AUXILIARY POTENTIALS IN CHIRAL MEDIA

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(**Received: December 23, 2004 - Accepted in Revised Form: March 18, 2007)**

Abstract In the present paper, the expressions for scalar and vector potentials in lossless isotropic chiral media are analyzed. Propagating eigenvalues of these potentials are then obtained. Furthermore by decomposition of sources and fields in a chiral medium, we introduce the auxiliary right-and left-handed potentials and find the associated fields. These potentials are used to solve the problem of a horizontal electric dipole (HED) above a chiral half space. Auxiliary right and left handed Hertzian vector potentials are introduced and $\vec{E}$ and $\vec{H}$ fields in terms of these potentials are obtained. The Hertzian potentials due to VED and/or VMD sources within a chiral half space are determined in terms of two-dimensional Fourier spectral domain and the expressions for EM fields with respect to these potentials are presented.

Keywords Scalar Potentials, Vector Potentials, Auxiliary Potentials, Chiral Media, Chirality

چکیده در مقاله حاضر پتانسیل های برداری و عددی کایرال بدون تلفات و مقادیر ویژه انتشار این پتانسیل ها در این محیط ها بدست آمده است. علاوه بر این با تجزیه ی منابع و میدانها در محیط کایرال پتانسیل های کمکی راستگرد و چپگرد معرفی شده و میدانهای وابسته به هر یک از آنها تعیین گردیده است. این پتانسیل ها برای حل مساله دو قطبی الکتریکی (HED) در بالای یک نیم فضای کایرال بکار گرفته شده است. پتانسیل های برداری راستگرد و چپگرد هرتز معرفی شده و میدانهای $\vec{E}$ و $\vec{H}$ بر حسب آنها بدست آمده اند. پتانسیل های هرتز مربوط به منابع VED و/ یا VMD داخل یک نیم فضای کایرال بر حسب حوزه ی طیفی فوریه دو بعدی تعیین گردیده و میدانهای الکترومغناطیسی بر حسب این پتانسیل ها بیان شده است.

1. INTRODUCTION

In recent decades, much attention has been given to the electromagnetic properties of chiral materials. One of the important properties of chiral media is optical activity. In such media, the EM waves propagate with two different wave numbers which correspond to right and left handed circularly polarized eigenmodes. With regard to radiation and propagation of EM waves in these media, several important problems have been investigated. Among which are Dyadic Green's functions in chiral media [1,2], reflection and refraction of EM waves in chiral-achiral interface [3,4], propagating properties of EM in chiral media [5] and Radiation in chiral media [6-9]. While EM properties of chiral media are extensively investigated, an adequate treatment of auxiliary potentials is not reported. Some applications of auxiliary potentials are also studied for uniaxial chiral omega media [10] and pseudochiral omega media [11]. In the present paper, scalar and vector potentials are introduced in order to investigate the problem of sources and fields in chiral media. The decomposition of sources and fields in chiral media is used for this purpose [8]. As an example the application of these auxiliary potentials in the analysis of chiral-achiral planar interface problems involving a horizontal electric dipole (HED) is studied. Then the auxiliary right and left handed potentials are introduced and the fields according to an electric and/or magnetic source in infinite chiral media are determined by decomposing sources and fields [8]. The expressions for these potentials due to localized electric and magnetic sources are obtained. The EM fields due to a vertical electric dipole (VED) and/or a magnetic source (VMD) located inside a half space chiral medium is carried out. The method used is based on the decomposition of sources and fields using the introduced auxiliary potentials followed by a two-dimensional Fourier transformation. An application of interface conditions finally yields these potentials in the spectral domain. The expressions for the EM fields in terms of auxiliary potentials have been achieved. Finally the approximate relations for these potentials and the small chirality conditions are derived.

2. ELECTRIC AND MAGNETIC VECTOR POTENTIALS

Consider a lossless isotropic chiral media in the presence of an electric current source, $\vec{J}$. Our purpose is to obtain the expression for the magnetic vector potential $\vec{A}$ and the relations of the electric and magnetic fields $\vec{E}$ and $\vec{H}$ in terms of $\vec{A}$. The constitutive relations in chiral media can be considered as [8]

$$\vec{D} = \varepsilon \vec{E} - j\kappa_r \sqrt{\mu\varepsilon}\, \vec{H} \tag{1}$$

$$\vec{B} = \mu \vec{H} + j\kappa_r \sqrt{\mu\varepsilon}\, \vec{E} \tag{2}$$

Where κ_r is the chirality parameter (dimensionless).

We introduce the potentials $\vec{A}_c$ and Φ_{ec} as auxiliary potentials in the chiral media. Equations 1 and 2 yield the so called Lorentz gauge in chiral form:

$$\nabla \cdot \vec{A}_c = -j\omega\mu\varepsilon (1-\kappa_r^2)\Phi_{ec} \tag{3}$$

Where A_c satisfies Equation 4

$$\nabla^2 \vec{A}_c + \omega^2 \mu\varepsilon (1-\kappa_r^2)\vec{A}_c + 2\omega\kappa_r \sqrt{\mu\varepsilon}\, \nabla \times \vec{A}_c = -\mu \vec{J} \tag{4}$$

Equation 4 yields $\vec{A}_c$ for a chiral media and reduces to the conventional equation for $\vec{A}$, when chirality is set to zero. The scalar electric and magnetic fields are thus given by:

$$\vec{E} = -j\omega \left[\vec{A}_c + \frac{(\nabla\nabla \cdot \vec{A}_c)}{\omega^2 \mu\, \varepsilon (1-\kappa_r^2)} \right], \quad \vec{H} = \nabla \times \frac{\vec{A}_c}{\mu} - \left(\frac{\omega\kappa_r}{\eta}\right) \left[\vec{A}_c + \frac{(\nabla\nabla \cdot \vec{A}_c)}{\omega^2 \mu\varepsilon (1-\kappa_r^2)} \right] \tag{5}$$

Where:

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}.$$

As for the electric vector potential in chiral media, consider a magnetic current source $\vec{M}$ within a chiral medium. In a similar manner as the previous section by considering the electric vector potential in the chiral medium, $\vec{F}_c$, with $\vec{D} = -\nabla \times \vec{F}_c$ and applying the duality properties between electric and magnetic fields, the Lorentz gauge for the potential $\vec{F}_c$ is considered as

$$\nabla \cdot \vec{F}_c = -j\omega\mu\varepsilon (1-\kappa_r^2)\, \Phi_{mc} \tag{6}$$

Where $\vec{F}_c$ satisfies Equation 7

$$\nabla^2 \vec{F}_c + \omega^2 \mu\varepsilon (1-\kappa_r^2)\, \vec{F}_c + 2\omega\kappa_r \sqrt{\mu\varepsilon}\, \nabla \times \vec{F}_c = -\varepsilon \vec{M} \tag{7}$$

In view of Equation 5, and in the presence of both electric and magnetic sources, generalized expressions for $\vec{E}$ and $\vec{H}$ fields in terms of electric and magnetic chiral potentials can be obtained as follows:

$$\vec{E} = -j\omega\left[\vec{A}_c + \frac{(\nabla\nabla\cdot\vec{A}_c)}{\omega^2\mu\varepsilon(1-\kappa_r^2)}\right] - \frac{\nabla\times\vec{F}_c}{\varepsilon} + \omega\,\kappa_r\,\eta\left[\vec{F}_c + \frac{(\nabla\nabla\cdot\vec{F}_c)}{\omega^2\mu\varepsilon(1-\kappa_r^2)}\right] \quad (8)$$

and

$$\vec{H} = \frac{\nabla\times\vec{A}_c}{\mu} - \left(\frac{\omega\,\kappa_r}{\eta}\right)\left[\vec{A}_c + \frac{(\nabla\nabla\cdot\vec{A}_c)}{\omega^2\mu\,\varepsilon\,(1-\kappa_r^2)}\right] - j\omega\left[\vec{F}_c + \frac{(\nabla\nabla\cdot\vec{F}_c)}{\omega^2\mu\,\varepsilon\,(1-\kappa_r^2)}\right] \quad (9)$$

The electric and magnetic scalar potentials for the chiral media readily follow from Equation 1 through Equation 7:

$$\nabla^2\Phi_{ec} + \omega^2\mu\,\varepsilon\,(1-\kappa_r^2)\,\Phi_{ec} = -\frac{j\nabla\cdot\vec{J}}{\omega\,\varepsilon\,(1-\kappa_r^2)} = -\frac{\rho_e}{\varepsilon\,(1-\kappa_r^2)} \quad (10)$$

$$\nabla^2\Phi_{mc} + \omega^2\mu\,\varepsilon\,(1-\kappa_r^2)\,\Phi_{mc} = -\frac{j\nabla\cdot\vec{M}}{\omega\,\mu\,(1-\kappa_r^2)} = -\frac{\rho_m}{\mu\,(1-\kappa_r^2)} \quad (11)$$

Where ρ_e and ρ_m are the electric and magnetic volume charge densities respectively.

2.1. Eigenvalues for Vector Potentials

In a source free chiral medium Equation 4 reduces to:

$$\nabla^2\vec{A}_c + \omega^2\mu\,\varepsilon\,(1-\kappa_r^2)\,\vec{A}_c + 2\,\omega\,\kappa_r\sqrt{\mu\,\varepsilon}\;\nabla\times\vec{A}_c = 0 \quad (12)$$

Without loss of generality we assume that the wave propagates along the z axis and the wave number in z direction is denoted by h. Thus

$$\vec{A}_c = (A_{cx}\,\hat{x} + A_{cy}\,\hat{y} + A_{cz}\,\hat{z})\,\exp(-j\,h\,z) \quad (13)$$

By substituting Expression 13 in 12, to have a propagating wave along the z axis with nonzero components in $\hat{x}$ and $\hat{y}$ directions, the following determinant must vanish;

$$\begin{vmatrix} \omega^2\mu\varepsilon\,(1-\kappa_r^2) - h^2 & j\,2\,\omega\kappa_r\sqrt{\mu\varepsilon}\,h \\ -j\,2\,\omega\kappa_r\sqrt{\mu\varepsilon}\,h & \omega^2\mu\varepsilon\,(1-\kappa_r^2) - h^2 \end{vmatrix} = 0 \quad (14)$$

Which yields

$$h_{1,2} = \pm\,\omega\sqrt{\mu\varepsilon}\;(1+\kappa_r)\;, \qquad h_{3,4} = \pm\,\omega\sqrt{\mu\varepsilon}\;(1-\kappa_r). \quad (15a,b)$$

The solutions $h_{1,2}$ in Equation 15a and $h_{3,4}$ in Equation 15b correspond to the right and left handed circularly polarized waves respectively, which propagate along the z axis in both directions. In this case, vector $\vec{A}_c$ involves only two components in $\hat{x}$ and $\hat{y}$ directions with different eigenvalues $h_{1,2}$ and $h_{3,4}$, each of which are as follows

$$A_{ic} = \sum_{k=1}^{4} A_{ik}\exp(-jh_k z)\,, \quad i = x, y \quad (16)$$

In a similar manner the same eigenvalues for $\vec{F}_c$ can be deduced.

3. ANALYSIS OF POTENTIALS IN CHIRAL MEDIA BY DECOMPOSITION OF SOURCES

3.1. Auxiliary Right and Left Handed Scalar and Vector Potentials in Chiral Media

Let us consider a chiral medium in the presence of a time harmonic electric current source $\vec{J}$ and a magnetic current source $\vec{M}$. The constitutive equations of such a chiral medium are given by Equations 1 and 2. By decomposing $\vec{E}$ and $\vec{H}$ fields into two wave fields $\vec{E}_+$, $\vec{H}_+$ and $\vec{E}_-$, $\vec{H}_-$, we treat the chiral medium as two nonchiral media with respective effective parameters μ_+, ε_+ and μ_-, ε_-. The wave fields are defined by [8]

$$\vec{E}_{\pm} = \frac{1}{2}(\vec{E} \mp j\eta \vec{H})$$

and

$$\vec{E} = \vec{E}_{+} + \vec{E}_{-} \qquad (17a,b)$$

$$\vec{H}_{\pm} = \frac{1}{2}(\vec{H} \pm \frac{j}{\eta}\vec{E})$$

and

$$\vec{H} = \vec{H}_{+} + \vec{H}_{-} \qquad (18a,b)$$

Where,

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} .$$

Since the wave fields do not couple in an isotropic chiral medium, Maxwell's equations can be written as a pair of double equations as follows

$$\nabla \times \vec{E}_{\pm} = -j\omega\mu_{\pm}\vec{H}_{\pm} - \vec{M}_{\pm} , \quad \mu_{\pm} = \mu(1 \pm \kappa_r) \qquad (19)$$

$$\nabla \times \vec{H}_{\pm} = j\omega\varepsilon_{\pm}\vec{E}_{\pm} + \vec{J}_{\pm} , \quad \varepsilon_{\pm} = \varepsilon(1 \pm \kappa_r) \qquad (20)$$

