MATHEMATICAL MODELLING OF BLOOD FLOW IN A STENOSED ARTERY UNDER MHD EFFECT THROUGH POROUS MEDIUM

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Abstract In this investigation, a mathematical model for studying oscillatory flow of blood in a stenosed artery under the influence of transverse magnetic field through porous medium has been developed. The equations of motion of blood flow are solved analytically. The analytical expressions for axial velocity, volumetric flow rate, pressure gradient, resistance to blood flow and shear stress have been derived. These expressions reveal significant alterations in blood flow due to stenosis. It is seen that magnetic field significantly controls the flow patterns. We have incorporated the magnetic field perpendicular to the flow of blood. The concept of porous medium is also taken into consideration which takes care of the suction factor. The effects of various parameters particularly magnetic number and porosity constant on the blood flow through stenosis have been examined. To validate the analytical results, numerical experiment is performed. The results obtained in the investigation are in reasonably good agreement with experimental findings existing in the literature.

Keywords*:* MHD flow, Stenosis, Magnetic number, Porosity, Axial velocity, Volumetric flow rate, Wall shear stress

چکیده در این تحقیق یک مدل ریاضی برای مطالعه جریان نوسانی خون در یک شریان تنگ شده، تحت تاثیر میدان مغناطیسی عرضی از میان یک مدیای متخلخل توسعه داده شد. معادلات حرکت جریان خون به صورت تحلیلی حل شدند. معادلات تحلیلی برای سرعت محوری، شدت جریان حجمی، گرادیان فشار، مقاومت نسبت به جریان خون و تنش برشی به دست آمدند. این معادلات تغییرات مهمی را در جریان خون با توجه به تنگی مجرا نشان دادند. از میدان مغناطیسی عمود بر جریان خون استفاده شد. همچنین مفهوم مدیای متخلخل برای تضمین ضریب خلا مورد توجه قرار گرفت. تاثیر پارامترهای مختلف به خصوص عدد مغناطیسی و ثابت تخلخل روی جریان خون در مجرای تنگ شده بررسی شد. برای توجیه نتایج تحلیلی آزمایش عددی انجام شد. نتایج به دست آمده در این تحقیق سازگاری خوب و منطقی با نتایج آزمایشگاهی موجود در متون دارد.

1. INTRODUCTION

One of the leading causes of deaths in the world is due to heart related diseases. The heart diseases mainly occur due to temporary deficiency of oxygen or blood supply to the heart. This deficiency may be due to a constriction or obstruction in the blood supply to that part; the constriction involves the deposition of some fatty substances like cholesterol, cellular waste product, calcium, etc.. This deposition is called stenosis. This stenosis disturbs the flow of blood from its normal state which leads to the development of atherosclerosis. The atherosclerosis may cause the heart attack.

Many researchers have studied blood flow in artery by considering blood as either Newtonian or non-Newtonian fluids. The study of magnetic field with porous medium is very important both from theoretical and practical point of view; because of most of natural flow problems are connected with porous medium. Ahmadi and Manvi [1] derived a general equation of motion for the flow of viscous fluid through a porous medium. Shukla et al [2] studied the effects of stenosis in an artery by considering the blood as power-law and Casson-model fluids. Shukla et al [3] discussed biorheological aspect of blood flow through artery with mild stenosis. Shukla et al [4] also presented a mathematical model to study the effects of peripheral layer viscosity on the physiological characteristics of blood flow through artery with mild stenosis. The effects of the viscosity-concentration dependence and the concentration profile on blood flow through a vessel with stenosis have been studied by Perkkiö and Keskinen [5]. Haldar [6] studied the problem of blood flow through an artery by considering blood as non-Newtonian. He also discussed the effects of shape of stenosis on blood flow. Misra and Singh [7] considered the mathematical model to investigate the pulsatile flow of blood through arteries by treating the blood vessel as thin-walled, non-linearly viscoelastic and incompressible circular shell. Misra and Patra [8] developed a mathematical model to study non-Newtonian nature of blood flow through arteries under stenosis. Ghalichi et al [9] proposed a mathematical model for blood flow studies in certain areas of arterial tree wherein both laminar and turbulent flow coexist. Oshima et al [10] developed a patient-specific modeling and simulation system to investigate the effects of vascular morphology on cerebral hemodynamics. Varghese and Frankel [11] analyzed numerically the pulsatile turbulent flow in stenotic vessels. Bhuyan and Hazarika [12] obtained an approximation solution for the pulsatile flow of blood in a porous channel in the presence of transverse magnetic field by assuming blood as Newtonian fluid. The study of blood flow through porous medium is another field of great interest in physiological problems. In some physiological blood flow situations, the distribution of fatty substances which deposits on the walls of an artery can be considered as equivalent to a porous medium. Peristalsis is one of the major mechanisms for fluid transport in many biological systems and peristaltic pumping occurs in many practical applications involving biomechanical systems. Mekheimer and Al-Arabi [13] studied the mathematical model to determine the characteristics of peristaltic transport of magnetohydrodynamic flow through a porous medium. Übeyli and Güler [14] presented a new technique which is based on neuro-fuzzy inference system for the detection of internal carotid artery stenosis and its occulation. Oshima and Torii [15] developed a medical image based simulation and database system to investigate the effects of wall deformation on blood flow. Lubbers et al [16] constructed a computer based model to explore the effects of varying size of stenosis on blood flow.

