BI-LEVEL CONTROL POLICY FOR REDUNDANT REPAIRABLE MULTICOMPONENT SYSTEM WITH RENEGING, SET UP AND VACATION

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Abstract This paper deals with bi-level control policy and queuing analysis of a machine repair problem. The model is developed by incorporating mixed standbys (cold and warm), reneging, set up and vacation time. The repair facility consists of two heterogeneous repairmen in the system. The life and repair time of the failed units and also their set up times are assumed to be exponentially distributed. The steady state queue size distribution is obtained by using recursive method. Expressions are derived for the number of failed units in the queue and the average waiting time for repair, throughput, etc. By setting appropriate parameters, we deduce some special models.

Keywords Machine Repair, Mixed Standbys, Set Up Time, Queue Size, Reneging, Vacation, Recursive Technique

چکیده این مقاله به سیاست کنترلی دوسطحی و تحلیل صف یک مسئلهٔ تعمیر ماشین می‌پردازد. مدل ارائه شده با استفاده از حالت رزرو (گرم و سرد) ترکیبی، reneging، زمان های آماده سازی و زمان های استراحت، طراحی شده است. سیستم واحد تعمیرات متشکل از دو تعمیرکار مشابه است. همچنین فرض شده است که طول عمر قطعات، زمان تعمیر قطعات خراب و زمان‌های آماده‌سازی از توزیع نمائی پیروی می‌کنند. توزیع احتمالی طول صف درحالت پایدار بااستفاده‌از روش بازگشتی به دست می‌آید. همچنین تعداد قطعات خراب در صف، متوسط زمان انتظار برای تعمیر، نرخ خروجی و غیره بررسی شده و با احتساب و تنظیم پارامترهای مناسب، مدل‌های ویژه‌ای به دست آمده است.

1. INTRODUCTION

With todays' advanced technology, machine repair system is inevitable in real life. The failure occurs frequently in computer, communication, production, manufacturing, service and distribution systems, etc. In machining system on the one hand the failed units wait for repairman to be free, on the other hand the repairman is idle until the next unit fails. First situation is known as machine interference and is encountered in almost all real time systems, working in machining environments. In the present investigation, we study a multi component machining system with the provision of mixed spares and two heterogeneous repairmen. When there are less failed units than a pre-assigned level (N), the repairman (R_1) can not be employed by the systems' administration due to cost constraints. If the number of failed units is greater than a threshold value (L), extra repairman (R_2) is recommended to reduce the work load. Thus there are two repairmen for the smooth running of the system. When an operating unit fails and if a repairman is available, it is sent for repair, otherwise failed unit waits in the queue for repair. The available cold spare unit replaces the failed unit; in case if cold spares are exhausted, the warm spares are used. The vacation period of first repairman (R_1) starts when there is no failed unit in the system and the vacation period of repairman R_2 starts when the number of failed units is less than or equal to a threshold level (L) in the system.

In real life, it is economical and of common experience to have optimal control policy for machine repair facility. In case of bi-level policy, the servers may be used for some ancillary work during vacation. Some important works in optimal control field and vacation models are given below. A queue of batch arrival with a vacation time under single server vacation policy was investigated by Choudhary, et al [1]. Niu, et al [2] suggested a vacation queue with set up and closed down times for the batch Markovian arrival processes. Ke, et al [3] obtained bi-level control for batch arrival queue with an early start-up and unreliable server. Choudhary, et al [4] analyzed a two state batch arrival queuing system with a modified Bernoulli schedule vacation under N-policy. Har, et al [5] developed the batch arrival queue with vacation and server set up. The convexity of two thresholds policy for an M/G/1 queue with vacations was examined by Zhang [6].

In some situations, the failed unit may renege, when it is in the queue due to impatience of care taker. The analysis of a repairable system with warm standbys, balking and reneging was made by Wang, et al [7]. Wang, et al [8] did the profit analysis of the M/M/R machine repair problem with balking, reneging and standby switching failure.

In many manufacturing/production systems when the operating unit fails, it is replaced by a standby. Gupta, et al [9] analyzed machine interference problem with warm spares, server vacation and exhaustive service. Analysis of a bulk queue under N-policy, multiple vacation and set up time was made by Reddy, et al [10]. Hur, et al [11] suggested the performance measures for state dependent M/G/1 queue with set up under N policy. Bi-level control of degraded machining system with warm standbys, set up and vacation time was obtained by Jain, et al [12]. The steady state analysis of a bulk queue with multiple vacations, set up times, N-policy and close-down times was made by Arumaganathan, et al [13].

