EXPERIMENTAL STUDY AND MODELING OF REINFORCED CONCRETE BEAMS STRENGTHENED BY POST-TENSIONED EXTERNAL REINFORCING BARS

M. Naghipour*, M. Nemati

Babol University of Technology, Department of civil Engineering, Babol, Iran
m-naghi@nit.ac.ir , marzieh_nemati62@yahoo.com

H. M. Doostdar

National Research Institute for Research Policy
Hossein1949@gmail.com

*Corresponding Author

(Received: August 11, 2009 – Accepted in Revised Form: March 11, 2010)

Abstract The utilization of unbounded external reinforcing bars is one of the strengthening methods used after loading stage and before failure. The method has been used in different forms to strengthen members of reinforced concrete structures. To investigate the effect of utilization of post-tensioned reinforcing bars in this method of strengthening, a number of reinforced concrete beams was tested. Strengthening was carried out by attaching external bars on both outside faces of the beam in the level of internal flexural tension reinforcement. The behavior of strengthened beams was then studied both by experiment and modeling using ANSYS finite element structural software. In post-tensioning of external reinforcing bars, hydraulic jack was used. The results showed that this method of strengthening has increased the flexural capacity, and decreased the ductility of the beams. It was also shown that the increase in flexural strength caused by the utilization of unbounded external post-tensioned reinforcing bars was in reverse proportion with the percentage of internal flexural tension reinforcement. It was also concluded that the method is very effective for beams with lower percentages of internal flexural tension reinforcement.

Key words: Strengthening, Reinforced concrete beam, Post-tensioning, External reinforcing bar, Nonlinear finite element analysis, ANSYS.

چکیده استفاده از آرماتورهای خارجی یکی از روش‌های مقاوم‌سازی در مرحله بعد از بارگذاری و قبل از شکست می‌باشد که تاکنون به طرق مختلف کار مقاوم‌سازی با استفاده از آرماتورهای خارجی صورت گرفته است. به منظور بررسی تاثیر استفاده از آرماتور خارجی پس‌کشیده در این روش مقاوم‌سازی، تعدادی تیر بتن‌آرمه مورد بررسی قرار گرفتند. مقاوم‌سازی توسط نصب آرماتور خارجی در طرفین تیر در ترازی برابر با آرماتور کششی صورت گرفته است. بررسی در دو فاز آزمایشگاهی و مدل‌سازی به وسیله نرم‌افزار صورت گرفته است. که در بررسی آزمایشگاهی، برای ایجاد پس‌کشیدگی در آرماتور خارجی از جک هیدرولیکی استفاده شده است. نتایج نشان می‌دهد که‌این روش مقاوم‌سازی باعث افزایش ظرفیت خمشی و کاهش شکل‌پذیری می‌شود و افزایش مقاومت خمشی در اثر استفاده از آرماتور غیرچسبیده پس‌کشیده خارجی با درصد فولاد کششی نسبت معکوس دارد و برای تیرهایی که درصد فولاد کششی کمی دارند، روش بسیار موثری می‌باشد.

1. INTRODUCTION

Different causes like design errors, change of application, lack of proper construction practices, and damages due to aging, environmental effect, war and/or earthquakes may require strengthening of structures during their life time span. However, because of economic and cultural considerations, strengthening of structural members has advantages over substitution or reconstruction of these members. Strengthening of buildings include strengthening of columns, beams, wall joints, and structural frames.

One of the strengthening methods used for reinforced concrete beams, is the utilization of external reinforcing bars. In this method, external reinforcing bars are attached on both outside faces of the beam in the level of internal flexural tension reinforcement. External reinforcing bars are secured to the beam by means of u-shaped steel connectors, deflectors, at specific locations along the beam length, to achieve the approximate flexural displacement profile (deflections) of external reinforcing bars and the beam. The idea of the utilization of external reinforcing bars for strengthening reinforced concrete beams was first introduced by Farooq in 1997 [1]. He studied the behavior of 30 full scale simply supported strengthened reinforced concrete beams using unbounded external reinforcing bars loaded to failure. The behavior of strengthened beams were related to different parameters, namely the percentages of internal flexural tension and external strengthening reinforcing bars, effective depth of external strengthening reinforcing bars, the usage of deflectors, 28 day cylindrical compressive strength of concrete, shear span, and beam span. The results of the experiments show that strengthening by means of unbonded external reinforcing bars, increases ultimate flexural strength of reinforced concrete beams.

In 2001, Mohammad Abdollahi [2] studied experimental results of Farooq. He used ANSYS finite element structural software to model reinforced concrete beams strengthened by unbounded external reinforcing bars, and used nonlinear analysis to calculate ultimate capacity and ductility of the beams. In 2002, Reza Erfanain [3], conducted an experimental study on the behavior of reinforced concrete beams strengthened by external reinforcing bars at the university of Mazandaran. In Farooq's experiment the external unbounded reinforcing bars were attached on both sides of the beams at the level of internal flexural tensile reinforcement, whereas in Erfanian's experiment external bars were located at the bottom of the beams. In the latter method the results showed a marked increase in flexural capacity and decrease in ductility of the beam.

In 2003, Reza Fooladvand [4], carried out an experimental investigation on the effect of this method of strengthening on cracked pre-loaded simply supported reinforced concrete beams. He observed that this method of strengthening, even in the presence of pre-loading and dead load at the time of strengthening, is effective in increasing the strength of the beams.

External post tensioning is defined as post tensioning that are created by cables that were installed from outside in a large length of the structure and is not jointed to the structure except in deflectors and ends of structure. The history of external post tensioning belongs to the Second World War but it was applicable as a comprehensive method in France. [5]

In 1969 Pannell studied the behavior of 38 concrete beams that were post tensioned with unbounded cables and showed that beams with large initial force in the steel, act the same as bonded post tensioned beams and have a special ductility with series of cracks in tension region, but in the beams with small initial force, only two or three deep cracks were established and by increasing the load, the width of only one crack were increased [6].

Mattock et al investigated the behavior of seven

Table 1. Specifications of rebars used in modeled beams

Bar name	Diameter *kg*	Tensile yield strength *kg/cm*^{2}	$E \times 10^5$ *Mpa*
Tension bar	12	3263	2
	18	4859	2
Compression bar & Deflector	10	5092	2
Shear tie	8	5092	2
External bar	18	4859	2

simply supported and three continued beams that completely loaded with post tensioned reinforcement and extra bonded ordinary reinforcements and concluded that their ductility and strength are the same or better than the bonded post tensioned beams. [7].

Harjli and Kanj represented the results of a comprehensive research on the ultimate stress of post tensioned steel for beams with unbounded cables. It was shown that when post tensioned beams were under two point single loads, some cracks were extended in the bending span but in the higher loads only one or two cracks were specially extended and entered in the compression region. [8]

The applicability of external post tensioned cables for strengthening of unbended concrete beams was experimentally investigated by Harjli [8]. It was shown that external post tensioning is a suitable way for strengthening of concrete members and depends on amount of external post tensioning and amount of tension reinforcements, the strength increases from 9 to 143 percent [5].

The results of experiment and from the analysis of modeled beams using ANSYS showed that the flexural capacity of beams was increased, the amount of strengthening was in reverse proportion with the amount of internal flexural tension reinforcement, and the deformability of strengthened beams was in reverse proportion with the percentage of dead load and extent of cracks in beams. It was concluded that an increase in the post-tensioning force applied to external strengthening reinforcing bars results in an increase in flexural capacity of beams. Moreover, it was concluded that this method of strengthening is a proper method of strengthening reinforced concrete beams. The advantages of the method are; speedy application, simplicity of employment, almost no increase in weight of the structure, and economic advantages.

2. ANALYTICAL EXPRESSIONS GOVERNING THE STRUCTURAL BEHAVIOR OF STRENGTHENED BEAMS

When an unbounded post-tensioned reinforcing external bar are used to strengthen reinforced concrete beams, a primary stress resulting from the attachment of these bars is induced. These external bars are also under stress after loading, due to the deflection of the beam [9].

