OPTIMIZATION OF TREE-STRUCTURED GAS DISTRIBUTION NETWORK USING ANT COLONY OPTIMIZATION: A CASE STUDY

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Abstract An Ant Colony Optimization (ACO) algorithm is proposed for optimal tree-structured natural gas distribution network. Design of pipelines, facilities, and equipment systems are necessary tasks to configure an optimal natural gas network. A mixed integer programming model is formulated to minimize the total cost in the network. The aim is to optimize pipe diameter sizes so that the location-allocation cost is minimized. Pipeline systems in natural gas network must be designed based on gas flow rate, length of pipe, gas maximum pressure drop allowance, and gas maximum velocity allowance. We use the information regarding gas flow rates and pipe diameter sizes considering the gas pressure and velocity restrictions. We apply the Minimum Spanning Tree (MST) technique to obtain a network with minimum number of arcs, spanning all the nodes with no cycle. As a main contribution here, we present and use an ant colony optimization algorithm for solving the problem. The proposed method is applied to a real life situation. Our obtained results are compared to the ones obtained by an exact method. The results show that ACO is an effective approach for gas distribution network optimization. A case study in Mazandaran Gas Company in Iran is conducted to illustrate the validity and effectiveness of the proposed approach.

Keywords Natural-gas network; Minimum spanning tree ; Location-allocation; Ant colony optimization.

چکیده در این مقاله، از الگوریتم بهینه سازی مورچگان برای بهینه سازی شبکه گازرسانی آنتنی استفاده می شود. طراحی لوله ها، تسهیلات و تجهیزات جز وظایف ضروری برای طراحی یک شبکه گازرسانی بهینه محسوب می شوند. یک مدل برنامه ریزی عدد صحیح آمیخته به منظور می نیمم سازی کلیه هزینه ها در شبکه ارائه می شود. هدف این مدل، بهینه سازی اندازه قطر لوله ها همراه با می نیمم سازی هزینه مکان یابی– تخصیص می باشد. سیستم لوله گذاری در یک شبکه گازرسانی می بایست منطبق بر نرخ جریان گاز، طول لوله، ماکزیمم افت فشار مجاز گاز و ماکزیمم سرعت مجاز گاز باشد. در این مقاله، از اطلاعاتی مرتبط با نرخ جریان گاز و اندازه قطر لوله همراه با در نظر گرفتن محدودیت های سرعت و فشار گاز استفاده می شود. همچنین از تکنیک می نیمم درخت پوششی برای بدست آوردن شبکه ای با کمترین یال و پوشش تمامی گره ها بدون شکل گیری مسیر بسته ای بهره گرفته می شود. سهم عمده این مقاله در استفاده از الگوریتم بهینه سازی مورچگان برای حل این گونه مسایل است. نتایج بدست آمده از این الگوریتم با نتایج بدست آمده از روش حل دقیق مقایسه می شوند. نتایج نشان می دهند که الگوریتم پیشنهادی بهینه سازی مورچگان، رویکردی موثردر بهینه سازی شبکه گازرسانی می باشد. مطالعه موردی انجام شده در شرکت گاز استان مازندران، صحت رویکرد پیشنهادی را تایید می کند.

1. INTRODUCTION

Natural gas is one of the most important sources of energy. Exploration, extraction, production, and transportation are stages which natural gas goes through to secure consumers. The network contains three phases. The transmission of gas from central production facility to City Gate Station (CGS) is called a transmission phase, while forwarding gas from CGS to Town Board Station (TBS) is called a feeding phase. Securement of gas to consumers is performed in the distribution phase, where TBS is responsible for supplying the desirable gas pressure based on consumer's viewpoint. Due to the movement of a large volume of gas at high pressures over long distances, transmission and distribution planning are basic elements of a natural gas network. While gas pressure is reinforced by compressors in the transmission network, it is reduced by pressure reduction stations in the distribution network. Gas pressure is lessened twice in the distribution network. CGS is defined as a pressure reduction station in which gas pressure is reduced from 1000 psi to 250 psi. In order to maintain the desired gas pressures based on consumers' viewpoints, gas pressure should be fractured for a second time. A TBS is a pressure reduction station reducing gas pressure from 250 psi to 60 psi. Optimal types and locations of pressure reduction stations play key roles in minimizing the total cost in the network. Various topologies of natural gas network exist such as linear, tree-structured, and cyclic. The tree-structured gas network is based on the minimum spanning tree (MST) model. This model is a combinational optimization problem. The model configures a connected graph with no cycles, spanning all the nodes in the network with the aim to minimize the total cost. This model has plentiful direct applications in computer design, telecommunication, transportation, pipeline network, etc. Also, indirect applications are in network reliability, clustering and classification problems.

The design of pipeline, facility, and equipment systems for securing consumers' demands are necessary actions in the process of achieving an optimal natural gas network. There are factors influencing the total cost of a gas network such as pipe diameter, pressure, length, etc. Pipelines in a natural gas network must be designed based on gas flow rate, length of pipe, gas maximum pressure drop allowance, and gas maximum velocity allowance. Gas minimum pressure allowance in the network is set to be 30 psi. In order to avoid the erosion phenomenon in a pipe, gas velocity should not exceed 20 m/sec. Consideration of all factors together for the design of an optimal gas network constitutes an impressive study. Gas companies usually apply heuristic methods which are based on human's judgment and experience to find an optimal network. Trial and error procedures are common for such methods. But, for such methods to generate an optimal solution, one often needs an excessive computing work. Optimization methods, however, are suitable tools guaranteeing obtainment of optimal solutions with reasonable computing costs.

In the present paper, we focus on distribution networks in which gas flow is transferred from the TBS to consumers by pipeline systems. In addition to determine the optimal types and locations of the TBS, our objective here is to construct an optimal distribution network of gas pipelines having various pipe diameter sizes. Optimal design of pipe diameter sizes in the network has an important effect on cost reduction. There are various factors influencing the pipe diameter sizes such as gas flow rate, gas pressure, and gas velocity. Pipe diameter sizes are selected from a set of standard sizes available in the market. The gas company provides the data concerning pipe diameter sizes and gas flow rates with the corresponding costs. Our study addresses the gas distribution problem on a tree-structured topology. A tree-structured topology is a graph with minimum number of arcs, no cycles, with all nodes being spanned in the network. In this study, we do not determine optimum pipe size for predefined gas distribution network with fixed locations of TBSs. Consequently, the optimal tree-structured gas distribution network is constituted. The objective function is linear, and all constraints are linear.

Many efforts for optimization of natural gas network have been expended in the past decades. The mainline system is a vital part of natural gas network systems. The investment required for the mainline system is enormous, usually accounting for 80% of the total investment for this system. The work of Ruan et al. [1] represents one of the attempts to design the mainline system in natural gas network. They considered all the factors influencing the total investment of a gas network system (e.g. pipe diameter, thickness, pressure, length, compression ratio, etc). Hamedi et al. [2] presented an integrated multi-period optimization model for the distribution planning in different stages of natural gas supply chain. They formulated a mixed integer non-linear programming model with the aim to minimize direct or indirect distribution costs. In their work, a real-world case study of a natural gas supply chain was investigated. Optimization of gas transport in networks was investigated by Domschke et al. [3]. They considered networks containing pipes and various other components like compressor stations and valves with the aim to minimize the running costs of the compressor stations. Besides, they used different equations based on the Euler equation to formulate the gas flow through the pipes. Andre et al. [4] presented techniques for solving the problem of minimizing investment costs on an existing gas transportation network. They found the optimal location of pipeline segments to be reinforced and the optimal sizes and developed new heuristics for solving a non-linear problem. Numerous studies related to the case study on natural gas exist. Nørstebø et al. [5] optimized operation of the Norwegian natural gas system. GassOpt is a general tool for gas network optimization, but they applied it on the Norwegian gas transport network specifically. Erturk and Turut-Asik [6] analyzed the performance of 38 Turkish natural gas distribution companies by Data Envelopment Analysis (DEA). The results were used to detect the important criteria affecting the efficiency levels, and to find the common characteristics of the most inefficient firms.

Our proposed natural gas network is focused on the gas distribution phase and is optimized by the Minimum Spanning Tree (MST) model and then is solved by a proposed ACO algorithm for large size problems. To solve optimization problems for gas networks, in recent years, several methods were developed and none of them have considered all aspects of the problem. The majority of them are based on the dynamic programming (DP) or gradient search techniques. The type of the methods used for solving optimization problems are influenced by problem's nature. The work of Carter [7] represents one of attempts to optimize networks with general structures and with fixed flow rates using DP techniques. Borraz-Sanchez and Haugland [8] proposed a solution method based on dynamic programming to address the fuel cost minimization problem to transport natural gas in a general class of transmission networks. They considered two types of continuous decision variables: mass flow rate along each arc and gas pressure level at each node and formulated a non-linear mathematical model. Wu et al. [9] formulated a model for fuel cost minimization. The model employs a gradient search technique for the gas network. Percell and Ryan [10] proposed a gradient search technique for minimization of fuel consumption in gas transmission networks.

