A NEW ACCEPTANCE SAMPLING DESIGN USING BAYESIAN MODELING AND BACKWARD INDUCTION

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Abstract In acceptance sampling plans, the decisions on either accepting or rejecting a specific batch is still a challenging problem. In order to provide a desired level of protection for customers as well as manufacturers, in this paper, a new acceptance sampling design is proposed to accept or reject a batch based on Bayesian modeling to update the distribution function of the percentage of nonconforming items. Moreover, to determine the required sample size the backwards induction methodology of the decision tree approach is utilized. A sensitivity analysis that is carried out on the parameters of the proposed methodology shows the optimal solution is affected by initial values of the parameters. Furthermore, an optimal (n,c) design is determined when there is a limited time and budget available and hence the maximum sample size is specified in advance.

Keywords Acceptance Sampling; Bayesian Inference; Decision Tree; Backwards Induction

چکیده تصمیم گیری در مورد پذیرش یا رد یک انباشتهٔ تولیدی در طرح های نمونه گیری پذیرش کماکان یک مسئلهٔ چالش برانگیز است. در این مقاله، به منظور فراهم کردن سطح مطلوبی از اطمینان برای مشتریان و برای تولید کنندگان یک محصول، یک طرح نمونه گیری پذیرش پیشنهاد شده است که از روش بیزی برای تعیین تابع توزیع احتمالی پسین درصد قطعات ناسالم استفاده می‌کند. علاوه بر این، برای تعیین اندازهٔ مناسب نمونه از روش استنتاج رو به عقب که در درخت تصمیم گیری مطرح است استفاده شده است. تحلیل حساسیت روی بعضی از پارامترهای روش پیشنهادی نشان می دهد که طرح بهینه به مقادیر ابتدایی پارامترها وابسته است. به علاوه، در مواقعی که به دلایل محدودیت بودجه و زمان، حداکثر اندازهٔ نمونه از قبل مشخص است نیز طرح (n,c) بهینه ارائه شده است.

1. INTRODUCTION

Acceptance sampling plan is a practical tool used in quality control to provide decision rules for lot acceptance regarding its desired level of quality. The decision, based on counting the number of nonconforming items in a sample, can be to accept the lot, reject the lot, or for multiple and sequential sampling schemes, to take another sample and then repeat the decision process. It protects consumers from getting unacceptable nonconforming product, and encouraging producers in the use of process quality control in two ways: (1) by varying the quantity and severity of acceptance inspections in direct relation to the importance of the characteristics inspected, and (2) in the inverse relation to the goodness of the quality level as indication by those inspections. The plans are employed when testing is destructive, the cost of 100% inspection is high, and/or 100% inspection takes too long.

McWilliams et al. [1] provided a method of finding exact designs for single sample acceptance sampling plans. William et al. [2] developed mathematical models that can be used to design both 100% inspection and single sampling plans. In their research, inspection error is explicitly included in the model as the ability to mitigate the consequences by expending resources. Aminzadeh [3] derived Bayesian economic acceptance sampling plans using the inverse Gaussian model and step-loss function. He used inverse Gaussian (IG) distribution as a lifetime model to obtain optimal values for sample size and decision limit for employing economic variable acceptance-sampling plans based on step-loss function.

Pearn and Wu [4] proposed a variables sampling plan based on C_{pm} index and process loss to handle processes requiring very low parts per million (PPM) fractions of defectives. They developed an effective method for obtaining the required sample sizes n and the critical acceptance value c by solving simultaneously two nonlinear equations. Aminzadeh [5] proposed acceptance-sampling plans based on the assumption that consecutive observations on a quality characteristic are autocorrelated. He obtained the sampling plans based on the autoregressive moving average (ARMA) model and suggested two types of acceptance sampling plans: (1) non-sequential acceptance sampling and (2) sequential acceptance sampling based on the concept of sequential probability ratio test (SPRT). Niaki and Fallahnezhad [6] used Bayesian inference concept to design an optimum-acceptance-sampling-plan in quality control environments. They formulated the problem into a stochastic dynamic programming model; aiming to minimize the ratio of the total discounted system cost to the discounted system correct choice probability. Fallahnezhad and Hosseininasab [7] proposed a single-stage acceptance-sampling plan based on the control threshold policy. In their model, decision is made based on the number of defectives items on an inspected batch. The objective of their model is to find a constant control threshold that minimizes the total costs, including the cost of rejecting the batch, the cost of inspection and the cost of nonconforming items.