The decomposed electric and magnetic sources $\vec{J}_{\pm}$ and $\vec{M}_{\pm}$ are defined as

$$\vec{J}_{\pm} = \frac{1}{2}(\vec{J} \mp \frac{j}{\eta}\vec{M}) , \quad \vec{M}_{\pm} = \frac{1}{2}(\vec{M} \pm j\eta\vec{J}) = \pm j\eta\vec{J}_{\pm} \qquad (21)$$

Due to the relations between $\vec{E}_{\pm}$ and $\vec{H}_{\pm}$ on one hand and $\vec{J}_{\pm}$ and $\vec{M}_{\pm}$ on the other, Equation 20 is actually the same pair of equations as Equation 19 and they can also be expressed as

$$\nabla \times \vec{E}_{\pm} \mp k_{\pm}\vec{E}_{\pm} = \mp j\eta\vec{J}_{\pm} \qquad (22)$$

With regard to Expressions 19 and 20, we can observe that the wave fields $\vec{E}_{\pm}$ and $\vec{H}_{\pm}$ satisfy Maxwell's equations corresponding to a pair of nonchiral isotropic media with effective parameters $\mu_{\pm}$, $\varepsilon_{\pm}$. Therefore, in this case we can use all the potentials commonly applied in isotropic media. With this respect, we can define the above mentioned wave fields in terms of the right and left handed potentials $\vec{A}_{\pm}$ and $\vec{F}_{\pm}$ and the corresponding scalar potentials $\Phi_{e\pm}$ and $\Phi_{m\pm}$ respectively [8]. The wave fields $\vec{E}_{\pm}$ and $\vec{H}_{\pm}$ can therefore be written in terms of the above mentioned right and left handed potentials. In view of Lorentz's conditions, these fields finally reduce to

$$\vec{E}_{\pm} = -\frac{\nabla \times \vec{F}_{\pm}}{\varepsilon_{\pm}} - j\omega\left[\vec{A}_{\pm} + \frac{1}{k_{\pm}^2}\nabla\nabla \cdot \vec{A}_{\pm}\right] ,$$
$$\vec{H}_{\pm} = \frac{\nabla \times \vec{A}_{\pm}}{\mu_{\pm}} - j\omega\left[\vec{F}_{\pm} + \frac{1}{\kappa_{\pm}^2}\nabla\nabla \cdot \vec{F}_{\pm}\right] \qquad (23)$$

Where: $k_{\pm}^2 = \omega^2\mu_{\pm}\varepsilon_{\pm}$, $\varepsilon_{\pm} = \varepsilon(1 \pm \kappa_r)$ and $\mu_{\pm} = \mu(1 \pm \kappa_r)$. Therefore,

$$\left|k_{\pm}\right| = \omega\sqrt{\mu_{\pm}\,\varepsilon_{\pm}} = \omega\sqrt{\mu\varepsilon}\ (1 \pm \kappa_r) \qquad (24)$$

The above wave numbers are in agreement with Equations 15a and 15b, and the eigenvalues given in [8].

3.2. Right and Left Handed Green Functions

Consider a general case of a chiral medium in the presence of localized electric and magnetic current sources; $\vec{J} = \delta(\vec{R} - \vec{R}')\,\hat{n}$ and $\vec{M} = \delta(\vec{R} - \vec{R}')\hat{n}$ respectively. Due to the linearity property of the decomposed nonchiral media and Maxwell's Equations, we can deduce $\vec{G}_A$ and $\vec{G}_F$ as:

$$\vec{G}_{A\pm} = \vec{G}^{e}_{A\pm} = \frac{\mu_{\pm}}{8\pi}(1 \pm \frac{j}{\eta})\frac{\exp(-jk_{\mp}\left|\vec{R} - \vec{R}'\right|)}{\left|\vec{R} - \vec{R}'\right|}\,\hat{n} \qquad (25)$$

and

$$\vec{G}_{F\pm} = \vec{G}^{e}_{F\pm} = \frac{\varepsilon_{\pm}}{8\pi}(1 \pm j\eta)\frac{\exp(-jk_{\mp}|\vec{R}-\vec{R}'|)}{|\vec{R}-\vec{R}'|}\hat{n} \quad (26)$$

The $\hat{z}$ component of Green's electric field function due to an electric source oriented in $\hat{z}$ direction is obtained as,

$$\vec{G}^{e}_{z\pm} = -j\omega\frac{\mu_{\pm}}{8\pi}\left[1 \pm \frac{1}{k_{\pm}^{2}}\frac{d^{2}}{dz^{2}}\right]\frac{\exp(-jk_{\pm}|\vec{R}-\vec{R}'|)}{|\vec{R}-\vec{R}'|}\hat{z} \quad (27)$$

Equation 27 is in agreement with [9].

3.3. A Horizontal Electric Dipole Source Above a Chiral Half Space Consider a HED source, $\vec{J} = IL\,\delta(z-h)\,\hat{x}$ located in air, $z > 0$, a distance of h above the interface, $z = 0$, with the isotropic chiral medium in region 2, $z < 0$; (Figure 1).

