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RESEARCH NOTE

Determining an Economically Optimal (N,C) Design via Using Loss Functions

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A B S T R A C T

In this paper, we introduce a new sampling plan based on the defective proportion of batch. The proposed sampling plan is based on the distribution function of the proportion defective. A continuous loss function is used to quantify deviations between the proportion defective and its acceptance quality level (AQL). For practical purpose, a sensitivity analysis is carried out on the different values of the required sample sizes that allow practitioners to design near optimal inspection plans. A numerical example is presented to illustrate how the proposed procedure can be applied to design acceptance plans.

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1. INTRODUCTION

Acceptance sampling plans are one of the practical tools in statistical quality control applications. Sampling plans are used for decision making about accepting or rejecting a lot of products. With attribute sampling plans, these accept/reject decisions are based on the number of defectives items, while in variables sampling plans, decisions are based on the sample average and standard deviation [1].

Economic design of sampling plans is not widely addressed in literature. Ferrell and Chhoker [2] proposed a method to determine economically optimal acceptance sampling plans. Their approach is based on the Taguchi loss function. Pearn and Wu [3] introduced a variable sampling plan for unilateral processes based on one-sided process capability indices to deal with lot sentencing problem with very low fraction of defectives. Niaki and Fallahnezhad [4] used Bayesian inferences and dynamic programming concept to design an optimal sampling plan. Guenther [5] developed a search method for binomial, hypergeometric, and Poisson distributions to determine the optimal acceptance sampling plans. Moskowitz and Tang [6] proposed acceptance-sampling plans based on Taguchi loss function and Bayesian approach. Hailey [7] developed a method to obtain minimum sample size for single sampling plans based on the Poisson or binomial distribution. Stephens [8] developed a single sample acceptance sampling plans via using an approximation of the normal distribution with the binomial distribution. Fallahnezhad and Hosseininasab [9] proposed a single stage acceptance sampling plan based on the control threshold policy. The objective of their model is to find a constant control level that minimizes the total costs, including the cost of rejecting the batch, the cost of inspection and the cost of defective items. Fallahnezhad and Niaki [10] proposed a new acceptance sampling policy based on number of successive conforming items. Fallahnezhad and Niaki [11] proposed a Markov chain approach in acceptance sampling plans based on the cumulative sum of the number of successive conforming items. Fallahnezhad [12] proposed Markov analysis of acceptance sampling plans. Also, Fallahnezhad et al. [13] proposed Bayesian acceptance sampling plan. Jain and Kumar [14] proposed queuing analysis of a machine repair problem based on bi-level control policy.

In this research, optimization models are developed to design both 100% inspection and single sampling plans. The results reported in this paper are based on designing economically optimal acceptance sampling plans via using loss functions.

The consumer loss function decreases as the defective proportion of the items approaches the acceptance quality level and increases vice versa. Also,

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this loss can be modeled as a continuous function. The objective function is to minimize the total loss, that is, the loss to the producer plus the loss to the consumer.

The rest of the paper is organized as follows. 100% inspection model is presented in Section 2. The Single sampling model comes in Section 3. The numerical demonstration on the application of the proposed methodology comes in section 4. We conclude the paper in Section 5.

2. 100% INSPECTION MODEL

In this model, the objective is to determine the producer's control threshold that minimizes total loss to the producer and consumer in the case of 100% inspection. It is assumed the consumer's loss associated with a batch which its defective proportion exceeds the control threshold, and the producer's loss to inspect and replace an item are known.

Following notations are used in the rest of the paper,

c' : Cost of one inspection

p : Defective proportion of the batch

$f(p)$: Probability distribution function of p

Using Bayesian inference, the posterior probability distribution of p is determined as follows [15]:

$$f(p)=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}p^{\alpha-1}(1-p)^{\beta-1} \tag{1}$$

where, α denotes the number of defective items and β denotes the number of non-defective items.

$c_p(p)$ is the cost of one defective item for producer. The producer's cost to inspect and repair or replace a defective item is defined as follows:

$$c_p(p)=B \tag{2}$$

The consumer's loss follows a continuous function, here represented by quadratic function,

$$c_c(p)=A(p-AQL)^2 \tag{3}$$

It means if the consumer accepts a batch which its defective proportion is equal to p then he will have a loss equal to $A(p-AQL)^2$ where AQL is the accepted quality level.

The methodology used in this research is to consider the expected total loss, $E(L)$, for an acceptance sampling that its defective proportion has a probability density function, $f(p)$. The equation for $E(L)$ is,

$$E(L)=\int_0^{\delta}A(p-AQL)^2 f(p)dp+\int_{\delta}^{1}Bf(p)dp \tag{4}$$

The value of δ^* denotes an upper bound for acceptable values of defective proportions. It means if the defective proportion was more than δ^* then the batch should be rejected and the producer's cost to inspect and repair or replace a defective item is B, also if the defective proportion was less than δ^* then the batch should be accepted and the consumer loss will be $A(p-AQL)^2$. The equation of these two events is evaluated in Equation (4).