The principal advantages of DP are that a global optimum is guaranteed and that the non-linearity can be easily treated. The disadvantages of DP are that its application is practically limited to the simple network topologies and that computation increases in an exponential way with problem dimension. The advantages of GRG method consist of that dimension is not a problem and, it could be applied to cyclic network schemes. However, as GRG method is based on a search gradient method, there is no guarantee to find a global optimum. In fact, with discrete decision variables, it can be fixed with the local optimum.

The main original contribution proposed in this paper, is that we use the ant colony optimization (ACO) algorithm to solve our proposed natural gas network problem. The results obtained with the suggested approach are excellent with a strong computation time saving compared to those obtained with an exact method. This will enable us to design a fast, effective and robust decision aid tool based on the suggested method. This tool will assist operators to make the most appropriate decision within a short time.

ACO was introduced in the early 1990s. The inspiring source of ACO algorithms is the actual ant colony behavior. Solving discrete optimization problems, continuous optimization problems, and other important problems in communication networks such as routing and line balancing have been considered using ACO. At the core of ants' behavior is the indirect communication amongst the ants by means of chemical pheromone trails, enabling the ants to find minimal paths between food sources and their nest. Dorigo and Stützle [11] used this ants' peculiarity in their proposed ant colony algorithm to solve problems such as discrete optimization problems. Blum and Roli [12] investigated the ACO algorithm from an Operations Research (OR) viewpoint, classifying the algorithm as a metaheuristic one. Ant System (AS) was the first type of an ACO algorithm that was used for the Traveling Salesman Problem (TSP). This algorithm was introduced by Dorigo [13]. The ACO algorithm has plentiful applications to discrete optimization problems. Beside its use for the TSP problem, this algorithm is used for several other problems such as assignment, scheduling, graph coloring, and vehicle routing. Design of an ACO algorithm for MST problems was first proposed by Reimann and Laumanns [14]. The authors used the algorithm for the capacitated minimum spanning tree problems where capacity restrictions existed on the arcs. Complexity of gas network problems necessitates the employment of heuristic and metaheuristic algorithms. Chebouba et al. [15] optimized the natural gas pipeline transportation using an ACO algorithm. The authors proposed an ACO algorithm for the gas pipeline systems having fixed flow rate. The study focused on gas transmission networks where the compressor stations reinforced the gas pressure. The decision variables used in the study were the number of compressor stations and the amount of pressure discharged in each station. The aim of the proposed model was to optimize the consumed power in the network.

While most studies deal with transmission of natural gas, our study focuses on gas distribution networks. In recent years, some studies optimized pipe sizes in predefined structure of gas distribution networks. Conversely, we intend to construct an optimal gas distribution network having various pipe sizes along with the optimal types and locations of TBSs. Overall, our main original contribution proposed in this paper, is that we present the linear model to construct an optimal gas distribution network and propose the ACO algorithm to solve the large size problems. Here, we intend to conduct a case study of the natural gas network in Mazandaran Gas Company in Iran.

The paper is organized as follows. A description of the natural gas network is given in Section 2. In Section 3, we present our proposed ACO algorithm. Section 4 discusses a case study conducted in Mazandaran Gas Company in Iran. Finally, conclusions are given in Section 5.

2. THE PROBLEM STATEMENT

2.1. Description Here, we focus on the gas distribution phase of the natural gas network comprising of the pipelines having various sizes, TBSs, and consumers. The pipeline and TBS belong to the two indispensable components. The pipeline is responsible to connect the two places and to transmit a gas between them, while the TBS provides the motive force to maintain the natural gas flow through the network systems. The gas flow originates from the TBS and moves in a downward direction (i.e., it moves from a place with a higher pressure towards a place with a lower pressure). The amount of gas flow transferred between the two places is set to be the aggregate value of both demand and gas flow exited from a place with lower pressure. So, the consumers being farther away from the TBS have smaller pipe diameter size.

A gas distribution network is defined by the set of all nodes V and the set of all arcs A. In this network, consumers and the TBS are defined as a set of nodes and connected pipelines are defined as a set of arcs. A tree-structured natural gas network is described for the TBS and consumers. A tree is a connected graph with no cycles and all nodes spanned. Candidate locations for the TBS are given by the gas company. Our approach proceeds in two steps: (a) determination of the locations and types of the TBS, and (b) choice of the size of the pipelines. It is necessary to take into account several constraints. Some of these are restrictions of network infrastructure. Others are technical constraints such as the capacity constraints. We assume that pressure of gas should not be lower than 30 psi in the network. Also, the upper bound of gas velocity in the network is set to be 20 m/sec. Our model minimizes the total cost in the gas distribution network using the Minimum Spanning Tree (MST) technique.

2.2. Formulation This section presents a mixed-integer linear programming (MILP) optimization model to minimize the total construction cost of this network. Costs in the objective function include the establishing and transportation costs. The transportation cost as presented in the problem is an average cost where material, construction, administrative overhead and maintenance costs are all aggregated in a total cost per unit length. The cost values are calculated by gas company. This study uses the data available in the historical data of the company. The model is a single period planning tool used for short, medium, and long term planning.

A mathematical modelling of the gas transportation problem in networks was previously presented elsewhere. The model is general enough to take into account equations which govern the flow of the gas through pipes. The flow equation describes the relationship between pressure difference and gas flow in pipes. The relationship between pressure and flow rate exhibits a high degree of nonlinearity. In order to overcome this problem, we use the provided information based on the relationships among the gas flow rates and pipe diameter sizes by the gas company. They computed ratio of gas flow rates and pipe diameter sizes, keeping constant velocity and gas pressure. Velocity and gas pressure considered to be 20 m/sec and 30 psi, respectively. Table 1 shows the relationships among gas flow rates and pipe diameter sizes.

Here, a tree-structured gas distribution network is modeled efficiently using MST method. However, in order to guarantee the reliability of the gas system, a tree-structured network among the TBS is introduced into the pipeline network.

In this problem, the main objective in dealing with tree-structured pipelines is to determine the flow rate of pipelines in the network. As an impressive study, we show how to formulate the relationships among gas flow rates and pipe diameter sizes to minimize the flow cost. The model description follows here.

Notations:

I = set of candidate TBSs

T = set of TBS types

Z = set of consumer/demand zones

Parameters:

C = cost of piping per distance unit with respect to the gas flow rate

CT = average cost of piping per distance unit among the TBS

S_t = establishing the cost for TBS of type t

q_z = demand of consumer zone z

Q_{it} = capacity of TBS i of type t

d_{iz} = distance between TBS i and consumer zone z

$d'_{ii'}$ = distance between TBS i and TBS i'

$d''_{zz'}$ = distance between consumer zone z and consumer zone z'

M = a large number

Decision variables:

$$r_i = \begin{cases} 1, & \text{if TBS } i \text{ is located} \\ 0, & \text{o.w.} \end{cases}$$

$$h_{it} = \begin{cases} 1, & \text{if TBS } i \text{ of type } t \text{ is selected} \\ 0, & \text{o.w.} \end{cases}$$

$$y_{itz} = \begin{cases} 1, & \text{if consumer zone } z \text{ is connected to TBS } i \text{ of type } t \\ 0, & \text{o.w.} \end{cases}$$

$$w_{zz'} = \begin{cases} 1, & \text{if there is a direct link between consumer zone } z \text{ to consumer zone } z' \\ 0, & \text{o.w.} \end{cases}$$

$$u_i = \begin{cases} 1, & \text{if TBS } i \text{ is a root} \\ 0, & \text{o.w.} \end{cases}$$

$$x_{ii'} = \begin{cases} 1, & \text{if there is a direct link between TBS } i \text{ to TBS } i' \\ 0, & \text{o.w.} \end{cases}$$

N_z =allocated number of consumers to consumer zone z

$f_{ii'}$ = amount of flow from TBS i to TBS i'

$f'_{zz'}$ = amount of flow from consumer zone z to consumer zone z'