By applying the two-dimensional Fourier transform and Sommerfeld identity, one can find, similar to [8]:

$$\tilde{\vec{A}}^{i(1)} = \frac{\mu_0 IL}{2j} e^{-j\beta_0|z-h|}\,\hat{x} \quad \text{for } z > 0 \quad (28)$$

Where; $\beta_0 = \sqrt{k_0^2 - (k_x^2 + k_y^2)}$, $k_0 = \omega\sqrt{\mu_0\varepsilon_0}$ and superscript 1 is used for region 1. The incident potential $\tilde{\vec{F}}^{i(1)}$ is simply zero; therefore the reflected potentials are:

$$\tilde{A}_x^{r(1)} = R_e^{(1)} A^i e^{-j\beta_0(z+h)}$$

and

$$\tilde{F}_x^{r(1)} = R_m^{(1)} A^i e^{-j\beta_0(z+h)}\,. \quad (29)$$

While the transmitted potentials are given by:

$$\tilde{A}_{x\pm}^{t(2)} = T_{\pm} A^i e^{j(\beta_{\pm} z - \beta_0 h)}$$

and

$$\tilde{F}_{x\pm}^{t(2)} = \pm\frac{j}{\eta}\tilde{A}_{x\pm}^{t(2)} \quad (30)$$

Where;

$\beta_{\pm} = \sqrt{k_{\pm}^2 - (k_x^2 + k_y^2)}$, $k_{\pm} = \omega\sqrt{\mu_{\pm}\,\varepsilon_{\pm}}$, $\mu_{\pm} = \mu(1 \pm k_r)$, $\varepsilon_{\pm} = (1 \pm k_r)$ and superscripts 1 and 2 denote regions 1 and 2 respectively.

The electric and magnetic field components in terms of the vector potentials are given in Appendix A. In view of the tangential components of $\vec{E}$ and $\vec{H}$ fields given in Appendix A across the interface z = 0 and using the expressions 28-30 for vector potentials, and some mathematical manipulations, the reflection and transmission coefficients ($R_e^{(1)}, R_m^{(1)}, T_+$ and T_-) can be obtained from a system of four simultaneous equations, i.e.

$$R_e^{(1)} = -1 + \frac{2\beta_0(D_+ a_- - D_- a_+)}{\mu_0 a_0 (L_+ D_- - L_- D_+)}, \quad R_m^{(1)} = -j\frac{2\beta_0(D_+ a_- + D_- a_+)}{\eta\mu_0 a_0 (L_+ D_- - L_- D_+)} \quad (31)$$

$$T_+ = -\frac{2\beta_0 D_-}{\mu_0(L_+ D_- - L_- D_+)}, \quad T_- = \frac{2\beta_0 D_+}{\mu_0(L_+ D_- - L_- D_+)} \quad (32)$$

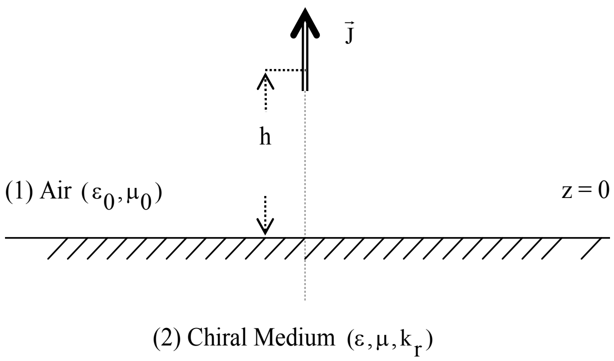

Figure 1. A localized current source a distance "h" above air-chiral medium interface.

Where a_0, a_+, a_-, $D_\pm$ and $L_\pm$ are given in Appendix B. The z components of E_z and H_z in both regions in terms of auxiliary potentials are:

$$\tilde{E}_z^{(1)} = -\frac{k_x}{\omega\mu_0\varepsilon_0}\frac{\partial\tilde{A}_x^{(1)}}{\partial z} - j\frac{k_y}{\varepsilon_0}\tilde{F}_x^{(1)} \quad (33a)$$

$$\tilde{H}_z^{(1)} = -\frac{k_x}{\omega\mu_0\varepsilon_0}\frac{\partial\tilde{F}_x^{(1)}}{\partial z} + j\frac{k_y}{\mu_0}\tilde{A}_x^{(1)} \quad (33b)$$

$$\tilde{E}_{z\pm}^{(2)} = -\frac{k_x}{\omega\mu_\pm\varepsilon_\pm}\frac{\partial\tilde{A}_{x\pm}^{(2)}}{\partial z} - j\frac{k_y}{\varepsilon_\pm}\tilde{F}_{x\pm}^{(2)} \quad (34a)$$

$$\tilde{H}_{z\pm}^{(2)} = -\frac{k_x}{\omega\mu_\pm\varepsilon_\pm}\frac{\partial\tilde{F}_{x\pm}^{(2)}}{\partial z} + j\frac{k_y}{\mu_\pm}\tilde{A}_{x\pm}^{(2)} \quad (34b)$$

$$\tilde{E}_z^{(2)} = \tilde{E}_{z+}^{(2)} + \tilde{E}_{z-}^{(2)}, \quad \tilde{H}_z^{(2)} = \tilde{H}_{z+}^{(2)} + \tilde{H}_{z-}^{(2)} \quad (35)$$

Having determined the reflection and transmission coefficients, and using the auxiliary potentials, it is possible to determine the EM fields $\vec{E}$ and $\vec{H}$ in both regions. Let us consider the special case $\kappa_r \to 0$, which approaches a nonchiral medium in which $K_\pm \to K$; $\beta_\pm \to \beta$ with $k = \omega\sqrt{\mu\varepsilon}$ and $\beta = \sqrt{k^2 - (k_x^2 + k_y^2)}$. Then the reflection coefficients are reduced to:

$$R_e^{(1)} = \frac{\varepsilon_0\beta - \varepsilon\beta_0}{\varepsilon_0\beta + \varepsilon\beta_0}$$

and

$$R_m^{(1)} = 0. \quad (36)$$

It is observed that with zero chirality there is no coupling between electric and magnetic fields, which is true for an ordinary nonchiral half space. Furthermore in the limiting case of a perfect electric conductor (PEC) with $\varepsilon_+ = \varepsilon_- \to \infty$, and for $k_y = 0$, one will conclude $R_e^{(1)} \to -1$ and $R_m^{(1)} \to 0$ as expected.