Since the external strengthening reinforcing bars are unbounded, there is no interaction between reinforcement and concrete, and therefore, the existing assumptions for the behavior of reinforced concrete sections are not applicable. Therefore, the stress in the external strengthening bars is related to total deformation of the member. To calculate the stress in the external strengthening bars on the basis of total deformation of the member, the analysis of the base element is necessary [10]. To simplify the analysis the following assumptions are made: The behavior of concrete in compression is linear elastic and the tensile strength of concrete is negligible.

The equilibrium condition for loads and compatibility condition for deformations should also be satisfied. To satisfy the equilibrium condition, the net force in any section of the beam under pure flexure should be zero. This is shown by equation (1). The internal forces should also be in equilibrium with applied flexural moment (equation (2)). In addition, the compatibility of deformations along the beam length should be satisfied according to equation (3). From equilibrium of loads and moments and compatibility of deformations along the beam length we have:

$$A_{sb}f_{sb}+A_{sub}f_{sub}+12f_cbx=0 \quad (1)$$

$$A_{sb}f_{sb}+A_{sub}f_{sub}=M \quad (2)$$

$$\int_0^1 \varepsilon_{sb}dl=\int_0^1 \varepsilon_{sub}dl=\int_0^1 \varepsilon_c dl \quad (3)$$

Where:

f_{sub}, f_{sb} = stress in unbounded and bonded reinforcement respectively (N/mm^2),

A_{sub}, A_{sb} = cross-sectional area of unbounded and bonded reinforcement respectively (mm^2)،

f_c = cylindrical compressive strength of concrete (N/mm^2)،

x = the distance of the extreme compressive fiber of concrete from neutral axis of the beam (mm).

z = the internal level arm (mm).

M= flexural moment in specified section (N-mm).

$\varepsilon_c, \varepsilon_{sub}, \varepsilon_{sb}$ = strains in concrete at the level of reinforcement, in unbounded reinforcement, and in bonded reinforcement, respectively

L = length of beam (mm) [10].

Finally from the above three equations stress in bonded reinforcement at service load, the amount of deflection at service load, ultimate stress in unbounded reinforcement, and ultimate flexural moment of the section can be calculated [10]. The algorithm of the calculation of ultimate load and deflection at service load in these beams can also be found in reference [1].

Ultimate flexural capacity of section can be calculated from the following expression [9]:

$$M_u = \acute{M}_u f_{cu} b d^2 \tag{4}$$

Where:

$$\acute{M}_u = Q_{ub}\left[\frac{d_{ub}}{d} - \frac{K_2}{K_1}(Q_{ub} + 2Q_b - 2Q_c)\right] + Q_b\left[1 - \frac{K_2}{K_1}(Q_b - 2Q_c)\right] - Q_c\left[\frac{K_2}{K_1}Q_c + \frac{\acute{d}}{d}\right] \tag{5}$$

$$Q_c = \rho_c \cdot f_{yc}/f_{cu} \tag{6}$$

$$Q_b = \rho_b \cdot f_{yb}/f_{cu} \tag{7}$$

$$Q_{ub} = \frac{Q_t + K_3}{\left(1 + \frac{K_2}{K_1}\cdot\frac{d}{d_{ub}}\right)} - \frac{K_3 d Q_b}{K_3 d_{ub} + K_3 d} + \frac{K_3 d Q_c}{K_3 d_{ub} + K_3 d} \tag{8}$$

$$K_3 = \frac{E_s \beta \varepsilon_c \rho_{ub} d_{ub}}{L f_{cu}} \tag{9}$$

$$Q_t = \rho_{ub} \cdot f_{st}/f_{cu} \tag{10}$$

$$\beta = \frac{f_{sub} - f_{st}}{\frac{\varepsilon_c E_s d_{ub}}{L}\left[1 - \frac{d}{d_{ub} K_3 f_{cu}}(\rho_{ub} f_{sub} + \rho_b f_{yb} - \rho_c f_{yc})\right]} \tag{11}$$

K1 and K2 can be determined from both BS8110 and ACI and the parameters used in the above equations can be found in reference [9].

Knowing Mu, the values of Pu and Δ can be determined from the following expressions.

$$P_u = \frac{2M_u}{a_v} \tag{12}$$

$$\Delta = \frac{k l_1^2 M_u}{E_c I} \tag{13}$$

The above analytical expressions were not used in our research for estimation of failure load .The derived formulations may need to develop for nonlinear structures subjected in failure load.

3. EXPERIMENTAL SPECIMEN AND PROCEDURE

12 beams with 1.8m span length were designed and fabricated for loading and recording the test results, in order to study the strength and ductility of reinforced concrete beams strengthened by external unbounded post-tensioned reinforcing bars. The beams were grouped into two six beam groups, according to percentages of their internal flexural reinforcing bars. The value of the percentage of tension reinforcement, ρ, was 0.014 in group one and 0.0318 in group two. All beams were reinforced by shear ties to prevent shear failure. The result of tensile strength test of reinforcement which was used in modeled beams was shown in the table (1). In each group one beam was used as a reference beam without external reinforcing bars, one was with external strengthening reinforcing bar without post-tensioning, and all the remaining four beams were with external strengthening reinforcing bars post-tensioned to different extent.

All beams had two number 10 internal compression reinforcing bars [5]. Details of cross sections and reinforcement of strengthened beams are shown in figure (1).

Strengthening was exercised by installing external reinforcing bars threaded at both ends, on outside faces of beams at the level of internal flexural tension reinforcement. Two end steel boxes separated from beams, were used to hold external bars in place. Since the external bars were only attached to the beam at ends by means of these boxes, to ensure the approximate similar deflection profile of beam and external reinforcing bars along the beam span, four deflectors located at specific locations, as shown in figure (2), were used. All beams had simple supports at ends, and were loaded at two points to failure [9] as shown. Figure (3) shows the details of deflectors.

The external reinforcing bars were post-tensioned using hydraulic jack, and the compressive force was applied to beam ends via end boxes [9]. These boxes were located at the lower half of the end beam section (figure (4)). In this type of strengthening after loading the induced deflection in the beam will cause the external bars to experience tensile strains and therefore, an additional compressive force to be applied to the ends of the beam via end boxes.

4. NUMERICAL MODELING OF STRENGTHENED BEAMS

The ANSYS finite element structural software can be used to conduct both simple analysis, like linear or elastic analysis, and more complex analysis, like nonlinear or dynamic analysis. Because of the applicability of the software to different engineering branches, and in order to increase the speed of the process and reduce the space needed, the program is divided into groups and subgroups with their own finite elements, specifications and instructions. This software, like other similar software, has three major sections: 1 – construction of the model, 2 – loading and analysis, and 3 – observing the results [11].

The most essential part of the model construction is the selection of proper elements. This program has180 elements each with a certain specifications, and therefore, the selection of the element with needed specifications can be done rather easily. For loading and analysis parts, analysis type, loading cases and the conditions for analysis should be entered. The type of analysis is dependent upon loading and considered response. This program includes static, modal, harmonic, transient, spectrum, semi-structural, and flexural analysis. The results of analysis can be observed in two ways. One choice is to see the results of the whole model or part of it in the form of deformation of the model, table and/or colored curve, and the other choice is to obtain the results corresponding to a specific point in the model in the form of curves with respect to tables [12].