$f''_{zz'}$ = amount of gas flow from consumer zone z to consumer zone z'

ew_z = amount of congested gas flow to be supplied to each consumer zone z

ew'_{itz} = amount of gas flow from TBS i of type t to consumer zone z

Objective function:

$$\min f = f_1 + f_2 + f_3 + f_4$$

where,

$$f_1 = \sum_{i \in I} \sum_{t \in T} h_{it} S_t, \tag{1}$$

$$f_2 = \sum_{i \in I} \sum_{t \in T} \sum_{z \in Z} y_{itz} d_{iz} C, \tag{2}$$

$$f_3 = \sum_{i \in I} \sum_{j \in J} x_{ii'} d'_{ii'} CT, \tag{3}$$

$$f_4 = \sum_{z \in Z} \sum_{z' \in Z} w_{zz'} d''_{zz'} C. \tag{4}$$

Constraints:

$$u_i \leq r_i\ , \qquad \forall i \in I, \quad (5)$$

$$\sum_{i \in I} u_i = 1, \quad (6)$$

$$r_i - M\left(r_{i'} - 1\right) \geq x_{ii'}, \qquad \forall i, i' \in I, \quad (7)$$

$$r_{i'} \geq x_{ii'}, \qquad \forall i, i' \in I, \quad (8)$$

$$\sum_{i \in I} x_{ii'} \geq \left(1 - u_{i'}\right) + M\left(r_{i'} - 1\right), \qquad \forall i' \in I, \quad (9)$$

$$\sum_{i \in I} x_{ii'} \leq \left(1 - u_{i'}\right) - M\left(r_{i'} - 1\right), \qquad \forall i' \in I, \quad (10)$$

$$\sum_{i \in I} x_{ii'} \leq r_{i'}, \qquad \forall i' \in I, \quad (11)$$

$$x_{ii'} \leq f_{ii'}, \qquad \forall i, i' \in I, \quad (12)$$

$$f_{ii'} \leq x_{ii'} M, \qquad \forall i, i' \in I, \quad (13)$$

$$\sum_{i \in I} f_{ii'} - \sum_{i'' \in I} f_{i'i''} \geq \left(\left(-u_{i'} M\right) + 1\right) + \left(r_{i'} - 1\right), \ \forall i' \in I, \quad (14)$$

$$\sum_{i \in I} f_{ii'} - \sum_{i'' \in I} f_{i'i''} \leq \left(\left(u_{i'} M\right) + 1\right) + \left(r_{i'} - 1\right), \qquad \forall i' \in I, \quad (15)$$

$$\sum_{t \in T} h_{it} = r_i, \qquad \forall i \in I, \quad (16)$$

$$\sum_{z \in Z} y_{itz} \geq h_{it}, \qquad \forall i \in I, \forall t \in T, \quad (17)$$

$$\sum_{z \in Z} y_{itz} \leq h_{it} M, \qquad \forall i \in I, \forall t \in T, \quad (18)$$

$$\sum_{z \in Z} w_{zz'} + \sum_{i \in I} \sum_{t \in T} y_{itz'} = 1\ , \qquad \forall z' \in Z, \quad (19)$$

$$N_z - \sum_{z' \in Z} f'_{zz'} = 1\ , \qquad \forall z \in Z, \quad (20)$$

$$N_{z'} + M\left(w_{zz'} - 1\right) \leq f'_{zz'}\ , \qquad \forall z, z' \in Z, \quad (21)$$

$$N_{z'} - M\left(w_{zz'} - 1\right) \geq f'_{zz'}\ , \qquad \forall z, z' \in Z, \quad (22)$$

$$f'_{zz'} \leq w_{zz'} M\ , \qquad \forall z, z' \in Z, \quad (23)$$

$$f'_{zz'} \geq w_{zz'}\ , \qquad \forall z, z' \in Z, \quad (24)$$

$$\left(w_{zz'} - 1\right) M + \left(q_{z'} + ew_{z'}\right) \leq f''_{zz'}, \qquad \forall z, z' \in Z, \quad (25)$$

$$\left(w_{zz'} - 1\right)\left(-M\right) + \left(q_{z'} + ew_{z'}\right) > f''_{zz'}, \qquad \forall z, z' \in Z, \quad (26)$$

$$f''_{zz'} \leq w_{zz'} M, \qquad \forall z, z' \in Z, \quad (27)$$

$$f''_{zz'} \geq w_{zz'}, \qquad \forall z, z' \in Z, \quad (28)$$

$$\sum_{z' \in Z} f''_{zz'} = ew_z, \qquad \forall z \in Z, \quad (29)$$

$$\left(y_{itz} - 1\right) M + \left(q_z + ew_z\right) \leq ew'_{itz}, \ \forall i \in I, \forall t \in T, \forall z \in Z, \quad (30)$$

$$\left(y_{itz} - 1\right)\left(-M\right) + \left(q_z + ew_z\right) \geq ew'_{itz}, \forall i \in I, \forall t \in T, \forall z \in Z \quad (31)$$

$$ew'_{itz} \leq y_{itz} M, \qquad \forall i \in I, \forall t \in T, \forall z \in Z, \quad (32)$$

$$ew'_{itz} \geq y_{itz}, \qquad \forall i \in I, \forall t \in T, \forall z \in Z, \quad (33)$$

$$\sum_{z \in Z} ew_{itz} \leq Q_{it}, \qquad \forall i \in I, \forall t \in T, \quad (34)$$

$$r_i, h_{it}, y_{itz}, u_i, x_{ii'}, w_{zz'}, \in \{0,1\}, \quad \forall i, i' \in I, \forall t \in T, \ \forall z, z' \in Z, \quad (35)$$

$$N_z, f_{ii'}, f'_{zz'}, f''_{zz'}, ew_z, ew'_{itz} \geq 0, \quad \forall i, i' \in I, \forall t \in T, \ \forall z, z' \in Z. \quad (36)$$

Equations (1)-(4) are the cost functions corresponding to the location-allocation costs. Constraint (5) indicates that exactly one TBS must be defined as a root. Constraint (6) ensures that there is exactly one TBS as the root in the network. Constraints (7) and (8) show the link between two TBSs. Constraints (9)-(11) impose that each TBS receive exactly one link from other TBSs if it is not the root node. The amount of flow between each TBS i and TBS i' is represented by constraints (12) and (13). Constraints (14) and (15) guarantee that there is no closed loop in the network. Constraint (16) shows that each TBS can adopt only one type when it is selected to service consumers. Constraints (17) and (18) ensure that each TBS covers at least one consumer. Constraint (19) represents that each consumer receives service from one consumer or one TBS. Constraint (20) determines the allocated number of consumers to consumer zones. Constraints (21)-(24) express the flow between two consumers. Constraints (25)-(28) represent the amount of gas flow from consumer zone z to consumer zone z'. Constraint (29) indicates the amount of congested gas flow for supplying other consumers by each consumer. The amount of gas flow from TBS to consumer is shown by constraints (30)-(33). Capacity restriction is shown by constraint (34). Constraint (35) imposes that the variables be binary. Non-negativity of the variables is represented by constraint (36).

3. THE PROPOSED ACO ALGORITHM

Our proposed model is based on minimum spanning tree (MST) method which belongs to the NP-hard class of problem (Garey and Johnson, [16]). Because of the complexity of these problems, exact methods need excessive computation time. So, heuristic and metaheuristic algorithms are essential tools for solving such problems in reasonable amounts of time. However, such algorithms do not necessarily always find global optimum solutions, but yield good solutions in reasonable times.

From the optimization model, it can be discerned that our problem belongs to the discrete optimization problems. Since, the ACO algorithm has plentiful applications to discrete optimization problems (e.g. TSP problem, MST problem, Vehicle Routing problem, Network Routing problem, etc) and converges to solutions rapidly keeping quality of solutions, the optimization model is solved by a proposed ACO algorithm for large size problems. The ACO algorithm is proposed to attain near optimized solution, decrease computational complexity, and overcome the computer memory limitation.

3.1. Introduction Metaheuristic algorithms are designed for solving optimization problems, taking their inspiration from nature. Dorigo [13] proposed an ACO algorithm to solve the combinatorial optimization problems.

The ACO algorithm is based on population and memory. A population-based approach produces different cycles in the algorithm. Each cycle contains a number of solutions. A memory-based approach saves the obtained information in each cycle. So, each cycle uses the obtained information in the previous cycles and gains solutions better than the previous ones. The algorithm cleans up the produced solutions at the end of each cycle.

In an implementation of an ACO algorithm consecutive, cycles are generated in which a number of solutions are produced by ants. At the end of each cycle, a pheromone trail matrix is updated using the obtained solutions. This process continues until the algorithm is terminated. There are essential elements to be considered in the design and implementation of an ACO algorithm. The elements are:

- A solution generation mode of ants.
- An updating mode for pheromone trail matrix.
- Stopping conditions.

3.2. A Solution Generation Mode A solution for the tree-based natural gas network problem is composed of a tree which spans all nodes and has no cycle. In this structure, there exists a variety of candidate TBS combinations to secure consumers. Naturally, we are faced with networks with different sizes. So, each ant is responsible to produce a solution structure for each plan. First, each ant adopts a node from its set of TBSs and consumer nodes, randomly. Then, the next node is selected using a probability. The process continues until all the nodes are spanned. A solution structure is designed based on the following assumptions:

- There is at least one pipe exiting a TBS
- There is an uttermost *np* pipes exiting a TBS
- Connection links among the TBSs are tree-structured

The number of produced solution structures at the end of each cycle is to be equal to the number of different candidate TBS combinations.

