OPTIMAL SLIDING-MODE GUIDANCE LAW FOR FIXED-INTERVAL PROPULSIVE MANEUVERS

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Abstract An optimal strategy based on minimum effort control and also with terminal position constraint is developed for an exoatmospheric interceptor with a fixed- interval guidance time. It is then integrated with sliding-mode control theory to drive an optimal sliding-mode guidance law for fixed-interval guidance time. In addition, this guidance law is generalized for intercepting an arbitrarily time-varying target maneuver. Robustness of the new guidance method against disturbances and good miss distance performance are achieved by the second method of Lyapunov and simulation results. The presented guidance law is simple to implement in practical applications.

Keywords Optimal, Sliding-Mode, Guidance, Robust, Fixed-Interval

چکیده در این مقاله قانون هدایت بهینه‌ای برای رهگیری با قید بردار موقعیت نهائی و محدودبت زمانی مانور مبتنی برحداقل تلاش کنترلی ارائه می‌گردد. سپس قانون هدایتی فوق با تئوری کنترل مد لغزشی تلفیق شده و قانون هدایت بهینه مقاوم ایجاد می گردد و برای رهگیری اهداف دارای مانور متغیر با زمان تعمیم داده می‌شود. در ادامه مقاومت قانون هدایتی ارائه شده در مقابل اغتشاشات و چگونگی عملکرد آن با استفاده از روش دوم لیاپانوف و نتایج شبیه سازی نشان داده می‌شود. قانون هدایت بهینه و مقاوم ارائه شده برای اجرا در عمل نیز ساده می‌باشد.

1. INTRODUCTION

The optimal control and estimation theory have been used to derive modern guidance laws with improved performance. Most optimal guidance laws (OGL) have been derived from linear-quadratic optimal control theory to obtain feedback solutions. Optimal guidance law and nonlinear estimation for interception of decelerating target [2] and accelerating target [3] have been extended for highly maneuvering target scenarios. Closed-form optimal guidance law has been applied for missile system considering time-varying velocity [5] and internal dynamics with uncertain time lag [6]. Optimal guidance time-to-go [7] and OGL subject to various constraints have been studied in [4,8]. Optimal midcourse fixed-interval guidance has been developed for intercepting a target with constant acceleration vector in [9]. An extensive literature review on guidance laws, in general, and optimal guidance laws, in particular has been performed in [10,16].

Although the optimal guidance is accurate and economical in energy consumption, it is difficult to implement due to its dependence on the information of relative range, relative velocity, and even target acceleration. Optimal control theory assumes that the future maneuver strategy of the target is completely defined, so any small change in assumptions produces undesired results. Integrating optimal control with sliding-mode allows the interceptor to correct itself for inaccurate predictions of target maneuvers and unmodeled dynamics.

The sliding-mode control method provides a systematic approach to the problem of consistent performance in the face of modeling imprecision. The main advantage of sliding-mode control is that the system response remains insensitive to model uncertainties and disturbances [11,12]. Although the technique has good robustness properties, pure sliding-mode control presents drawbacks that include large control authority requirements. The performance of pure sliding-mode can be improved by making the states to reach the sliding surface in a manner as optimal as possible. Sliding-mode control law has been developed for a nonlinear system considering air-to-air missile-target interception scenario [13]. It has been proved that the performance of the feedback controller is robust to certain parameter variations in the model by assuming that the information for the maximum target acceleration is available. Systematic approach to the design of guidance law using variable structure control has been studied in [14]. In this paper decreasing boundary-layer scheme has been introduced for high-gain sliding mode control. Integration of optimal control and sliding-mode control based on guidance law has been proposed for a homing-missile against target maneuvers [15].

This work deals with integrating three-dimensional optimal guidance with sliding-mode theory to obtain a new guidance law with robustness against disturbances, good dynamic performance, energy saving properties, and terminal accuracy for fixed-interval propulsive maneuvers.

2. PROBLEM FORMULATION

2.1. Equations of Motion The governing equations of motion for particle P (interceptor or target) with a given acceleration vector, $a_{\mathbf{p}}(t)$, are given by

$$\begin{cases} \dot{r}_p = v_p \\ \dot{v}_p = a_p(t) \end{cases} \tag{1}$$

in which r, v and a denote position, velocity, and acceleration vectors, respectively. The subscript “p” represents particle P. Integrating the preceding equations with respect to time, the final displacement and velocity at final time t_f are obtained as follows:

$$v_p(t_f) = v_p(t) + \int_t^{t_f} a_p(\xi)\, d\xi \tag{2a}$$

$$r_p(t_f) = r_p(t) + v_p\, t_g + \int_t^{t_f} (t_f - \xi) a_p(\xi)\, d\xi \tag{2b}$$

Where t_g is time-to-go until intercept (i.e. $t_g = t_f - t$).