4. AUXILIARY HERTZIAN POTENTIALS

Let us consider a chiral medium in the presence of a time harmonic electric current source $\vec{J}$ and a magnetic current source $\vec{M}$. The constitutive relations for such a chiral medium are given by Equations 1 and 2. Similar to the previous section, by decomposing the $\vec{E}$ and $\vec{H}$ fields into two wave fields $\vec{E}_\pm$ and $\vec{H}_\pm$, which treats the chiral medium as two nonchiral media with effective parameters $\mu_\pm$, $\varepsilon_\pm$ [8], we can define the above mentioned wave fields in terms of the auxiliary Hertzian vector potentials $\vec{\Pi}_\pm$ and $\vec{\Pi}_\pm^*$. Thus, electric and magnetic fields in terms of Hertzian vector potentials are expressed as,

$$\vec{E}_\pm = -j\omega\mu_\pm\nabla\times\vec{\Pi}_\pm^* + k_\pm^2\vec{\Pi}_\pm + \nabla\nabla\cdot\vec{\Pi}_\pm \quad (37a)$$

$$\vec{H}_\pm = j\omega\varepsilon_\pm\nabla\times\vec{\Pi}_\pm + k_\pm^2\vec{\Pi}_\pm^* + \nabla\nabla\cdot\vec{\Pi}_\pm^* \quad (37a)$$

Where $\vec{\Pi}_\pm$ and $\vec{\Pi}_\pm^*$ satisfy Helmholtz equations;

$$\nabla^2\vec{\Pi}_\pm + k_\pm^2\vec{\Pi}_\pm + \frac{-\vec{J}_\pm}{j\omega\varepsilon}, \qquad \nabla^2\vec{\Pi}_\pm^* + k_\pm^2\vec{\Pi}_\pm^* = \frac{-\vec{M}_\pm}{j\omega\mu_\pm} \quad (38)$$

In which

$$k_\pm = \pm\omega\sqrt{\mu_\pm\varepsilon_\pm}$$

In view of 38 $\vec{E}_\pm$ and $\vec{H}_\pm$ can be determined using 37a,b.

4.1. Potentials for Dipole Sources

Let us consider a vertical electric dipole (VED) and a vertical magnetic dipole (VMD) with respective moments, IL and I_mL as follows:

$$\vec{J}(z) = IL\delta(x)\,\delta(y)\,\delta(z)\,\hat{z}$$

and

$$\vec{M}(z) = I_m L\ \delta(x)\delta(y)\delta(z)\hat{z} \qquad (39)$$

In order to calculate the potentials $\vec{\Pi}_{\pm}$ and $\vec{\Pi}^*_{\pm}$ at first we substitute the expression of Equation 39 in Equation 21. In view of Equation 21 the decomposed electric and magnetic sources will be;

$$\vec{J}_{\pm} = \frac{1}{2}(IL \mp \frac{j}{\eta} I_m L)\ \delta(x)\delta(y)\delta(z)\hat{z} \qquad (40a)$$

$$\vec{M}_{\pm} = \frac{1}{2}(I_m L \pm j\eta IL)\ \delta(x)\delta(y)\delta(z)\hat{z} = \pm j\eta J_{\pm} \qquad (40b)$$

$\vec{\Pi}_{\pm}$ and $\vec{\Pi}^*_{\pm}$ are therefore expressed as

$$\vec{\Pi}_{z\pm} = \frac{(IL \mp \frac{j}{\eta} I_m L)}{8\pi j\omega\varepsilon_{\pm}} \frac{e^{-jk_{\pm}R}}{R}\hat{z}\ ,\quad \vec{\Pi}^*_{z\pm} \frac{(I_m L \pm j\eta IL)}{8\pi j\omega\mu_{\pm}} \frac{e^{-jk_{\pm}R}}{R}\hat{z} \qquad (41)$$

Where R is the variable of the spherical coordinate system, and $\hat{z}$ is the unit vector along the z axis. From Equation 41, the ratio $\frac{\vec{\Pi}^*_{z\pm}}{\vec{\Pi}_{z\pm}}$ is determined as

$$\vec{\Pi}^*_{z\pm} = \pm\frac{j}{\eta}\vec{\Pi}_{z\pm} \qquad (42)$$

4.2. The Half Space Problem Let us now consider the case where the sources are in a chiral medium; $z < 0$. region 1 is air; with VED or VMD as sources (Figure 2) i.e.,

$$\vec{J}(z) = IL\delta(z+h)\ \hat{z}\ ,\quad \vec{M}(z) = I_m L\delta(z+h)\hat{z} \qquad (43)$$

Applying expressions of Equation 21 the vertical sources $\vec{J}_{\pm}$ and $\vec{M}_{\pm}$ are;

$$\vec{J}_{\pm} = \frac{1}{2}(IL \mp \frac{j}{\eta} I_m L)\delta(z+h)\ \hat{z}\ ,\quad \vec{M}_{\pm} = \frac{1}{2}(I_m L \pm j\eta IL)\delta(z+h)\hat{z} \qquad (44)$$

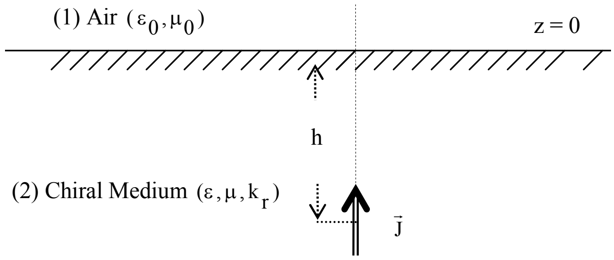

Figure 2. The half space problem with the source in the chiral medium.