Abbas et al [17] investigated two dimensional magnetohydrodynamic (MHD) flow of upper-convected Maxwell fluid in a porous channel. Arterial wall shear stress is considered to be an important factor in the localization of atherosclerotic. The height of the stenosis is a key factor influencing blood flow than tapering. Since high wall shear stress causes the innermost membrane of an artery or a vein thickening, but may also activate platelets, cause platelet aggregation, and finally may result in the formation of a thrombus. A mathematical model to study the effect of porous parameter and height of stenosis on the wall shear stress has been studied by Misra and Verma [18]. Chapman [19] developed a mathematical model for the blood flow through the leaky neovasculature and porous interstitium of a solid tumor. Weinberg and Mofrad [20] considered a three dimensional model to examine the effects of geometric factor on multiscale valve mechanics. A multiphase kinetic theory for the computation of viscosity of red blood cells and their migration from vessel walls has been discussed by Huang [21]. Jain et al [22] developed a mathematical model to study the blood flow problem through narrow blood vessels in the presence of mild stenosis. A mathematical analysis of MHD flow of blood in very narrow capillaries in the presence stenosis has been studied by Jain et al [23]. Rathod and Tanveer [24] studied the plusatile flow of blood through a porous medium under the influence of periodic body acceleration by considering blood as a couple stress, incompressible, electrical conducting fluid in presence of magnetic field.

In this paper, we have made an attempt to see the effects of magnetic field on the blood flow through stenosis under porous medium. Here the artery is considered as a porous medium; because in some pathological situation the deposition of fatty material or cholesterol and artery-clogging blood clots in the lumen of the coronary artery can be considered as equivalent to fictitious porous medium. The geometry of the stenosis also affects the blood flow. There are many types of stenosis like cosine shaped stenosis, bell-shaped stenosis, overlapping stenosis, irregular stenosis, multi-irregular stenosis etc.. But in this study, we consider the most common cosine shaped geometry of the stenosis.

The organization of paper is as follows. The mathematical formulation of the problem along with requisite assumptions and notations has been provided in section. 2. Section 3 presents the analysis of the problem. The sensitivity analysis is carried out in section 4. Conclusion is given in last section 5.

2. MATHEMATICAL MODEL

In this paper, the application of porous medium for the study of blood flow through stenosed artery in the presence of transverse magnetic field is made. The application of magnetohydrodynamics in physiological problems is of growing interest. The flow of blood can be controlled by applying sufficient quantity of magnetic field. The Reynolds number is assumed to be very small and therefore induced magnetic field has been neglected. Two important physical factors occur when the fluid moves into magnetic field. The first one electric field E produced in the flow. We assume that there is no excess charge density, so that $\nabla \cdot E = 0$. Also neglecting induced magnetic field implies that $\nabla \times E = 0$. The second factor is Lorentz force $(J \times B)$, where J the current density acting on the fluid. Therefore there is transfer of energy $(J \cdot B)$ from electromagnetic field to the fluid. In this paper, the relativistic effects are neglected and J is given by Ohm's law as $J = \sigma(q \times B)$. Here blood is considered to be laminar, incompressible and Newtonian in nature, through an artery with mild constriction. The density and viscosity of the fluid is assumed to be constant. The geometry of stenosis is considered to be symmetrical and cosine shaped. Let the length of tube be L and z be the axis along which the blood flows. The geometry of stenosis is given by the following relation

$$\frac{R}{R_0} = 1 - \frac{\varepsilon}{2R_0}\left(1 + \cos\frac{\pi z}{L_0}\right)$$

where R and R_0 are the radii of tube and of constricted region of tube due to stenosis, respectively. Here L_0 and ε are the length and maximum height of the stenosis, respectively. The schematic diagram of the stenosis is depicted in figure. 1.