2. 1. Proposition 1 The producer's control threshold that minimizes $E(L)$, is

$$\delta^*=AQL+\sqrt{\frac{B}{A}} \tag{5}$$

Proof.

$$E(L)=\int_0^{\delta}A(p-AQL)^2 f(p)dp+\int_{\delta}^{1}Bf(p)dp \tag{6}$$

The first derivative is equated to zero as follows:

$$\frac{dE(L)}{d\delta}=0\rightarrow A(\delta-AQL)^2 f(\delta)=Bf(\delta) \tag{7}$$

Thus,

$$\delta=AQL\pm\sqrt{\frac{B}{A}} \tag{8}$$

Since theoretically the optimal value of δ is more than AQL, therefore:

$$\delta^*=AQL+\sqrt{\frac{B}{A}} \tag{9}$$

Further,

$$\frac{d^2E(L)}{d\delta^2}\Big|_{\delta=\delta^*}=2A\left(\sqrt{\frac{B}{A}}\right)f(\delta^*)+A\left(\sqrt{\frac{B}{A}}\right)^2 f'(\delta^*) -Bf'(\delta^*)=2\left(\sqrt{BA}\right)f(\delta^*)>0 \tag{10}$$

Therefore, $E(L)$ is convex and a minimum is determined.

3. SINGLE SAMPLING MODEL

This above modeling can be extended to single sampling models. Assume that the lot size (N) is arrived and a sample of n items is taken for inspection. Assume the results of inspection denoted that α defective items and $\beta=n-\alpha$ non-defective items existed in the sample. Then $f(p)$ that is the probability density function of p, is derived via Bayesian inference (Equation (1)). Decision making strategy is defined as follows:

If defective proportion of items in sample was more than the control threshold like δ then batch is rejected. In the case of rejecting a batch, 100% inspection should be carried out. Following the strategy used in 100% inspection model, the total expected loss associated with the single sampling scheme can be represented as:

$$E(L)=n\left(c'+\int_0^{\delta}A(p-AQL)^2 f(p)dp+\int_{\delta}^1 Bf(p)dp\right)+ \Pr\{\text{Accept the lot}\}(N-n)\int_0^1 A(p-AQL)^2 f(p)dp+ \Pr\{\text{Reject the lot}\}(N-n)\times \left(c'+\int_0^{\delta}A(p-AQL)^2 f(p)dp+\int_{\delta}^1 Bf(p)dp\right) \tag{11}$$

where, $\Pr\{\text{Accept the lot}\}$ is derived as follows

$$\Pr\{\text{Accept the lot}\}=\int_0^{\delta} f(p)dp=F(\delta) \qquad \Pr\{\text{Reject the lot}\}=1-F(\delta) \tag{12}$$

The equation $n\left(c'+\int_0^{\delta}A(p-AQL)^2 f(p)dp+\int_{\delta}^1 Bf(p)dp\right)$ denotes the cost of n inspected items that includes the cost of inspection, c' and producer and consumer loss functions. If the lot is accepted, then total cost is consumer loss function that is $(N-n)\int_0^1 A(p-AQL)^2 f(p)dp$ and if the lot is rejected then, 100% inspection should be performed thus the cost is the summation of inspection cost and loss functions for each item as follows:

$$(N-n)\times\left(c'+\int_0^{\delta}A(p-AQL)^2 f(p)dp+\int_{\delta}^1 Bf(p)dp\right) \tag{13}$$

By differentiating Equation (11) with respect to δ and setting it zero, we get

$$\frac{dE(L)}{d\delta}=0\rightarrow n\left(A(\delta-AQL)^2 f(\delta)-Bf(\delta)\right)+ (-f(\delta))(N-n)\times \left(c'+\int_0^{\delta}A(p-AQL)^2 f(p)dp+\int_{\delta}^1 Bf(p)dp\right)+ (1-F(\delta))(N-n)\left(A(\delta-AQL)^2 f(\delta)-Bf(\delta)\right)=0 \tag{14}$$

Thus,

$$\left(n+(1-F(\delta))(N-n)\right)\left(A(\delta-AQL)^2-B\right)= (N-n)\left(c'+\int_0^{\delta}A(p-AQL)^2 f(p)dp+\int_{\delta}^1 Bf(p)dp\right) \tag{15}$$

By solving the above equation, the optimal value of δ will be obtained. To solve above equation, numerical search procedures are suggested.