With these sources, the incident potentials are assumed to be; $\vec{\Pi}^{(2)i}_{\pm}$ and $\vec{\Pi}^{*(2)i}_{\pm}$ respectively. Either of these potentials create a pair of reflected and a pair of transmitted potentials below and above the interface respectively denoted by $\vec{\Pi}^{(2)r}_{z\pm}$, $\vec{\Pi}^{*(2)r}_{z\pm}$ and $\vec{\Pi}^{(1)t}_{z\pm}$, $\vec{\Pi}^{*(1)t}_{z\pm}$. Considering the sources and Equation 41 the incident potentials are seen to be spherical waves. Applying a two-dimensional Fourier transform and Sommerfeld identity, one can deduce [8];

$$\tilde{\Pi}^{(2)i}_{z\pm} = \tilde{\Pi}^{i}_{\pm} e^{-j\beta_{\pm}|z+h|}\ ,\quad \tilde{\Pi}^{i}_{\pm} = \frac{1}{2\omega\varepsilon_{\pm}\beta_{\pm}} I_{\pm} \qquad (45)$$

$$\tilde{\Pi}^{*(2)i}_{z\pm} = \tilde{\Pi}^{*i}_{\pm} e^{-j\beta_{\pm}|z+h|}\ ,\quad \tilde{\Pi}^{*i}_{\pm} = \frac{1}{2\omega\mu_{\pm}\beta_{\pm}} I_{m\pm} \qquad (46)$$

Where;

$$k_\rho^2 = k_x^2 + k_y^2, k_{\pm} = \omega\sqrt{\mu_{\pm}\varepsilon_{\pm}}\ ,\ I_{\pm} = \frac{1}{2}(IL \mp \frac{j}{\eta} I_m L)$$

and

$$I_{m\pm} = \frac{1}{2}(I_m L \pm j\eta IL) = \pm j\eta I_{\pm},\ \beta_{\pm} = \sqrt{k_{\pm}^2 - k_\rho^2}\ .$$

The reflected and transmitted potentials are given in Appendix C. The reflection and transmission coefficients can be obtained from the interface conditions at z = 0, valid for arbitrary Π^i_+ and Π^i_-. In order to obtain the interface conditions for these auxiliary potentials, first we use the Fourier transform of Equation 37a by applying vector analysis and the transformations $\frac{\partial}{\partial x} \to -jk_x$ and $\frac{\partial}{\partial x}\frac{\partial}{\partial y} \to -jk_y$.This yields [11]

$$\tilde{\vec{E}}_+ = (-jk_+k_y\tilde{\Pi}_{z+} - jk_x \frac{\partial \tilde{\Pi}_{z+}}{\partial z})\hat{x} + (jk_+k_x \tilde{\Pi}_{z+} - jk_y \frac{\partial \tilde{\Pi}_{z+}}{\partial z})\hat{y} + (k_+^2\tilde{\Pi}_{z+} + \frac{\partial^2 \tilde{\Pi}_{z+}}{\partial z^2})\hat{z} \quad (47)$$

and

$$\tilde{\vec{E}}_- = (-jk_-k_y\tilde{\Pi}_{z-} - jk_x \frac{\partial \tilde{\Pi}_{z-}}{\partial z})\hat{x} + (jk_-k_x \tilde{\Pi}_{z-} - jk_y \frac{\partial \tilde{\Pi}_{z-}}{\partial z})\hat{y} + (k_-^2\tilde{\Pi}_{z-} + \frac{\partial^2 \tilde{\Pi}_{z-}}{\partial z^2})\hat{z} \quad (48)$$

From the z component of the wave fields, we have:

$$\tilde{E}_{z\pm} = k_\rho^2 \tilde{\Pi}_{z\pm}, \quad k_\rho^2 = k_x^2 + k_y^2 \quad (49)$$

In this case by noting k_ρ^2 in Equation 49, we see that the value of k_ρ is identical in both regions. Therefore, the same interface conditions apply for both $\tilde{\Pi}_{z\pm}$ and $\tilde{E}_{z\pm}$. So we can use the interface conditions for $\tilde{\Pi}_{z\pm}$ as for the vertical wave field component in [8].

$$\frac{\partial}{\partial z}\tilde{\Pi}_{z+} + \frac{\partial}{\partial z}\tilde{\Pi}_{z-} = \text{cont}, \quad \frac{1}{\eta}\frac{\partial}{\partial z}\tilde{\Pi}_{z+} - \frac{1}{\eta}\frac{\partial}{\partial z}\tilde{\Pi}_{z-} = \text{cont} \quad (50)$$

$$k_+\tilde{\Pi}_{z+} - k_-\tilde{\Pi}_{z-} = \text{cont}, \quad \frac{k_+\tilde{\Pi}_{z+}}{\eta} + \frac{k_-\tilde{\Pi}_{z-}}{\eta} = \text{cont} \quad (51)$$

Where, "cont" denotes continuity at the interface. By applying the interface conditions Equations 50-51 at z = 0 for any arbitrary values of Π^i_+ and Π^i_- one can find systems of equations in terms of reflection and transmission coefficients. By solving these systems of equations the reflection and transmissions coefficients can be determined [12].

Using these coefficients and Equations 45-46, it is possible to determine the potentials $\tilde{\Pi}^{(2)r}_{z\pm}$ and $\tilde{\Pi}^{(1)}_{z\pm}$ from Appendix C. The potentials in region 2 are thus derived from the following equation:

$$\tilde{\Pi}^{(2)}_{z\pm} = \tilde{\Pi}^{(2)i}_{z\pm} + \tilde{\Pi}^{(2)r}_{z\pm} \quad (52)$$

Where $\tilde{\Pi}^{(2)i}_{z\pm}$ is determined from Equation 45. In view of Equation 42, the potentials $\tilde{\Pi}^*_{z\pm}$ are derived in terms of $\tilde{\Pi}_{z\pm}$ in both media;

$$\tilde{\Pi}^{*(1)}_{z\pm} = \pm\frac{j}{\eta_0}\tilde{\Pi}^{(1)}_{z\pm}, \quad \tilde{\Pi}^{*(2)}_{z\pm} = \pm\frac{j}{\eta}\tilde{\Pi}^{(2)}_{z\pm} \quad (53)$$

Now by applying relations Equation 53 and using Equations 47 and 48 it will be possible to obtain the wave fields, $\tilde{\vec{E}}_\pm$ and $\tilde{\vec{H}}_\pm$ in both regions. Finally, the total field corresponding to both regions is given by Equations 17b and 18b.