Moreover, Fallahnezhad et al. [8] proposed a Markov model for a single sampling plan based on the control threshold policy taking into consideration the run-lengths of successive conforming items as an indicator of process performance. Recently, Fallahnezhad and Niaki [9] extended this approach to the sum of run-lengths of successive conforming items. Aslam et al. [10] presented acceptance-sampling plans for generalized exponential distribution when the product lifetime is truncated at a pre-determined time.

One of the challenging issues in acceptance sampling plans is that they do not guarantee detection of all defective items, i.e. there is a risk of not achieving the exact quality level of the lot. Acceptance sampling methods are mostly designed based on the desired probabilities of the first and the second type of errors. Nonetheless, designing economically optimal acceptance sampling plans has not been widely addressed even though sampling remains a commonly used technique in certain quality engineering systems.

In this research, a new selection approach on the choices between accepting or rejecting a batch based on Bayesian modeling and backwards induction is proposed. The Bayesian modeling is utilized to model the uncertainty involved in the probability distribution of the nonconforming percentage of the items and the backwards induction method is employed to determine the sample size. Moreover, when the decision on accepting or rejecting a batch cannot be made, we assume additional observations can be gathered with a cost to update the probability distribution of the nonconforming percentage of the batch. In other words, a mathematical model is developed in this research to design optimal single sampling plans. This model finds the optimum sampling design whereas its optimality is resulted using the decision tree approach. As a result, the main contribution of the paper is to model the acceptance-sampling problem as a cost optimization model so that the optimal solution can be achieved via using the decision tree approach. In this approach, the required probabilities of decision tree are determined employing the Bayesian Inference. To do this, the probability distribution function of nonconforming proportion of items is first determined by Bayesian inference using a non-informative prior distribution. Then, the required probabilities are determined by applying Bayesian inference in the backward induction method of the decision tree approach. Since this model is completely designed based on the Bayesian inference and no approximation is needed, it can be viewed as a new tool to be used by practitioners in real case problems to design an economically optimal acceptance-sampling plan. However, the main limitation of the proposed methodology is that it can only be applied to items not requiring very low fractions of nonconformities.

The rest of the paper is organized as follows. After introducing notations in Section 2, the problem is defined in Section 3. The Bayesian modeling comes in Section 4. Section 5 contains the proposed backward induction method. The numerical demonstration on the application of the proposed methodology comes in Section 6. Sensitivity analyses are carried out in Section 7. The general decision making framework comes in Section 8. We conclude the paper in Section 9.

2. NOTATIONS

The following notations are used throughout the paper.

Set of decisions: $A=\{a_1,a_2\}$ is defined the set of possible decisions where a_1 and a_2 refer to accepting and rejecting the batch, respectively.

State space: $P=\{p_i\,;\, i=1,2,...;\, 0<p_i<1\}$ is defined the state of the process where p_i represents nonconforming proportion items of the batch in ith state of the process. The decision maker believes the consequences of selecting decision a_1 or a_2 depend on P that cannot be determined with certainty. However, the probability distribution function of the random variable p can be obtained using Bayesian inference.

Set of experiments: $E=\{e_i\,;i=1,2,...\}$ is the set of experiments to gather more information on p and consequently to update the probability distribution of p. Further, e_i is defined an experiment in which, i items of the batch are inspected.

Sample space: $Z=\{z_j;\, j=0,1,2,...,i\}$ denotes the outcomes of experiment e_i where, z_j shows the number of nonconforming items in e_i.

Cost function: The function $u(e,z,a,p)$ on $E\times Z\times A\times P$ denotes the cost associated with performing experiment e, observing z, making decision a, and finding p.