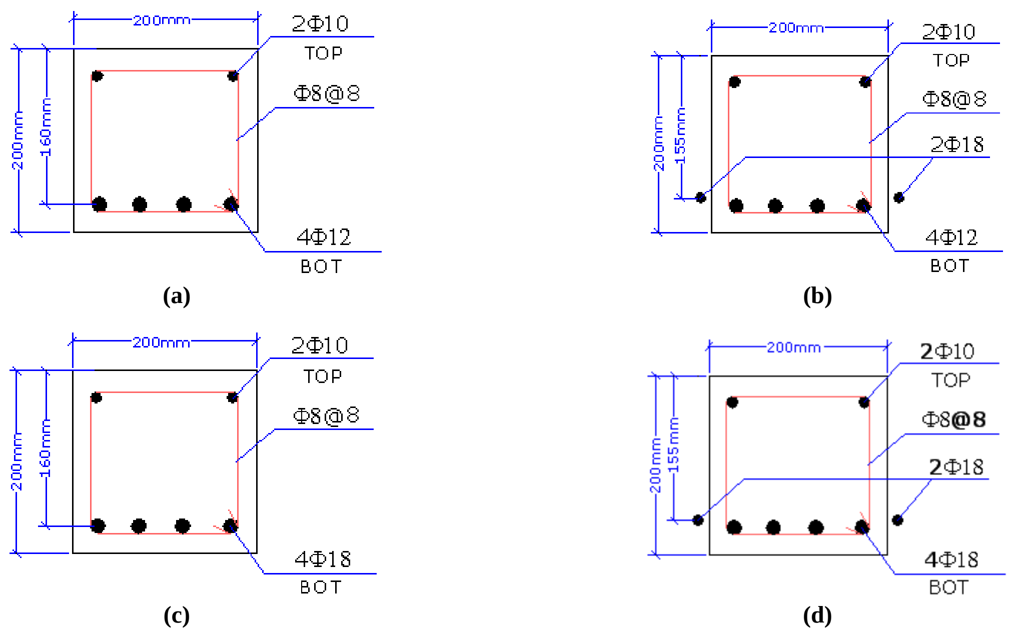


Figure 1. Cross sections of test beams, (a) beam A1;(b) beams A2 to A6;(c) beams B1; (d) beams B2 to B6

Table 2. Specifications of group one beams

Beam name	Internal tension bar Number	Internal tension bar Diameter	ρ_{min}	ρ_{max}	28 day cylindrical compressive strength of concrete *kg/cm²*	Tensile strength of concrete *kg/cm²*	Post-tensioning load *kg*	Comments
A_1	4	12	0.0045	0.0393	345	30	*0*	Reference
A_2	4	12	0.0046	0.04	360	33	*0*	strengthened by external reinforcing bar
A_3	4	12	0.0049	0.0431	400	35	*3460*	strengthened by post-tensioned external bar
A_4	4	12	0.0044	0.0381	328	32	*4580*	strengthened by post-tensioned external bar
A_5	4	12	0.00503	0.0445	423	37	*5100*	strengthened by post-tensioned external bar
A_6	4	12	0.00493	0.0435	406	36	*6500*	strengthened by post-tensioned external bar

Table 3. Specifications of group two beams

Beam name	Internal tension bar Number	Internal tension bar Diameter	ρ_{min}	ρ_{max}	28 day cylindrical compressive strength of concrete *kg/cm²*	Tensile strength of concrete *kg/cm²*	Post-tensioning load *kg*	Comments
B_1	4	18	0.00307	0.0231	350	33	*0*	Reference
B_2	4	18	0.00296	0.0219	324	32	*0*	strengthened by external reinforcing bar
B_3	4	18	0.00308	0.0231	352	34	*3257*	strengthened by post-tensioned external bar
B_4	4	18	0.0032	0.0243	380	34.5	*4580*	strengthened by post-tensioned external bar
B_5	4	18	0.0032	0.0244	382	35	*5089*	strengthened by post-tensioned external bar
B_6	4	18	.00324	0.0246	388	36	*6328*	strengthened by post-tensioned external bar

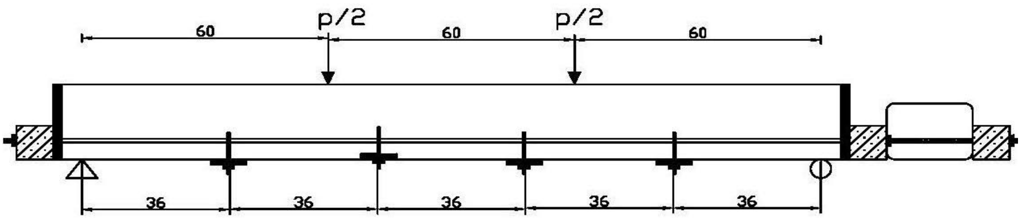
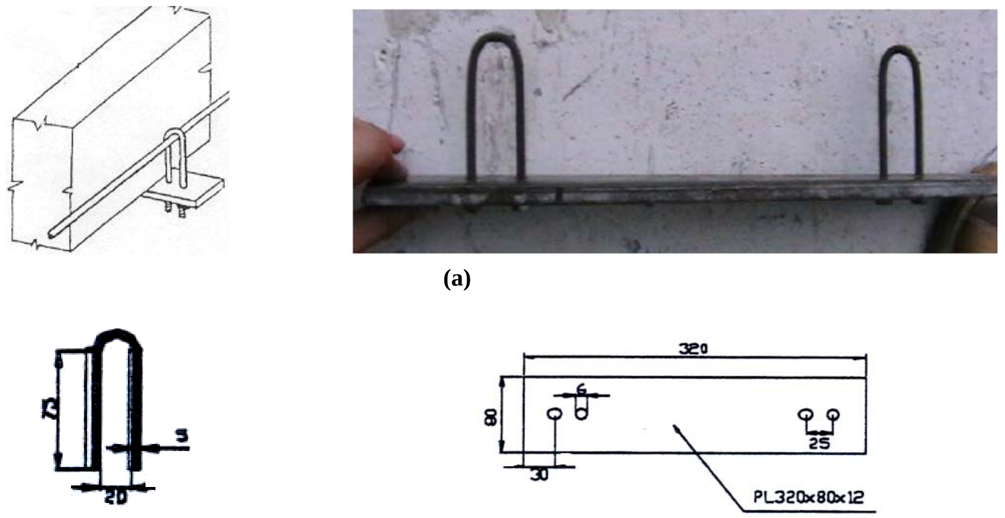


Figure 2. The position of deflectors of external reinforcing bars and the position of applied loads



(a)

(b) (c)

Figure 3. Details of deflectors; (a) Used in the lab; (b) Deflector; (c) MS strip

