MINIMIZING THE SUPPLYING COST OF LEVERAGE ITEMS: A MATHEMATICAL APPROACH

J. Razmi* and A. Keramati

Department of Industrial Engineering, College of Engineering, University of Tehran
P.O. Box 11155-4563, Tehran, Iran
jrazmi@ut.ac.ir, keramati@ut.ac.ir

*Corresponding Author

Abstract   In the new competitive environment, selecting and planning of the supply chain is very crucial and involves evaluation of many factors. Different approaches have been applied to assess the supplier/s. Most of these studies are based upon the supplier/s' capabilities. It may be neither rational nor economical to deal with each item via a generic material control system. Furthermore, supplier/s selection not only depends on supplier/s' capability, but also depends on the nature and characteristics of the parts. Little attention is given to decisions on the appropriate selecting of supplier/s, and assigning order quantities to them, especially, in case of multiple sources considering their performance and materials' characteristics. In this paper two models are presented which locate the optimal supplier/s regarding supplier/s' capabilities and materials' characteristics for leverage items based on references 1 and 2. The first one is a G.P. model which considers constraints of lead-time, quality, demand, supplier/s capacity and budgets in a dynamic condition. The second one considers the problem in a contingent environment and considers criteria with normal distributions. Each model is followed an illustrative numerical example and sensitivity analysis which demonstrate the effectiveness and validation of the model.

Keywords   Supply Chain Management, Supplier Selection, Goal Programming, Materials and Inventory Management, Leverage Items

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چکیده   در دنیای رقابتی کنونی، مساله ارزیابی، انتخاب و برنامه ریزی زنجیره تامین بالا دستی از اهمیت ویژه‌ای برخوردار می‌باشد. انتخاب تامین کنندگان ارزیابی متغیرهای بسیاری مانند قیمت اقلام، هزینه های حمل و نقل، کیفیت، ویژگی های تحویل و غیره است. مدل های مختلفی در این زمینه ارائه شده است. عمده مطالعات انجام شده مبتنی بر توانایی های تامین کنندگان استوار است. باید به این نکته توجه نمود که هر کدام از اقلام خریداری شده دارای کاراکتری منحصر بفرد می باشد. در چنین حالتی منطقی نیست که مدلی عمومی برای برنامه ریزی و کنترل کلیه اقلام تهیه گردد. لذا طبیعی است که مساله انتخاب تامین کنندگان هم با توانایی آنها و هم به کاراکتر و مشخصات اقلام خریداری ارتباط داشته باشد. متاسفانه به این پارامتر کمتر مورد توجه توسط محققان قرار گرفته و غالباً مساله توجه به مشخصات اقلام مغفول مانده است. این مقاله بر مبنای کارهای انجام شده توسط کرالجیک [۱] و رزمی و کرباسیان [۲]، دو مدل جهت انتخاب بهترین تامین کنندگان و برنامه ریزی بهینه خرید را مبتنی بر مشخصات اقلام اهرمی ارائه می کند. اولین مدل یک مدل برنامه ریزی آرمانی در فضای پویاست که به محدودیت های زمان های تحویل، کیفیت، تقاضا، ظرفیت های محدود تامین کنندگان و همین طور بودجه محدود خریداران توجه دارد. مدل دوم توجه به عدم قطعیت نموده و لذا در آن نرخ کیفیت و زمان های تحویل از توزیع نرمال تبعیت می کند. برای هر دو مدل مثالی عددی ارائه شده و علاوه بر حل آن ها تحلیل حساسیت نیز ارائه شده است. هر دو مدل یک سری برتری هائی بر یکدیگر دارند.

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1. INTRODUCTION

In most industries, cost of raw materials and components constitute the major portion of the product cost. According to Neely and Byrne [3]

fifty percent of all British companies' costs relate to the management of material resources and in some cases it is accounted for up to 60% [4]. Therefore, there has always been a considerable attempt to minimize the cost of production related

to material procurement. In such circumstances, purchasing department can play a key role in cost reduction and supplier selection. This is one of the most important policies to achieve the above goal. In the current globalized environment, companies have to optimize not only their internal process but also manage their procurement, distribution processes and even their members in the supply chain. Procurement management is one of the most important subjects in supply chain management which involves evaluation, selection and control of the suppliers as well as setting up their development programs. In addition, in this paper it is shown that the relationship between suppliers and manufacturers cannot be unique in all the circumstances.

Gaball [5] is the first author who applied a mixed-integer programming model to minimize the total discounted price of allocated items to the vendors based upon vendor's capacity and demand satisfaction constraints. Anthony and Buffa [6] developed a single objective linear programming model to minimize total cost by considering limitations of purchasing budegt, vendor capacities and buyer's demand. Narasimhan and Stoynoff [7] applied a single objective, mixed integer programming model to optimize the allocation of procurement among a group of vendors. Razmi, et al [8] have developed a fuzzy ANP model to evaluate the potential suppliers. The authors have developed a non-linear programming model to elicit eigenvectors from fuzzy comparison matrices. Rosenthat, et al [9] developed a mixed integer linear program that investigates the purchasing strategy for the buyer to minmize the total purchasing cost. Ghodsypour and O'Brien [10] and Razmi and Rafie [11] present a mixed integer non-linear programming model to solve the multiple sourcing problems, which considers the total cost of logistics, including net price, storage, transportation, and ordering costs. All the above models do not consider materials/parts characteristics and select and plan the suppliers only based upon suppliers' capabilities and their previous performances.