3.3. A Selection Mode for Next Mode At first, an initial node is selected by each ant, randomly. Then, each ant selects the other nodes using a transition probability function. This function allocates a transition probability to each node. To prevent construction of closed loops in the network, a list of nodes is maintained as a tabu list. A tabu list contains the nodes visited by an ant. A transition probability of nodes available in the tabu list is set to be zero. In contrast, the unvisited nodes constitute an allowed list of nodes for each ant. These are nodes not met previously by an ant. Let $Tabu_k$ be the tabu list corresponding to ant *k* and $Allowed_k$ be the allowed list of ant *k*. A transition probability function is defined as follows:

$$p_{ij} = \begin{cases} \dfrac{[\tau_{ij}(t)]^{\alpha}[\eta_{ij}]^{\beta}}{\sum_{k \in Allowed_k}[\tau_{ik}(t)]^{\alpha}[\eta_{ik}]^{\beta}}, & \\ \qquad \text{if } (j \in Allowed_k, i \in Tabu_k) & \\ 0, \qquad \text{otherwise,} & \end{cases} \tag{37}$$

where, p_{ij} is the transition probability of node j corresponding to ant located at node i, η_{ij} is set to be $1/d_{ij}$ with d_{ij} being the distance between nodes i and j, $\tau_{ij}(t)$ is the pheromone trail on arc (i, j) in period t, β and α are relative importance of distance and pheromone trail, respectively.
Considering the selection mode for the next node, generation of infeasible solutions is possible. To prevent this, a solution structure should be adopted following the assumptions pointed out in the previous section.

3.4. An Updating Mode for Pheromone Trail Matrix Information is saved by the pheromone trail matrix in the ACO algorithm. Updating of pheromone trail matrix at the end of each cycle is an essential task for the obtainment of new solutions. Let $\tau_{ij}(t)$ be a pheromone trail on arc (i, j) in period t. Each ant moves from the current node to the next node in period $t+1$. An iteration is defined as a movement of ant from the current position to another position. A tree structure is completed after n iterations. Note that n iterations configure a cycle. Updating of pheromone trail on arcs at the end of each cycle is followed here:

$$\tau_{ij}(t+n) = \rho \cdot \tau_{ij}(t) + \Delta\tau_{ij}, \tag{38}$$

where, $\tau_{ij}(t)$ is a pheromone trail on arc (i, j) in period $t, \tau_{ij}(t+n)$ is a pheromone trail corresponding to the similar arc in the next cycle, ρ is the resistance coefficient of pheromone trail, and so $1-\rho$ is an evaporation rate of pheromone trail, and $\Delta\tau_{ij}$ is the pheromone trail on arc (i, j), secreted from each ant in interval $(t, t+n)$, as specified below:

$$\Delta\tau_{ij} = \begin{cases} \dfrac{Q}{L}, & \text{if ant uses arc } (i, j) \text{ in} \\ & \text{its tour } (\text{between } t \text{ and } t+n) \\ 0, & \text{otherwise,} \end{cases} \tag{39}$$

where, Q is a constant and L is the length of the traveled path by ant at the end of each cycle.

We design a mutation operator to prevent stagnation of the algorithm at the final step. After constitution of the updated pheromone trail matrix, each ant selects an arc, randomly. The pheromone trail corresponding to the arc is calculated as follows:

$$\begin{gathered} \tau_{kz}(t+n) = \underset{i,j}{Max}\, \tau_{ij}(t+n) \cdot (1+\delta), \\ \text{with } k \text{ and } z, \text{random numbers chosen uniformly} \\ \text{from the interval } [1, N] \end{gathered} \tag{40}$$

where, δ is a mutation coefficient and N is the total sum of number of TBSs and number of consumer nodes.

3.5. Stopping Condition Stopping condition can be introduced in various forms. Here, our stopping condition for our proposed ACO algorithm is the maximum number of cycles allowed in the algorithm.

3.6. Algorithm Here, the proposed ACO algorithm is presented as follows.

Algorithm 1. ACO algorithm

Initialize maxiter, inittrail, $\alpha, \beta, \delta, \rho$.
Set $\tau_{i,j}$ =inittrail, for all $(i, j) \in I$, noiter $= 0$.
Generate a solution x using a construction procedure.
Update the pheromone values and set $x^* = x$.
repeat
  Compute x using a construction procedure.
    if $f(x) \le f(x^*)$ then
    update the pheromone values.
    set $x^* = x$.
    end if
      noiter = noiter+1.
until stopping condition is met (i.e., noiter=maxiter)

Note that at the initial step, the parameters are initialized. Maxiter is used in the stopping condition for the proposed algorithm. Inittrail is an initial amount of pheromone trail for each arc. At the second step, a construction procedure is used to form a solution. Then, the pheromone trail matrix is updated and fitness function is evaluated. A fitness function for the proposed ACO algorithm contains two types of costs. The costs are:

- Establishing cost for TBS with respect to its type
- The cost of piping with respect to gas flow rate among the nodes

At the next cycles, if the fitness function is better than previous one, a pheromone trail matrix is updated. The main steps in the proposed ACO algorithm are shown in Figure 1 and include:

- Ant builds its tour as it moves from one decision point to the next until all decision points have been covered
- The cost of the tour generated is calculated, pheromone is updated
- After the completion of one iteration (t), if the fitness function is better than previous one, pheromone is updated.

A construction procedure of solution is given by Algorithm 2. This algorithm is tree-structured. A tree is a connected graph with no cycle, spanning all the nodes with a minimal number of arcs.

Algorithm 2. Construction procedure (G,τ,η)

(0) Give the input set of nodes $V=\{v_1,v_2,...,v_n\}$;
Set $T=\phi$ {tabu list}; $A=V$ {allowed list}; $P=\phi$ {set of probabilities}; $path=[0]_{n\times n}$ {path matrix}.
(1) Choose $s\in V$ arbitrarily.
(2) Set $T=\{s\}$ and $A=\{V\}-\{s\}$.
(3) Compute $R_u=\sum_{v\in A}[\tau_{(u,v)}]^{\alpha}\cdot[\eta_{(u,v)}]^{\beta},\ \forall u\in T$.
(4) Compute $R'=\sum_{u\in T}R_u$.
(5) Compute the probabilities:

$$p_{(u,v)}=\frac{\left[\tau_{(u,v)}\right]^{\alpha}\cdot\left[\eta_{(u,v)}\right]^{\beta}}{R'},\quad \forall u\in T,\forall v\in A,$$

and $P=P\bigcup_{\substack{u\in T\\ v\in A}}p_{(u,v)}$.
(6) Find maximal probability value in P:
$p(u^*,v^*)=\max_{p(u,v)\in P}p(u,v)$.
(7) Update the path matrix:
$path(u^*,v^*)=1$.
(8) Update T and A:
$T=T\cup\{v^*\},\ A=A-\{v^*\},\quad P=\phi$.
(9) If $A\neq\phi$ then go to (3)
else, stop {the matrix path contains the optimal path}.

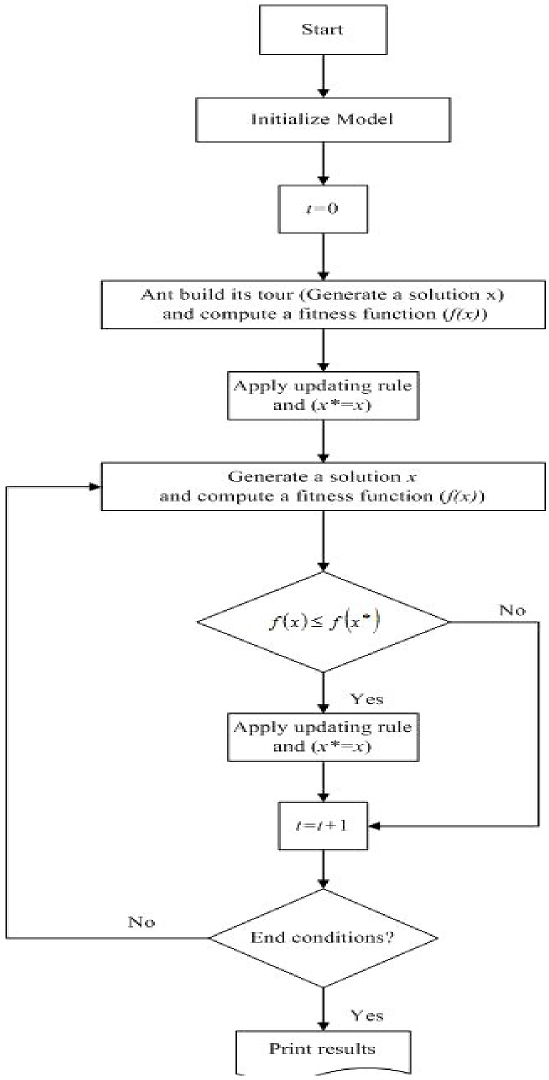

Figure 1. Steps in the proposed ACO algorithm.

Note that the minimum spanning tree approach makes use of a graph *G= (V, E)* as a connected network, where *V* is a set of nodes and *E* is a set of arcs. *T* is a Tabu list and *A* is an Allowed list. *P* is a set of probabilities. The Path matrix contains 0 and 1 values. If each ant uses arc (i,j) in its tour, the related element is set to be 1; otherwise, 0. This procedure continues until all nodes are spanned in the network. In other words, until the Allowed list for each ant becomes empty.

We now give an example to illustrate the steps of the above algorithm.

Example: Let $V=\{1,2,3,4,5\}$, following step (0), we have: $T=\phi$, $A=\{1,2,3,4,5\}$, $P=\phi$, $path=[0]_{5\times 5}$.