The three-dimensional intercept geometry with respect to an inertial reference (Oxyz) is shown in Figure 1, in which interceptor M is moving toward its desired final position F.

The relative displacement r and velocity v for the intercept problem are defined as

$$r = r_t - r_m \tag{3a}$$

$$v = v_t - v_m \tag{3b}$$

Where subscripts “m” and “t” denote the missile (interceptor) and target, respectively.

2.2. Zero-Effort Errors The zero-effort miss at time t, zem(t), is the distance that the interceptor will miss its target if the interceptor makes no corrective maneuvers after time t; therefore,

$$zem(t) = r_{m_f} - r_m(t_f), \quad u(\xi) = 0 \ \text{ for } \ t < \xi < t_f \tag{4}$$

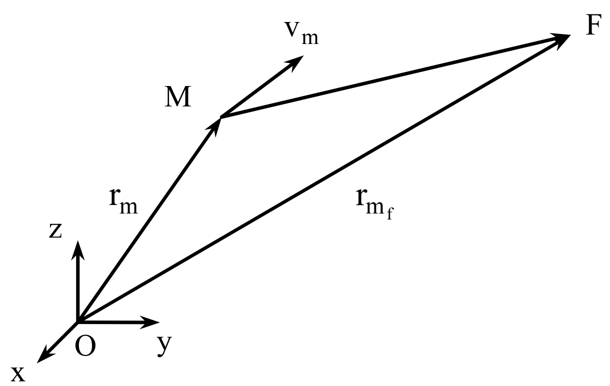

Figure 1. Interceptor geometry.

Where r_{m_f} is the desired final position of interceptor and u is the acceleration command.

Assume the interceptor acceleration as $a_m = c(t) + u$ in which c(t) is the acceleration exerting on the interceptor excluding acceleration command, like gravitational acceleration. We assume that c(t) is given as a function of time.

Using Equations 2, the final position vector of the interceptor, in the absence of corrective maneuvers can be calculated as

$$v_m(t_f) = v_m(t) + \int_t^{t_f} c(\xi)\, d\xi \tag{5}$$

$$r_m(t_f) = r_m(t) + v_m \cdot t_g + \int_t^{t_f} (t_f - \xi)\, c(\xi)\, d\xi \tag{6}$$

Hence,

$$zem(t) = r_{m_f} - r_m(t) - v_m \cdot t_g - \int_t^{t_f} (t_f - \xi)\, c(\xi)\, d\xi \tag{7}$$

Consider the case in which we are to intercept a maneuvering target with a given acceleration $a_t(t)$. By using Equations 2-3, the expression for the zem in terms of relative position and velocity can be expressed as

$$zem(t) = r(t) + v(t).t_g + \int_t^{t_f} (t_f - \xi)\left[a_t(\xi) - c(\xi)\right] d\xi \tag{8}$$

3. OPTIMAL FIXED-INTERVAL GUIDANCE LAW

Our objective is to obtain the optimal guidance law in the time period $0 < t < t_b$ (i.e. until burnout) and making the interceptor to reach the final position r_{m_f} satisfying following state equations.

$$\begin{cases} \dot{r}_m = v_m \\ \dot{v}_m = a_m = c(t) + u \end{cases} \tag{9}$$

For $t > t_b$ we have no corrective maneuvers. Thus, u should be such that it guides the interceptor to the final position over the whole period of motion.

The desired performance index for guidance law in the exoatmospheric interception is the amount of fuel consumption required for corrective maneuvers to have a successful intercept. The lateral divert, $\Delta V = \int_0^{t_f} |u|\, dt$, imparted to an interceptor during intercept, is directly related to the amount of fuel required by the interceptor [1]. Since there is no analytical solution for this objective function, we must minimize the following performance index instead as follows:

$$PI = \frac{\omega}{2} miss^T miss + \frac{1}{2}\int_0^{t_b} u^T(\tau)\, u(\tau)\, d\tau \tag{10}$$

With $miss = r_{mf} - r_m(t_f)$ and weighting factor ω. By choosing very large values for ω, we can guarantee that miss distance takes on small values and interception takes place undoubtedly. Since, we have no corrective maneuver for $t > t_b$, final position of interceptor can be written as

$$r_m(t_f) = r_{m_b}(t_b) + v_{mb} \cdot (t_f - t_b) + \int_{t_b}^{t_f} (t_f - \xi)\, c(\xi)\, d\xi \tag{11}$$