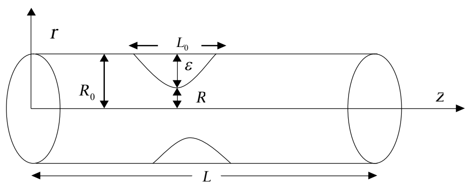

Figure 1. Geometry of cosine shaped stenosis in artery

2.1. Governing Equations Since the blood is considered as an incompressible Newtonian and the flow is axially symmetric with negligible body forces, the equation of continuity and the Navier-Strokes equations which govern the motion in the cylindrical coordinates (r, θ, z) are as follows:

$$\frac{\partial w}{\partial z} = 0 \tag{1}$$

$$\frac{\partial p}{\partial r} = 0 \tag{2}$$

$$\frac{1}{r}\frac{\partial p}{\partial \theta}=0 \tag{3}$$

$$\rho\frac{\partial w}{\partial t}=-\frac{\partial p}{\partial z}+\mu\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right)-\left(\sigma B_0^{\,2}-\frac{\mu}{k}\right)w \tag{4}$$

where w is the velocity in axial direction, p be the fluid pressure, ρ be the density of fluid, μ be the viscosity, k is the porous parameter, σ is the magnetic intensity and B_0 is the applied magnetic field.

The boundary conditions are

$$\frac{\partial w}{\partial r}=0 \qquad \text{at} \quad r=0 \tag{4a}$$

$$w=0 \qquad \text{at} \quad r=R \tag{4b}$$

$$p=p_0 \qquad \text{at} \quad z=0 \tag{4c}$$

$$p=p_L \qquad \text{at} \quad z=L \tag{4d}$$

3. THE ANALYSIS

To solve the governing equations of the problem, it is convenient to introduce a transformation as

$$y=\frac{r}{R_0}.$$

On substituting the above transformation in the main equation of motion, it takes the form

$$\frac{\partial^2 w}{\partial y^2}+\frac{1}{y}\frac{\partial w}{\partial y}-\left(\frac{\sigma B_0^{\,2}R_0^{\,2}}{\mu}-\frac{R_0^{\,2}}{k}\right)w-\frac{\rho R_0^{\,2}}{\mu}\frac{\partial w}{\partial t}=\frac{R_0^{\,2}}{\mu}\frac{\partial p}{\partial z} \tag{5}$$

The corresponding boundary conditions are:

$$\frac{\partial w}{\partial y}=0 \qquad \text{at} \qquad y=0 \tag{5a}$$

$$w=0 \qquad \text{at} \qquad y=\frac{R}{R_0} \tag{5b}$$

$$p=p_0 \qquad \text{at} \qquad z=0 \tag{5c}$$

$$p=p_L \qquad \text{at} \qquad z=L \tag{5d}$$

It is evident from equation (1) that w is independent of z. Also from (2) and (3), it is clear that p is independent of r and θ both. Therefore it is justified to make the following substitutions:

$$w(y,t)=\overline{w}(y)e^{i\omega t}$$

$$\frac{\partial p}{\partial z}=-Pe^{i\omega t}$$

Putting above values in equation (5), we obtain

$$\frac{d^2\overline{w}}{dy^2}+\frac{1}{y}\frac{d\overline{w}}{dy}-\left(\frac{\sigma B_0^{\,2}R_0^{\,2}}{\mu}-\frac{R_0^{\,2}}{k}+\frac{\rho R_0^{\,2}i\omega}{\mu}\right)\overline{w}-\frac{R_0^{\,2}}{\mu}P=0 \tag{6}$$

The solution of equation (6) with boundary conditions (5a)-(5b) is given as

$$\overline{w}(y)=\frac{P}{i\mu\left(\frac{\sigma B_0^{\,2}}{\mu}-\frac{1}{k}+\frac{\rho i\omega}{\mu}\right)}\left[1-\frac{J_0\left\{\alpha^2\frac{y}{R_0}i^{3/2}\right\}}{J_0\left\{\alpha^2\frac{R}{R_0}i^{3/2}\right\}}\right]e^{i\omega t} \tag{7}$$

where $\alpha^2=\left(\frac{\sigma B_0^{\,2}R_0^{\,2}}{\mu}-\frac{R_0^{\,2}}{k}+\frac{\rho R_0^{\,2}i\omega}{\mu}\right)$ and J_0 is the Bessel's function of zero order with complex argument.