N : The total number of items in a batch

R : The cost of rejecting a batch

C : The cost of one nonconforming item

S : The cost of inspecting one item

m : The sample size

n : An upper bound on the number of inspected items

α : The number of nonconforming items in an inspected sample

β : The number of conforming items in an inspected sample

c : The optimum value of acceptance threshold in the resulted (n,c) design

3. PROBLEM DEFINITION

Consider a batch of size N with an unknown percentage of nonconforming p and assume m items are randomly selected for inspection. Based on the outcome of the inspection process in terms of the observed number of nonconforming items, the decision-maker desires to accept the batch, reject it, or to perform more inspections by taking more samples. As Raiffa & Schlaifer [11] stated "the problem is how the decision maker chose an e and then, having observed z, choose an a such that $u(e,z,a,p)$ is minimized. Although the decision maker has full control over his choice of e and a, he has neither control over the choices of z nor p. However, we can assume he is able to assign probability distribution function over these choices." They formulated this problem in the framework of the decision tree approach, the one that is partially adapted in this research as well.

4. BAYESIAN MODELING

For a nonconforming proportion p, referring to Jeffrey’s prior [12], we first take a Beta prior distribution with parameters $\alpha_0=0.5$ and $\beta_0=0.5$ to model the absolute uncertainty. Then, the posterior probability density function of p using a sample of $\alpha+\beta$ inspected items is

$$f(p)=Beta\left(\alpha+0.5,\beta+0.5\right)=\frac{\Gamma(\alpha+\beta+1)}{\Gamma(\alpha+0.5)\Gamma(\beta+0.5)}p^{\alpha-0.5}(1-p)^{\beta-0.5} \quad (1)$$

where, α is the number of nonconforming items and β is the number of conforming items in the sample. Moreover, to allow for more flexibility in representing prior uncertainty it is convenient to define a discrete distribution by discretization of the Beta density [13]. In other words, we define the prior distribution for p_1 as

$$Pr\{p = p_1\} = \int_{p_1-\delta/2}^{p_1+\delta/2} f(p)\,dp \tag{2}$$

where, $p_1 = (2l - 1/2)\delta$ and $\delta = \frac{1}{m}$ for $l = 1,2,...,m$. Now, define $(j,i); i = 1,2,...,m$ and $j = 0,1,2,...,i$ the experiment in which j nonconforming items are found when i items are inspected. Then, the sample space Z becomes $Z = \{(j,i): 0 \le j \le i \le m\}$, resulting in the cost function representation of $u[e_i,(j,i),a_k,p_1]$; $k = 1,2$ that is associated with taking a sample of i items, observing j nonconforming, and adopting a_1 or a_2 when the defective proportion is p_1. Using the notations defined in Section 2, the cost function is determined by the following equations:

$$\begin{aligned} &\text{1. For accepted batch: } u(e_i,(j,i),a_1,p_1) = CNp_1 + Se_i \\ &\text{2. For rejected batch: } u(e_i,(j,i),a_2,p_1) = R + Se_i \end{aligned} \tag{3}$$

Moreover, the probability of finding j defective items in a sample of i inspected items, i.e., $\Pr\{z = z_j \mid p = p_1, e = e_i\}$, can be obtained using a binomial distribution with parameters $(i, p = p_l)$ as:

$$\Pr\{z = z_j \mid p = p_1, e = e_i\} = C_j^i p_1^j (1 - p_1)^{i-j} \tag{4}$$

Hence, the probability $\Pr\{p = p_1, z = z_j \mid e = e_i\}$ can be calculated as

$$\begin{aligned} &\Pr\{p = p_1, z = z_j \mid e = e_i\} = \Pr\{z = z_j \mid p = p_1, e = e_i\} \\ &\Pr\{p = p_1\} = C_j^i p_1^j (1 - p_1)^{i-j} \int_{p_1-\delta/2}^{p_1+\delta/2} f(p)\,dp \end{aligned} \tag{5}$$

Thus,

$$\begin{aligned} &\Pr\{z = z_j \mid e = e_i\} = \sum_{l=1}^{m} \Pr\{p = p_1, z = z_j \mid e = e_i\} \\ &\Pr\{p = p_l\} = \sum_{l=1}^{m} \left(C_j^i p_1^j (1 - p_1)^{i-j} \int_{p_1-\delta/2}^{p_1+\delta/2} f(p)\,dp \right) \end{aligned} \tag{6}$$