In this paper we investigate bi-level policy for machine repair problem with mixed standbys, set up and vacation time along with reneging. There are two heterogeneous repairmen in the repair facility. The repairman (R_1) turns on and starts repair after a set up time, when N-failed units are accumulated in the system. The first repairman (R_1) leaves for a vacation when there is no failed unit in the system. The second repairman (R_2) turns on and starts to repair, when there is greater than a threshold number (L) of failed units in the system, and goes for vacation as soon as the number of failed units ceases to L. The set up time, life and repair time are assumed to be exponentially distributed. Using recursive method, we obtain the expressions for various performance indices, namely average number of failed units in the queue, throughput, average waiting time, etc. The rest of the paper is structured as follows: In Section 2 we have presented model description and notations. In Section 3 queue size distribution is given. Performance indices are given in section 4. Section 5 provides some special cases. Finally conclusion is given in Section 6 followed by references in Section 7.

2. MODEL DESCRIPTION

We consider bi-level policy for multi component machine repair system, cared by the repair facilitiy having two heterogeneous repairmen. If an operating unit fails, it is replaced first by cold standby, and if cold standbys are exhausted, it is replaced by warm standby if available. It is assumed that failed unit may renege exponentially when it is in the queue. The repair of failed units is performed according to a bi-level control policy. In the beginning when there are less than N failed units in the system, both repairmen are in idle state. As soon as N number of failed units is accumulated in the system, the first repairman (R_1) turns on and starts to repair after a set up time; the repairman (R_1) returns vacation, when there is no failed unit in the system. If the number of failed units is greater than a threshold number L, then repairman (R_2) turns on and starts repair; it returns back for vacation when there are less than or equal to L failed units in the system.

For model description the following notations are used:

M	Number of operating units in the system functioning in normal mode
S(Y)	Number of warm spare (cold) units in the system
K	Number of total units in the system (K=M+Y+S)
N (L)	Threshold number of failed units when repairman R_1 (R_2) turns on, R_2 goes for vacation if numbers of failed units are less than L.

$P_{i,j}$	Steady state probability that there are i (0<i<k) failed units in the system and j (j=0,1,2) busy repairman in the system
$\lambda(\alpha)$	Failure rate of operating (warm) units
μ_1 (μ_2)	Repair rate of repairman R_1 (R_2)
β	Set up rate of repairman (R_1) when there are greater than or equal to N failed units
γ_1	Rate from which first repairman (R_1) returns from a vacation and finds that there is no failed unit in the system
γ_2	Rate from which second repairman (R_2) returns from a vacation when there are L failed units in the system
ε_1	Rate at which first repairman (R_1) goes back for another vacation if he finds that there is no failed units in the system
ε_2	Rate at which second repairman (R_2) goes back for another vacation if he finds less than L failed units after returning back from a vacation
α_1	Failure rate for R_1
α_2	Failure rate for R_2

3. QUEUE SIZE DISTRIBUTION

To obtain steady state queue size distribution, we consider the following two cases.

Case 1. $Y+S<N$

In this case the steady state equations governing the model are as follows:

$$(M\lambda+S\alpha+\gamma_1)P_{0,0}=\varepsilon_1 P_{0,1}+\mu_1 P_{1,1} \tag{1}$$

$$(M\lambda+S\alpha)P_{i,0}=(M\lambda+S\alpha)P_{i-1,0}\ ,\ 1\le i\le Y \tag{2}$$

$$[M\lambda+(Y+S-i)\alpha]\,P_{i,0}=[(M\lambda+(Y+S-i+1)\alpha]P_{i-1,0},\quad Y\le i\le Y+S \tag{3}$$

$$[(K-i)\lambda]P_{i,0}=[(K-i+1)\lambda]P_{i-1,0}\ ,\ Y+S\le i<N \tag{4}$$

$$[(K-i)\lambda+\beta]P_{i,0}=[(K-i+1)\lambda]P_{i-1,0}\ ,\ N\le i<L \tag{5}$$

$$\beta P_{L,0}=\left[(K-L+1)\lambda\right]P_{L-1,0} \tag{6}$$

$$(M\lambda+S\alpha+\varepsilon_1)P_{0,1}=\gamma_1 P_{0,0} \tag{7}$$

$$[(M\lambda+S\alpha)+\mu_1+(i-1)\alpha_1]P_{i,1}=(M\lambda+S\alpha)P_{i-1,1}+(\mu_1+i\alpha_1)P_{i+1,1},\ 1<i\le Y \tag{8}$$

$$[M\lambda+(Y+S-i)\alpha+\mu_1+(i-1)\alpha_1]P_{i,1}=[M\lambda+(Y+S-i+1)\alpha]P_{i-1,1}+(\mu_1+i\alpha_1)P_{i+1,1},\ Y+1\le i<Y+S \tag{9}$$

$$[(K-i)\lambda+\mu_1+(i-1)\alpha_1]P_{i,1}=[(K-i+1)\lambda]P_{i-1,1}+(\mu_1+i\alpha_1)P_{i+1,1},\ Y+S\le i<N \tag{10}$$