3.1. Axial Velocity The axial velocity of the blood flow in the artery is given by the following expression

$$w(r,t)=\frac{P}{i\mu\left(\frac{\sigma B_0^2}{\mu}-\frac{1}{k}+\frac{\rho i\omega}{\mu}\right)}\left[1-\frac{J_0\left\{\alpha^2\frac{r}{R_0}i^{3/2}\right\}}{J_0\left\{\alpha^2\frac{R}{R_0}i^{3/2}\right\}}\right]e^{i\omega t} \tag{8}$$

3.2. Volumetric Flow Rate (Q) The volumetric flow rate (Q) is given by

$$Q=\int_0^R 2\pi r w dr \tag{9}$$

Therefore

$$Q=\frac{RR_0^2P}{i\mu\left(\frac{\sigma B_0^2}{\mu}-\frac{1}{k}+\frac{\rho i\omega}{\mu}\right)}\left[\frac{R}{R_0}-\frac{2J_1\left\{\alpha^2\frac{r}{R_0}i^{3/2}\right\}}{i^{3/2}\alpha^2J_0\left\{\alpha^2\frac{R}{R_0}i^{3/2}\right\}}\right]e^{i\omega t} \tag{9a}$$

where J_1 is the Bessel function of first order with complex argument.

3.3. Pressure Gradient Pressure gradient is expressed as

$$\frac{\partial p}{\partial z}=-\left\{\frac{i\mu R_0\left(\frac{\sigma B_0^2}{\mu}-\frac{1}{k}+\frac{\rho i\omega}{\mu}\right)J_0\left\{\alpha^2\frac{R}{R_0}i^{3/2}\right\}}{RJ_0\left\{\alpha^2\frac{R}{R_0}i^{3/2}\right\}-2J_1\left\{\alpha^2\frac{r}{R_0}i^{3/2}\right\}}\right\}Qe^{-i\omega t} \tag{10}$$

3.4. Resistance to Flow On integrating equation (10) and using equations (4c)-(4d), we obtain

$$\Delta p=p_0-p_L=$$

$$\left\{\frac{i\mu R_0\left(\frac{\sigma B_0^2}{\mu}-\frac{1}{k}+\frac{\rho i\omega}{\mu}\right)J_0\left\{\alpha^2\frac{R}{R_0}i^{3/2}\right\}}{R\alpha J_0\left\{\alpha^2\frac{R}{R_0}i^{3/2}\right\}-2J_1\left\{\alpha^2\frac{r}{R_0}i^{3/2}\right\}}\right\}\times$$

$$Qe^{-i\omega t}\left[L-L_0\right] \tag{11}$$

The resistive impedance to the flow (cf. Young [25]) is given by the relation

$$\lambda=\frac{p_0-p_L}{Q} \tag{11a}$$

Now equations (10) and (11) yield

$$\lambda=\left\{\frac{i\mu R_0\left(\frac{\sigma B_0^2}{\mu}-\frac{1}{k}+\frac{\rho i\omega}{\mu}\right)J_0\left\{\alpha^2\frac{R}{R_0}i^{3/2}\right\}}{RJ_0\left\{\alpha^2\frac{R}{R_0}i^{3/2}\right\}-2J_1\left\{\alpha^2\frac{r}{R_0}i^{3/2}\right\}}\right\}\times$$

$$e^{-i\omega t}\left[L-L_0\right] \tag{11b}$$

3.5. Wall Shear Stress The shearing stress at wall $r=R$ is given as follows;

$$\tau_s=-\left(\mu\frac{\partial w}{\partial r}\right)_{r=R} \tag{12}$$

$$=\frac{P}{i\mu\left(\frac{\sigma B_0^2}{\mu}-\frac{1}{k}+\frac{\rho i\omega}{\mu}\right)}\left[1-\frac{J_1\left\{\alpha^2\frac{1}{R_0}i^{3/2}\right\}}{J_0\left\{\alpha^2\frac{R}{R_0}i^{3/2}\right\}}\right]e^{i\omega t} \tag{12a}$$