In other words, applying the Bayesian rule, the probability $\Pr\{p = p_l \mid z = z_j, e = e_i\}$ can be obtained by:

$$\begin{aligned} \Pr\{p = p_1 \mid z = z_j, e = e_i\} &= \frac{\Pr\{p = p_1, z = z_j \mid e = e_i\}}{\Pr\{z = z_j \mid e = e_i\}} \\ &= \frac{C_j^i p_1^j (1 - p_1)^{i-j} \int_{p_l-\delta/2}^{p_l+\delta/2} f(p)\,dp}{\sum_{k=1}^{m} C_j^i p_k^j (1 - p_k)^{i-j} \int_{p_k-\delta/2}^{p_k+\delta/2} f(p)\,dp} \end{aligned} \tag{7}$$

In the next Section, a backward induction approach is taken to determine the optimal sample size.

5. BACKWARD INDUCTION

The analysis continues by working backwards from the terminal decisions of the decision tree to the base of the tree, instead of starting by asking which experiment e the decision maker should select when he does not know the outcomes of the random events. This method of working back from the outermost branches of the decision tree to the initial starting point is often called "backwards induction" [11]. As a result, the steps involved in the solution algorithm of the problem at hand using the backwards induction becomes;

1. Probabilities $\Pr\{p = p_l\}$ and $\Pr\{(j,i) \mid p = p_l\}$ are determined using Equations (2) and (4), respectively.
2. The conditional probability $\Pr\{p = p_l \mid z = z_j, e = e_i\}$ is determined using Equation (7).
3. With a known history (e,z), since p is a random variable, the costs of various possible terminal decisions are uncertain. Therefore, the cost of any decision a for the given (e,z) is set as a random variable $u(e,z,a,p)$.

Applying the conditional expectation , $E_{p|z}$, which takes the expected value of $u(e,z,a,p)$ with respect to the conditional probability $P_{p|z}$ (Equation (7)), the conditional expected value of the cost function on state variable $p_1 = \Pr\{p = p_1\}$ is determined by the following equation:

$$u^*(e_i, z_j, a_k) = \sum_{l=1}^{m} \begin{pmatrix} u^*(e_i, z_j, a_k, p_1) \\ \Pr\{p = p_1 \mid z = z_j, e = e_i\} \end{pmatrix} \quad (8)$$

4. Since the objective is to minimize the expected cost, the cost of having history (e,z) and the choice of decision (accepting or rejecting) can be determined by:

$$u^*(e_i, z_j) = \min_{a_k} u^*(e_i, z_j, a_k) \quad (9)$$

5. The conditional probability $\Pr\{z = z_j \mid e = e_i\}$ is determined using Equation (6).
6. The costs of various possible experiments are random because the outcome z is a random variable. Defining a probability distribution function over the results of experiments and taking expected values, we can determine the expected cost of each experiment. The conditional expected value of function $u^*(e_i, z_j)$ on the variable z_j is determined by the following equation:

$$u^*(e_i) = \sum_{j=0}^{i} \left\{ \begin{matrix} u^*(e_i, z_j) \\ \sum_{l=1}^{m} \left(C_j^i p_1^j (1-p_1)^{i-j} \int_{p_l-\delta/2}^{p_l+\delta/2} f(p)\,dp \right) \end{matrix} \right\} \quad (10)$$

7. Now the minimum of the values $u^*(e_i)$ would be the optimal decision, which leads to an optimal sample size.

$$u^* = \min_e u^*(e_i) = \min_e E_{z|e} \min_a E_{p|z} u(e_i, z_j, a_k, p_1) \quad (11)$$

In the next Section, a numerical example is given to illustrate the application of the proposed methodology.