$$[(K-i)\lambda+\mu_1+(i-1)\alpha_1]P_{i,1}=[(K-i+1)\lambda]P_{i-1,1}+(\mu_1+i\alpha_1)P_{i+1,1}+\beta P_{i,0},\ N\le i\le L \tag{11}$$

$$[\mu_1+(L-1)\alpha_1+\gamma_2]P_{L,1}=[(K-L+1)\lambda]P_{L-1,1}+\beta P_{L,0}+[\mu_1+\mu_2+(L-1)\alpha_2]P_{L+1,2}+\varepsilon_2 P_{L,2} \tag{12}$$

$$[(K-L)\lambda+\varepsilon_2]P_{L,2}=\gamma_2 P_{L,1} \tag{13}$$

$$[(K-i)\lambda+\mu_1+\mu_2++(i-2)\alpha_2]P_{i,2}=\left[(K-i+1)\lambda\right]P_{i-1,2}+[\mu_1+\mu_2+(i-1)\alpha_2]P_{i+1,2},\ L<i<K \tag{14}$$

$$[\mu_1+\mu_2+(K-2)\alpha_2]P_{K,2}=\lambda P_{K-2} \tag{15}$$

Equations 1-15 can be solved by using recursive method and assuming

$$P_{i,j}=q_{i,j}P_{0,0},\qquad 0\le i\le K\ ,\ 0\le j\le 2 \tag{16}$$

Where $P_{0,0}$ is obtained by using normalizing condition

$$\sum_{i=0}^{K}\sum_{j=0}^{2}P_{i,j}=1.\ \ \text{Thus}\ \ P_{0,0}=\left[\sum_{i=0}^{K}\sum_{j=0}^{2}q_{i,j}\right]^{-1} \tag{17}$$

From Equation 16, we obtain

$$q_{0,0}=1 \tag{18}$$

Solving Equations 2-5, we get

$P_{i,0} = P_{0,0}$; $L \le i \le Y$

$$P_{i,0} = \frac{\sigma_0}{\sigma_{i-Y}} - P_{0,0} \; ; \quad Y+1 \le i \le Y+S$$

$$P_{i,0} = \frac{[k-(Y+S)]\lambda}{[k-i]\lambda} \frac{\sigma_0}{\sigma_S} P_{0,0}; \quad Y+s+1 \le i \le N-1$$

$$P_{i,0} = \frac{\left[\prod_{j=N}^{i-1}(k-j)\lambda\right][k-(Y+S)]\lambda\sigma_0}{\prod_{j=n}^{i}[(k-j)\lambda+\beta]\sigma_s} P_{0,0}; \quad N \le i \le L-1$$

Where

$$\sigma_0 = M\lambda + S\alpha$$
$$\sigma_{i-Y} = M\lambda + (Y+S-i)\ \alpha,$$

From Equations 7 and 8, we find

$$q_{L,0} = \phi_{L-1} q_{L-1,0} \tag{19}$$

Where

$$\phi_{L-1} = \frac{(K-L+1)\ \lambda}{\beta} \quad q_{L,0} = \frac{\gamma_1}{M\lambda + S\alpha + \varepsilon_1}$$

Denote $Z = q_{0,1} + 1 = \dfrac{M\lambda + S\alpha + \varepsilon_1 + \gamma_1}{M\lambda + S\alpha + \varepsilon_1}$ (20)

Solving Equations 9-11, we obtain

$$q_{i,1} = \begin{cases} \theta_0 Z, & i=1 \\ \dfrac{\mu_1\theta_0}{\mu_1+(i-1)\alpha_1} + \\ \left[1 + \dfrac{\sum_{L=1}^{i-3}(\mu_1\theta_0)^L}{\prod_{m=2}^{i-2}(\mu_1+m\alpha_1)} + \dfrac{(\mu_1\theta_0)^{i-2}(1+\theta_0 Z)}{\prod_{j=2}^{i-2}(\mu_1+j\alpha_1)}\right], \\ i < i \le Y \end{cases}$$