4.3. Approximate Solutions for Small Chirality

Let us assume that the chirality parameter κ_r is small (κ_r << 1) which is often the practical case. By neglecting κ_r^2 and higher order terms, and after tedious mathematical manipulations the reflection and transmission coefficients reduce to:

$$R_{++} \approx \frac{\eta\eta_0\beta(k_0^2\beta^2 - k^2\beta_0^2) + k_r k k_0 \beta_0 (\eta^2+\eta_0^2)(k^2-\beta^2)}{\beta\Delta'} \tag{54}$$

$$R_{--} \approx \frac{\eta\eta_0\beta(k_0^2\beta^2 - k^2\beta_0^2) - k_r k k_0 \beta_0 (\eta^2+\eta_0^2)(k^2-\beta^2)}{\beta\Delta'} \tag{55}$$

$$R_{+-} \approx \frac{k k_0 \beta_0 (\eta^2-\eta_0^2)\left[\beta^2 - k_r(\beta^2+k^2)\right]}{\beta\Delta'} \tag{56}$$

$$R_{-+} \approx \frac{k k_0 \beta_0 (\eta^2-\eta_0^2)\left[\beta^2 + k_r(\beta^2+k^2)\right]}{\beta\Delta'} \tag{57}$$

$$T_{++} \approx \frac{\eta_0 k(\eta+\eta_0)\left[(k\beta_0 + k_0\beta) + k_r\beta(k_0\beta^3 + k^3\beta_0)\right]}{\beta^2\Delta'} \tag{58}$$

$$T_{--} \approx \frac{\eta_0 k(\eta+\eta_0)\left[(k\beta_0 + k_0\beta) - k_r\beta(k_0\beta^3 + k^3\beta_0)\right]}{\beta^2\Delta'} \tag{59}$$

$$T_{+-} = \frac{\eta_0 k(\eta-\eta_0)}{\beta^2\Delta'} \times \frac{\left[\beta^2(k\beta_0 - k_0\beta) + k_r\beta(k_0\beta^3 - 2k\beta_0\beta^2 + k^3\beta_0)\right]}{\beta^2\Delta'} \tag{60}$$

$$T_{-+} = \frac{\eta_0 k(\eta-\eta_0)}{\beta^2\Delta'} \times \frac{\left[\beta^2(k\beta_0 - k_0\beta) - k_r\beta(k_0\beta^3 - 2k\beta_0\beta^2 + k^3\beta_0)\right]}{\beta^2\Delta'} \tag{61}$$

Where;

$$\Delta' = k k_0 \beta\beta_0(\eta^2+\eta_0^2) + \eta\eta_0(k^2\beta_0^2 + k_0^2\beta^2),\ k = \omega\sqrt{\mu\varepsilon},\ k_0 = \omega\sqrt{\mu_0\varepsilon_0},\ \beta^2 = k^2 - k_\rho^2,\ \beta_0^2 = k_0^2 - k_\rho^2,\ k_\rho^2 = k_x^2 + k_y^2,\ \eta = \sqrt{\frac{\mu}{\varepsilon}},\ \eta_0 = \sqrt{\frac{\mu_0}{\varepsilon_0}} \tag{62}$$

5. VED AND VMD SOURCES

In the case where only a single VED or VMD source is present, even though in the relations of Appendix C the absolute values of I_+ and I_- are equal, but due to the difference between the values of β_+ and β_- on one hand and the values of ε_+ and ε_- on the other, the absolute value of $\tilde{\Pi}_+^i$ (or $\tilde{\Pi}_+^{*i}$) is not the same as the values of $\tilde{\Pi}_-^i$ (or $\tilde{\Pi}_-^{*i}$). Thus with regard to the above explanation and Equation 42 it is seen that the potentials $\tilde{\Pi}_{z+}^{(2)r}$ and $\tilde{\Pi}_{z-}^{(2)r}$ involve different exponential functions of z with different wave numbers β_+ and β_- respectively. Similarly the above result is valid for $\tilde{\Pi}_{z+}^{(2)i}$ and $\tilde{\Pi}_{z-}^{(2)i}$ in Equation 45. Therefore the reflected wave fields $\tilde{E}_{z+}^{(2)r}$ and $\tilde{E}_{z-}^{(2)r}$ (or $\tilde{H}_{z+}^{(2)r}$ and $\tilde{H}_{z-}^{(2)r}$) include different exponential functions of z with different wave numbers β_+ and β_- respectively. Consequently in region 2 we can not define the reflection coefficients R_e, R_{em}, R_m and R_{me} as is applicable in air. Consequently, the exact image method given by Lindell [8], can not be applied in the present problem where the source is located in the chiral medium. As a special case, we set $k_r = 0$, in Equation 45 and relations of Appendix C and subsequent equations, R_e and R_m are reduced to:

$$R_e = \frac{\varepsilon_0\beta - \varepsilon\beta_0}{\varepsilon_0\beta + \varepsilon\beta_0}$$

and

$$R_m = \frac{\mu_0 \beta - \mu \beta_0}{\mu_0 \beta + \mu \beta_0} \tag{63}$$

and the problem reduces to a simple dielectric half space.

6. CONCLUSION

The scalar and vector potentials in chiral media $\vec{A}_c$, $\vec{F}_c$, Φ_{ec} and Φ_{mc} were introduced in this work and the $\vec{E}$ and $\vec{H}$ fields were obtained using these potentials. It is observed that either of the fields $\vec{E}$ and $\vec{H}$ contain a vector component in the directions of both $\vec{F}_c$ and $\vec{A}_c$. This is different from the nonchiral media, where the vectors $\vec{E}$ and $\vec{H}$ are perpendicular to the vector $\vec{F}_c$ (or $\vec{A}_c$) in the presence of an electric source (or a magnetic source). It was shown that in the case of a chiral media, and in the presence of only an electric or a magnetic source, both the electric and magnetic potentials are nonzero. Therefore unlike the nonchiral media, there is coupling between the electric and magnetic potentials in a chiral media. It is seen that the right and left handed potentials presented here, simplify the analysis of EM fields in two layered problems involving chiral media. To do this, we write the fields in a pair of equivalent ordinary nonchiral media with the effective parameters of $\varepsilon_\pm$ and $\mu_\pm$. Then by using a decomposition scheme for sources and fields in an infinite chiral medium, the uncoupled right and left handed auxiliary Hertzian vector potentials were derived. These potentials correspond to the right and left handed features of chiral media of the Helmholtz equation with different effective parameters $(\varepsilon_\pm, \mu_\pm)$ or (ε_-, μ_-). These parameters depend on the chirality parameter of the medium. These auxiliary potentials are then used to obtain the EM fields in the spectral domain for the chiral medium. The problem of a VED and/or VMD within a chiral half space in the vicinity of free space was investigated. It was observed that due to the properties of the chiral media the right and left handed incident waves with different wave numbers were generated. This case is completely different from the case in which a VED and/or a VMD is located in air where the corresponding right and left handed incident waves have identical wave numbers. Therefore the physical behavior when the source is in the chiral medium, is different from the case with a source in air [8]. In addition the exact image method [8], can not be used in the present scheme with the source located in the chiral medium.