Figure 4. Post-tensioning using hydraulic jack


Figure5. Loading the beam

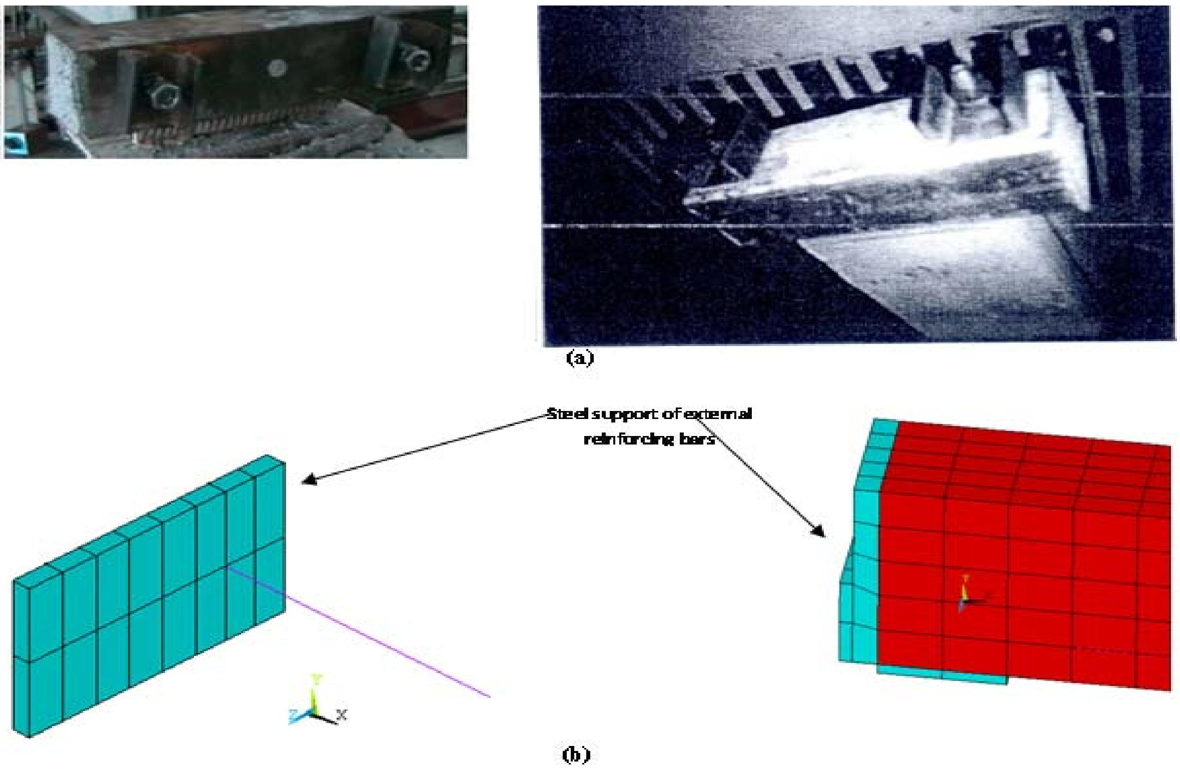


Figure 6. Details of steel support of external reinforcing bar; (a) in test beam; (b) in the model

- Modeling

In this section the geometry of the model is drawn and the types of elements are determined. A reinforced concrete beam strengthened with external reinforcement includes different elements for which the type of element, material properties, real constants, should be defined. The constituent elements of strengthened reinforced concrete beam, using external reinforcing bars are: [13]

Steel support of external reinforcing bars, End plate, Steel support, MS strip, steel loading plate, Concrete beam, Compression reinforcement, Tension reinforcement, External reinforcing bars, Shear reinforcement, Deflectors.

Steel support of external reinforcing bars is a steel box filed with concrete for more strength, and bolts are used to anchor the external bars to the beam in the lab. For modeling of this set in Ansys, a rectangular steel plate is used which plays a role of anchorage for external reinforcing bar.

Details of this anchorage are shown in figure (6) for two conditions.

End plate is a steel plate with cross-section equal to that of beam cross-section, and the thickness of 2 mm, as shown in figure (7), which was used to prevent stress concentration at both ends of the beam, caused by external reinforcement attachment.

At the supports, the beam was placed on the steel plates to prevent stress concentration we call these plates steel support.

Also deflectors were used to achieve the same deflection profile in the beam and external reinforcing bar. For external reinforcing bars to attain the same deflection profile of the beam under the load, deflectors as shown in figure (3) were used in the lab. As shown in figure (3), a deflector was composed of two parts, a plate which is located under the beam at specific locations, and a u-shaped bar which is hanged on the external reinforcement and attached to the plate. These are called deflectors. To model the deflectors a plate and two reinforcing bars were used. These bars which were used to model the connector bars are called deflectors. Modeling of steel plate and its attachment to external reinforcing bars is shown infigure (8).

To prevent concrete crack in the vicinity of the applied load, a load was placed on the steel plate which prevents stress concentration at that point and we call that steel loading plate. External reinforcing bars are strengthening reinforcing bars with threads at the ends which were attached to both sides of the beam, by means of deflectors, at the level of internal flexural reinforcement. To prevent shear failure of the beams and to insure flexural failure, all beams were reinforced by shear ties. All the details are shown in figure (9)

vicinity of the applied load, a load was placed on the steel plate which prevents stress concentration at that point and we call that steel loading plate. External reinforcing bars are strengthening reinforcing bars with threads at the ends which were attached to both sides of the beam, by means of deflectors, at the level of internal flexural reinforcement. To prevent shear failure of the beams and to insure flexural failure, all beams were reinforced by shear ties. All the details are shown in figure (9). The type of element, material specification, and real constants of constituent elements of the strengthened reinforced concrete beam are shown in table (4). The element Solid65 which is used to model concrete, is a three dimensional element which is capable of cracking in tension and crushing in compression. According to the reference [14] this element requires linear isotropic and multilinear isotropic material properties to properly model concrete. The multilinear isotropic material uses the Von Mises failure criterion along with the William and Warnke (1974) model to define the failure of the concrete. Implementation of the William and Warnke (1974) material model in ANSYS requires that different constants be defined. These 9 constants are:

1. Shear transfer coefficients for open crack;
2. Shear transfer coefficients for closed crack;
3. Uniaxial tensile cracking stress;
4. Uniaxial crushing stress;
5. Biaxial crushing stress;
6. Ambient hydrostatic stress state for use with constant 7 and 8;
7. Biaxial crushing stress under the ambient hydrostatic stress state;
8. Uniaxial crushing stress under the ambient hydrostatic stress state;
9. Stiffness multiplier for cracked tensile condition.

Typical shear transfer coefficients range from 00 to 10 which representing a smooth crack and rough

crack respectively. And as recommended by [11], for open crack its range is from 0.15 to 0.3 and for closed crack is from 0.7 to 1.00. The uniaxial cracking stress was based upon the modulus of rupture. Also the uniaxial crushing stress in this model was based on the uniaxial unconfined compressive
strength
fc
and is denoted as ft. It was entered as -1 to turn of the crushing capability of the concrete element as suggested by past researchers [15]. Convergence problems have been repeated when the crushing capability was turned on. Other coefficients entered 0 as suggested by reference [12].

In this element the reinforcement can be modeled by one dimensional rod with compressive and tensile behavior. This bar can be introduced in the middle of the element in all three directions [16]. As recommended by reference [11], Solid65 with zero percent reinforcement is used to model reinforced concrete beam. Tension and compression reinforcement and shear ties in the beam are separately modeled using element Link8. To ensure the necessary conditions for transfer of load Link8 element should be located between two or more elements modeled by Solid65, otherwise, the forces would be only transferred at the nodes of Solid65 and the element Link8 is not effective in representing a reinforcing bar, as if it does not exist. [11]

On the other hand since all the beams considered are symmetrical in two directions, as it is recommended in reference (3), only a quarter of the beam is modeled due to symmetry. This would save both process time and the memory capacity.

A modeled beam is shown in figure (10).