For the first time Kraljic [1] developed a conceptual model in order to determine the purchasing strategies for an organization based on parts' characteristics. This simple model is a two dimensional figure in which profit impact factor has been allocated on the first side and supply risk factor on the other side. Several large companies like Shell, Alcatel, Philips, Akzo, Nobel and Siemens have applied this model. The survey by Boodie [12] illustrates that fifty percent of the purchasing management offices have implemented this model to establish their purchasing strategy, and eighty five percent of the managements have applied this concept to their organizations. Recently, the supply management subjects have been extended, and new issues have been introduced based upon Kraljic's work such as product development [13], supplier development [14], web-base procurement [15], engineering-purchasing interactions [16], change joint venture [17], and supplier segmentation [18]. Others like [19-30] also employed Kraljic model in their studies.

2. KRALJIC PERSPECTIVE ABOUT SUPPLY MANAGEMENT

Kraljic's notion is based upon the idea of minimizing risk of supply and maximizing power of supply. In essence, his model is a lattice-space model which one side projects the *profit impact* and the other side the *supply risk*. In this model a point or a region within the lattice-space represents four different circumstances of purchased materials. At the origin of the state-space model the amount of both factors are at minimum level and when one deviates from the origin of the model and reaches to the extremity of the model, these values increase. In this state-space model four circumstances have been defined in which all purchased items can be classified into these categories (please see Figure 1). The first category relates to the *routine* items. The *routine* items are non-critical items which are produced in standard configuration. The best method of controlling these items is to keep the level of the inventory in optimal level, and there isno need to consider other attributes. The second groups are *leverage items*. The *leverage* items are the materials which buyer has big maneuver to bargain and it is easy for him to find the best price by calling for tenders. The *bottleneck items* are those which their supply involves various risks and problems. In this

situation the guaranty of the contract, supplier control, and strategy to keep enough inventory are recommended. Finally, the *strategic* items are the group of materials which needs a strategic/long-term relationship between buyer and supplier in order to maintain safe business. It is clear that different characteristics of the materials lead buyer to apply different methods of inventory control policies (Razmi and Ahmed [32]). In addition, buyer has different levels of authority and maneuver capability to buy particular materials (for detail discussion, please refer to Razmi and Karbasian [2]).

Based on a similar point of view, Dubios and Pederson [31] assigned four different criteria for the above four items (Routine items, Leverage items, Strategic items, and Bottleneck items). As it is illustrated in Figures 2 and 3, all items must be controlled by different policies and one should consider different criteria in order to have the optimal solution. The evaluation and selection
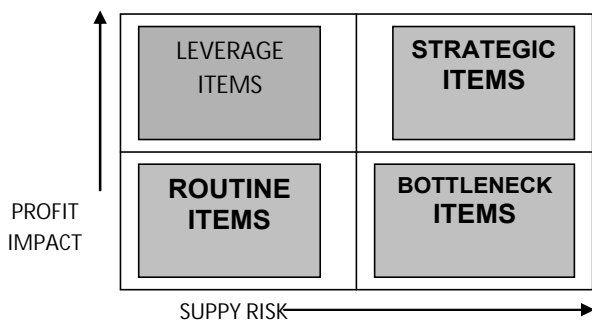

Figure 1. Classification of the purchasing items by Kraljic.
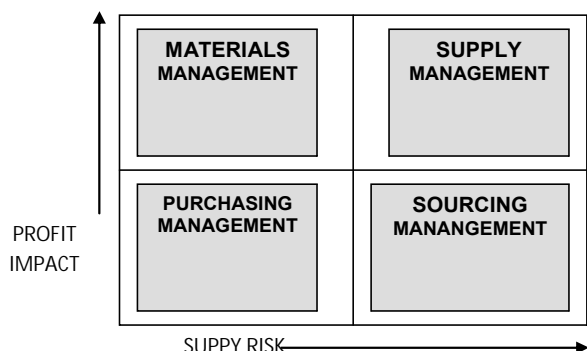

Figure 2. Classification of the management criteria by Dubios.

process differ depending on whether the product is classified as "quality" or "commodity". A quality product is one that is critical in the manufacturing process and needs hi-tech manufacturing techniques. These products are usually candidate for long-term supplier partnerships. A commodity product (or service) is generally non-critical in the manufacturing process and has a routine nature in market. Among the four categories introduced by Kraljic the leverage items and strategic items pose more motivation to optimize compared to the other two categories of routine items and bottleneck items since they encompass tremendous impact on profit. As Razmi and Karbasian have illustrated [2], the leverage items outlet with more suppliers compared to strategic items and also more value compared to total finished products. Therefore, they are still in strategic position. However, buyer should search for the best bargain and consider price as the most important criterion for long term contracts. In this situation, buyer should make sure that the quality of the materials and batches of materials received are in a regular manner. It must be mentioned that because of the important role of these items in the company's products, the manufacturer must be on its suppliers' performance list otherwise all production, distribution, and marketing plans will be faced with big challenges. Therefore, motivation to assess the suppliers and optimizing the purchasing schedule from the candidate suppliers is more apparent for leverage items. Due to little impact on supply risk in leverage items, there is more room to maneuver for optimization of the purchased items. As a result, in order to complete the idea introduced by Razmi and Karbasian [2] at the first step, the study is focused on optimizing procurement plan for leverage items. In this paper, two mathematical models are introduced for leverage items in order to solve supplier selection problem. The first model is a goal programming that considers deterministic condition and includes the following primary goals:

- Minimizing net price, net rejection, and on time deliveries in a dynamic mode subject to the constraints regarding buyer's demands, vendor's capacity, and the buyer's limitations on budget, quality, on time delivery, etc. This model considers vendor selections for multiple items, multiple time periods and multiple sourcing.
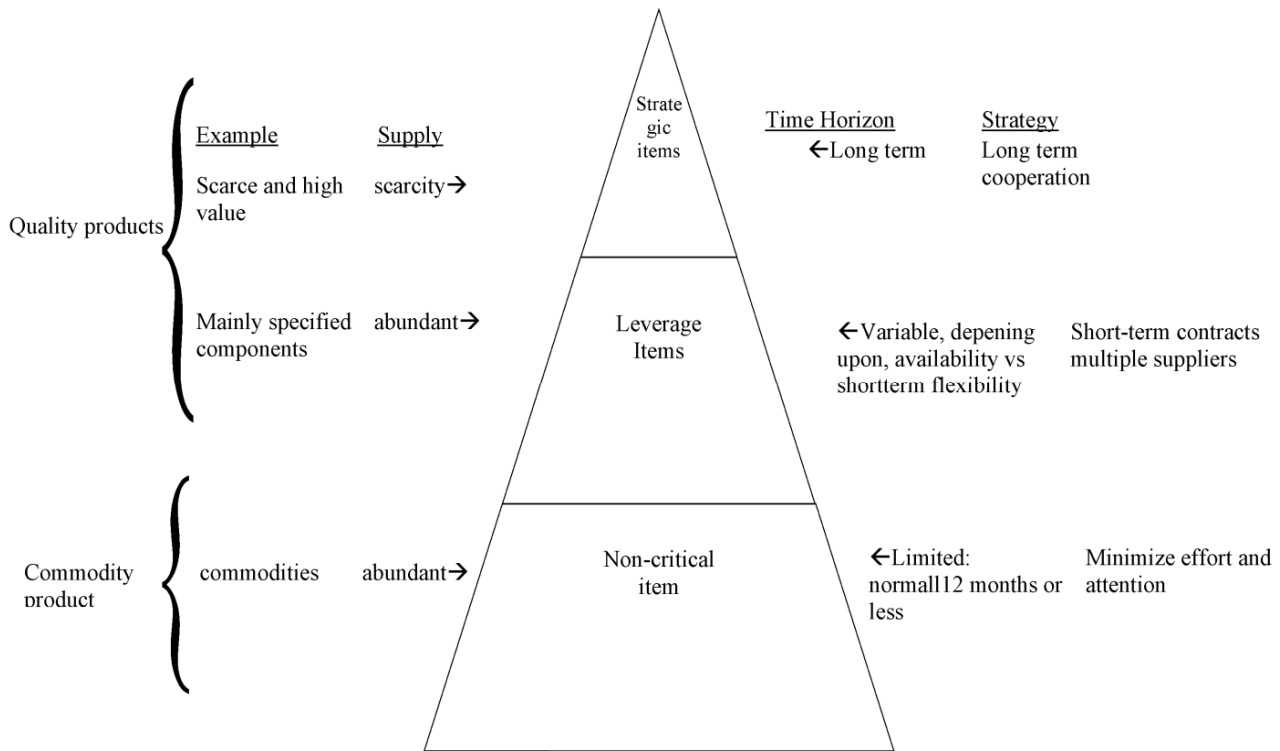
Figure 3. A Hierarchy of strategic importance.

The second model is a goal programming in probabilistic condition which includes the following primary goals:

- Minimizing purchasing cost, rejection, and late deliveries (tardiness) subject to constraints regarding buyer's demand, vendor's capacity, and buyer's limitations on budget, quality, delivery time, etc.

3. SUPPLIER SELECTION IN DETERMINISTIC CONDITION

First of all, consider the following notations used in this paper.

Assumptions:

- Demand of the item is constant and known with certainty.
- Every supplier must supply at least one item.
- Multiple sourcing, multiple items, multiple time periods are considered.

Indices:

i: index for product, for all i=1…..n
j.   index for supplier for all j=1…..s_i
t.   index for time

Variables:

Xijt: is a zero/one variable that shows the allocation of purchased i_{th} item from j_{th} supplier in period t

Y_{ijt}: percent of D_{it} assigned to j_{th} supplier over a period t.

Parameters:

D_{it}: demand of i_{th} item in period t (constant).
C_{ijt}: j_{th} supplier's capacity for supplying the i_{th} item in period t.
q_{ia}: minimum acceptance rate of incoming i_{th} item (constant for all time periods)
q_{ijt}: acceptance rate of j_{th} supplier for i_{th} item in period t.
P_{ijt}: price of j_{th} supplier for i_{th} item in period t.
TB_{it}: total bodget for purchasing item i in period t

S_i: number of candidates for purchasing i_{th} item.

U_{it}: maximum accepted earliest due date rate for i_{th} item in period t. (It is known and constant)

K_{it}: maximum accepted delay time rate for i_{th} item in period t. (It is known and constant)

M_{ijt}: earliest due date or delay time rate of j_{th} supplier for i_{th} item in period t.

