the minimum eccentricity difference to the maximum eccentricity difference.

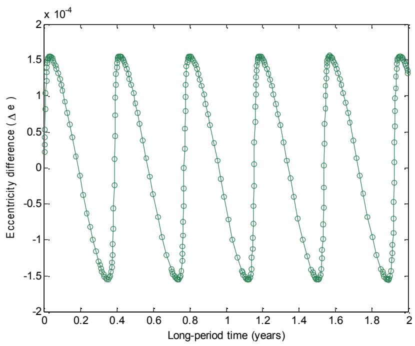
The value differences in argument of perigee between Satellites I and II are shown in Figures 6. Similar to previous conditions, the plots repeat periodically to the same magnitudes. Moreover, for the problem (c), even though the maximum difference in argument of perigee is two orders of magnitude larger than the other cases, this maximum magnitude is passed through very quickly. This means that for the majority of the time, the two satellites have similar argument of perigee values, when are near to frozen conditions.



Problem (a)

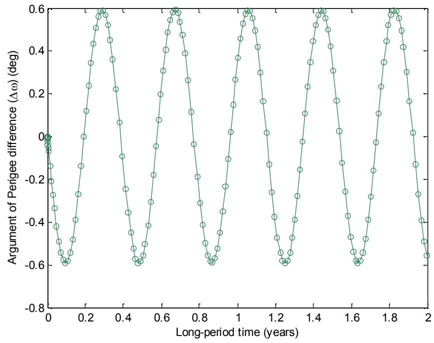


Problem (b)

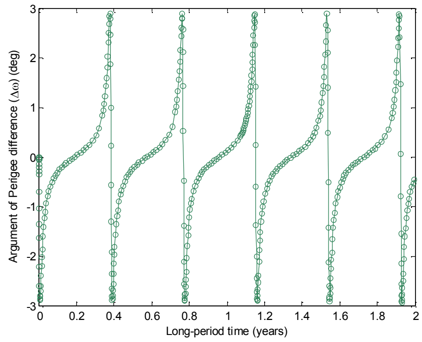


Problem (c)

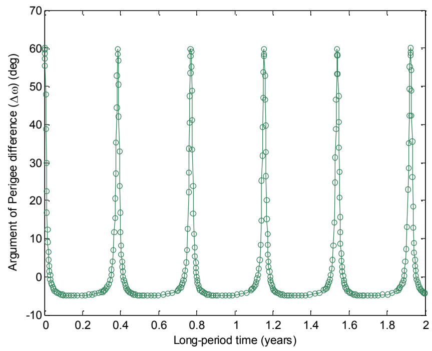
Figure 5. Eccentricity difference vs. time.



Problem (a)



Problem (b)



Problem (c)

Figure 6. Argument of perigee difference vs. time

4. CONCLUSION

In this research, effects of application the particular earth orbits in dynamical modeling problem of spacecraft relative motion are analyzed. One challenge in implementing these motions is maintaining the relations as it experiences orbital perturbations (zonal harmonics J_2 and J_3), most notably due to the non-spherical Earth. A natural phenomenon exists called a frozen orbit, for which the orbital elements of argument of perigee and eccentricity remain virtually fixed over extended periods of time. Simulation results show that, using frozen orbits conditions in relative motion dynamics results can be reduced the amount of required propellant for orbit correction maneuvers. This result is due to the perturbations over the course of a year and can extended the duration in orbital mission.

5. FURTHER WORKS

In the future, first the effects of other perturbations, such as atmospheric drag, solar radiation pressure, and lunar and solar gravity on the frozen orbit results could be studied. In addition, the effects of higher-level zonal harmonic perturbations and nonlinear models to elliptical reference orbits about relative motion dynamics could be investigated.