$$q_{i,1} = \begin{cases} \dfrac{\prod_{n=1}^{i-Y-1}\sigma_n + \prod_{m=0}^{i-Y-1}\varphi_m}{\prod_{n=1}^{i-Y-1}\varphi_K} q_{Y+1,1} \\ + \dfrac{\sum_{i=0}^{i-Y-3,i-Y-2}\prod_{j=i+1}\sigma_j\prod_{K=0}^{i}\varphi_K}{\prod_{n=1}^{i-Y-1}\varphi_K} q_{Y+1,1} + \\ \dfrac{\left(\sum_{i=0}^{i-Y-1}\prod_{P=i+3}^{Y-1}\sigma_P\prod_{K=1}^{i+1}\varphi_K\right)\sigma_0}{\prod_{n=1}^{i-Y-1}\varphi_n} q_{Y,1}, \\ Y < i < Y+S \\ \dfrac{\prod_{n=1}^{i-Y-1}\psi_n + \prod_{m=0}^{i-Y-2}\varphi_m}{\prod_{n=1}^{i-Y-1}\varphi_n} \\ + \dfrac{\sum_{r=0}^{i-Y-3,i-Y-1}\prod_{j=i+1}\psi_j\prod_{K=0}^{i}\varphi_K}{\prod_{n=1}^{i-Y-1}\varphi_n} q_{Y+1,1} \\ + \dfrac{\left(\sum_{i=0}^{i-Y-1,i-Y-1}\prod_{P=i+1}\psi_P\prod_{K=1}^{i+1}\varphi_K\right)\psi_0}{\prod_{n=1}^{i-Y-1}\varphi_n} q_{Y,1}, \\ Y+S \le i \le N \\ \dfrac{\psi_{i-1}+\upsilon_{i-2}}{\upsilon_{i-2}} q_{i-2,1} - \dfrac{\psi_{i-2}}{\upsilon_{i-3}} q_{i-2,1} \\ - \dfrac{\beta}{\upsilon_{i-3}} q_{i-1,0}, \quad N < i < L \end{cases} \tag{21}$$

$$\sigma_n = M\lambda + (Y+S-n)$$
$$\psi_n = (K-n)\alpha, \qquad \phi_n = (\mu_1 + n\alpha_1)$$

To obtain $q_{i,2}$(i=L,L+1,…,K), using Equations 12-15, we obtain

$$q_{L,2} = \delta\, q_{L,1} \tag{22}$$

$$q_{L+1,2} = \frac{\upsilon_{L+1}+\gamma_2}{\xi_{L+1}} - \delta\varepsilon_2 q_{L,1} - \frac{\psi_{L-1}}{\xi_{L+1}} q_{L-1} - \frac{\beta}{\xi_{L+1}} q_{L,0} \tag{23}$$

$$q_{i,2} =$$

$$\frac{\psi_{i-1}+\xi_{i-3}}{\xi_{i-4}} q_{i-1,2} - \frac{\psi_{i-2}}{\xi_{i-4}} q_{i-2,2}, \quad L < i < K \tag{24}$$

Where

$$\delta = \frac{\gamma_2}{(K-L)\lambda + \varepsilon_2}, \qquad \xi_{L+1} = \mu_1 + \mu_2 + (L-1)\alpha_2,$$

$$\upsilon_{L+1} = \mu_1 + (L-1)\alpha_2, \qquad \psi_{L-1} = (K-L+1)\lambda,$$

If P_i denotes the probability that there are i failed units in the system, then we have

$P_i = P_{i,0} + P_{i,1} + P_{i,2}$,

Where

$P_{i,0} + P_{i,1} = 0, \quad L < i < K+1$

and

$P_{i,2} = 0, \quad i < L.$

From Equation 16, substituting the value of $P_{i,j}$, we get

$P_i = f_i P_{0,0}, \quad 0 \le i \le K$

Also

$$P_{0,0} = \left[\sum_{i=0}^{K} f_i\right]^{-1}, \tag{25}$$

Thus we have

$$f_i = q_{i,0} + q_{i,1} + q_{i,2} \tag{26}$$

From Equations 18-21, we find

$$f_0 = 1 + q_0 = Z \tag{27}$$

$$f_1 = q_{1,0} + q_{1,1} = 1 + \theta_0 Z \tag{28}$$

$$f_i = 1 + \frac{\mu_1\theta_0}{\mu_1 + (i-1)\alpha_1}$$

$$\left[1 + \sum_{l=1}^{i-3} \frac{(\mu_1\theta_0)^l}{\prod_{m=2}^{i-1}(\mu_1 + m\alpha_1)} + \frac{(\mu_1\theta_0)^{i-2}(1+\theta_0 Z)}{\prod_{j=2}^{i-2}(\mu_1 + j\alpha_1)}\right],$$

$2 \le i \le Y$

(29)

$$f_{Y+1} = q_{Y+1,0} + q_{Y+1,1} \tag{30}$$

$$f_{Y+1} = \frac{\sigma_0}{\sigma_1} + \frac{\mu_1\theta_0}{\mu_1 + Y\alpha_1)}$$

$$\left[1 + \sum_{l=1}^{Y-2} \frac{(\mu_1\theta_0)^l}{\prod_{m=2}^{Y-2}(\mu_1 + m\alpha_1)} + \frac{(\mu_1\theta_0)^{Y-1}(1+\theta_0 Z)}{\prod_{j=2}^{Y-1}(\mu_1 + j\alpha_1)}\right]$$