6. NUMERICAl ILLUSTRATION

Assuming a lot of $N = 100$ items is received, the number of inspected items is $m = 3$, the cost of each inspection is $S = 1$, the cost of accepting a nonconforming item is $C = 2$, and the cost of rejecting a batch is $R = 45$. Moreover, assume 2 out of 10 inspected items are nonconforming, i.e., $\alpha = 2$ and $\beta = 8$. Then, the distribution function of nonconforming proportion is obtained as;

$$f(p) = Beta(2.5, 8.5) \quad (12)$$

By discretization of the Beta density function with $m = 3$, the discrete values of the nonconforming proportion will be $p_1 = 0.17,\ p_2 = 0.50,\ p_3 = 0.83$, where the elements of different spaces are given in Table 1. Furthermore, the conditional probabilities $P_{z|e,p}$ for all possible (e,p) pairs are shown in Table 2, where the marginal probabilities $\Pr\{p = p_l\}$ are given in Table 3.

TABLE 1. Elements of the spaces

Space	Elements	Interpretation
A	$\begin{cases} a_1 \\ a_2 \end{cases}$	Accept Reject
P	$\begin{cases} p_1 = 0.17 \\ p_2 = 0.50 \\ p_3 = 0.83 \end{cases}$	Different Values for Nonconforming Proportion
E	$\{e_i = i \quad i = 1,2,3$	Inspecting i items
Z	$\{z_j = j \quad j = 0,1,2,3$	Finding j nonconforming items

TABLE 2. Conditional measures on Z

Z	E								
	e_1			e_2			e_3		
	p_1	p_2	p_3	p_1	p_2	p_3	p_1	p_2	p_3
z_0	0.83	0.50	0.17	0.69	0.25	0.03	0.58	0.13	0.00
z_1	0.17	0.50	0.83	0.28	0.50	0.28	0.35	0.38	0.07
z_2				0.03	0.25	0.69	0.07	0.38	0.35
z_3							0.00	0.13	0.58

TABLE 3. Marginal measures on P

p	P_p
p_1	0.8115
p_2	0.1873
p_3	0.0012
	1

Based on these probabilities, we first obtain the joint probability $P_{p,z|e}$ for each of the experiment $e_i\,; i=1,2,3$. The results are shown in Tables 4 to 6, respectively. From the joint probabilities, we then compute the marginal probability $P_{z|e}$ for each e and obtain the results given in Table 4 to 6. The results of conditional probabilities $P_{p|z,e}$ for each z are shown in Table 7.

TABLE 4. Measures associated with e_1

Z	Joint Measures on $P \times Z$			Marginal Measures on Z
	p_1	p_2	p_3	
z_0	0.6735	0.0937	0.0002	0.77
z_1	0.1380	0.0937	0.0000	0.23
z_2	0.0000	0.0000	0.0000	0.00
z_3	0.0000	0.0000	0.0000	0.00
Marginal Measures on P	0.8115	0.1873	0.0002	

TABLE 5. Measures associated with e_2

Z	Joint Measures on $P \times Z$			Marginal Measures on Z
	p_1	p_2	p_3	
z_0	0.5599	0.0468	0.0000	0.61
z_1	0.2272	0.0937	0.0003	0.32
z_2	0.0243	0.0468	0.0008	0.07
z_3	0.0000	0.0000	0.0000	0.00
Marginal Measures on P	0.8115	0.1873	0.0012	

TABLE 6. Measures associated with e_3

Z	Joint Measures on $P \times Z$			Marginal Measure on Z
	p_1	p_2	p_3	
z_0	0.4707	0.0244	0.0000	0.4950
z_1	0.2840	0.0712	0.0001	0.3553
z_2	0.0568	0.0712	0.0004	0.1284
z_3	0.0000	0.0244	0.0007	0.0250
Marginal Measures on P	0.8115	0.1911	0.0012	

The first step of the decision tree analysis is to start from the end of the tree and to use the data to evaluate $u^*(e,z,a)$ for all values of (e,z,a). Different values of $u(e_i,z_j,a_1,p_l)$ are shown in Table 8.