3.1. Constraints

The most important constraints of the problem are buyer's *demand* and *quality*, supplier *capacity* and *lead time*. These constraints are formulated as follows:

3.1.1. Capacity constraint

The vendor j can provide up to c_j units of the product over a period t. Therefore:

$$Y_{ijt} D_{it} \le C_{ijt} \quad \text{for all } i, j, t \tag{1}$$

3.1.2. Demand constraint

Demand is equal to D_{it} and it is assumed that S_i vendors can meet the buyer's demand, then:

$$\sum_{j=1}^{s_i} Y_{ijt} D_{it} = D_{it} \quad \text{for all } i, t \; \rightarrow \; \sum_{j=1}^{s_i} Y_{ijt} = 1 \tag{2}$$

3.1.3. Quality constraint

q_{ia} is minimum acceptance rate of incoming i_{th} item and q_{ijt} is acceptance rate of j_{th} supplier for i_{th} item in period t and purchased volume is $Y_{ijt} D_{it}$ the quality constraint can be formulated as follow:

$$\sum_{j=1}^{s_i} Y_{ijt} q_{ijt} D_{it} \ge q_{ia} D_{it} \; \rightarrow \; \sum_{j=1}^{s_i} Y_{ijt} q_{ijt_{it}} \ge q_{ia} \tag{3}$$

3.1.4. Lead time constraint

U_{it} is assumed as the maximum accepted earliest due date rate for i_{th} item in period t and K_{it} is maximum accepted delay time rate for i_{th} item in period t. M_{ijt} is earliest due date or delay time rate of j_{th} supplier for i_{th} item in period t. Therefore, the lead time constraint can be formulated as follows:

$$\sum_{j} M_{ijt} Y_{ijt} D_{it} \le Z_{ijt} U_{it} D_{it} + (1 - Z_{ijt}) K_{it} D_{it} \tag{4}$$

$$Z_{ijt} = \begin{cases} 1 & : \text{if } M_{ijt} \text{ is assigned as earliest due date} \\ 0 & : \text{if } M_{ijt} \text{ is assigned as delay time rate} \end{cases}$$

It is obvious that if X_{ijt} is zero, Y_{ijt} is also zero, and if X_{ijt} is equal to 1, Y_{ijt} must be bigger than zero. Therefore, since Y_{ijt} cannot be more than 1, the following constraints can satisfy the above conditions:

$$\begin{aligned} (Y_{ijt} \ge \varepsilon X_{ijt}) \\ (Y_{ijt} \le X_{ijt}) \end{aligned} \quad \text{for all } i, j, t$$

Where ε is always slightly greater than zero.

3.1.5. The final model

$$\min z = \Sigma_i \Sigma_j \Sigma_t P_{ijt} Y_{ijt} D_{it} + \Sigma_i \Sigma_t dp_{i1t} + \Sigma_i \Sigma_j \Sigma_t (dn_{ij2t}) \tag{5}$$

$$Y_{ijt} D_{it} \le C_{ijt}$$

$$\sum_{j=1}^{s_i} Y_{ijt} q_{ijt} D_{it} \ge q_{ia} D_{it} \rightarrow$$

$$\sum_{j=1}^{s_i} Y_{ijt} q_{i1t} + dn_{i1t} - dp_{i1t} = q_{ia}$$

for all i, t

$$\sum_{j=1}^{s_i} Y_{ijt} = 1 \quad \text{for all } i, t$$

$$X_{ijt} = \begin{cases} 1: & \text{if } Y_{ijt} \text{ is more than zero} \\ 0: & \text{if } Y_{ijt} \text{ is equal to zero} Y_{ijt} \end{cases}$$

$$(Y_{ijt} \ge \varepsilon X_{ijt})$$

$$(Y_{ijt} \le X_{ijt})$$

$$\sum_{j} P_{ijt} Y_{ijt} D_{it} \le TB_{it}$$

$$Z_{ijt} = \begin{cases} 1: & \text{if } M_{ijt} \text{ is assigned as earliest due date} \\ 0: & \text{if } M_{ijt} \text{ is assigned as delay time rate} \end{cases}$$

$$\sum_j \begin{aligned} M_{ijt}Y_{ijt}D_{it} + dn_{ij2t} - dp_{ij2t} = \\ Z_{ijt}U_{it}D_{it} + (1 - Z_{ijt})K_{it}D_{it} \end{aligned} \qquad (6)$$

3.2. Numerical Example It is assumed that the manager decides to minimize the purchasing cost for the leverage items in two periods by selecting and planning its suppliers. Tables 1- 4 illustrate the required information of the suppliers for these two periods.

TABLE 1. Supplier's Information in Period (1) for Item (A).

		$D_{11}=1000$, $K_{11}=0.7$, $U_{11}=0.1$, $TB_{11}=16000$, $q_1a=0.92$		
	Price	Defect Free Rate	M_{ijt}	Capacity
Sup.1	9	0.92	0.6 (delay time rate)	600
Sup.2	12	0.95	0.4 (earliest due date rate)	700
Sup.3	14	0.98	0.4 (delay time rate)	500

TABLE 2. Supplier's Information in Period (2) for Item (A).

		$D_{12}=800$, $K_{12}=0.5$, $U_{12}=0.4$, $TB_{12}=16800$, $q_1a=0.92$		
	Price	Defect Free Rate	M_{ijt}	Capacity
Sup.1	22	0.95	0.5 (Delay Time Rate)	400
Sup.2	20	0.92	0.0 (on-time)	600
Sup.3	18	0.90	0.4 (Earliest Due Date Rate)	700

TABLE 3. Supplier's Information in Period (1) for Item (B).

		$D_{21}=500$, $K_{13}=0.2$, $U_{21}=0.1$, $TB_{21}=15000$		
	Price	Defect Free Rate	M_{ijt}	Capacity
Sup.4	4	0.93	0.6(Delay Time Rate)	400
Sup.5	6	0.96	0.0 (on-time)	300

TABLE 4. Supplier's Information in Period (2) for Item (B).

		$D_{22}=600$, $K_{22}=0.2$, $U_{22}=0.1$, $TB_{22}=18000$, $q_2a=0.93$		
	Price	Defect Free Rate	M_{ijt}	Capacity
Sup.4	3	0.98	0.5 (Delay Time Rate)	500
Sup.5	5	0.92	0.5 (Earliest Due Date Rate)	600

3.2.1. Review of the cases By evaluating the suppliers, it can be understood that for item (A), there are 8 combinations of assigning suppliers of which only four possibilities should be evaluated, since other combinations are not feasible. Regarding item (B), there are 4 combinations of assigning suppliers of which only two possibilities must be assessed.