(31)

$$f_{Y+S} = \frac{\sigma_0}{\psi_{Y+S}} + \frac{\left(\prod_{n=1}^{i}\psi_n + \prod_{m=0}^{S-2}\varphi_m\right)}{\prod_{n=1}^{S-1}\varphi_n}$$

$$+ \frac{\left(\sum_{j=Y+S+1}^{S-3}\ \prod_{j=Y+S+1}^{S-2}\psi_j \prod_{K=0}^{Y+S}\varphi_K\right)}{\prod_{n=1}^{S-1}\varphi_n} q_{Y+1,1}$$

$$+ \frac{\left(\sum_{i=0}^{S-3}\ \prod_{P=Y+S+1}^{Y-1}\psi_P \prod_{K=1}^{Y+S+1}\varphi_K\right)\psi_0}{\prod_{n=1}^{S-1}\varphi_n} q_{Y,1}$$

(32)

$$f_i = \frac{\sigma_0}{\psi_i} + \frac{\left(\prod_{n=1}^{i}\psi_n + \prod_{m=0}^{i-Y-2}\varphi_m\right)}{\prod_{n=1}^{S-1}\varphi_n} +$$

$$\frac{\left(\sum_{i=0}^{i-Y-3,}\ \prod_{j=i+1}^{i-Y-2}\psi_j \prod_{K=0}^{i}\phi_K\right)}{\prod_{n=1}^{i-Y-1}\phi_n} q_{Y+1,1} +$$

$$\frac{\left(\sum_{i=0}^{i-Y-3}\ \prod_{P=i+1}^{Y-1}\psi_P + \prod_{K=1}^{i+1}\phi_K\right)\psi_0}{\prod_{n=1}^{i-Y-3}\phi_n} q_{Y,1}$$

$Y < i \le Y + S$

(33)

$$f_N = \frac{\sigma_0}{\psi_{Y+\beta}} + \frac{\psi_{N-1} + \upsilon_{N-2}}{\upsilon_{i-3}} q_{i-1,1} - \frac{\psi_{N-2}}{\upsilon_{i-3}} q_{i-2,1} + \frac{\beta}{\upsilon_{i-3}} q_{i-1,0} \quad (34)$$

$$f_{N+1} = \frac{\sigma_0 \left(\prod_{l=N}^{i-1} \psi_l \right)}{\prod_{j=N}^{i} (\psi_{j+\beta})} + \frac{(\psi_N + \upsilon_{N-1})}{\upsilon_{-1}} q_{N,1} - \frac{\psi_{N-1}}{\upsilon_{N-2}} q_{N-1,1} - \frac{\beta}{\upsilon_{N-2}} q_{N,0}, \quad (35)$$

$$f_i = \frac{\sigma_0 \left(\prod_{l=N}^{i-1} \psi_l \right)}{\prod_{j=N}^{i} (\psi_{j+\beta})} + \frac{\psi_{i-1} + \upsilon_{i-2}}{\upsilon_{i-2}} q_{i-2,1} + \frac{\psi_{i-2}}{\upsilon_{i-3}} q_{i-3,1} - \frac{\beta}{\upsilon_{i-3}} q_{i-1,0}, \quad N+1 \le i < L \quad (36)$$

$$f_L = \phi_{L-1} q_{L-1} + \frac{(\psi_{L-1} + \upsilon_{L-1})}{\upsilon_{L-2}} q_{L-1,2} - \frac{\psi_{i-2}}{\upsilon_{i-3}} q_{L-2,1} \frac{\beta}{\upsilon_{i-3}} q_{L-1,0} + \upsilon q_{L,0} \quad (37)$$

$$f_{L+1} = \frac{\upsilon_{L+1} + \gamma_2 + \delta\varepsilon_2}{\xi_{L+1}} q_{L,1} - \frac{\beta}{\xi_{L+1}} q_{L,1} - \frac{\psi_{L-1}}{\xi_{L+1}} q_{L-1,1} \quad (38)$$

$$f_i = \frac{\psi_{i-1} + \xi_{i-2}}{\xi_{i-4}} q_{i-1,2} - \frac{\sigma_{i-2}}{\xi_{i-4}} q_{i-2,1}, \quad L \le i \le K \quad (39)$$

Case 2. N < Y

In this case the steady state equations are given as:

$$(M\lambda + S\alpha + \gamma_1) P_{0,0} = \varepsilon_2 P_{0,1} + \mu_2 P_{1,1}, \quad (40)$$

$$(M\lambda + S\alpha) P_{i,0} = (M\lambda + S\alpha) P_{i-1,0}, \quad 1 < i < N \quad (41)$$

$$(M\lambda + S\alpha + \beta) P_{N,0} = (M\lambda + S\alpha) P_{N-1,0}, \quad i = N \quad (42)$$

$$(M\lambda + S\alpha + \beta) P_{i,0} = (M\lambda + S\alpha) P_{i-1,0}, \quad N < i \le Y \quad (43)$$

$$[M\lambda + (Y+S-i)\alpha + \beta] P_{i,0} = [M\lambda + (Y+S-i+1)\alpha] P_{i-1,0}, \quad Y < i \le Y+S \quad (44)$$