6. REFERENCES

1. Smith, J.C., "Analysis and application of frozen orbits for the TOPEX mission", ***AIAA and AAS, Astrodynamics Conference***, Williamsburg, VA, (1986), AIAA-1986-2069.
2. Frauenholz, R.B., Bhat, R.S. and Shapiro, B.E., "An analysis of the TOPEX/Poseidon operational orbit: observed variations and why", ***Advances in the Astronautical Sciences,*** Vol. 90, No. 2, (1996), 1127-1144.
3. Aorpimai, M. and Palmer, P.L., "Analysis of frozen conditions and optimal frozen orbit insertion", ***Journal of Guidance, Control and Dynamics***, Vol. 26 No. 5, (2003), 786-793.
4. Chao, C.C., Pollard, J.E., and Janson, S.W., "Dynamics and control of cluster orbits for distributed space missions", ***AAS/AIAA Spaceflight Mechanics Meeting***, Breckenridge, Vol. 102, (1999), 355-374.
5. Lara, M., Deprit, A. and Elipse, A., "Numerical continuation of families of frozen orbits in the zonal problem of artificial satellite theory", ***Journal of Celestial Mechanics and Dynamical Astronomy***, Vol. 62 No. 2, (1995), 167-181.
6. Wang, G., Meng, Y. and Zheng, W., "Artificial frozen orbit control scheme based on J_2 perturbation", ***Journal of Science China Technological Sciences,*** Vol. 53 No. 11, (2010), 3138-3144.
7. Shapiro, B.E., "Phase plane analysis and observed frozen orbit for the TOPEX mission", ***Sixth International Space Conference of Pacific Basin Societies***, Marina Del Rey, California, December 6-8, 1995 (1995).
8. Zhou, J.B., Yuan, J.P. and Luo, J.J., "Study of control strategy for frozen orbits with arbitrary orbital element by using radial low-thrust", ***Journal of Astronomy***, Vol. 29, (2008), 1536-1539.
9. Vallado, D.A. and McClain, W.D., "Fundamentals of Astrodynamics and Applications", McGraw-Hill, New York (1997).
10. Bate, R., Mueller, D. and White, J.E., "Fundamentals of Astrodynamics", Dover Publications, New York, (1971).
11. Chobotov, V.A., "Orbital Mechanics", AIAA Published, 3rd ed., Reston, VA, (2002).
12. Battin, R., "An Introduction to the Mathematics and Methods of Astrodynamics", AIAA Published, Reston, VA, (1999).

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Received in revised form 20 May 2012
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Zonal Harmonic Perturbation

چکیده

هدف این تحقیق، تحلیل کاربرد مؤثر مدارهای زمینی خاص در مدل‌سازی مسئله دینامیک نسبی بین دو فضاپیما و نقش آنها در کاهش محسوس مانورهای تصحیح مدار در مدت زمان یک ماموریت مداری نسبی می‌باشد. یکی از چالش‌ها در پیاده‌سازی این حرکات، حفظ شرایط نسبی بین دو وسیله بوده که بیشتر به علت ناهمگن بودن زمین دچار اغتشاشات (هارمونیک منطقه‌ای J_2 و J_3) می‌گردد. جنبه‌های معینی از شکل هندسی مدار می‌تواند عملا در بازه وسیعی از زمان ثابت بمانند که این ناشی از پدیده‌ای طبیعی تحت عنوان مدار منجمد می‌باشد. از جمله المان‌های مداری ثابت در این پدیده می‌توان به نرخ آرگومان حضیض (ω) و خروج از مرکز (e) اشاره نمود. نتایج شبیه سازی صورت گرفته، اهمیت بکارگیری شرایط مدارهای منجمد در بحث مدل‌سازی دینامیک نسبی، نقش مؤثر آن در کاهش خطای موقعیت نسبی و بالطبع افزایش عملکرد سیستم را بازگو می‌نماید. در همین راستا، متدی عرضه شده، که مشخص می‌سازد که آیا شرایط مدار هیل با توجه به تفاوت‌های اولیه در میزان خروج از مرکز و آرگومان حضیض برای دو فضاپیما رعایت خواهد شد. با بکارگیری شرایط منجمد در دینامیک حرکت نسبی، میزان پیشران موردنیاز فضاپیما جهت مانورهای تصحیح مدار ناشی از اغتشاشات هارمونیک در طی یک دوره مداری یکساله میتواند بطور محسوس کاهش داده شود.

doi: *10.5829/idosi.ije.2012.25.04a.01*