Tables 5-8 illustrate the above facts and show the feasible allocation situations. Regarding Tables 5-8, we have 64 feasible combinations (4*4*2*2=64) to calculate the optimal solution.

TABLE 5. Zero/ One Variables Represent Supplier Selection and Feasible Cases for Part 1 in Period 1.

Case	X_{111}	X_{121}	X_{131}	Capacity	Feasible/ unfeasible
1	1	1	1	1800	Feasible
2	1	1	0	1300	Feasible
3	0	1	1	1200	Feasible
4	1	0	1	1100	Feasible
5	1	0	0	600	unfeasible
6	0	1	0	700	unfeasible
7	0	0	1	500	unfeasible
8	0	0	0	0	unfeasible

TABLE 6. Zero/ One Variables Represent Supplier Selection and Feasible Cases for Part 1 in Period 2.

Case	X_{112}	X_{122}	X_{132}	Capacity	Feasible/ unfeasible
1	1	1	1	1700	Feasible
2	1	1	0	1000	Feasible
3	0	1	1	1300	Feasible
4	1	0	1	1100	Feasible
5	1	0	0	400	unfeasible
6	0	1	0	600	unfeasible
7	0	0	1	700	unfeasible
8	0	0	0	0	unfeasible

TABLE 7. Zero/ One Variables Represent Supplier Selection and Feasible Cases for Part 2 in Period 1.

Case	X_{241}	X_{251}	Capacity	Feasible/ unfeasible
1	1	1	700	Feasible
2	1	0	400	unfeasible
3	0	1	300	Feasible
4	0	0	0	unfeasible

TABLE 8. Zero/ One Variables Represent Supplier Selection and Feasible Cases for Part 2 in Period 2.

Case	X_{242}	X_{252}	Capacity	Feasible/unfeasible
1	1	1	1100	Feasible
2	1	0	500	unfeasible
3	0	1	600	Feasible
4	0	0	0	unfeasible

The optimum answers for all situations are shown in Table 9. It can be seen from Table 9 that case 3 is an infeasible situation and case 1 has the minimum cost. Therefore, case 1 is the optimum solution for the above example with the minimum cost of 21752.03 money units. As it can be seen from the above example, the approach introduced in this paper can easily distinguish between feasible and infeasible situations of suppliers combinations. This is very important when we deal with the real cases in which practitioners face with numerous periods and suppliers. In addition, it is shown that for the first time the concept of budget constraint is put into purchasing plan in all individual periods of the purchasing plan.

3.2.2. Sensitivity analysis For the above example if demand changes to [D_{11}=1200, D_{12}=700, D_{21}=400, D_{22}=500], Tables 5-8 will be modified as follows. As it can be observed for item (A), there are 8 combinations of assigning suppliers in each period of time, which in periods one and two only three and five possibilities should be evaluated respectively, since the other combinations of assigning the suppliers are not feasible. Regarding item (B), there are 4 combinations of assigning suppliers which only two possibilities must be assessed. Tables 10-13 illustrate the above facts and show the feasible allocation situations. Regarding these tables we have 60 feasible combinations (5*3*2*2=60) to calculate the optimal solution. Table 14 illustrates the optimal result for this case. In the second period, if the proposed prices of suppliers 4 and 5 respectively change to 5 and 2 money units, we expect that the allocated purchased materials will be modified in favour of supplier 5. Based upon the above modification, the model is once again executed and the results are shown in Table 15 which illustrates logical answer to the above changes.

4. SUPPLIER SELECTION IN PROBABILISTIC CONDITION

The proposed second model attempts to minimize the cost of purchasing in probabilistic condition considering the following assumptions. It must be mentioned that solving the problem in the probabilistic condition makes the second model more complicated and therefore, in this paper the elements of time has been excluded and the proposed model is not dynamic.

Assumptions:

(1)   The periods of planning is known.
(2)   The demand for each leverage items is known.
(3)   The supply base (maximum number of suppliers) for all items is constant and known

Variables:

X ij :   this is a zero/one variable that shows the allocation of purchased i_{th} item from j_{th} supplier

Y_{ij}:   the quantity of materials purchased from supplier j if it has been selected

Parameters:

D_i: demand of i_{th} item (constant).
C_{ijt}: j_{th} supplier's capacity for supply the i_{th} item
q_{ij}:   failure rate of materials dispatched by supplier j_{th} for i_{th} item (this is a random variable with normal distribution)
q_{ia}:   maximum acceptance rate of failure for i_{th} item (constant and known)
L_{ij}:   rate of tardiness related to j_{th} supplier which has been selected to supply i_{th} item that is a random variable with normal distribution
L_{ia}:   maximum acceptance rate of tardiness to supply i_{th} item which is a constant and known
P_{ij}:   price of j_{th} supplier for i_{th} item
F_{ij}:   constant cost offer by supplier j for procure item i
TBP: total budget need to purchase all required items
l-A: probability of obtaining appropriate quality which is known
l-B: probability of delivering batch of materials in appropriate lead-time which is known

4.1. Mathematical Models, Considering Risk Factor The risk factor can be directly interfered in the problem solution as follows:

TABLE 9. Optimum Solution for Best of Cases**.**

Case	Y_{111}	Y_{121}	Y_{131}	Y_{112}	Y_{122}	Y_{132}	Y_{241}	Y_{251}	Y_{242}	Y_{252}	Z
A	0.6	0.4	0	0	0.14	0.87	0.8	0.2	0.8	0.2	29048.02

TABLE 10. Zero/ One Variables Represent Supplier Selection and Feasible Cases for Part 1 in Period 1.