Moreover, different values of $u^*(e_i,z_j,a_k)$ are shown in Table 9. As a single example of the computations involved in this table, since $u(e_i,z_j,a_1,p_l)=CNp_l+Si$ and $u(e_i,z_j,a_2,p_l)=R+Si$ we have

$$u^*(e_1,z_1,a_1)=u(e_1,z_1,a_1,p_1)Pr\{p_1|z_1,e_1\}$$
$$+u(e_1,z_1,a_1,p_2)Pr\{p_2|z_1,e_1\}+$$
$$u(e_1,z_1,a_1,p_3)Pr\{p_1|z_1,e_1\}=$$
$$34.33(0.596)+101(0.404)+167.67(0)=61.26$$

Now, we are ready to obtain $u^*(e_i,z_j)$. The results are given in Table 10, where as an example, $u^*(e_1,z_1)$ is obtained as follows.

$$u^*(e_1,z_1)=min\left\{\begin{matrix}u^*(e_1,z_1,a_1),\\ u^*(e_1,z_1,a_2)\end{matrix}\right\}=min\{61.26,46\}=46$$

Then, $u^*(e_i)$ for all values of e_i are computed. For example,

$$u^*(e_1)=u^*(e_1,z_0)P(z_0|e_1)+u^*(e_1,z_1)P(z_1|e_1)$$
$$=42.47(0.77)+46(0.23)=43.28$$

The results are shown in Table 11.

TABLE 7. Conditional measures on P associated with $e_i ; i = 1,2,3$

Z	E											
	e_1				e_2				e_3			
	p_1	p_2	p_3	sum	p_1	p_2	p_3	sum	p_1	p_2	p_3	sum
z_0	0.878	0.122	0.000	1	0.9228	0.0772	0.0001	1	0.9508	0.0492	0.0000	1
z_1	0.596	0.404	0.000	1	0.7074	0.2916	0.0010	1	0.7994	0.2004	0.0002	1
z_2	0.811	0.187	0.001	1	0.3381	0.6505	0.0114	1	0.4424	0.5544	0.0032	1
z_3	0.811	0.187	0.001	1	0.8115	0.1873	0.0012	1	0.0000	0.9726	0.0274	1

TABLE 8. Different values of the end points in the decision tree, $u\left(e_i, z_j, a_1, p_l\right)$

Z	$u\left(e_i, z_j, a_1, p_l\right)$								
	e_1			e_2			e_3		
	p_1	p_2	p_3	p_1	p_2	p_3	p_1	p_2	p_3
z_0	34.33	101.00	167.67	35.33	102.00	168.67	36.33	103.00	169.67
z_1	34.33	101.00	167.67	35.33	102.00	168.67	36.33	103.00	169.67
z_2				35.33	102.00	168.67	36.33	103.00	169.67
z_3							36.33	103.00	169.67

TABLE 9. Different values of $u^*\left(e_i, z_j, a_k\right)$

Z	$u^*\left(e_i, z_j, a_k\right)$					
	e_1		e_2		e_3	
	a_1	a_2	a_1	a_2	a_1	a_2
z_0	42.47	46.00	40.48	47.00	39.61	48.00
z_1	61.26	46.00	54.74	47.00	49.69	48.00
z_2			78.30	47.00	73.18	48.00
z_3					100.18	48.00

TABLE 10. The $u^*\left(e_i, z_j\right)$ values

Z	$u^*\left(e_i, z_j\right)$		
	e_1	e_2	e_3
z_0	42.47	40.48	39.61
z_1	46.00	47.00	48.00
z_2		47.00	48.00
z_3			48.00

TABLE 11. The final values of $u^*\left(e_i\right)$

$u^*\left(e_i\right)$		
e_1	e_2	e_3
43.28	43.02	44.02

Finally, we compute:

$$u^* = min\begin{Bmatrix} u^*(e_1) = 43.28 & ,u^*(e_2) = \\ 43.02, u^*(e_3) = 44.02 & \end{Bmatrix} = 43.02$$

It means that two more items need to be inspected. After inspecting the items, the optimal decision is obtained as given in Table 12. It is obvious that when the outcome of experiment is z_0 , we should accept the batch. Otherwise, the batch is rejected.