Substituting the preceding relation into Equation 10, the objective function can be rewritten as

$$PI = \varphi(r_{m_b}, v_{m_b}, t_b) + \frac{1}{2}\int_t^{t_b} u^T(\tau)\, u(\tau)\, d\tau \tag{12}$$

The optimal control can then be found as

$$u = At - B \tag{13}$$

In which A and B are found to be

$$A = c_2\left[r_{m_b} - r_{m_f} + \int_{t_b}^{t_f} (t_f - \xi)\, c(\xi)\, d\xi\right] + c_1 v_{m_b} \tag{14a}$$

$$B = (c_1 + c_2 t_b)\left[r_{m_b} - r_{m_f} + \int_{t_b}^{t_f} (t_f - \xi)\, c(\xi)\, d\xi\right] + (c_3 + c_1 t_b) v_{m_b} \tag{14b}$$

From $\lambda_r^T(t_b) = \frac{\partial \varphi}{\partial r_m}(t_b)$ and $\lambda_v^T(t_b) = \frac{\partial \varphi}{\partial v_m}(t_b)$, where λ_r and λ_v are Lagrange multipliers and $c_1 = \omega(t_f - t_b)$, $c_2 = \omega$, and $c_3 = \omega(t_f - t_b)^2$. Substituting Equations 14 into Equation 13, we obtain

$$u = -(c_1 + c_2 t_{bg})\left[r_{m_b} - r_{m_f} + \int_{t_b}^{t_f} f(t_f - \xi)\, c(\xi)\, d\xi \right] - (c_3 + c_1 t_{bg}) v_{m_b} \tag{15}$$

By replacing Equation 13 into state Equation 9 and solving it for $r_m(t_b)$ and $v_m(t_b)$, we will have

$$v_m(t_b) = v_m + \frac{A}{2}(t_b^2 - t^2) - B(t_b - t) + \int_t^{t_b} c(\xi)\, d\xi \tag{16}$$

$$r_m(t_b) = r_m + v_m(t_b)\cdot(t_b - t) - \int_t^{t_b} (\xi - t) c(\xi)\, d\xi - \frac{A}{6}(2t_b^3 - 3t_b^2 t + t^3) + \frac{B}{2}(t_b - t)^2 \tag{17}$$

Now if we replace A and B from Equations 14 in the above equations and solve the set of equations for $r_m(t_b)$ and $v_m(t_b)$ we can find them in terms of r_m and v_m. By substituting the resulted relations into Equation 15, it can be rewritten as

$$u = \Lambda \cdot \left[r_{m_f} - r_m - v_m t_g - \int_{t_b}^{t_f} (t_f - \xi)\, c(\xi)\, d\xi \right] \tag{18}$$

In which

$$\Lambda = \frac{3t_g}{\frac{3}{\omega} + t_g^3 - (t_g - t_{bg})^3} \tag{19}$$

For the sake of small miss distance for interception, the value for ω is assumed to be very large and its effect on Equation 19 is discarded. If we were to intercept a maneuvering target with a given $a_t(t)$, we should use Equation 2b for the target. Hence, the optimal command in terms of relative position and velocity are obtained as

$$u = \Lambda \cdot \{ r + v t_g + \int_t^{t_f} f(t_f - \xi)\left[a_t(\xi) - c(\xi) \right] d\xi \} \tag{20}$$

If the only acceleration acting on interceptor, excluding acceleration command considered gravitational acceleration, i.e. $c(t) = g_m$ and target moves under gravitational acceleration $(a_t(t) = g_t)$, then Equation 20 will be conducted as

$$u = \Lambda \cdot \{ r + v t_g + \int_t^{t_f} f(t_f - \xi)\left[g_t(\xi) - g_m(\xi) \right] d\xi \} \tag{21}$$

If we assume that the difference between target and interceptor positions is small enough so that the gravitational acceleration can be taken approximately equal for the interceptor and target, then the resulted expression for u will be reduced to [9]

$$u = \Lambda \cdot (r + v t_g) \tag{22}$$

For the case in which the positional difference is considerable, the integrand of Equation 21 must be estimated.

4. OPTIMAL SLIDING-MODE GUIDANCE FOR FIXED-INTERVAL PROPULSIVE MANEUVERS

Optimal guidance law by itself can not be robust against disturbance and modeling inaccuracies and they can have strong adverse effects on the performance of the guidance law. To overcome this drawback, it is proposed that the optimal guidance integrated with the sliding-mode control theory to produce a new guidance method.