Case	X_{111}	X_{121}	X_{131}	Capacity	Feasible/ unfeasible
1	1	1	1	1800	Feasible
2	1	1	0	1300	Feasible
3	0	1	1	1200	Feasible
4	1	0	1	1100	unfeasible
5	1	0	0	600	unfeasible
6	0	1	0	700	unfeasible
7	0	0	1	500	unfeasible
8	0	0	0	0	unfeasible

TABLE 11. Zero/ One Variables Represent Supplier Selection and Feasible Cases for Part 1 in Period 2.

Case	X_{112}	X_{122}	X_{132}	Capacity	Feasible/ unfeasible
1	1	1	1	1700	Feasible
2	1	1	0	1000	Feasible
3	0	1	1	1300	Feasible
4	1	0	1	1100	Feasible
5	1	0	0	400	unfeasible
6	0	1	0	600	unfeasible
7	0	0	1	700	feasible
8	0	0	0	0	unfeasible

TABLE 12. Zero/ One Variables Represent Supplier Selection and Feasible Cases for Part 2 in Period 1.

Case	X_{241}	X_{251}	Capacity	Feasible/ unfeasible
1	1	1	700	Feasible
2	1	0	400	feasible
3	0	1	300	unfeasible
4	0	0	0	unfeasible

TABLE 13. Zero/One Variables Represent Supplier Selection and Feasible Cases for Part 2 in Period 2.

Case	X_{242}	X_{252}	Capacity	Feasible/ unfeasible
1	1	1	1100	Feasible
2	1	0	500	unfeasible
3	0	1	600	Feasible
4	0	0	0	unfeasible

TABLE 14. Optimum Solution for Best of Cases.

Case	Y_{111}	Y_{121}	Y_{131}	Y_{112}	Y_{122}	Y_{132}	Y_{241}	Y_{251}	Y_{242}	Y_{252}	Z
A	0.5	0.5	0.0	0.0	0.0	1.0	1.0	0.0	0.8	0.2	42916.01

TABLE 15. Optimum Solution for Best of Cases.

Case	Y_{111}	Y_{121}	Y_{131}	Y_{112}	Y_{122}	Y_{132}	Y_{241}	Y_{251}	Y_{242}	Y_{252}	Z
A	0.5	0.5	0.0	0.0	0.0	1.0	1.0	0.0	0.4	0.6	42936.01

4.1.1. Being probable only for a single or multi-goal function in a multi-goal problem:

$$f_i(x) = \sum_{j=1}^{n} \widetilde{C}_{ij} x_j \tag{7}$$

$$p(f_i(x) \ge b_i) \ge 1 - \alpha \qquad \alpha \in [0,1] \tag{8}$$

The above constraint could be a way to interfere the risk into the problem i.e. the more probable the above relation is (smaller the α), the more accessible b_i (the time parameter of the goal function) would be. α is known and specified by the decision maker.

In this particular case, if the all distributions become normal, then the distribution $f_i(x)$ is normal; hence:

$$p\left(z \ge \frac{b_i - \sum\limits_{J=1}^{n} E(\widetilde{C}_{ij})X_j}{[VAR(f_i(x))]^{1/2}} \right) \ge 1 - \alpha \tag{9}$$

$$\phi\left[\frac{b_i - \sum\limits_{j=1}^{n} E(\widetilde{C}_{ij})X_j}{[VAR(f_i(X))]^{1/2}} \right] \le \alpha \tag{10}$$

Since the accumulated normal function is absolutely ascending function, hence:

$$Optimum\, E(f_i(x)) = \sum_{j=1}^{n} E(\widetilde{C}_{ij})X_j$$

$$st: \tag{11}$$

$$\sum E(\widetilde{C}_{ij})X_j + \phi^{-1}(\alpha)[VAR(f_i(x))]^{1/2} \ge b_i$$

4.2. Constraints In this situation the most important constraints of the problem are: buyer demand, desired quality and lead-time, supplier capacity, supply base constraint and finally budget constraint. These constraints are formulated as follows:

4.2.1. Capacity constraint The vendor j can provide up to c_j units of the item i over period t. Therefore:

$$Y_{ij} \le C_{ij} X_{ij} \tag{12}$$

4.2.2. Quality constraint It is mentioned that the parameter of q_{ia} is a maximum acceptance failure rate for i_{th} item. In this case, q_{ij} is the failure rate of item i offered by the j_{th} supplier. In this model q_{ij} is a random variable with normal distribution.

$$P(\sum_{j=1}^{S_i} X_{ij} q_{ij} \le q_{ia}) \ge 1 - A \Rightarrow$$

$$P\left(z \le \frac{q_{ia} - \sum_{j=1}^{S_i} X_{ij}\mu_{ij}}{\left(\sum_{j=1}^{S_i} X_{ij}{}^2 \delta_{ij}{}^2 \right)^{1/2}} \right) \ge 1 - A \Rightarrow \tag{13}$$