$$[(K-i)\lambda + \beta] P_{i,0} = [(K-i+1)\lambda] P_{i-1,0}, \quad Y+S < i < N \quad (45)$$

$$\beta P_{L,0} = [(K-L+1)\lambda] P_{L-1,0} \quad (46)$$

$$(M\lambda + S\alpha + \varepsilon_1) P_{0,1} = \gamma_1 P_{0,0} \quad (47)$$

$$\left[M\lambda + S\alpha + \mu_1 + (i-1)\alpha_1 \right] P_{i,1} = (M\lambda + S\alpha) P_{i-1,1} + (\mu_1 + i\alpha_1) P_{i+1,1}, \quad 1 < i \le N-1 \quad (48)$$

$$\left[M\lambda + S\alpha + \mu_1 + (i-1)\alpha_1 \right] P_{i,1} = (M\lambda + S\alpha) P_{i-1,1} + (\mu_1 + i\alpha) P_{i-1,1} + (\mu_1 + i\alpha) P_{i+1,2} + \beta P_{i,0}, \quad N \le i < Y \quad (49)$$

$$\left[M\lambda + (Y+S-i)\alpha + \mu_1 + (i-1)\alpha_1 \right] P_{i,1} = [M\lambda + (Y+S-i+1)\alpha_1) P_{i-1,1} + (\mu_1 + i\alpha_1) P_{i+1,1} + \beta P_{L,0}, \quad Y < i \le Y+S \quad (50)$$

$$\left[(K-i)\lambda + \mu_1 + (i-1)\alpha_1 \right] P_{i,1} = [(K-i+1)\lambda] P_{i-1,1} + (\mu_1 + i\alpha_1) P_{i+1,1} + \beta P_{i,0}, \quad Y+S \le i \le L-1 \quad (51)$$

$$\left[\mu_1 + (L-1)\alpha + \gamma_2 \right] P_{L,1} = [(K-L+1)\lambda)] P_{L-1,1} + \beta P_{L,0} + [\mu_1 + \mu_2 + (L-1)\alpha_2] P_{L+1,2} \quad (52)$$

$$\left[(K-L)\lambda + \varepsilon_2 \right] P_{L,2} = \gamma_2 P_{L,1} \quad (53)$$

$$[(K-L)\lambda + \mu_1 + \mu_2 + (i-2)\alpha_2] P_{i,2} = [(K-L+1)\lambda] P_{i+1,2} + [\mu_1 + \mu_2 + (i-1)\alpha_2] P_{i+1,2} \quad L \le i \le K \quad (54)$$

$$[\mu_1 + \mu_2 + (K-2)\alpha_2] P_{K,2} = \lambda P_{K-1,2} \quad (55)$$

Equations 40-55 can be solved by using recursive method as

$$P_{i,j} = q_{i,j}\, P_{0,0} \tag{56}$$

Where $P_{0,0}$ is obtained by using normalizing condition

$\sum_{i=0}^{K}\sum_{j=0}^{2} P_{i,j} = 1$, as

$$P_{00}\left[\sum_{i=0}^{K}\sum_{j=0}^{2} q_{i,j}\right]^{-1}, \tag{57}$$

From Equation 64, we have

$q_{0,0} = 1$

The solution of Equations 42-45 is given by

$$q_{i,0} = \begin{cases} 1, & 1 \le i < N \\ \left(\dfrac{\sigma_0}{\sigma_0+\beta}\right)^{i+1-N}, & N \le i \le Y \\ \dfrac{\prod_{n=1}^{i-Y}\sigma_n}{\prod_{m=1}^{i-Y}(\sigma_m+\beta)} \cdot \left(\dfrac{\sigma_0}{\sigma_0+\beta}\right)^{i+1-N}, & Y < i < Y+S \\ \dfrac{\prod_{n=Y+s}^{i-1}\psi_n}{\left(\prod_{n=Y+S+1}^{i}\psi_n+\beta\right)} \cdot \dfrac{\prod_{P=0}^{i-Y-1}\sigma_P}{\prod_{r=1}^{i}\sigma_r+\beta} \cdot \left(\dfrac{\sigma_0}{\sigma_0+\beta}\right)^{i+1-N}, & Y+S \le i < L-1 \end{cases} \tag{58}$$

From Equations 46-47 we obtain:

$$q_{L,0} = \phi_{L-1}\, q_{L-1,0}$$

$$q_{0,1} = \frac{\gamma_1}{(M\lambda + S\alpha + \varepsilon_1)} \tag{59}$$

$$\phi_{L-1} = \frac{(K-L+1)\,\lambda}{\beta} \tag{60}$$

$$Z = q_{0,1} + 1 = \frac{(M\lambda + s\alpha + \gamma_1 + \varepsilon_1)}{(M\lambda + s\alpha + \varepsilon_1)} \tag{61}$$