The optimal guidance law for fixed-interval propulsive maneuvers can be expressed in the following form:

$$u = \Lambda \cdot \text{zem} \tag{23}$$

Where zem is calculated using Equation 8. Now we define a new state zem. To zero out the zero effort miss, the switching surface is chosen as

$$s = zem = 0 \tag{24}$$

To ensure that the new state zem approaches the sliding-mode s = 0, the result in section three is used to constitute an optimal reaching law of sliding-mode. Differentiating zem with respect to time produces

$$\begin{aligned} \dot{zem} &= -\dot{r}_m - \dot{v}_m.t_g + v_m + c(t).t_g \\ &= -\dot{r}_m - [u + c(t)].t_g + v_m + c(t).t_g \\ &= -u.t_g \\ &= -\Lambda \cdot t_g \cdot zem \end{aligned} \tag{25}$$

Due to Equation 25, the optimal reaching law is constructed as

$$\dot{s} = -K(t)s - E(t)\text{sign}(s) \tag{26}$$

Where $K(t) = \Lambda.t_g$, $E(t) = \varepsilon \cdot t_g$, and $\varepsilon = \text{const.} > 0$.

Substituting Equation 23 into Equation 25 and Equation 24 into Equation 26 yields the optimal sliding-mode guidance law as

$$u = \Lambda \cdot zem + \varepsilon\,\text{sign}(zem) \tag{27}$$

During the motion outside the sliding-mode, the switching term in the guidance law is liable to cancel the influence of disturbance, and try to make the motion as optimal as possible.

Now, robustness of this guidance law against disturbance should be verified to ensure $zem \to 0$. We consider a bounded disturbance $d(t)$ is exerted on the system as follows

$$\begin{cases} \dot{r}_m = v_m \\ \dot{v}_m = a_m = c(t) + u + d(t) \end{cases} \tag{28}$$

Where we assume that its upper limit is known as d_{max}. By using the second method of Lyapunov, let the Lyapunov function be $V = \frac{1}{2} zem^2$. Substituting Equation 23 into Equation 25 gives

$$\dot{zem} = -t_g \cdot u + D(t) \tag{29}$$

Where $D(t) = t_g.d(t)$ and defined as disturbance in the zem coordinates. Differentiating V with respect to time and considering Equation 29 results in

$$\dot{V} = t_g \cdot zem[-u + d(t)] \tag{30}$$

Substituting Equation 27 into Equation 30 gives

$$\begin{aligned} \dot{V} &= -\Lambda \cdot t_g \cdot zem^2 - t_g.zem\left[\varepsilon\,\text{sign}(zem) - d(t)\right] \le \\ &- \Lambda \cdot t_g \cdot zem^2 - t_g.zem\left[\varepsilon.\text{sign}(zem) - d_{max}\right] \end{aligned} \tag{31}$$

Substituting Equation 19 into Equation 31 and noting that $t_g > 0$ and $t_g^3 - (t_g - t_{bg})^3 \ge 0$, it is apparent that the first term in Equation 31 is negative. Obviously, if $\varepsilon > |d_{max}|$ is satisfied, then the second term in Equation 31 is also negative. Thus $\dot{V} < 0$ is sufficiently ensured.

In practical guidance, the sign function sign (zem) in Equation 27 can be replaced by a continuous function zem/(|zem|+γ) to alleviate chattering, where γ is a vector with positive real components. The guidance law then becomes

$$u = \Lambda \cdot zem + \varepsilon \frac{zem}{|zem| + \gamma} \tag{32}$$

5. SIMULATION STUDY

Suppose an interceptor at the origin with initial velocity of 1000 m/s in the vertical direction. The desired final position is (35, 35, 30 km). The interceptor must reach the final position at $t_f = 30$s, while its guidance period is $t_b = 30$s. We consider a disturbance exerted on the system as D(t) = D sin(Ωt) where $D_i = 10$, i = 1,2,3 and Ω = π/5. It is noted that for moving and maneuvering target with estimated acceleration $a_t(t)$, zem must be calculated by Equation 8.

We use Equation 32 to implement the optimal sliding-mode guidance law for fixed-interval propulsive maneuvers, where ε = 10, $\gamma_i = 10^{-4}$ and i = 1, 2, 3. The fourth-order Rung-Kutta algorithm is used to obtain the numerical solution of the target and missile motion equation. The guidance command is given out by a microcomputer onboard the interceptor and the sampling period as assumed to be 10 ms.