Iteration 1:

Step (1): Choose $4 \in V$.

Step (2): $T = \{4\}$, $A = \{1,2,3,5\}$.

Step (3): $R_4 = \sum_{v \in A} \left[\tau_{(4,v)}\right]^{\alpha} \cdot \left[\eta_{(4,v)}\right]^{\beta}$, $4 \in T$.

Step (4): $R' = R_4$.

Step(5): $p_{(4,1)} = 0.25, p_{(4,2)} = 0.35, p_{(4,3)} = 0.10,$
$p_{(4,5)} = 0.30,$

and

$$P = \{p_{(4,1)} = 0.25, p_{(4,2)} = 0.35, p_{(4,3)} = 0.10, p_{(4,5)} = 0.30\}.$$

Step (6): $p_{(u^*,v^*)} = p_{(4,2)} = 0.35.$

Step (7): $path_{(4,2)} = 1.$

Step (8): $T = \{4,2\}$, $A = \{1,3,5\}$, $P = \phi$.

Step (9): Since $A \neq \phi$, Then go to step (3).

Iteration 2:

Step(3):

$$R_4 = \sum_{v \in A} \left[\tau_{(4,v)}\right]^{\alpha} \cdot \left[\eta_{(4,v)}\right]^{\beta},\ R_2 = \sum_{v \in A} \left[\tau_{(2,v)}\right]^{\alpha} \cdot \left[\eta_{(2,v)}\right]^{\beta}.$$

Step (4): $R' = R_4 + R_2$.

Step (5): $p_{(4,1)} = 0.15,\ p_{(4,3)} = 0.15,\ p_{(4,5)} = 0.20,$
$p_{(2,1)} = 0.25,\ p_{(2,3)} = 0.15,\ p_{(2,5)} = 0.10,$

and

$$P = \begin{Bmatrix} p_{(4,1)} = 0.15,\ p_{(4,3)} = 0.15,\ p_{(4,5)} = 0.20,\ p_{(2,1)} = 0.25, \\ p_{(2,3)} = 0.15,\ p_{(2,5)} = 0.10 \end{Bmatrix}.$$

Step (6): $p_{(u^*,v^*)} = p_{(2,1)} = 0.25.$

Step (7): $path_{(2,1)} = 1.$

Step (8): $T = \{4,2,1\}$, $A = \{3,5\}$, $P = \phi$.

Step (9): Since $A \neq \phi$, Then go to step (3).

Iteration 3:

Step(3):

$$R_4 = \sum_{v \in A} \left[\tau_{(4,v)}\right]^{\alpha} \cdot \left[\eta_{(4,v)}\right]^{\beta},\ R_2 = \sum_{v \in A} \left[\tau_{(2,v)}\right]^{\alpha} \cdot \left[\eta_{(2,v)}\right]^{\beta},$$

$$R_1 = \sum_{v \in A} \left[\tau_{(1,v)}\right]^{\alpha} \cdot \left[\eta_{(1,v)}\right]^{\beta}.$$

Step (4): $R' = R_4 + R_2 + R_1$.

Step (5): $p_{(4,3)} = 0.3,\ p_{(4,5)} = 0.05,$
$p_{(2,3)} = 0.2,\ p_{(2,5)} = 0.15,$
$p_{(1,3)} = 0.15,\ p_{(1,5)} = 0.15,$

and

$$P = \begin{Bmatrix} p_{(4,3)} = 0.3,\ p_{(4,5)} = 0.05,\ p_{(2,3)} = 0.2, \\ p_{(2,5)} = 0.15,\ p_{(1,3)} = 0.15,\ p_{(1,5)} = 0.15 \end{Bmatrix}.$$

Step (6): $p_{(u^*,v^*)} = p_{(4,3)} = 0.3.$

Step (7): $path_{(4,3)} = 1.$

Step (8): $T = \{4,2,1,3\}$, $A = \{5\}$, $P = \phi$.

Step (9): Since $A \neq \phi$, Then go to step (3).

Iteration 4:

Step(3):

$$R_4 = \sum_{v \in A} \left[\tau_{(4,v)}\right]^{\alpha} \cdot \left[\eta_{(4,v)}\right]^{\beta},\ R_2 = \sum_{v \in A} \left[\tau_{(2,v)}\right]^{\alpha} \cdot \left[\eta_{(2,v)}\right]^{\beta},$$

$$R_1 = \sum_{v \in A} \left[\tau_{(1,v)}\right]^{\alpha} \cdot \left[\eta_{(1,v)}\right]^{\beta},\ R_3 = \sum_{v \in A} \left[\tau_{(3,v)}\right]^{\alpha} \cdot \left[\eta_{(3,v)}\right]^{\beta}.$$

Step (4): $R' = R_4 + R_2 + R_1 + R_3$.

Step (5): $p_{(4,5)} = 0.3,\ p_{(2,5)} = 0.35,\ p_{(1,5)} = 0.15,$
$p_{(3,5)} = 0.2,$

and

$$P = \{p_{(4,5)} = 0.3,\ p_{(2,5)} = 0.35,\ p_{(1,5)} = 0.15,\ p_{(3,5)} = 0.2\}.$$

Step (6): $p_{(u^*,v^*)} = p_{(2,5)} = 0.35.$

Step (7): $path_{(2,5)} = 1.$

Step (8): $T = \{4,2,1,3,5\}$, $A = \phi$, $P = \phi$.

Step (9): Since $A = \phi$, Then the algorithm stops and

we have:

$$path = \begin{bmatrix} 00000 \\ 01001 \\ 00000 \\ 01100 \\ 00000 \end{bmatrix}_{5\times 5}.$$

As many as different plans for candidate TBS combinations to secure consumers, the ants or solutions are formed. All ants are assumed to behave according to the above algorithms until convergence to solutions. The solutions are compared and the best is identified.

4. CASE STUDY

A natural gas network case study of Mazandaran Gas Company in Iran is conducted to verify the proposed model. Surveying on this case, nine potential locations for the TBS were decided. TBSs are selected to secure 119 consumers having definite demands. While the applied optimization software is not able to provide solutions for 119 consumers in a reasonable time, we categorized the customers into 11more comprehensive zones with aggregated demands. All the required information based on the relationships among gas flow rates and pipe diameter sizes are provided by the gas company. They computed ratio of gas flow rates and pipe diameter sizes, keeping constant velocity and gas pressure. Velocity and gas pressure were considered to be 20 m/sec and 30 psi, respectively. Table 1 shows the relationships among gas flow rates and pipe diameter sizes. Consumers' demands are presented in Table 2. The total sum of consumers' demands is 11195 m^3/h. Three types of TBSs with different capacities exist in the network. Table 3 represents the establishing cost and capacity of different TBSs. We use the Minimum Spanning Tree (MST) technique to find a spanning tree in the network with a minimal total distance of the links. There are three types of connection links in the system.

Possible connection links are as bellow:

- Connection link between the TBS and consumer
- Connection link among TBSs
- Connection link among consumers

Tables 4-6 show distances corresponding to the defined connection links. The average cost of piping per distance unit among the TBS is considered to be 38000 units.

We applied CPLEX 11.0 software package to facilitate computations in our Mixed Integer Programming (MIP) model. CPLEX is a tool developed for solving large-scale linear optimization problems. It can also be used for quadratic programs. The available CPLEX optimizers are the standard primal simplex, dual simplex, network, barrier and mixed-integer optimizers [17].

Now, we make further experiments by changing the model to restrict the total number of pipes exiting a TBS by predefined values. For this, we need to change constraints (18) to:

$$\sum_{z \in Z} y_{itz} \le np \cdot h_{it}, \qquad \forall i \in I, \forall t \in T, \tag{41}$$

where, *np* is the maximum number of exiting pipes. We have used values of *np* equal to 2 and 3 and reported the results in Table 7 along with the optimal solution obtained before, for comparison purposes. The validity of model is measured for gas distribution network case study of Mazandaran Gas Company in Iran as seen in Figures 2, 3, and 4, schematically. The results are summarized in Table 7. Table 7 presents objective function of three cases. As expected, the unrestricted case assures the global optimization and has the best objective function value i.e. the lowest cost. The objective function value in the *np*=2 and *np*=3 cases are little higher than the unrestricted case. Gas flow rates and pipe diameter sizes of selective paths are shown in Table 7. There are two types of connection links in the selective path column:

- Links connected between the TBS and consumers is indicated by a-b:[n] format; where, a and b indicate number of selected TBS and consumer, respectively. n is a number which indicates a selective path on the figure. [] is a symbol related to this kind of connection links.
- Links connected among the consumers is indicated by c-d:(n) format; where, c and d indicate number of selected consumers. () is a symbol related to this kind of connection links.