4. NUMERICAL ILLUSTRATIONS

In the present section, numerical results have been provided to explore the effects of various parameters on the axial velocity, wall shear stress etc.. For this purpose, we develop a program coded in MATLAB software. We use the defaults parameters fixed as

$$M = \sigma B_0^2 R_0^2 i = 1.5,\ R = 0.5,\ \varepsilon = 1,\quad \sigma = 0.05,\ \mathrm{k} = 0.8,\quad \mu = 0.06,\quad \rho = 0.08,\quad P = 0.4,\ \omega = \pi/2,\ J_0(\alpha^2) = 0.483,\ J_1(\alpha^2) = 0.564.$$

The values of Bessel's functions (zero order and first order) are calculated up to 3 decimals. Figures 2a-2e demonstrate the effects of magnetic field, porosity and height of stenosis on the axial velocity profiles with the variation in different parameters. Figure 2a depicts that increasing values of magnetic number have decreasing effects on the axial velocity while axial velocity decreases with radial distance. It has been noticed that at r=2 there is a stagnation point at which axial velocity becomes zero.

Figure 2b shows that the higher value of magnetic number leads to lower axial velocity. There is a stagnation point at r=2 for t=$\pi/2$. In figure 2c, the pattern is different for axial velocity; there is more unsteadiness in the flow for t=π. Figure 2d is graphical profile for axial velocity versus height of stenosis for different values of porous parameter. It has been observed that the porosity has reciprocal effect on the axial velocity and it decreases smoothly with the increase in the height of stenosis. Figure 2e represents the axial velocity versus radial distance for different values of stenosis height. It is seen that the axial velocity decreases with the increase in the height of the stenosis. It is also noticed from the figure that the axial velocity decreases sharply up to r=0.4, but beyond that the value of the axial velocity remains constant as radial distance increases.

Figures 3a-3d display the graphical representations of wall shear stress versus radial distance and height of stenosis for different parameters. Figure 3a depicts wall shear stress versus radial distance for fixed value of magnetic number at different times; it is evident from figure that the wall shear stress at wall varies with the time upstream. Figure 3b shows that increasing values of magnetic number are responsible for the increase in the shear stress at the wall of the artery.

We observe in figure 3c that as porous parameter increases the shear stress at wall increases. The wall shear stress also increases with the increase in radial distance. Figure 3d represents the wall shear stress versus height of stenosis for different values of length of stenosis. It is noticed from the figure that the wall shear stress is somewhat constant up to r=0.3, but it begins to increase after that. The lower values of the length of stenosis lead to lower value of wall shear stress.

Overall, we conclude that the magnetic field causes stagnation on the blood flow and enhances the unsteadiness of the flow. It also produces remarkable effects on velocity distribution. The axial velocity decreases towards the walls of artery. It is also noticed that the porosity produces positive (negative) effects on the wall shear stress (velocity distribution).

5. CONCLUSION

In this investigation, we have developed a mathematical model for magnetohydrodynamic (MHD) blood flow in a stenoses artery under porous medium by considering the cosine shaped geometry of the stenosis. Our study facilitates analytical expressions for the axial velocity, pressure gradient, volumetric flow rate and resistance to flow. The effects of magnetic field and porosity examined indicate that the height of the stenosis remarkably affects the velocity, pressure and flow rate of the fluid. It has also been established that the magnetic field affects the velocity and pressure gradient. The length of stenosis also positively affects the wall sheer stress. Our investigation may be helpful for the medical practitioners and Bio-mathematicians to understand the flow of blood in the presence of stenosis. The outcomes of investigation done may be useful for the treatment of hypertension patients through magnetic therapy.

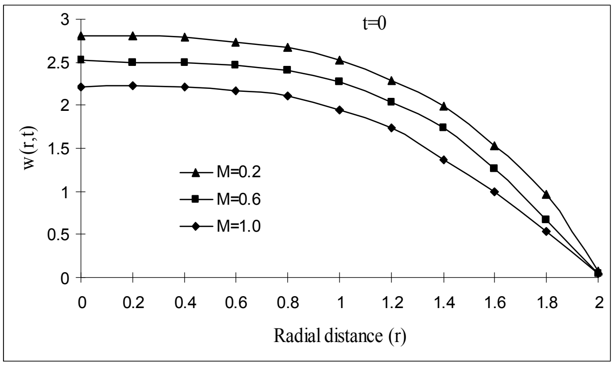

Figure 2a. Effect of magnetic number on axial velocity at t=0 with variation in r.