In the case of selection of the *jth* supplier, X*ij* will be equal to 1, otherwise X*ij*=0 expressed as follows:

$$X_{ij}(1 - X_{ij}) = 0 \Rightarrow X_{ij} - X_{ij}{}^2 = 0 \Rightarrow$$

$$X_{ij}{}^2 = X_{ij} \tag{14}$$

$$P\left(Z \le \frac{q_{ia} - \sum_{j=1}^{S_i} X_{ij}\mu_{ij}}{\left(\sum_{j=1}^{S_j} X_{ij}{}^2 \delta_{ij}{}^2 \right)^{1/2}} \right) \ge \varphi^{-1}(1-A) \Rightarrow (2)$$

$$\sum_{j=1}^{S_i} X_{ij}\delta_{ij}{}^2 = M^2$$

$$(2) \frac{q_{ia} - \sum_{j=1}^{S_i} X_{ij}\mu_{ij}}{M} \ge \varphi^{-1}(1-A) \Rightarrow \tag{15}$$

$$(2) \sum_{j=1}^{S_i} X_{ij}\mu_{ij} + M\varphi^{-1}(1-A) \le q_{ia}$$

4.2.3. Number of vendors constraint The number of vendors is the most important decision for single/multi resource/s problems. In traditional era, manufacturers thought that increasing the number of vendors can decrease the risk of supply. However, today, it is believed that this idea is not proficient any more. In a sense, decreasing the number of vendors leads to the following benefits: 1) decrease the total cost of product, 2) procure from the best supplier/s, 3) utilize all supplier/s' facilities, 4) minimize the management cost and ease of implementation of developing supply policies, and 5) capability to develop supplier/s. Therefore, companies would like to optimize the number of vendors. This can be accomplished by introducing the model to capture all quantitative and qualitative parameters which affect the risk of supply. In this study we assumed that the maximum number of vendors cannot be more than T (a constant parameter).

4.2.4. Lead-time constraint As mentioned before, L_{ia} is a maximum acceptance tardiness rate to supply i_{th} item, and L_{ij} is the tardiness rate of the supplier j regarding delivering i_{th} item which is a normal random variable. Hence, the following formula must be satisfied:

$$P(\sum_{j=1}^{S_i} X_{ij} L_{ij} \le L_{ia}) \ge 1 - B \Rightarrow$$

$$P\left(z \le \frac{L_{ia} - \sum_{j=1}^{S_i} X_{ij}\mu_{ij}}{\left(\sum_{j=1}^{S_i} X_{ij}{}^2 \delta_{ij}{}^2 \right)^{1/2}} \right) \ge 1 - B \Rightarrow \tag{16}$$

Since $\left(X_{ij}{}^2 = X_{ij} \right)$, like quality constraint we have:

$$\sum_{j=1}^{S_i} X_{ij}\delta_{ij}{}^2 = N^2$$

$$(3) \Rightarrow \frac{L_{ia} - \sum_{j=1}^{S_i} X_{ij}\mu_{ij}}{N} \ge \varphi^{-1}(1-B) \Rightarrow \tag{17}$$

$$\sum_{j=1}^{S_i} X_{ij}\mu_{ij} + N\varphi^{-1}(1-B) \le L_{ia}$$

4.3. The Final Model

$$Min \sum_{i=1}^{m} \sum_{j=1}^{S_i} \left(P_{ij}Y_{ij} + F_{ij}X_{ij} \right)$$

$$(1)Y_{ij} \leq C_{ij}X_{ij}, i = 1...m, j = 1...S_i;$$

$$(2)P\left(\sum_{j=1}^{S_i} X_{ij}q_{ij} \leq q_{ia} \right) \geq 1 - A, i = 1...m$$

$$(3)P\left(\sum_{j=1}^{S_i} X_{ij}L_{ij} \leq L_{ia} \right) \geq 1 - B, i = 1...m$$

$$(4) \sum_{j=1}^{S_i} X_{ij} = T \qquad i = 1...m$$

$$(5) \sum_{j=1}^{S_i} Y_{ij} \geq D$$

$$(6) \sum_{i=1}^{m} \sum_{j=1}^{S_i} P_{ij}Y_{ij} \leq TBP$$

$$(7)Y_{ij} \geq 10 * X_{ij}, i = 1...m, j = 1...S_i \qquad (18)$$

4.4. Numerical Example It is assumed that the manager decides to minimize the supply cost for two leverage items. The demand for the first item is 900, the maximum acceptance failure rate is 0.25, the maximum acceptance lead-time is 0.25, the probability of on-time delivery (1-B) is 0.75 and the probability of establishing the quality (1-A) is 0.75. The demand for the second item is 850, the maximum acceptance failure rate is 0.35, the maximum acceptance lead-time is 0.35, the probability of on-time delivery is 0.76, and the probability of establishing the quality is 0.76. Total budget for supplying these two items is 30,000 money units. The complementary information is illustrated in Tables 16 and 17. Furthermore, the optimal purchasing schedule proposed by the model is shown in Table 18. It is clear that in real situation, practitioners have to consider supplier/s development programs. This development can change the suppliers' situations and the practitioners do not need to wait and can highlight the future changes in their decisions.

As can be observed in Table 18, in both parts, suppliers with maximum total value are assigned.

TABLE 16. Supplier Information Regarding the First Item.

	Price	Tardiness rate	Failure rate	Capacity	Constant cost
Supplier No. 1	5	n (0.065,0.0036)	n (0.06,0.0036)	800	4
Supplier No. 2	7	n (0.065,0.0049)	n (0.06,0.0049)	850	8
Supplier No. 3	9	n (0.065,0.0016)	n (0.06,0.0016)	500	12
The maximum number of suppliers is 2 (T=2)					

TABLE 17. Supplier Information Regarding the Second Item.