7. ACKNOWLEDGMENT

The support of Center of Excellence on Applied Electromagnetic systems, university of Tehran is gratefully acknowledged.

8. APPENDICES

8.1. Appendix A

$$\tilde{E}_x^{(1)} = \frac{1}{j\omega\mu_0\varepsilon_0}(k_0^2 - k_x^2)\tilde{A}_x^{(1)} \; , \quad \tilde{E}_y^{(1)} = \frac{jk_x k_y}{\omega\mu_0\varepsilon_0}\tilde{A}_x^{(1)} - \frac{1}{\varepsilon_0}\frac{\partial \tilde{F}_x^{(1)}}{\partial z} \tag{1A}$$

$$\tilde{H}_x^{(1)} = \frac{1}{j\omega\mu_0\varepsilon_0}(k_0^2 - k_x^2)\tilde{F}_x^{(1)} \; , \quad \tilde{H}_y^{(1)} = \frac{jk_x k_y}{\omega\mu_0\varepsilon_0}\tilde{F}_x^{(1)} + \frac{1}{\mu_0}\frac{\partial \tilde{A}_x^{(1)}}{\partial z} \tag{2A}$$

$$\tilde{E}_x^{(2)} = \frac{1}{j\omega\mu_+\varepsilon_+}(k_+^2 - k_x^2)\tilde{A}_{x+}^{(2)} + \frac{1}{j\omega\mu_-\varepsilon_-}(k_-^2 - k_x^2)\tilde{A}_{x-}^{(2)} \tag{3A}$$

$$\tilde{E}_y^{(2)} = \frac{jk_x k_y}{\omega\mu_+\varepsilon_+}\tilde{A}_{x+}^{(2)} - \frac{1}{\varepsilon_+}\frac{\partial\tilde{F}_{x+}^{(2)}}{\partial z} + \frac{jk_x k_y}{\omega\mu_-\varepsilon_-}\tilde{A}_{x-}^{(2)} - \frac{1}{\varepsilon_-}\frac{\partial\tilde{F}_{x-}^{(2)}}{\partial z} \tag{4A}$$

$$\tilde{H}_x^{(2)} = \frac{1}{j\omega\mu_+\varepsilon_+}(k_+^2 - k_x^2)\tilde{F}_{x+}^{(2)} + \frac{1}{j\omega\mu_-\varepsilon_-}(k_-^2 - k_x^2)\tilde{F}_{x-}^{(2)} \tag{5A}$$

$$\tilde{H}_y^{(2)} = \frac{jk_x k_y}{\omega\mu_+\varepsilon_+}\tilde{F}_{x+}^{(2)} + \frac{1}{\mu_+}\frac{\partial\tilde{A}_{x+}^{(2)}}{\partial z} + \frac{jk_x k_y}{\omega\mu_-\varepsilon_-}\tilde{F}_{x-}^{(2)} + \frac{1}{\mu_-}\frac{\partial\tilde{A}_{x-}^{(2)}}{\partial z} \tag{6A}$$

8.2. Appendix B

$$a_0 = \frac{k_0^2 - k_x^2}{\mu_0\varepsilon_0},\ a_\pm = \frac{k_\pm^2 - k_x^2}{\mu_\pm\varepsilon_\pm},\ b_0 = \frac{k_x k_y}{\omega\mu_0\varepsilon_0},\ b_\pm = \frac{k_x k_y}{\omega\mu_\pm\varepsilon_\pm},\ B_\pm = -\frac{k_x k_y}{\omega\mu_\pm\varepsilon_\pm} \pm \frac{j\beta_\pm}{\eta\varepsilon_\pm} \tag{1B}$$

$$C_\pm = \mp j\frac{k_x k_y}{\eta\omega\mu_\pm\varepsilon_\pm} - \frac{\beta_\pm}{\mu_\pm},\ D_\pm = \frac{a_\pm}{a_0}(b_0 \pm j\frac{\beta_0}{\eta\varepsilon_0}) + B_\pm,\ L_\pm = -\frac{\beta_0 a_\pm}{\mu_0 a_0} \pm j\frac{b_0 a_\pm}{\eta a_0} \tag{2B}$$

8.3. Appendix C

$$\tilde{\Pi}_{z+}^{(2)r} = (R_{++}\Pi_+^i + R_{+-}\Pi_-^i)e^{j\beta_+(z-h)} \tag{1C}$$

$$\tilde{\Pi}_{z-}^{(2)r} = (R_{-+}\Pi_+^i + R_{--}\Pi_-^i)e^{j\beta_-(z-h)} \tag{2C}$$

$$\tilde{\Pi}_{z+}^{(1)t} = (T_{++}\Pi_+^i + T_{+-}\Pi_-^i)e^{-j(\beta_0 z + \beta_+ h)} \tag{3C}$$

$$\tilde{\Pi}_{z-}^{(1)t} = (T_{-+}\Pi_+^i + T_{--}\Pi_-^i)e^{-j(\beta_0 z + \beta_- h)} \tag{4C}$$

Where;

$$\beta_0 = \sqrt{k_0^2 - k_\rho^2}$$

with

$$k_0 = \omega\sqrt{\mu_0\varepsilon_0}\ .$$

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