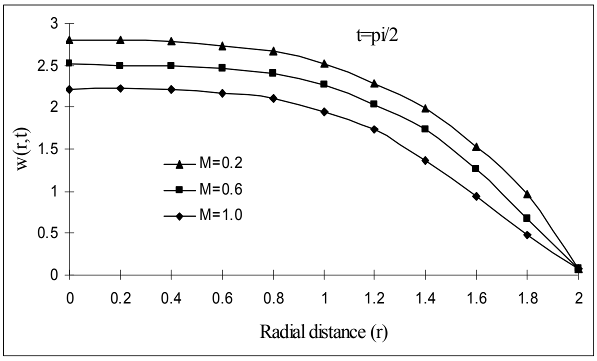

Figure 2b. Effect of magnetic number on axial velocity at t= $\pi/2$ with variation in r.

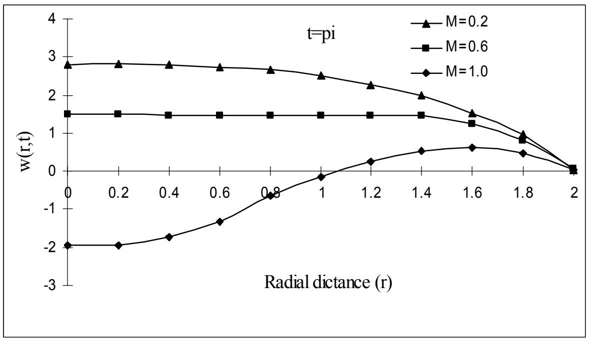

Figure 2c. Effect of magnetic number on axial velocity at t= π with variation in r.

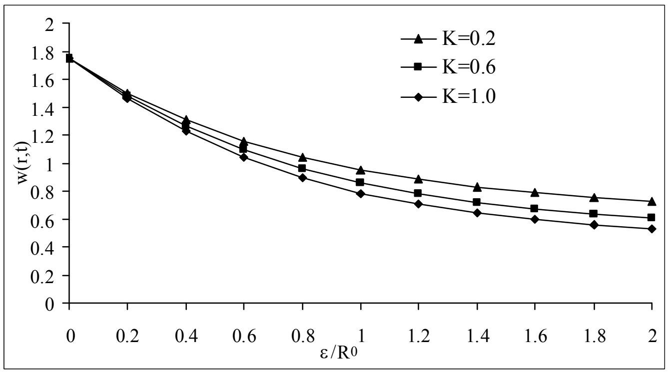

Figure 2d. Effects of porosity on axial velocity with variation in stenosis height ε/R_0.

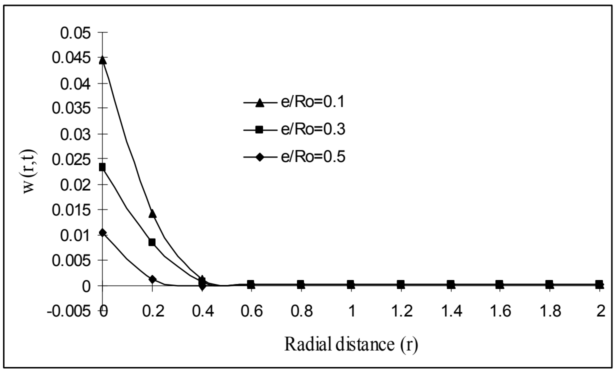

Figure 2e. Effects of stenosis height ε/R_0 on axial velocity with variation in r.

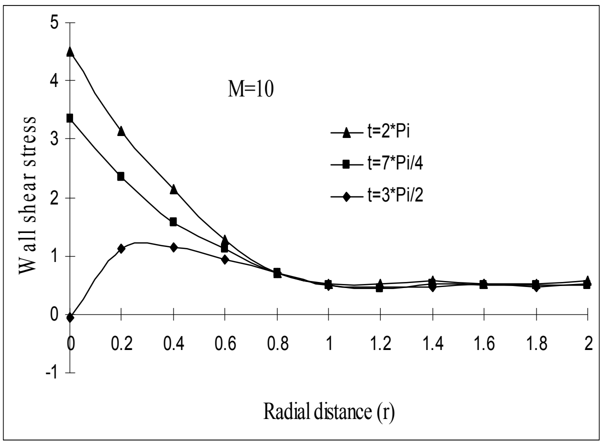

Figure 3a. Effect of magnetic number on wall shear stress for values of different t.