	Price	Delivery Tardiness Rate	Quality Failure Rate	Capacity	Constant cost
Supplier No. 1	18	n (0.075, 0.0016)	n (0.065, 0.0016)	750	12
Supplier No. 2	20	n (0.075, 0.0049)	n (0.065, 0.0049)	700	14
Supplier No. 3	15	n (0.75, 0.0036)	n (0.065, 0.0036)	750	10
Supplier No. 4	12	n (0.075, 0.0036)	n (0.065, 0.0036)	700	6
Supplier No. 5	14	n (0.075, 0.0049)	n (0.065, 0.0049)	800	8
The maximum number of suppliers is 3 (T=3)					

The value is calculated based upon average of suppliers' quality, average of lead time, variance, proposed price, and finally the capacity of suppliers in each period of time. Using the above model, one can easily accomplish the sensitivity analysis by illustrating the results when faces with changes in suppliers' parameters or buyer's preferences.

4.4.1. Sensitivity analysis   In the above numerical example l, first of all, we assumed that the capacity of suppliers 1 and 2 for supplying the first item changes from 800 to 900, and from 600 to 500 respectively. Also the capacity of suppliers 3 and 4 for supplying the second item changes from 700 to 850 and from 800 to 600 respectively. In this new condition, the complementary information will be changed as shown in Tables 19 and 20. The optimal purchasing schedule for the recent changes is shown in Table 21. Suppose suppliers 1, 2, and 3 change their proposed price for the first item from 5 to 10, 7 to 8, and 9 to 6 money units respectively. In addition, suppose suppliers 3, 4, and 5 change their proposed price for the second item from 15 to 14, 12 to 16, and 14 to 12 money units respectively. Then for the first item, it is logical to expect increase in the share of supplier 3 and for the second item we should see an enhancement in supplier's 5 purchased share. Therefore, the optimal solutions, shown in Table 22 confirm the rightness of the designed model.

TABLE 18. The Optimal Solution for These Two Leverage Items.

Y11	Y12	Y13	y21	y22	Y23	y24	Y25
800	100	0	0	0	10	700	140

TABLE 19. Supplier Information Regarding the First Item.

	Price	Tardiness Rate	Failure Rate	Capacity	Constant Cost
Supplier No. 1	5	n (0.065,0.0036)	n (0.06,0.0036)	900	4
Supplier No. 2	7	n (0.065,0.0049)	n (0.06,0.0049)	750	8
Supplier No. 3	9	n (0.065,0.0016)	n (0.06,0.0016)	600	12
The maximum number of suppliers is 2 (T=2)					

TABLE 20. Supplier Information Regarding the Second Item.

	Price	Delivery Tardiness Rate	Quality Failure Rate	Capacity	Constant Cost
Supplier No. 1	18	n (0.075,0.0016)	n (0.065,0.0016)	750	12
Supplier No. 2	20	n (0.075,0.0049)	n (0.065,0.0049)	700	14
Supplier No. 3	15	n (0.75,0.0036)	n (0.065,0.0036)	650	10
Supplier No. 4	12	n (0.075,0.0036)	n (0.065,0.0036)	850	6
Supplier No. 5	14	n (0.075, 0.0049)	n (0.065, 0.0049)	600	8
The maximum number of suppliers is 3 (T=3)					

TABLE 21. The Optimal Solution for These Two Leverage Items.

Y11	Y12	Y13	y21	y22	Y23	y24	Y25
890	10	0	0	0	10	830	10

TABLE 22. The Optimal Solution for These Two Leverage Items.

Y11	Y12	Y13	y21	y22	Y23	y24	Y25
300	600	0	0	0	240	10	600

5. CONCLUSIONS

Supplier selection is one of the most important aspects in supply chain management in which cost, quality, delivery performances, etc. should be considered. Based upon Kraljic finding, the characteristics of the required materials must be considered in addition to the capability and performance of the suppliers. Regarding this concept, all materials can be categorized into four categories of strategic, leverage, bottleneck, and routine items. Among these four groups of items leverage items possesses the highest motivation to optimize due to its share in profit impact and the potential for buyer maneuverability. This paper illustrates two mathematical models (goal programming) for leverage items in order to select the best supplier/s and procurement plan. In the first model supplier/s selection process is achieved in a dynamic state with the constant and deterministic rate of delivery tardiness and quality failure rate. The second model considers the selection of supplier/s and procurement of leverage items plan under risk condition. Furthermore, the second model studies the problem in a static state. It evaluates the situation considering rate of delivery tardiness and quality failure rate as probabilistic variables which is more realistic and close to practical circumstances. Both models are followed by relative numerical examples in order to illustrate how the proposed models work. In addition, sensitivity analyses are provided for both models.

For future research it is suggested to consider the cost parameter in more detailed structure. For example, in the developed model all costs are presented in one figure of material price. However, the cost factor can be divided into several aspects of transportation cost and ordering expenses. In addition, in order to catch up lean production concept, one can ponder materials return cost into the model. Regarding the second model, the current model has a static viewpoint. Moreover, it is suggested to develop a dynamic mathematical model. In order to make the model easier to solve, the authors presumed the quality and lead time as random variables with normal distributions. It is recommended to consider other distributions in the model. Furthermore, it is recognized that due to difficulty in solving the probabilistic model by mathematical programming, the time factor has not considered in the second model. It is recommend applying meta-heuristic methods such as Genetic Algorithm, Ant-Colony, etc. to construct the dynamic approach.

Since the strategic items have also an important impact on benefit and the bottleneck items have very vital affects on supply risk, it is recommended to study and introduce the model to optimize the procurement plan for these two items as well. When practitioners deal with strategic items, they must make sure about progress in quality and delivery performances in the time horizon; since company cannot change the supplier very easily (this figure is totally different as we are dealing with leverage items). Therefore, the proposed model should guaranty this continuous development in the model parameters.

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