From Table 7, it is concluded that:

1. For *np*=2, *np*=3 and unrestricted case (*np*=4), only one TBS of type three is selected (No. 5) to secure the total sum of consumers' demands.
2. The aggregate value of gas flow in exiting pipes a TBS is equivalent to the total sum of consumers' demands. That means, the whole of consumers' demands in the network are met.
3. For *np*=2, two pipelines with the same diameter sizes exit from the TBS (No. 5).
4. For *np*=3, three pipelines with different diameter sizes exit from the TBS (No.5).
5. For unrestricted optimal solution (*np*=4), an optimal gas distribution network with four pipelines exited from the TBS is reported in Table 7.

For *np*=2, *np*=3, and unrestricted optimal solution (*np*=4), an optimal gas distribution network is shown in Figures 2, 3, and 4, respectively. In these figures, we consider a particular color for

TABLE 1. The cost of piping per distance unit with respect to the gas flow rate

Gas flow rate (m^3/h)	Pipe diameters (mm-inch)	Cost (unit)
0-400	63 mm	11600
401-800	90 mm	14200
801-1500	110 mm	18500
1501-2000	125 mm	22000
2001-3800	160 mm	28000
3801-6000	6"	38000
6001-10000	8"	53000
10001-15000	10"	80000
15001-20000	12"	125000

TABLE 2. Consumers' demands

CONSUMER	1	2	3	4	5	6	7	8	9	10	11
Demand(m^3/h)	623	912.4	435.8	1903.1	1663.8	1095.3	1536.9	945.7	766.6	495.3	817.1

TABLE 3. The establishing cost and capacity of different TBSs

TBS types	**Capacity (m^3/h)**	**Cost (unit)**
TBS 1	5000	50000000
TBS 2	10000	65000000
TBS 3	20000	85000000

TABLE 4. Distance among the TBS and consumers

D	1	2	3	4	5	6	7	8	9	10	11
1	160.854	363.9794	307.3191	813.182	888.1306	1515.165	1883.351	2027.984	2535.591	2990.097	3294.432
2	392.6576	736.3077	317.1577	417.0048	430.1093	1194.531	1433.087	1621.247	2098.236	2542.003	2857.911
3	758.0402	1127.865	690.0101	219.6952	114.3897	930.8501	1036.738	1252.259	1705.42	2145.709	2464.808
4	1218.117	1630.224	1195.905	533.0225	530.9652	643.1835	565.6925	756.1296	1204.854	1660.834	1964.456
5	1324.387	1789.507	1379.622	652.6354	777.5873	358.4983	626.2348	569.1696	1122.834	1615.072	1865.811
6	510.1853	1008.793	667.8877	240.3373	530.4988	861.3629	1322.241	1388.144	1927.745	2404.167	2679.914
7	2179.566	2605.866	2169.883	1491.181	1489.071	1054.943	509.5724	378.9512	242.6623	743.9207	993.0519
8	1919.088	2376.945	1956.269	1241.923	1306.719	710.2288	526.135	28.3196	581.3028	1086.803	1286.459
9	3096.335	3496.422	3052.351	2407.967	2360.768	1967.751	1336.361	1287.292	690.6902	239.8708	220.5176

TABLE 5. Distance among the TBS

D	1	2	3	4	5	6	7	8	9
1	0	458.31	849.79	1332.1	1459.7	656.66	2302.3	2052.9	3211.1
2	458.31	0	396.35	894.23	1063.8	384.98	1869.8	1644.5	2766.3
3	849.79	396.35	0	506.17	718.77	424.47	1480	1273.8	2370.4
4	1332.1	894.23	506.17	0	304.7	756.9	975.64	775.79	1879.4
5	1459.7	1063.8	718.77	304.7	0	819.77	880.51	594.82	1813.5
6	656.66	384.98	424.47	756.9	819.77	0	1688.3	1414.2	2614.5
7	2302.3	1869.8	1480	975.64	880.51	1688.3	0	352.26	933.28
8	2052.9	1644.5	1273.8	775.79	594.82	1414.2	352.26	0	1259
9	3211.1	2766.3	2370.4	1879.4	1813.5	2614.5	933.28	1259	0

each pipe size. So, the selective paths given in Table 7 are indicated by different colors.

The objective function for unrestricted optimal solution is 217391366.76 units in 117573.93 seconds. Note that this computation time is needed to be spent for solving a problem with nine potential locations for the TBS and eleven consumer zones. Since, the real case at Mazandaran Gas Company in Iran contains 119 consumer zones with nine potential locations for the TBS, the applied optimization software is not able to provide solutions for 119 consumer zones in a reasonable time. Due to the complexity of these problems, exact methods need excessive computation time. So, the ACO algorithm is an essential tool for solving such problems in reasonable amounts of time. However, such algorithms do not necessarily always find global optimum solutions, but yield good solutions in reasonable times. The test problems are produced to show the validity and effectiveness of our proposed algorithm. These problems are based on two factors (number of consumers and number of candidate TBS). Since the proposed model has

TABLE 6. Distance among the consumers

D	1	2	3	4	5	6	7	8	9	10	11
1	0	524.6723	370.6926	688.91	813.7481	1362.428	1777.473	1893.554	2415.647	2878.951	3172.612
2	524.6723	0	447.3679	1136.895	1138.036	1870.13	2160.459	2353.061	2833.109	3269.325	3592.613
3	370.6926	447.3679	0	734.0981	692.0065	1507.03	1715.998	1933.537	2393.807	2824.082	3152.555
4	688.91	1136.895	734.0981	0	334.0733	786.3078	1097.57	1217.516	1726.766	2192.192	2484.126
5	813.7481	1138.036	692.0065	334.0733	0	1018.077	1024.471	1286.822	1707.445	2132.091	2464.468
6	1362.428	1870.13	1507.03	786.3078	1018.077	0	940.0303	682.0594	1290.607	1796.184	1982.158
7	1777.473	2160.459	1715.998	1097.57	1024.471	940.0303	0	527.928	696.4919	1109.249	1444.573
8	1893.554	2353.061	1933.537	1217.516	1286.822	682.0594	527.928	0	609.2003	1114.753	1314.463
9	2415.647	2833.109	2393.807	1726.766	1707.445	1290.607	696.4919	609.2003	0	505.6095	759.6848
10	2878.951	3269.325	2824.082	2192.192	2132.091	1796.184	1109.249	1114.753	505.6095	0	421.2861
11	3172.612	3592.613	3152.555	2484.126	2464.468	1982.158	1444.573	1314.463	759.6848	421.2861	0

TABLE 7. Results for different *np*

np=2			*np=3*			*Unrestricted optimal solution (np=4)*		
Selective path	Amount of gas flow	Pipe (type)	Selective path	Amount of gas flow	Pipe (type)	Selective path	Amount of gas flow	Pipe (type)
5-4:[1]	5538.1	6"	**5-4:[1]**	5538.1	6"	**5-4:[1]**	5538.1	6"
5-8:[2]	5656.89	6"	**5-6:[2]**	1095.29	110 mm	**5-6:[2]**	1095.3	110 mm
3-1:(1)	623	90 mm	**5-8:[3]**	4561.59	6"	**5-7:[3]**	1536.9	125 mm
3-2:(2)	912.4	110 mm	**3-1:(1)**	623	90 mm	**5-8:[4]**	3024.69	160 mm
4-3:(3)	1971.2	125 mm	**3-2:(2)**	912.4	110 mm	**3-2:(1)**	912.4	110 mm
4-5:(4)	1663.8	125 mm	**4-3:(3)**	1971.2	125 mm	**3-1:(2)**	623	90 mm
8-6:(5)	1095.3	110 mm	**4-5:(4)**	1663.8	125 mm	**4-3:(3)**	1971.2	125 mm
8-7:(6)	1536.9	125 mm	**8-7:(5)**	1536.9	125 mm	**4-5:(4)**	1663.8	125 mm
8-9:(7)	2078.99	160 mm	**8-9:(6)**	2079	160 mm	**8-9:(5)**	2079	160 mm
9-10:(8)	1312.39	110 mm	**9-10:(7)**	1312.39	110 mm	**9-10:(6)**	1312.39	110 mm
10-11:(9)	817.09	110 mm	**10-11:(8)**	817.09	110 mm	**10-11:(7)**	817.09	110 mm
	objective 226906194.67			objective 220920313.86			objective 217391366.76	

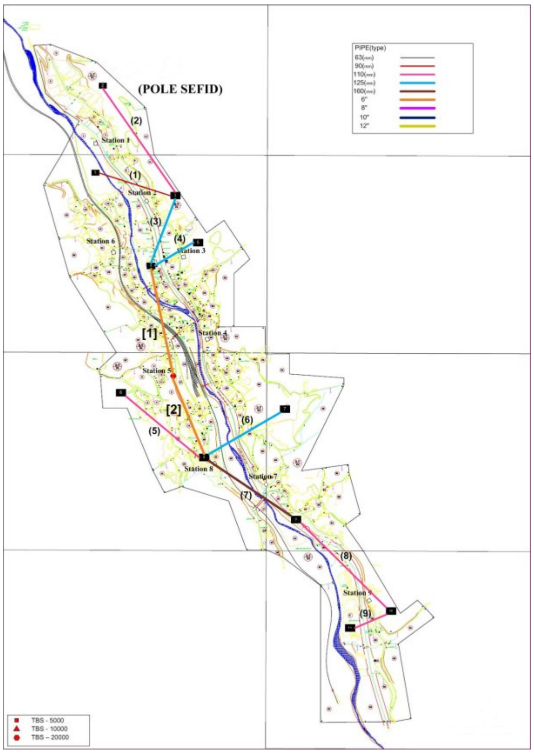

Figure 2. A configuration of an optimal distribution chain with two pipelines exited from the TBS