TABLE 12. The final values of $u^*(e_2, z_j)$ when two items are inspected

	$u^*(e_2, z_j)$	Optimal Decision
z_0	40.48	a_1
z_1	47.00	a_2
z_2	47.00	a_2

Based on $f(p)$, there are two nonconforming items among 10 inspected items. Further, from the decision tree we need to inspect two more items. If the number of nonconforming in the two inspected items is zero, the batch is accepted. Otherwise, the batch is rejected. In other words, as the whole, if in 12 inspected items the number of nonconforming items is less than 3, the batch is accepted. This results in an $(n = 12, c = 2)$ design for the existing sampling plan problem.

In the next Section, sensitivity analyses are performed on different values of α , β , and m .

7. SENSITIVITY ANALYSIS ON α AND β

Depending on different values of α and β, various acceptance sampling plans may be obtained. A sensitivity analysis on various values of (α, β) is carried on in this section and the results are given in Table 13.

Based on the results in Table 13, one can conclude that the design (n, c) is totally dependent on the parameters (α, β). It means the value of (α, β) affects the final design and therefore a sufficient initial data would be required. Further, the results in Table 13 show that as the expected proportion of nonconforming items ($\alpha/(\alpha+\beta)$) increases, the proportion c/n generally increases. It means that by reducing the quality of the batch, the value of the control threshold to accept the batch (c) increases, resulting in lower probability of accepting the batch as expected. This pattern can be better visualized in Figure 1, where the relation between $p = \alpha/(\alpha+\beta)$ and c/n is depicted. Moreover, Figure 2 shows that an increase in the expected proportion of nonconforming items causes the optimum value of system cost (u^*) to generally increase as expected.

TABLE 13. The results of a sensitivity analysis on different values of (α, β)

(α, β)	(n, c)	(u^*, i)
(2,8)	(12,2)	(43.3,2)
(2,15)	(17,3)	(35.81,1)
(3,10)	(17,4)	(44.89,2)
(3,20)	(24,4)	(34.6,1)

8. EXTENSION OF THE PROPOSED DECISION MAKING FRAMEWORK

The proposed methodology can be extended to the case in which the sampling budget is limited and hence the maximum number of inspected items is known priory. Assume the upper bound for the number of inspected samples is n .

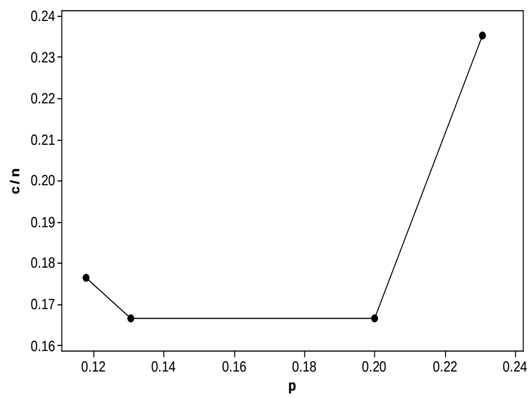

Figure 1. The plot of c/n vs. $p = \alpha/(\alpha+\beta)$

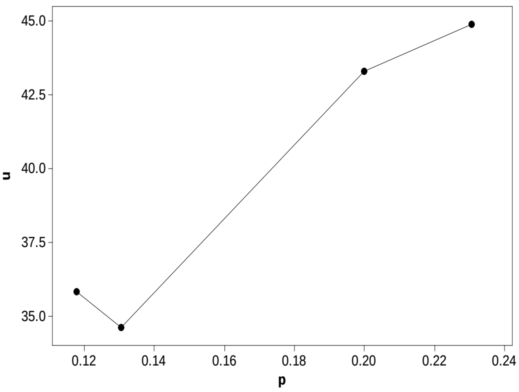

Figure 2: The plot of u^* vs. $p = \alpha/(\alpha + \ )$

Further, assume that if the number of nonconforming items in the sample is less or equal to c then batch is accepted. Then, the proposed methodology is extended and the following decision making framework is proposed to obtain the plan.

1. Select $m < n$ as the initial sample size
2. Obtain the values of α and β after inspecting the sample
3. Determine the probability distribution of p via Bayesian inference
4. Determine the optimal experiment e_i via the backwards induction approach
5. Inspect i more items and set $m = m + i$
6. If $m < n$ then go to stage 2 above. Otherwise, solve the decision tree and determine the optimal value of c .