- Support conditions and loading

To ensure that modeled beam behaves as test

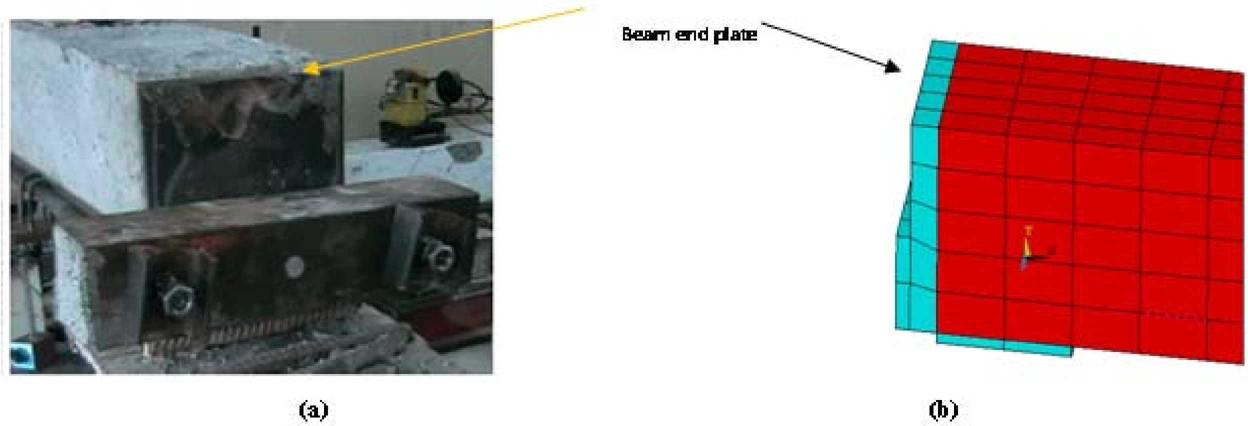


Figure7. Beam end plates; (a) in test beams; (b) in the model

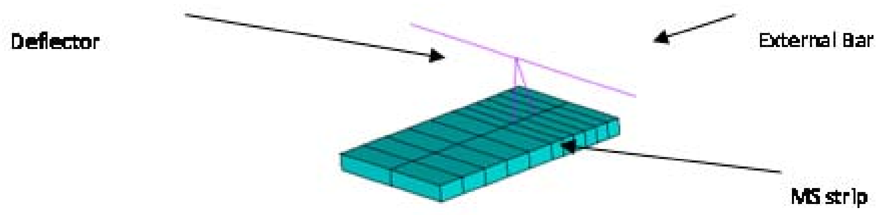


Figure8. ANSYS model of deflector

beam, boundary conditions at points of symmetry, supports and load points should be satisfied.
Boundary conditions for symmetry will be set first. The constructed model is symmetrical in two directions. To model the symmetry, points in the plane of symmetry should be constraint in the perpendicular direction. [12] The boundary conditions for both planes of symmetry are shown in figure (11-a).
Considering the connection between MS plate and concrete in the lab, it is seen that there was no connection between deflectors and concrete and they were just touched with each other so for modeling in ANSYS, firstly we created two volumes that have a common surface, then by using glue command, the connection was created. So no friction element was used.
To model the connection between external reinforcing bar and deflector, we consider a common node, since this node should only be displaced in vertical direction, therefore, all of the connecting nodes should be constraint in X and Z directions as shown in figure (11-b).
The support was modeled in such a way that a roller was created. A single line of nodes on the plate were given constraint in the UY, and UZ directions, and applied as constant values of 0. By doing this, the beam will be allowed to rotate at the support. The support condition is shown in figure (12-a).
The force, P, applied at the steel plate is applied across the entire centerline of the plate. Figure (12-b) illustrates the plate and applied loading.
The load is applied in one or two steps corresponding to the test beam type. The required number of steps of loading for each test beam is shown in table (5). Figure (13) shows the way the loads are applied.

- Analysis

To analyze the constructed models, static analysis is used and since the materials behavior is nonlinear, the nonlinear elastic analysis is carried out. Analysis is also carried out on the basis of small displacements. Finite element analysis is organized in a way that three different behaviors of material, primary crack of the beam, yielding of the internal flexural reinforcement, and ultimate capacity of the beam, can be determined. Newton-Raphson method is used to consider nonlinear response [16]. Failure of the beam occurs when convergence fails with this very small load increment. The load incremental trace produced by the analysis, confirms the failure load.

5. TEST AND ANALYTICAL RESULTS OF BEAMS

To verify the validity of modeling and analysis, analytical results obtained using ANSYS were compared to those obtained by test.

- Study of the strength of specimen

Table (6) and Table (7) also shows beam ultimate load for two groups of beams. It should be noted that during the test of beam number B6 in group two, the threads at one end of one of the external post-tensioned strengthening reinforcing bars was partly destroyed. The test was stopped and the recording of the ultimate load was not possible. The results of the analysis of this beam using ANSYS is available.

- Study of beam deflection and ductility

Figure (14) shows the load deflection diagrams for group A beams, and figure (15) shows the load deflection diagrams for group B beams. As it is apparent from above diagrams, the results from experiments show strengthening of reinforced concrete beams, using post-tensioned strengthening unbounded external reinforcing bars, increases the flexural rigidity, and therefore, reduces the deflections of beams. If the deflections of beams are compared for a constant beam load, it can be seen that the amount of deflection is in reverse proportion with the increase in post-tensioning load. From the study of the diagrams of modeled beams, it can be concluded that the utilization of this method of strengthening does not have a considerable effect on the increase in the rigidity of the beam, specially because the rigidity of all beams are approximately equal before the yielding of tension reinforcement. Therefore a meaningful relationship cannot be found between the beam deflection and post-tensioning load.