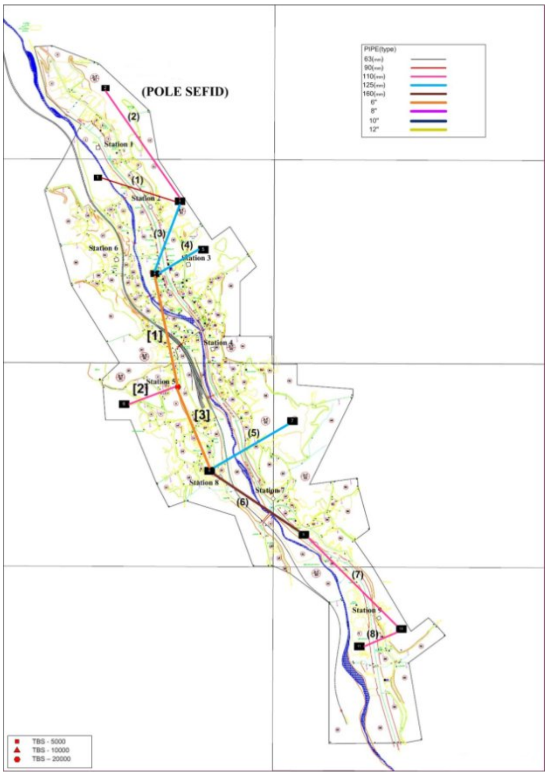

Figure 3. A configuration of an optimal distribution chain with three pipelines exited from the TBS

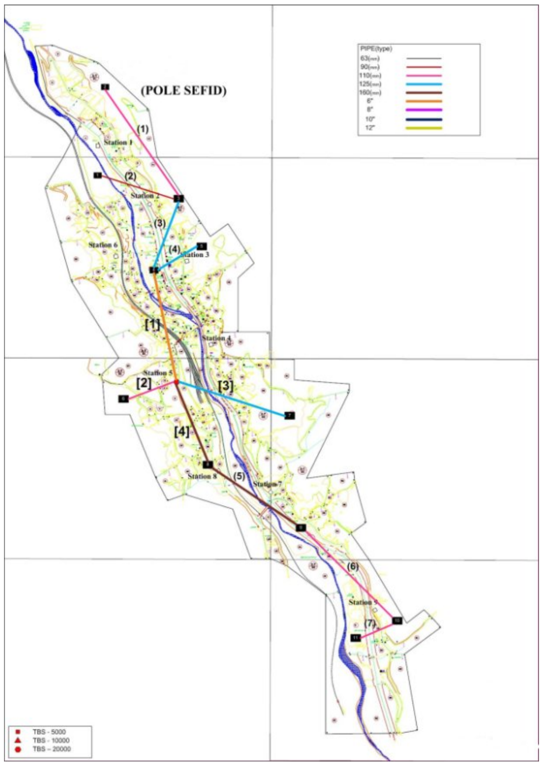

Figure 4. A configuration of an optimal distribution chain

been formulated for the case study of Mazandaran Gas Company in Iran, the generation of test problems are based on real data. Due to the limited capability of CPLEX 11.0 software package for solving our problems in a reasonable time, the test problems were produced in sizes convenient for the software to solve. The test problems are given as follows:

- One TBS and five consumers
- Two TBS and seven consumers
- Three TBS and six consumers
- Four TBS and nine consumers
- Five TBS and eleven consumers

The different values of algorithm's parameters influence the desirability of obtained solutions. So, there are solutions with different qualities based on different combinations of parameters. Parameter tuning plays a key role for the algorithm to produce desirable solutions. First, we introduce several levels for each parameter. A different combination of parameter's levels is defined as a test plan. Then, the test plans are implemented to determine a suitable level for each parameter. The levels of parameters are given as follows:

- Mutation coefficient (δ): two levels (0.2-0.5).
- Number of cycle (cyclenum): three levels (100-300-500).
- Resistance coefficient (ρ): two levels (0.3-0.5).
- Initial amount of pheromone trail (initial): two levels (50-100).
- Relative importance of pheromone trail (α): three levels (1-3-5).
- Relative importance of distance (β): two levels (2-4).

There are $2\times3\times2\times2\times3\times2=144$ test plans. Each test plan is implemented for each defined test problem and the obtained results are saved using the following relation:

$$Gap=\frac{\left|f_{ACO}-f_{optimal}\right|}{f_{optimal}}\times100, \tag{42}$$

where, f_{ACO} is a fitness function of the proposed ACO algorithm and $f_{optimal}$ is an optimal solution obtained by employing the exact method. The test problem's gap is considered to compute the mean of gap for each test plan. Minitab 14.0 software package is applied to analyze the impact of different parameters on gap value. The results are shown in Figure 5.

Corresponding to the obtained results, levels 0.5, 500, 0.3, 100, 5, and 2 were selected for (δ), (Cyclenum), (ρ), (inittrail), (α), and (β), respectively.

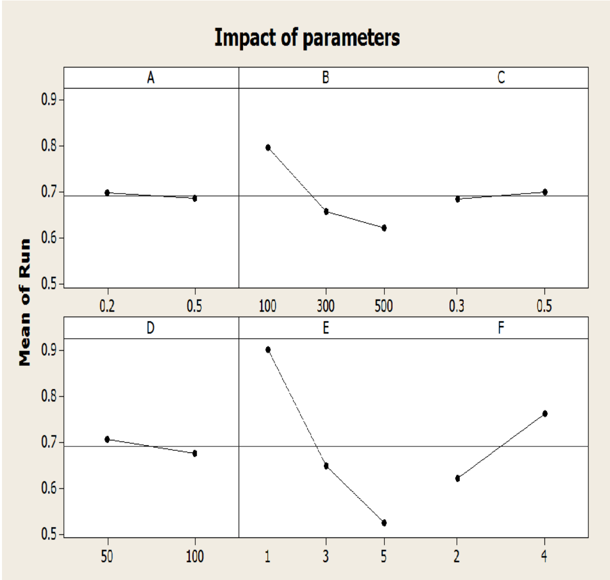

Figure 5. Impact of each parameter on mean of gap

4.1. Computational Results Here, the validity and effectiveness of the proposed ACO algorithm in comparison to the exact method is analyzed. The computational test was developed on a personal computer with Intel (R) Pentium (R) Dual with 2.2 GHz CPU / 4 GB RAM. The algorithm was coded using MATLAB R2009 software package. We considered five different problems for each test problem. Also, each problem contained three types (restricted to *np*=2, restricted to *np*=3, and unrestricted). Consequently, we generated 75 test problems to test our proposed algorithm. The results are given in Table 8.

The results show that our proposed ACO algorithm is effective for solving the problems. The algorithm obtains good solutions in reasonable times.

In comparison to the exact method, the algorithm obtains solutions closer to the optimal solutions with much less time than the time needed to be spent for obtaining exact optimal solution.

The applied ACO approach in the real case at Mazandaran Gas Company in Iran provide 324645641 unit of cost while without the proposed mathematical model and the solution approach the cost was estimated by Mazandaran Gas Company's experts to be 625562000 unit of cost showing about 50 percent improvement.

5. CONCLUSIONS

In this study, a comprehensive optimal design mathematical model of gas pipeline was established. The model considered all factors influencing the total cost of a gas network such as pipe diameter, length of pipe, etc. We considered a distribution network in which gas moves from pressure reduction stations to the consumers. The design of natural gas distribution networks is a complex problem. In order to gain a cost effective design solution, the use of optimization is necessary. A mixed integer programming model for the gas distribution network was formulated. In fact, with the suggested model, we obtained excellent results. This will enable us to design a fast, effective and robust decision aid tool based on the suggested model. In recent years, some studies optimized pipe sizes in predefined structure of gas distribution network. Our main original contribution proposed in this paper, is that we presented the linear model to construct an optimal gas distribution networks having various pipe sizes along with the optimal types and locations of TBSs. In this paper, we used a MST technique to minimize location-allocation costs. Also, we constructed an optimal pipeline system considering various pipe diameter sizes. Advantage of our study was to acquire relationship among gas flow rates and pipe diameter sizes based on expert's opinion and to show how to formulate it minimizing the flow cost. We used the actual data on Mazandaran Gas Company in Iran to conduct a case study. Optimal results were obtained applying CPLEX 11.0 software package. Due to the inability of this software to provide solutions for large size problems in a reasonable time, the ACO algorithm was proposed. The results obtained by the ACO algorithm were compared with those obtained by exact methods. Numerical results showed the effectiveness of the ACO algorithm for gas distribution network optimization. Therefore, the proposed ACO algorithm was applied in the real case at Mazandaran Gas Company in Iran having 119 consumers.