Based on the above methodology, a sensitivity analysis on different values of m is carried out, where the nonconforming proportion of the batch is assumed 0.2. The other parameter values of the proposed methodology are the same as the ones in the numerical example of Section 6. Moreover, assume an upper bound on the sample size (the number of inspected items) is 15.

In a sample of 15 generated Bernoulli observations, two nonconforming items were found in the second and tenth observations. Then, the proposed methodology was employed based on different values of $m =$ 8, 10, 12, and 14. Each run of the above framework is defined a decision making stage. The results are shown in Table 14.

The results in Table 14 show that a unique final solution is obtained for all values of m . This is due to the fact that for all values of m a unique final distribution function for the nonconforming proportion p is obtained in the final stage. Moreover, we note that as m gets closer to n , the amount of computational work decreases. Thus, in general, since m does not affect the final solution and there is less computational efforts associated with closer values of m to n , it is better to select m values closer to n .

9. CONCLUSIONS

Acceptance sampling plans have been widely used in industry to determine whether a specific batch of manufactured or purchased items satisfy a pre-specified quality. In this paper, based on the Bayesian modeling and the backwards induction method of the decision-tree approach, we developed a sampling plan to deal with the lot-sentencing problem; aiming to determine an optimal sample size to provide desired levels of protection for customers as well as manufacturers.
We made a logical analysis of the choices between accepting or rejecting a batch when the distribution function of nonconforming proportion could be updated by taking additional observations and using Bayesian modeling. The decision tree approach was used to evaluate the cost associated with different decisions. At the end, an analytical method to obtain the sample size required for inspection was developed that would lead to an optimal (n, c) design for acceptance sampling problem. The main result of this paper was to develop an optimal acceptance-sampling plan that minimizes the total expected system cost. To search for an optimal solution, different decisions with their probabilistic outcomes were determined, where the expected cost of each decision was evaluated based on probabilities of each outcome determined by Bayesian inference. Moreover, sensitivity analyses on different values of some parameters of the proposed methodology were carried out. The results showed the optimal solution is affected by initial values of the parameters.

TABLE 14. The results of a sensitivity analysis on different values of m

	m =8	m =10	m =12	m =14
Stage one	$(m=8,\alpha=1,\beta=7)$ $(u^*=40.79, i=1)$	$(m=10,\alpha=2,\beta=8)$ $(u^*=44.25, i=2)$	$(m=12,\alpha=2,\beta=10)$ $(u^*=40.75, i=1)$	$(m=14,\alpha=2,\beta=12)$ $(u^*=38.58, i=1)$
Stage two	$(m=9,\alpha=1,\beta=8)$ $(u^*=39.3, i=1, accept)$	$(m=12,\alpha=2,\beta=10)$ $(u^*=40.75, i=1, accept)$	$(m=13,\alpha=2,\beta=11)$ $(u^*=39.49, i=1, accept)$	$(m=15,\alpha=2,\beta=13)$ $(u^*=37.36, i=1, accept)$
Stage three	$(m=10,\alpha=2,\beta=8)$ $(u^*=44.25, i=2, reject)$	$(m=13,\alpha=2,\beta=11)$ $(u^*=39.49, i=1, accept)$	$(m=14,\alpha=2,\beta=12)$ $(u^*=38.58, i=1, accept)$	
Stage four	$(m=12,\alpha=2,\beta=10)$ $(u^*=40.75, i=1, accept)$	$(m=14,\alpha=2,\beta=12)$ $(u^*=38.58, i=1, accept)$	$(m=15,\alpha=2,\beta=13)$ $(u^*=37.36, i=1, accept)$	
Stage five	$(m=13,\alpha=2,\beta=11)$ $(u^*=39.49, i=1, accept)$	$(m=15,\alpha=2,\beta=13)$ $(u^*=37.36, i=1, accept)$		
Stage six	$(m=14,\alpha=2,\beta=12)$ $(u^*=38.58, i=1, accept)$			
Stage seven	$(m=15,\alpha=2,\beta=13)$ $(u^*=37.36, i=1, accept)$			
Resulted (n,c)	(15,3): Accept the batch	(15,3): Accept the batch	(15,3): Accept the batch	(15,3): Accept the batch

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