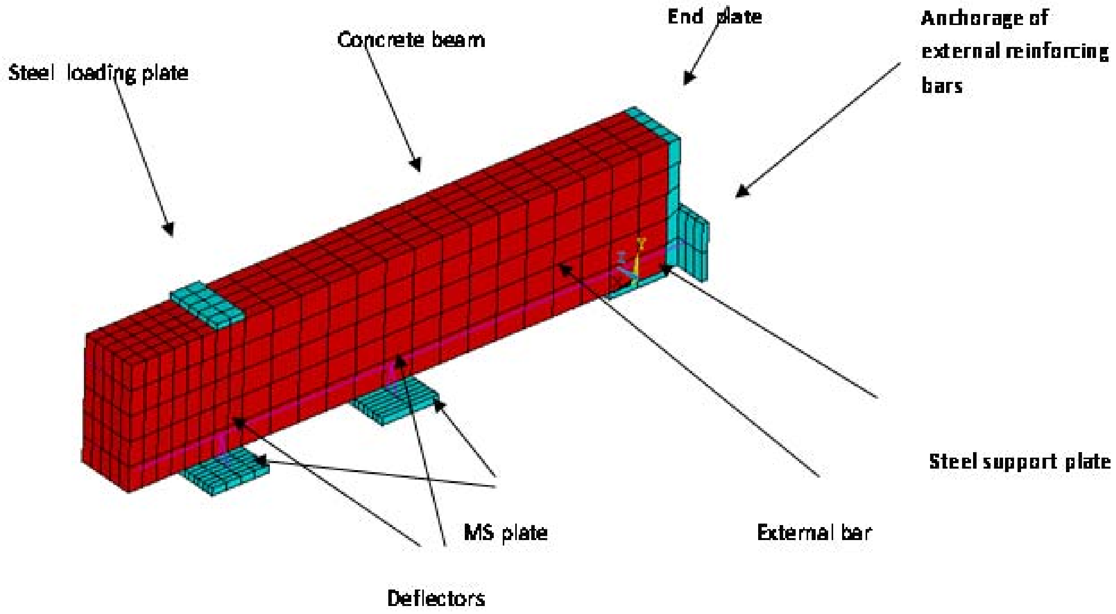


Figure9. Constituent elements of strengthened reinforced concrete beam in ANSYS

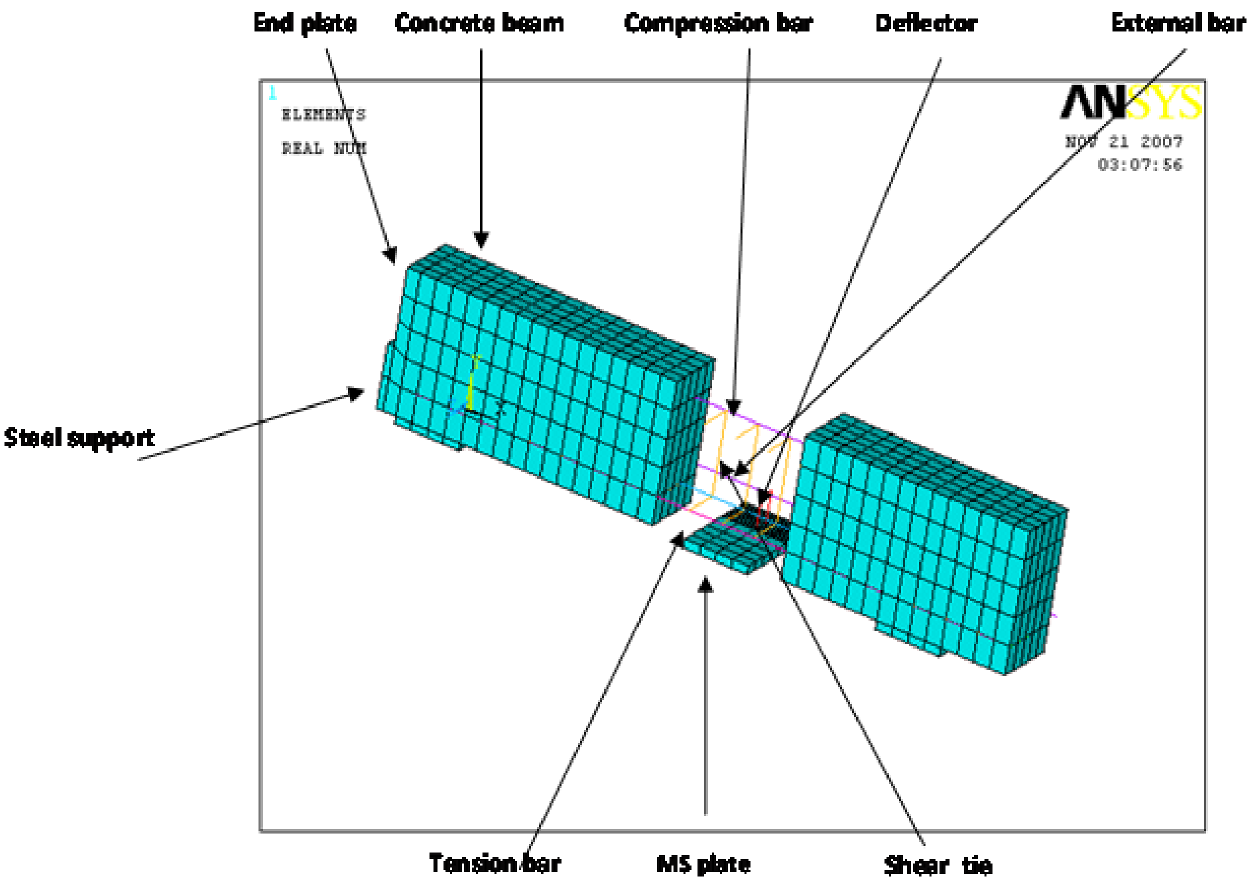


Figure 10. ANSYS beam model

Table 4. The type of element, material properties, and real constants of constituent elements of the strengthened reinforced concrete beam

Model parts	Element type	Material properties			Real constant	
Steal support of EX-bar	Solid45	Linear isotropic			-	
End plate	Soli`d45	Linear isotropic			-	
Steal support	Solid45	Linear isotropic			-	
MS plate	Solid45	Linear isotropic			-	
Steal loading plate	Solid45	Linear isotropic			-	
Concrete beam	Solid 65	Linear isotropic	, Concrete	, Multilinear isotropic	Properties of bar exist in solid65 element	
Compression bar	Link8	Linear isotropic	, Bilinear isotropic		Cross-sectional area	,initial strain
Tension bar	Link8	Linear isotropic	, Bilinear isotropic		Cross-sectional area	,initial strain
External bar	Link8	Linear isotropic	, Bilinear isotropic		Cross-sectional area	,initial strain
Shear tie	Link8	Linear isotropic	, Bilinear isotropic		Cross-sectional area	,initial strain
Deflector	Link8	Linear isotropic	, Bilinear isotropic		Cross-sectional area	,initial strain

Table 5. Number of steps of loading in each beam

Number of steps of loading	**One step loading**	**Two step loading**
Name of beam	Reference beam A_1 , B_1	Strengthened beam $A_2,\ldots,A_6$ $B_2,\ldots,B_6$

Table 6. Ultimate load for group A beams

Number of beam	**Ultimate load** **Exp** KN	**Ansys** KN
A_1	67.14	74.59
A_2	144.38	170.1
A_3	158.6	181.42
A_4	162	169.74
A_5	189.56	187.49
A_6	200	198.147

Table 7. Ultimate load for group B beams

Number of beam	**Ultimate load** **Exp** KN	**ANSYS** KN
B_1	210.44	214.21
B_2	246.22	261.49
B_3	255.48	281.16
B_4	262.93	284.25
B_5	274.85	282.19
B_6	-	281.94

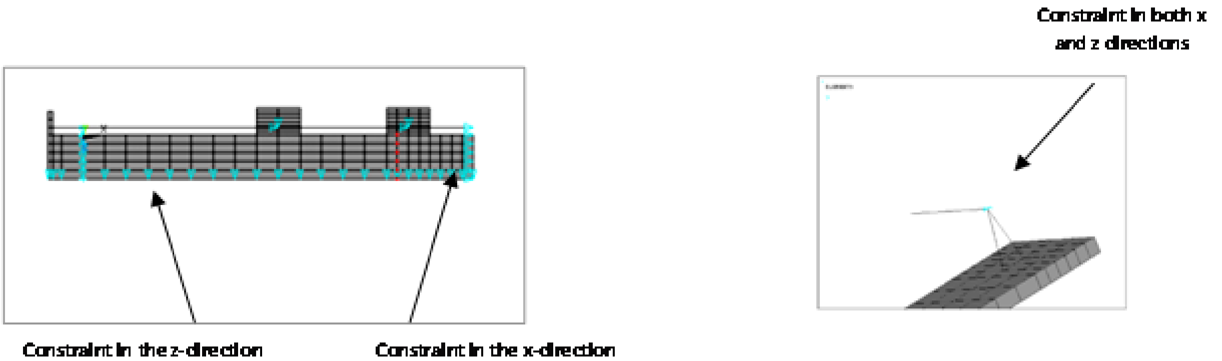
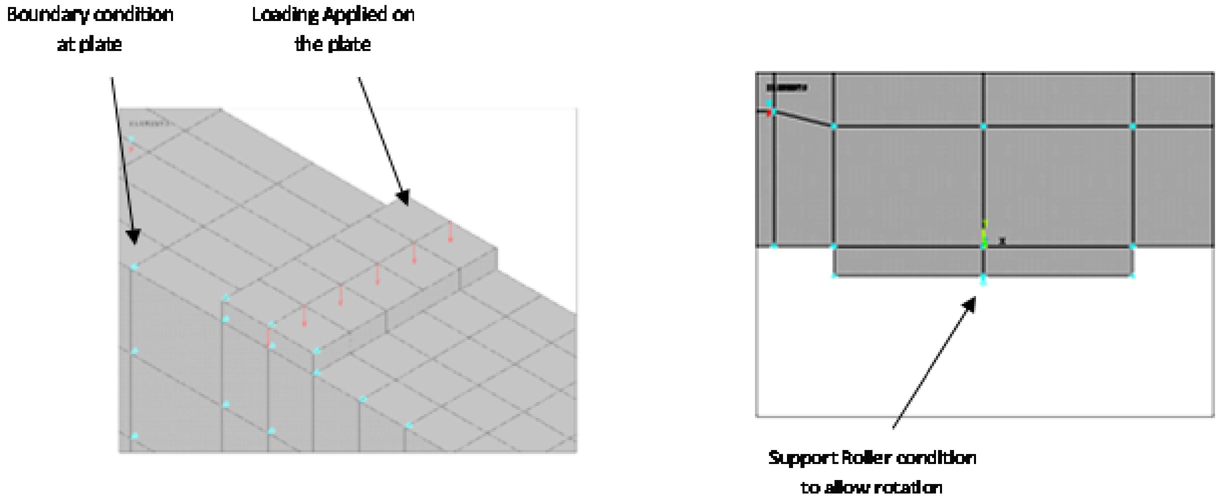


Figure 11. Boundary conditions for planes of symmetry and the point of attachment of external strengthening reinforcing bars and deflector



(a) (b)

Figure 12. (a) boundary conditions of the support; (b) loading on the steel plates under concentrated load

As can be seen from the diagram of figure (15-a) it can be concluded that this method of strengthening of reinforced concrete beams increases flexural rigidity, and therefore, reduces the deflections of the beams. If the deflections of beams are compared for a constant beam load, it can be seen that the amount of deflection is in reverse proportion with the increase in post-tensioning load, but its effect is not as considerable as those of beams in group A. From the study of the diagrams in figure (15-b), it can be concluded as in the case of group A beams, that the utilization of this method of strengthening does not have a considerable effect on the increase in the rigidity of the beam either, specially because the rigidity of all beams are approximately equal before the yielding of tension reinforcement.