From Equations 48-51, we get

$$\begin{cases} \theta_0 Z, & i = 1 \\ \dfrac{(\mu_1\theta_0)}{\mu_1 + (i-1)\alpha_1}\left[1 - \sum_{l=1}^{i-3}\dfrac{(\mu_1\theta_0)^l}{\prod_{m=2}^{i-2}(\mu_1 + m\alpha_1)}\right] + \left[\dfrac{(\mu_1\theta_0)^{i-2}(1+\theta_0 Z)}{\prod_{j=1}^{i-2}(\mu_1 + j\alpha_1)}\right] & 2 \le i < N \\ \dfrac{\sigma_0+\psi_i}{\psi_{i-1}} q_{i-1,1} - \dfrac{\sigma_0}{\psi_i} q_{i-2,1} - \dfrac{\beta}{\psi_{i-1}} q_{i-1,0}, & N \le i < Y \\ \dfrac{\sigma_0+\psi_i}{\psi_{i-1}} q_{i-1,1} - \dfrac{\sigma_0}{\psi_{i-1}} q_{i-2,1} - \dfrac{\beta}{\psi_{i-1}} q_{L,0}, & Y+S \le i \le L-1 \end{cases} \tag{62}$$

To obtain the value of $q_{i,2}$ ($i = L, L+1, \ldots, K$), we use Equations 53-55 and we obtain

$$q_{L,2} = \delta q_{L,1} \tag{63}$$

$$q_{L+1} = \frac{\upsilon_{L+1} + \gamma_1 - \delta\varepsilon_2}{\xi_{L+1}} q_{L-1} - \frac{\psi_{L-1}}{\xi_{L+1}} q_{L-1,1} - \frac{\beta}{\xi_{L+1}} q_{L,0} \tag{64}$$

$$q_{i,2} = \frac{\psi_{i-1}}{\xi_{i-4}} q_{i-4,2} - \frac{\psi_{i-2}}{\xi_{i-4}} q_{i-2,2}, \quad L+2 \le i \le K \tag{65}$$

Where

$\delta = \dfrac{\gamma_2}{(K-L)\lambda + \varepsilon_2}$, $\quad \xi_{L+1} = \dfrac{\mu_1 + \mu_2 + (L-1)\alpha_2}{}$,

$\upsilon_{L+1} = \mu_1 + L\alpha_1$, $\quad \psi_{L+2} = (K-L+2)\lambda$, From Equations 65-66, we get

$$f_i = q_{i,0} + q_{i,1} + q_{i,2} \tag{66}$$

In this case, we find the value of f_i, as

$$f_i = q_{i,0} + q_{i,1} + q_{i,2} \qquad 1 \le i \le K \tag{67}$$

$$f_0 = 1 + q_{0,1} = Z$$

$$f_1 = q_{1,0} + q_{1,1} \tag{68}$$

$$f_i = 1 + \frac{\mu_1\theta_0}{\mu_1 + (i-1)\alpha}\left[1 + \sum_{l=1}^{i-3}\frac{(\mu_1\theta_0)}{\prod\limits_{m=2}^{i-2}(\mu_1 + m\alpha_1)}\right] + \left[\frac{(\mu_1\theta_0)^{i-2}(1+\theta_0 Z)}{\prod\limits_{j=1}^{i-2}(\mu_1 + j\alpha_1)}\right], \qquad 1 \le i < N \tag{69}$$

$$f_N = \left(\frac{\sigma_0}{\sigma_0+\beta}\right)^{i+1-N} + \left[\frac{\sigma_0+\psi_N}{\psi_{N-1}}q_{N-1,1} - \frac{\sigma_0}{\psi_{N-1}}q_{N-2,1}\right]\left[-\frac{\beta}{\psi_{N-1}}q_{N-1,0}\right], \tag{70}$$

$$f_Y = \left(\frac{\sigma_0}{\sigma_0+\beta}\right)^{i+1-N} + \frac{\sigma_0+\upsilon_Y}{\upsilon_{Y-1}}q_{Y-1,1} - \frac{\upsilon_{-Y}}{\upsilon_{Y-1}}q_{Y-2,1} - \frac{\beta}{\upsilon_{Y-1}}q_{L,0}, \tag{71}$$

$$f_i = \frac{\prod\limits_{n=0}^{i-Y-1}\sigma_N}{\prod\limits_{m=0}^{i-Y}\sigma_m + \beta}\left(\frac{\sigma_0}{\sigma_0+\beta}\right)^{i+1-N} + \frac{\sigma_0+\upsilon_i}{\upsilon_{i-1}}q_{i-1,1} - \frac{\upsilon_{-1}}{\upsilon_{i-1}}q_{i-2,1} - \frac{\beta}{\upsilon_{i-1}}q_{L,0}, \qquad Y < i < Y+S \tag{72}$$

$$f_i = \frac{\left(\prod\limits_{n=Y+S}^{i-1}\psi_n\right)}{\left(\prod\psi_n + \beta\right)} \cdot \frac{\prod\limits_{P=0}^{i=-Y-1}\sigma_n}{\prod\limits_{r=1}^{i}\sigma_r + \beta} \cdot \left(\frac{\sigma_0}{\sigma_0+\beta}\right)^{i+1-N} + \frac{\psi_i+\upsilon_i}{\upsilon_{i-1}}q_{i-1,1} - \frac{\psi_{i-1}}{\upsilon_{i-1}}q_{i-2,1} - \frac{\beta}{\upsilon_{i-1}}q_{L,0}, \quad Y+S \le i \le L-1 \tag{73}$$