Finally, obtained results encourage us to study more complex structures (cyclic network topology). In this paper, it was assumed that the velocity and gas pressure are constant and the influence of them on the property of natural gas was neglected. The effects of these parameters on total cost were not included in the study, but would be continued in the future.

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TABLE 8. Computational results

			ACO Algorithm			CPLEX 11.0			GAP(%)		
Num	TBS No.	Consumer No.	Without cons.	With cons<=3	With cons <=2	Without cons.	With cons<=3	With cons <=2	Without cons.	With cons<=3	With cons <=2
1	1-(1)	5-(1,2,3,4,5)	1.09E+08 3.580002 sec	1.09E+08 3.560001 sec	1.10E+08 3.564496 sec	108500385.9 6.49 sec	108849094 5.91 sec	109748998.4 6.97 sec	2.55391E-09	5.2826E-10	3.4989E-09
2	1-(4)	5-(1,3,6,8,9)	1.15E+08 3.452745 sec	1.15E+08 3.575086 sec	1.19E+08 3.587468 sec	114572469.5 2.2 sec	114572469 2 sec	119053167.4 2.4 sec	1.06396E-08	1.06396E-08	5.7109E-09
3	1-(6)	5-(3,5,7,10,11)	1.36E+08 3.465900 sec	1.36E+08 3.569835 sec	1.37E+08 3.550768 sec	136492630.2 2.32 sec	136492630 2.04	136643042.8 2.21	5.17097E-09	5.17097E-09	1.8684E-09
4	1-(7)	5-(1,5,8,9,11)	1.28E+08 3.470619 sec	1.28E+08 3.600929 sec	1.28E+08 3.578847 sec	127589624.7 2.01 sec	127589625 1.92 sec	127589624.7 2.01 sec	8.28987E-09	8.28987E-09	8.2899E-09
5	1-(9)	5-(2,4,6,8,9)	1.82E+08 3.506841 sec	1.82E+08 3.610820 sec	1.82E+08 3.582128 sec	182136939.8 4.29 sec	182136940 4.12 sec	182136939.8 5.82	8.78458E-10	8.78458E-10	8.7846E-10
6	2-(2,5)	7-(1,2,3,4,5,6,7)	1.46E+08 18.141380 sec	1.48E+08 18.742526 sec	1.54E+08 18.803007 sec	145523703 30.73 sec	146321852 28.25 sec	153661723 35.6	1.44169E-09	0.931906784	4.8496E-09
7	2-(4,6)	7-(2,4,5,7,8,9,10)	1.58E+08 18.045650 sec	1.58E+08 18.963954 sec	1.59E+08 18.880413 sec	158309233.7 19.86 sec	158309234 23.28 sec	159250762.3 25.01 sec	3.03332E-09	3.03332E-09	2.9105E-09
8	2-(3,7)	7-(3,5,6,8,9,10,11)	1.61E+08 18.083770 sec	1.61E+08 18.782539 sec	1.61E+08 18.705875 sec	160772573.6 52.38 sec	160772574 49.30 sec	160772573.6 53.31 sec	2.18447E-09	2.18447E-09	2.1845E-09
9	2-(1,4)	7-(1,2,4,6,7,8,9)	1.52E+08 18.333495 sec	1.52E+08 18.756589 sec	1.60E+08 18.935196 sec	152182225.9 19.12 sec	152182226 21.09 sec	160160251.4 24.18 sec	5.74966E-10	5.74966E-10	9.0097E-10
10	2-(7,9)	7-(1,3,5,7,9,10,11)	1.55E+08 18.297460 sec	1.55E+08 18.958305 sec	1.55E+08 18.839943 sec	155056996.4 35.58 sec	155056996 34.34 sec	155056996.4 34.91 sec	1.59426E-09	1.59426E-09	1.5943E-09
11	3-(1,3,5)	6-(1,2,3,4,5,6)	1.17E+08 32.276179 sec	1.17E+08 32.899660 sec	1.21E+08 33.476269 sec	116935098.1 28.66 sec	116935098 34.99 sec	117665356.1 34.21 sec	1.23572E-09	1.23572E-09	2.44135006
12	3-(2,5,7)	6-(2,4,6,8,10,11)	1.55E+08 31.647400 sec	1.55E+08 33.040303 sec	1.61E+08 32.983540 sec	155292035 20.8 sec	155292035 26.01 sec	161277915.8 29.33 sec	2.30342E-09	2.30342E-09	3.2912E-09

TABLE 8. Continued

13	3-(4,5,6)	6-(5,6,7,8,9,10)	1.34E+08 31.523391 sec	1.34E+08 33.197395 sec	1.43E+08 33.148941 sec	134250942.4 24.23 sec	134250942 26.29 sec	143436673 33.68 sec	4.07298E-09	4.07298E-09	3.7069E-09
14	3-(2,6,8)	6-(3,5,7,9,10,11)	1.51E+08 31.648026 sec	1.51E+08 32.867413 sec	1.51E+08 32.955780 sec	151182312.4 34.13 sec	151182312 26.47 sec	151182312.4 37.35 sec	3.9621E-09	3.9621E-09	3.9621E-09
15	3-(5,8,9)	6-(1,4,5,6,8,9)	1.35E+08 31.920429 sec	1.35E+08 33.642459 sec	1.41E+08 33.235614 sec	134736874.4 33.35 sec	134736874 20.42 sec	140626183.7 28.67 sec	1.22165E-09	1.22165E-09	2.1013E-09
16	4-(1,2,3,4)	9-(1,2,3,4,5,6,7,8,9)	1.68E+08 173.263461 sec	1.73E+08 180.224726 sec	1.71E+08 188.281469 sec	165620606.7 176.48 sec	166350865 157.98 sec	168867128.7 183.49 sec	1.287481422	3.87331547	1.09145936
17	4-(2,4,6,8)	9-(1,3,4,5,7,8,9,10,11)	1.76E+08 176.695956 sec	1.79E+08 181.834077 sec	1.80E+08 182.413250 sec	176091665.3 321.99 sec	177069377 499.17 sec	178504571.7 577.36 sec	5.71856E-10	1.099656263	1.09081516
18	4-(1,5,7,8)	9-(2,4,5,6,7,8,9,10,11)	2.09E+08 175.025946 sec	2.12E+08 193.903513 sec	2.16E+08 185.575994 sec	208733624.2 4158.72 sec	212262571 12020.86 sec	215719528.1 4643.68 sec	4.48419E-10	1.24233E-09	1.1591E-11
19	4-(3,6,8,9)	9-(3,4,5,6,7,8,9,10,11)	1.85E+08 173.109131 sec	1.85E+08 181.018974 sec	1.85E+08 179.567040 sec	183824922.3 881.42 sec	184450971 854.44 sec	185111165.4 1213.58 sec	0.678251724	0.33653764	1.8746E-09
20	4-(1,4,5,8)	9-(1,2,3,6,7,8,9,10,11)	1.75E+08 172.823931 sec	1.75E+08 178.552422 sec	1.75E+08 182.870818 sec	175116505.8 555.89 sec	175116506 1578.69 sec	175144061.1 579.72 sec	2.5503E-09	2.5503E-09	0.00678983
21	5-(1,2,3,4,5)	11-(1,2,3,4,5,6,7,8,9,10,11)	2.18E+08 619.199491 sec	2.21E+08 646.618506 sec	2.27E+08 630.686102 sec	217391366.8 24921.53 sec	220920314 45457.6 sec	225870992.8 66759.58 sec	0.200557226	0.08214748	0.31985862
22	5-(1,3,5,7,9)	11-(1,2,3,4,5,6,7,8,9,10,11)	2.18E+08 619.702566 sec	2.21E+08 634.109322 sec	2.27E+08 621.673069 sec	217391366.8 29905.30 sec	220920314 80927.99 sec	225870992.8 86520.89 sec	0.200557226	0.197353555	0.65134345
23	5-(1,4,6,7,9)	11-(1,2,3,4,5,6,7,8,9,10,11)	2.19E+08 609.370853 sec	2.21E+08 641.559703 sec	2.31E+08 653.713545 sec	218733829 31231.71 sec	221101794 42027.06 sec	226593460.7 49513.47 sec	7.08632E-10	4.09314E-09	1.81313588
24	5-(2,4,6,8,9)	11-(1,2,3,4,5,6,7,8,9,10,11)	2.23E+08 616.522290 sec	2.21E+08 637.830749 sec	2.27E+08 688.579275 sec	218733829 37808.43 sec	221101794 63106.88 sec	226593460.7 84851.38 sec	1.878286202	4.29657E-10	0.24672373
25	5-(3,5,7,8,9)	11-(1,2,3,4,5,6,7,8,9,10,11)	2.18E+08 609.018370 sec	2.21E+08 644.222535 sec	2.28E+08 639.369670 sec	217391366.8 34004.69 sec	220920314 40457.6 sec	225870992.8 60845.92 sec	0.200557226	0.197353555	0.72388325