- study of the behavior of deflectors

The deflectors are used to ensure similar deflection profile in beam and external strengthening reinforcing bars when the load is applied. If the deflection of external bars and the beam are shown in one coordinate axis, it can be seen that there exists a good correlation between these two deflections. In figures (16) and (17) the diagrams of one beam in group A and a beam in group B are shown respectively.

- study the failure modes of specimens

In reference beams, load deflection trace has a linear behavior before first crack occurs, then the nonlinear region starts. For beam A1, after yielding of steel reinforcement, a large deflection occurs at the beam centerline and failure is happened. But in beam B1 failure occurs due to failure of concrete

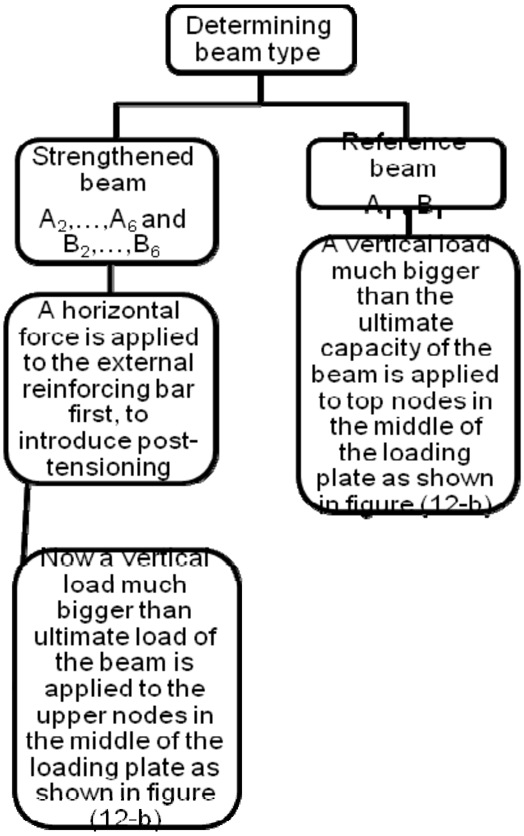


Figure 13. Algorithm of how applying loads

in compression region.

In strengthened beams of first group, till yielding of the internal tension reinforcement, beams show a similar behavior, but after yielding the reinforcement, due to existing of external bar the rigidity of beam increase. And because the amount of external used reinforcement is high, so before yielding these external bars, concrete in compression region is crushed, so failure is occurred.

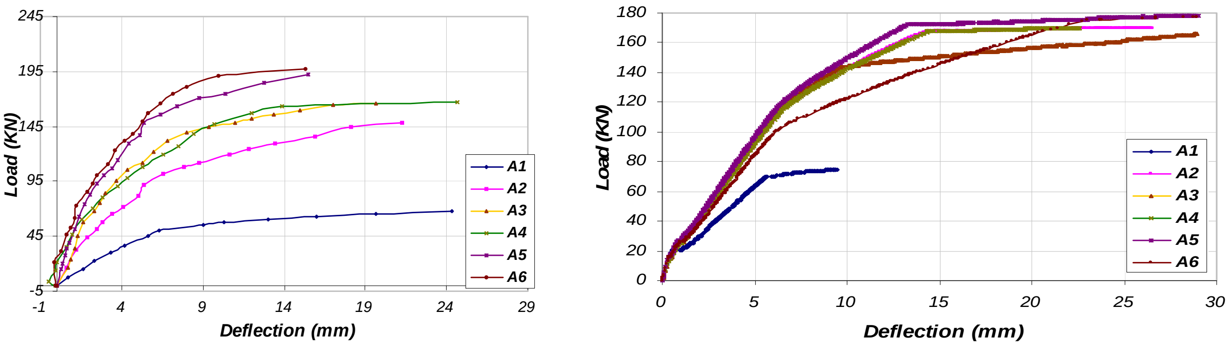
In strengthened beams of second group, using external bars did not make a remarkable effect on ultimate strength and internal reinforcement was yielded at the load near to ultimate load of the beam.

6. CONCLUSION

The results of the study are summarized below:

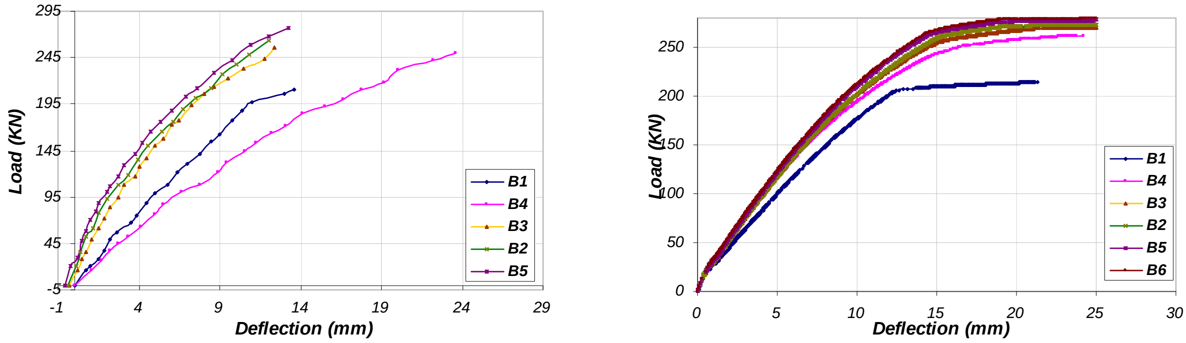
- Strengthening reinforced concrete beams by means of unbounded external post-tensioned strengthening reinforcing bars increases the flexural capacity of the beams. This increase is in reverse proportion with the percentage of internal flexural tension reinforcement. In the first group it makes more than 100 percent increase in strength while for the second group that has a high percent of internal reinforcement it just increases the strength of beams about 20 percent.
- This method of strengthening increases the rigidity of beams. The increase in rigidity is in reverse proportion with the percentage of internal flexural tension reinforcement. i. e., for higher percentages of internal flexural tension reinforcement the increase in rigidity is less.
- In this method of strengthening as the force of post-tensioning of external reinforcing bars increases the flexural capacity of beam increases.
- This method of strengthening decreases the ductility of beams.
- Strengthening of reinforced concrete beams using post-tensioned unbounded external strengthening reinforcing bars, can be utilized as a proper method of strengthening. The advantages of the method are; speedy application, simplicity of employment, almost no increase in weight of the structure, and economic advantages.
- Comparing load-deflection diagrams of reference beam in each group with the rest of the beams in the same group, shows that the existence of the strengthening bars causes a considerable decrease in deflection of the beams, i. e., it causes an increase in rigidity and decrease in ductility of the beams.
- Since there exists a close agreement between load-deflection diagrams of modeled and test beams, especially before the yielding of internal flexural tension reinforcement. The models constructed can be used for future research and the model is valid for modeling of reinforced concrete beams strengthened by external strengthening bars.

In load-deflection diagrams of reference beams, the beams show linear behavior up to the first crack of the beam, and a nonlinear behavior thereafter. The deflection of the beam at the middle of the span will increase considerably with yielding of longitudinal reinforcing bars.