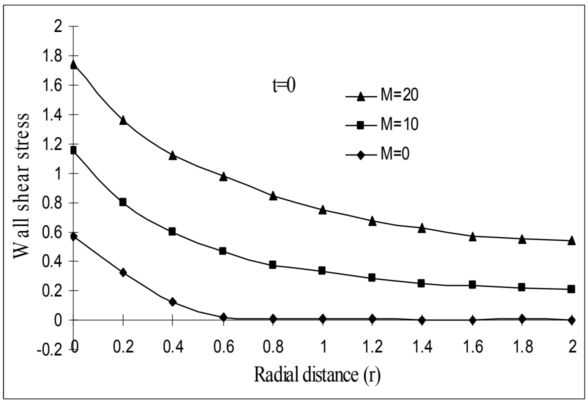

Figure 3b.Effect of magnetic number on wall shear stress for values of different M.

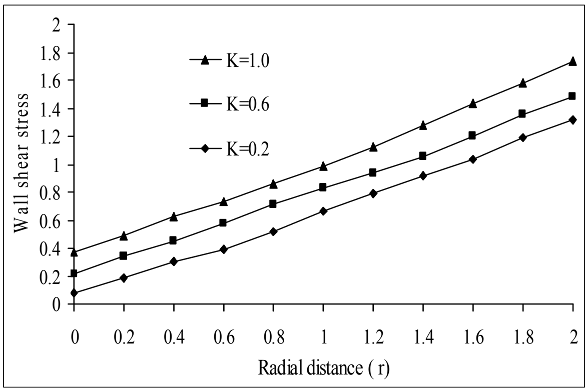

Figure 3c. Effect of porous parameter on wall shear stress with variation in r.

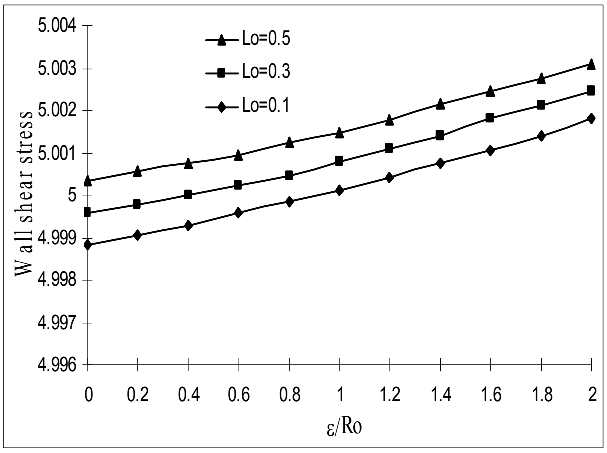

Figure 3d.Effect of stenosis length on wall shear stress with variation in stenosis height.