Simulation results show that the final miss distance is less than 0.17 m. For comparison purpose, applying pure optimal guidance law for fixed-interval propulsive maneuvers will induce miss distance of 4.93 m. This indicates that the switching term in the proposed guidance law is able to cancel out the influence of disturbances such that the motion outside of the sliding mode approaches the optimal motion. Figure 2 shows interceptor trajectory. Commanded accelerations of optimal sliding-mode guidance and pure optimal guidance for fixed-interval propulsive maneuvers are shown in Figure 3. Magnitude of zem for both guidance laws is demonstrated in Figure 4.

6. CONCLUSION

A new guidance law is proposed for fixed-interval propulsive maneuvers by integrating optimal and sliding-mode theories to achieve robustness against disturbances and terminal accuracy. The guidance command is derived based on three-dimensional engagement kinematics. The proposed guidance law requires a disturbance limit, and therefore exact information of disturbance is not necessary. The effectiveness of the presented method is established by the second method of Lyapanuv and the robustness of the guidance law against disturbances was demonstrated by simulation results. It was shown that the guidance law over whole period of guidance can be extracted from the proposed guidance law by canceling the coasting phase. Furthermore, the presented guidance law is simple to implement in practical applications.

7. NOMENCLATURE

a	Acceleration vector, a = |a|
g	Gravitational acceleration, g = |g|
PI	Performance Index, $\int_t^t f\, u(\tau)^T u(\tau)\, d(\tau)$
miss	Miss distance vector, miss = |miss|
r	Displacement vector, r = |r|
t	Current time
t_g	Time-to-go until intercept (t_f -t)
t_{bg}	Time-to-go until burnout (t_b -t)
v	Velocity vector, v = |v|
zem	Zero-effort miss, zem = |zem|
Λ	Guidance gain

Subscript

o	Initial Value
f	Final Value
m	Interceptor
t	Target

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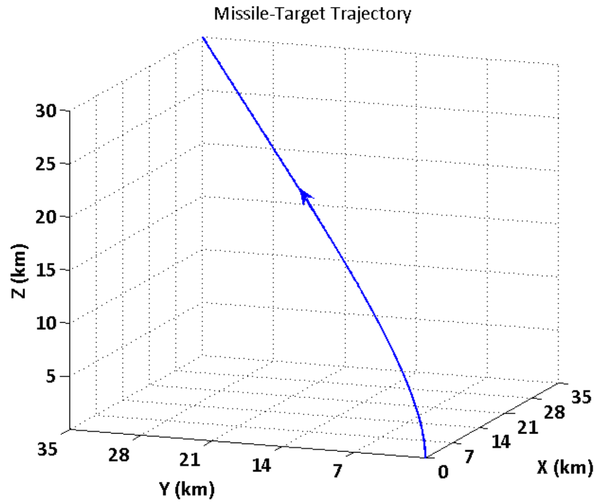

Figure 2. Interceptor trajectory.

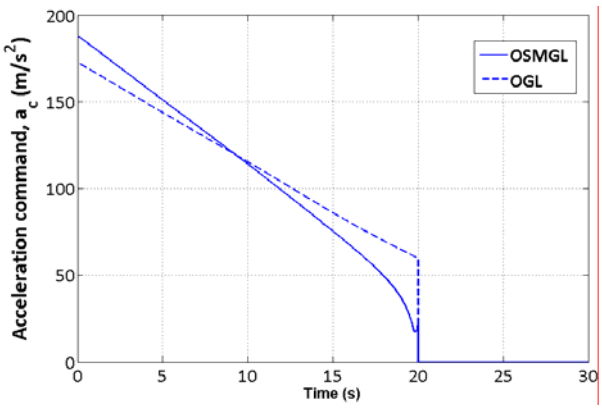

Figure 3. Acceleration commands; OSMGL and OGL for fixed-interval propulsive maneuvers.

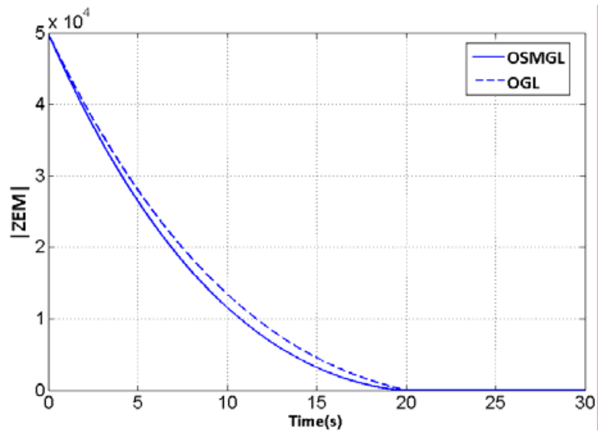

Figure 4. |ZEM| for fixed-interval propulsive maneuvers.

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