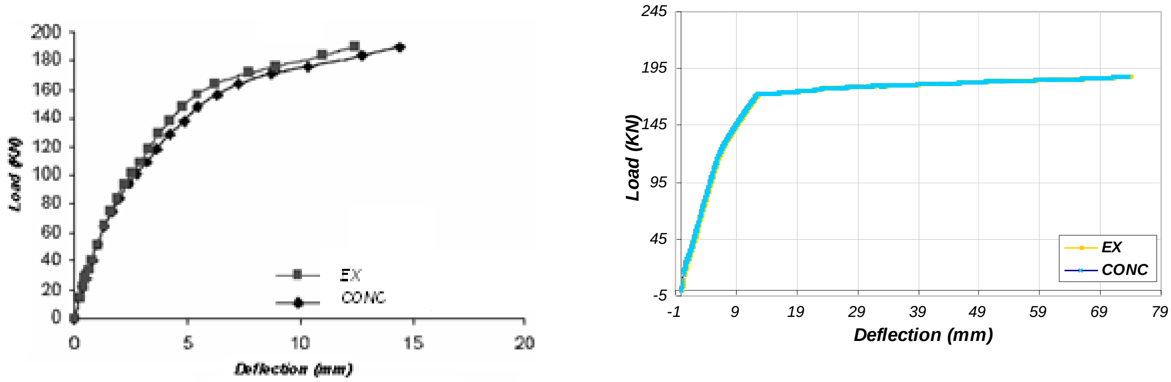
(a) (b)

Figure 14. The load-deflection diagram for group A beams; (a) test beams; (b) modeled beams



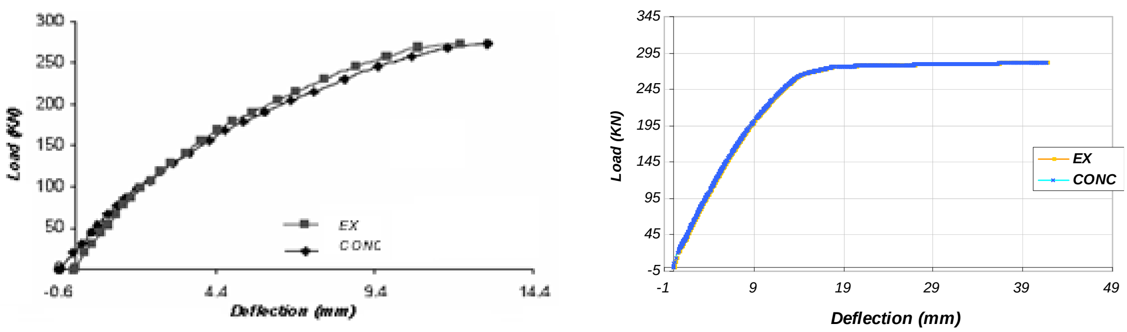
(a) (b)

Figure 15. The load-deflection diagram for group B beams; (a) test beams; (b) modeled beams



(a) (b)

Figure 16. Load-deflection diagram of the beam and external strengthening reinforcing bar for beam A5; (a) test beam; (b) modeled beam



(a) (b)

Figure 17. Load-deflection diagram of the beam and external strengthening reinforcing bar for beam B5; (a) test beam; (b) modeled beam

In strengthened beams, the load-deflection diagrams up to yielding of internal flexural tension reinforcement are similar to those of reference beams. But after yielding because of the existence of external strengthening bars, the rigidity of beams increases.

7. NOTATIONS

a_v	shear span	mm
A_{sb}	Cross sectional area of bonded reinforcement	mm^2
A_{sub}	Cross sectional area of unbounded reinforcement	mm^2
b	Width of beam	mm
d	Effective depth	mm
$\acute{d}$	Compression effective depth	mm
d_{ub}	Unbounded effective depth at ultimate load	mm
E_c	Elastic modulus of concrete	N/mm^2
E_S	Elastic modulus of steel	N/mm^2
f_c	Cylindrical compressive strength of concrete	N/mm^2
f_{cu}	Compressive strengths of concrete	N/mm^2
f_{sb}	Stress in bonded reinforcement	N/mm^2
f_{st}	Post tensioned stress	N/mm^2
f_{sub}	Stress in unbounded reinforcement	N/mm^2
f_{yb}	Yield strength of bonded tension reinforcement	N/mm^2
$\acute{f}_{yc}$	Yield strength of compression reinforcement	N/mm^2
I	Moment of inertia of cracked section	mm^4
K	Defined factor	
K_1	Defined factor	
K_2	Defined factor	
K_3	Defined factor	
L	Length of beam	mm
L_1	Beam span	mm
M	Flexural moment in specified section	$N.mm$
M_u	Ultimate moment	$N.mm$
$\acute{M}_u$	Defined factor	
P_u	Ultimate load	N
Q_b	Defined factor	
Q_c	Defined factor	
Q_t	Defined factor	
Q_{ub}	Defined factor	
x	The distance of the extreme compressive fiber of concrete from neutral axis of the beam	mm
z	The internal lever arm	mm
β	Defined factor	

Δ	Deflection at service load	mm
ε_c	Strain in concrete at the level of reinforcement	
ε_{sb}	Strain in bounded reinforcement	
ε_{sub}	Strain in unbounded reinforcement	
ρ_b	Bonded tension reinforcement ratio	
ρ_c	Compression reinforcement ratio	
ρ_{ub}	unbounded reinforcement ratio	

8. REFERENCES

1. Farooq. A.H.S. (May 1997), Flexural behavior of reinforced concrete beams strengthened by external unbounded reinforcement .PHD thesis, Heriot-Watt university, Edinburgh, Scotland, 358p.
2. Abdollahi, M., Naghipour, M., and Mohammadi Doostdar, H., Modeling and analysis of strengthened reinforced concrete beams by external strengthening bars by finite element method. The First Conference on Retrofitting of Structures, 2002.
3. Erfanian, R., Experimental investigation of beams strengthened by external reinforcement. M. Sc thesis, School of Engineering, University of Mazandaran, spring 2002.
4. Fooladvand, R., Naghipour, M., and Mohammadi Doostdar, H., Experimental studies of pre-loaded reinforced concrete cracked beams strengthened by external reinforcement. First National Congress of Civil Engineering, 2004.
5. Hosseini Abrishami, M., Naghipour, M., The experimental study of strengthening of reinforced concrete beams by unbounded post-tensioned external bars, 2005.
6. Pannell, F.N., The ultimate moment resistance of unbounded prestressed concrete, Magazine of concrete research ,Vol. 22, No. 66, PP. 43-53, March 1969.
7. Mattock .A.H., Yamazaki, j. And Kattula, B.T., Comparative study of prestressed concrete beam with and without bond, ACI journal, Vol. 68, No. 2, pp. 116-125, February 1971.
8. Harjli, M.H. and Kanj, M.A., ultimate flexural strength of concrete prestressed members with bonded tendons, ACI structural journal, Vol. 88, No. 6, PP. 663-673, November – December 1991.
9. Nemati, M., Analysis of pre-loaded reinforced concrete beams strengthened by external pre-tensioned reinforcing bars and their modeling by finite element method. M.Sc. Thesis, School of Engineering, University of Mazandaran, summer 2008.
10. Abdollahi, M., Modeling and analysis of reinforced concrete beams strengthened by external strengthening reinforcing bars using finite element method. M.Sc thesis, School of Engineering, University of Mazandaran, spring 2001.
11. Razaghi, J., Hosseini, A., and Hatami, F., Finite element method application in nonlinear analysis of reinforced concrete structures. Second National Congress of Civil Engineering, 2005.
12. Antony J.Wolanski, B.S. (may 2004), Flexural Behavior of reinforced and prestressed concrete beams using FRP element Analysis, A thesis submitted to the faculty of the graduated school, Marquetee university, in partial fulfillment of the requirement for the degree of master of science, p.118.
13. Nemati, M., and Naghipour, M., Modeling of pre-loaded reinforced concrete beams strengthened by external pre-tensioned reinforcing bars by finite element method. Fourth National Congress of Civil Engineering. 2008.
14. Fanning .p, Nonlinear models of reinforced and post tensioned concrete beams, Lecture, Department of Civil Engineering University College Dublin Earls fort Terrace, Dublin, Ireland, P.13, 2001.
15. Kachlakev, D., and Miller, T., Final report, Finite element modeling of reinforced concrete structures strengthened with FRP laminates, Oregon state University, May 2001, p. 105.
16. ANSYS Manual.ver 9.00.