6. REFERENCES

1. Ahmadi, G. and Manvi, R., "Equation of motion for viscous flow through a rigid porous medium", ***Indian, J. Tech.,*** Vol. 9, (1971), 441-444.
2. Shukla, J. B., Parihar, R. S. and Gupta, S. P., "Biorheological aspects of blood flow through artery with mild stenosis: Effects of peripheral layer", ***Biorh.***, Vol. 17, (1980a), 403-410.
3. Shukla, J. B., Parihar, R. S. and Gupta, S. P., "Effects of peripheral layer viscosity on blood flow through the artery with mild stenosis", ***Bull. Math. Biol.,*** Vol. 42, (1980b), 797-805.
4. Shukla, J. B., Parihar, R. S. and Rao, B. R. P., "Effects of stenosis on non-Newtonian flow of blood in artery", ***Bull. Math. Biol.,*** Vol. 42, (1980c), 283-294.
5. Perkkiö, J. and Keskinen, R., "On the effect of the concentration profile of red cells on blood flow in the artery with stenosis", ***Bull. Math. Biol.,*** Vol. 45, (1983), 259-267.
6. Haldar, K., "Effects of the stenosis on the resistance to blood flow through an artery", ***Bull. Math. Biol.,*** Vol. 47, (1985), 545-550.
7. Misra, J. C. and Singh, S. I., "A study on non-linear flow of blood through arteries", ***Bull. Math. Biol.,*** Vol. 49, (1987), 257-277.
8. Misra, J. C. and Patra, M. K., "A non-newtonian fluid model for blood flow through arteries under stenotic conditions", ***J. Biomech.***, Vol. 26, (1992), 1129-1141.
9. Ghalichi, F., Deng, X., Champlain, A. D., Douville, Y., King, M. and Guidoin, R., "Low Reynolds turbulence modeling of blood flow in arterial stenosis", ***Biorh.***, Vol. 35 (1998), 281-294.
10. Oshima, M., Torii, R., Kobayashi, T., Taniguchi, N. and Takagi, K., " Finite element simulation of blood flow in the cerebral artery", ***Comput. Methods Appl. Mech. Eng.,*** Vol. 191, (2001), 661-671.
11. Varghese, S. S. and Frankel, S. H., "Numerical modeling of pulsatile turbulent flow in stenotic vessels", ***J. Biomech. Eng.***, Vol. 125, (2003), 445-460.
12. Bhuyan, B. C. and Hazarika, G. C., "Effect of magnetic field on pulsatile flow of blood in a porous channel", ***Bio-Sci. Res. Bull.,*** Vol. 17, (2001), 105-112.
13. Mekheimer, Kh. S. and Al-Arabi, T. H., "Nonlinear peristaltic transport of MHD flow through a porous medium", ***Int. J. Math. Math. Sci.,*** Vol. 26, (2003), 1663-1682.
14. Übeyli, E. D. and Güler, İ., "Adaptive neuro-fuzzy inference system for analysis of internal carotid arterial dopple signals", ***Comput. Biol. Med.,*** Vol. 35, (2005), 687-702.
15. Oshima, M. and Torii, R., "Numerical evaluation of elastic models in blood flow-arterial wall interaction", ***Int. J. Comput. Fluid Dyn.***, Vol. 20, (2006), 223-228.

16. Lubbers, J., Vries, M. P. D., Veldman, G. J. V., "Influence of a downstream narrowing on the flow profile in a tube", ***J. Biomech***., Vol. 39, (2006), 70-77.
17. Abbas, Z., Sajid, M. and Hayat, T., "MHD boundary-layer flow of an upper-convected Maxwell fluid in a porous channel", ***Theor, Comput. Fluid Dyn.***, Vol. 20, (2006), 229-238.
18. Misra, B. K. and Verma, N., "Effect of porous parameter and stenosis on the wall sheer stress for the flow of blood in human body", ***Res. J. Med. Med. Sci.,*** Vol. 2, (2007), 98-101.
19. Chapman, J. S., Shipley, R. J. and Jawad, R., " Multiscale modeling of fluid transport in tumor", ***Bull. Math. Biol.,*** Vol. 70, (2008), 2334-2357.
20. Weinberg, E. J. and Mofrad, M. R. K., "A multiscale computational comparison of the bicuspid and tricuspid aortic valves in relation to calcific aoretic stenosis", ***J. Biomech.,*** Vol. 41, (2008), 3482-3487.
21. Huang, J., Lyczkowski, R. W. and Gidaspow, D., "Pulsatile flow in a coronary artery using multiphase kinetic theory", ***J. Biomech***., Vol. 42, (2009), 743-754.
22. Jain, M., Sharma, G. C. and Sharma, S. K., "A mathematical model for blood flow through narrow vessels with mild stenosis", ***Int. J. Engg., Sci.,*** Vol. 22, (2009), 99-106.
23. Jain, M., Sharma, G. C. and Singh, A., "Mathematical analysis of MHD flow of blood in very narrow capillaries, International", ***Journal of Engineering, Trans. B: Application,*** Vol. 22, (2009), 307-315.
24. Rathod, V.P. and Tanveer, S., "Pulsatile of couple stress fluid through a porous medium with a periodic body acceleration and magnetic field", ***Bull. Malay. Math. Sci. Soc.,*** Vol. 32, (2009), 245-259.
25. Young, D. F., "Fluid mechanics of arterial stenosis", ***J. Biomech. Eng. Trans. ASME***", Vol. 101, (1968), 157-175.