The minimum is insured by checking $\frac{d^2E(L)}{d\delta^2}>0$ that is resulted from the following inequality,

$$\frac{d^2E(\delta)}{d\delta^2}>0\rightarrow(N-n)\left((1-f(\delta))\left(A(\delta-AQL)^2-B\right)+ \left(n+(1-F(\delta))\right)2A(\delta-AQL)\right)>0 \tag{16}$$

If δ^* satisfies above inequality then it is concluded that $E(L)$ is convex and a minimum is assured. Since $\left(A(\delta-AQL)^2-B\right)$ is positive (Equation (15)), it is concluded that

$$\frac{d^2E(\delta)}{d\delta^2}>0 \tag{17}$$

4. NUMERICAL EXAMPLE

Consider a 100% inspection plan with a sample size of $n=10$. Assume 8 items are accepted in quality control inspection $(\alpha=2,\beta=8)$. The probability distribution function of defective proportion is derived as follows:

$$f(p)=Beta(2,8) \tag{18}$$

Assume consumer and producer loss functions are defined by the following equations,

$$c_p(p)=4, \qquad c_c(p)=400(p-0.1)^2 \tag{19}$$

4. 1. 100% Inspection The data in this numerical example for 100% inspection results in following model:

$$E(L)=\int_0^{\delta}400(p-0.1)^2 f(p)dp+\int_{\delta}^1 4f(p)dp \tag{20}$$

Thus,

$$\delta^*=0.1+\sqrt{\frac{4}{400}}=0.2 \tag{21}$$

And,

$$E(L)\big|_{\delta^*}=1.81$$

4. 2. Single Sampling Consider a single sampling plan where the lot size is $N=100$, the cost of each inspection is $c'=4$ and consumer and producer loss functions are defined in Equation (19). Combining all information in Equation (11) and (12) results in following model:

$$E(L)=10\left(4+\int_0^{\delta}400(p-0.1)^2 f(p)dp+\int_{\delta}^1 4f(p)dp\right) +F(\delta)(100-10)\int_0^1 400(p-0.1)^2 f(p)dp+ (1-F(\delta))(100-10)\left(\int_0^{\delta}400(p-0.1)^2 f(p)dp +\int_{\delta}^1 4f(p)dp+4\right) \tag{22}$$

By differentiating Equation (22) with respect to δ

and setting it zero yields $\delta^* = 0.365$. Since the value of δ^* denotes the upper bound for acceptable value of the defective proportion therefore the value of $n\delta^*$ denotes the acceptance number so that if the number of defective items in sample of n items was more than $n\delta^*$ then the batch is rejected otherwise it is accepted. Since the value of $n\delta^*$ is not necessarily an integer number therefore the parameter c in the optimal (n,c) design for acceptance plan is $c = \left[n\delta^*\right]$. Therefore in this numerical example $c = [10 \times 0.365] = 3$ and since $\alpha = 2 \le c = 3$ thus the batch should be accepted.

The model can be used to construct a sensitivity analysis. Assume that the proportion of defective items in the lot is 0.2, therefore the number of defective items in the sample size of n is $0.2n$ and the probability distribution function of p is $f(p) = Beta(0.2n, 0.8n)$ Table 1 denotes the optimal values of δ^* for different values of sample size, n (the lot size is assumed to be $N = 100$).

It is concluded from Table 1, when the value of sample size increases then the optimal values of δ^* converges to 0.2 that is equal to the optimal value of δ^* in 100% inspection plan. This result can be justified from Equation (15). For example when $N = n$, following result is concluded from Equation (15)

$$N\left(A(\delta - AQL)^2 - B\right) = 0 \rightarrow \delta^* = AQL + \sqrt{\frac{B}{A}} \tag{23}$$

Thus, the result of single sampling model coincides with the result of 100% inspection plan. Also in Table 1, minimum value of the objective function, $E(L)$ occurs at $n = 20$ therefore the optimal design is($n = 20$, $c = 6$).

5. CONCLUSION

In this paper, we develop a new sampling plan based on the defective proportion of the lot to deal with 100% inspection and single sampling when a continuous loss function is used to consider the cost associated with deviation between the actual value of a defective proportion and its accepted quality level. Also a method is developed for obtaining required sample size for inspection and corresponding critical acceptance values based on the probability distribution function of defective proportion, which minimize the loss function for both producers and consumers. In numerical example, it is illustrated that how the models can be used to determine the optimal design.

TABLE 1. The results of sensitivity analysis for various values of sample size

sample size, n	optimal values of δ^*	Optimal (n,c) design	$E(L)$
10	0.365	(10,3)	208.4169
20	0.305	(20,6)	175.6217
30	0.275	(30,8)	208.88
40	0.255	(40,10)	256.3835
50	0.242	(50,12)	306.3277
60	0.214	(60,12)	414.01

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چکیده

در مطالعه حاضر، یک روش نمونه‌گیری پذیرش بر اساس نسبت قطعات معیوب معرفی شده است. طرح نمونه‌گیری پیشنهادی بر اساس تابع توزیع نسبت قطعات معیوب است. از یک تابع زیان پیوسته برای اندازه‌گیری انحرافات بین نسبت قطعات معیوب و حد کیفی مورد پذیرش استفاده شده است. یک تحلیل حساسیت بر روی مقادیر محتلف اندازه نمونه صورت گرفته و یک مثال عددی در مورد نحوه عملکرد روش پیشنهادی نیز ارائه شده است.

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