Equations 37-39 are same for this case as taken for case I for $L \le i \le K$.

4. SOME PERFORMANCE INDICES

In this section, we derive expressions for various performance indices in terms of probabilities as follows:

- The average number of failed units in the system

$$L_S = \sum_{i=0}^{K} iP_i = \sum_{i=0}^{K} i(q_{i,0} + q_{i,1} + q_{i,2})P_{0,0} \tag{74}$$

- The average number of failed units in the queue

$$L_q = \left[\sum_{i=0}^{N} iq_{i,0} + \sum_{i=1}^{L}(i-1)q_{i,1}\right] + \left[\sum_{i=L+1}^{K}(i-2)q_{i,2}\right]P_{0,0}, \tag{75}$$

- The effective failure rate is given by

$$\lambda_{eff} = \sum_{i=0}^{S-1}[M\lambda + (S-i)\alpha]P_i + \sum_{i=S}^{K}(K-i)\lambda P_{i.} \tag{76}$$

- The waiting time in the system

$$w = \frac{L_S}{\lambda_{eff.}} \tag{77}$$

- The waiting time in the queue

$$w_q = \frac{L_q}{\lambda_{eff.}} \tag{78}$$

- The throughput of the system is obtained as

$$T = \left[\left\{\sum_{i=1}^{L} \mu_1 + (i-1)\alpha_1\right\} q_{i,1}\right] + \left[\left\{\sum_{i=L+1}^{K} \mu_1 + \mu_2 + (i-2)\alpha_2\right\} q_{i,2}\right] P_{0,0} \qquad (79)$$

5. SOME SPECIAL CASES

Case 1.

N-policy for single repairman model with warm spares and degraded failure To deduce results for this queueing model, we set parameters corresponding to second repairman $\varepsilon_2 = \gamma_2 = \mu_2 = 0$ and L = K. In particular when set up time is also zero, we substitute $\beta_1 = \infty$, $\gamma_1 = \varepsilon_1 = 0$ However when spares are cold, we substitute failure rate of warm spares $\alpha = 0$ in the corresponding results.

Case 2.

Machine repair problem with spares and multiple vacations By substituting N = 1, $\gamma_1 = \gamma_2 = \varepsilon_1 = \varepsilon_2 = \mu_2 = 0$ our model reduces to machine repair problem with multiple vacation. Further to deduce results for hot and cold spares, we fix $\alpha = \lambda_0 = \lambda_{i-\delta}\ (i = \delta+1, \delta+2, ..., K)$ and $\alpha = 0$ respectively. In order to convert the results for classical machine repairman problem (without vacation), we have to substitute $\beta_1 = \infty$.

Case 3.

Consider N=1, $\alpha = \lambda_0 = \lambda_{i-s}(i=S+1, S+2,\ldots,K)$, $\gamma_1 = \gamma_2 = \varepsilon_1 = \varepsilon_2 = \mu_2 = 0$. The model gives results for machine interference problem with multiple vacation and hot spares.

Case 4.

We put in our model $\lambda_{i-s} = \lambda$, (i=1,2,…,K), and $\mu_1 = \mu_1 + (i-1)\alpha_1$, $\mu_2 = \mu_1 + \mu_2 + (i-2)\alpha_2$, the model gives results for Bi-level control policy of degraded machining system with warm standbys, set up and vacation.

6. CONCLUSION

We have considered bi-level machining system with mixed standbys, set up, reneging and vacation. There is the provision of repair facilities consisting of two repairmen in the system. The incorporation of reneging parameter makes our model closer to real life situations as line delay causes impatient caretakers of failed units to renege. The provision of second repairman (R_2) in case of many failed units, are accumulated in the system, may be helpful in reducing the work load of the first regular repairmen, and it is recommended for smooth running of the system. We have considered that both repairmen go for a vacation according to a pre-specified policy; this concept is common in real time system, and may be an appropriate tool to reduce the cost incurred on repairmen since they may be assigned for some secondary jobs during vacation. Using recursive method, we have obtained steady state queue size distribution, performances indices such as average number of failed units in the systems, waiting time, throughput and etc. The quantitative prediction of these indices may be important in the development of machining system.

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