THE FINITE HORIZON ECONOMIC LOT SCHEDULING IN FLEXIBLE FLOW LINES

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Abstract This paper addresses the common cycle multi-product lot-scheduling problem in flexible flow lines (FFL) where the product demands are deterministic and constant over a finite planning horizon. Objective is minimizing the sum of setup costs, work-in-process and final products inventory holding costs per time unit while satisfying the demands without backlogging. This problem consists of a combinatorial part (machine assignment and sequencing sub-problems) and a continuous part (lot sizing and scheduling sub-problems). To account for these two elements, a new mixed integer nonlinear program (MINLP) is developed which simultaneously determines machine allocation, sequencing, lot-sizing and scheduling decisions. In order to reduce computational complexity, instead of solving this MINLP directly, we propose an efficient enumeration method to determine optimal solution of the model. Moreover, the performance of the proposed method is evaluated by some numerical experiments. Two other applicable cases (zero setup costs and Lot streaming) are studied and required modifications in the model formulation and the solution method are described. Finally, a case example in a PCB assembly system is presented to illustrate applicability of the mathematical model and the proposed solution method.

Keywords: Lot-scheduling, Flexible flow line, Common cycle approach, Finite horizon

چکیده در این مقاله، مسئله زمان بندی تولید در سیستم های جریان کارگاهی انعطاف پذیر مورد بررسی قرار گرفته و یک مدل مختلط صفر و یک غیر خطی جدید برای تعیین حل بهینه آن ارائه شده است. طول افق برنامه ریز محدود بوده و تقاضای محصولات در واحد زمان قطعی و ثابت فرض شده است. هدف مدل، حداقل نمودن مجموع هزینه های راه اندازی و نگهداری محصولات نیمه ساخته و نهایی در واحد زمان است. برای حل این مدل ریاضی، فرض شده است که زمان سیکل محصولات یکسان بوده و افق زمانی مضرب صحیحی از زمان سیکل تولید است. به منظور کاهش پیچیدگی حل مسئله، به جای حل مستقیم مدل مختلط غیر خطی، یک روش شمارشی کارا در تعیین جواب بهینه توسعه یافته است. همچنین دو حالت دیگر کاربردی از مسئله (شامل هزینه های راه اندازی صفر و انتقال زیر انباشته ها) مورد مطالعه قرار گرفته و تغییرات مورد نیاز در مدل ریاضی اولیه برای هر یک از این حالات بیان شده است. یک مثال کاربردی نیز در صنعت مونتاژ بردهای مدار چاپی برای بیان کاربردی بودن مدل پیشنهادی و روش حل آن ارائه شده است.

1. INTRODUCTION AND PROBLEM DEFINITION

The production facility considered in this paper, is a flexible flow line (or hybrid flow shop) consisting of several work centers (stages), which are arranged serially as a flow shop. Each stage has one or more parallel identical machines. There is a unidirectional flow across production stages and each product must be processed by at most one machine at each stage, but some products may skip some stages. Such systems are one of the most usual production systems in manufacturing discrete parts that can be considered as an extension of two classical systems, namely the flow line and the parallel shop.

In this paper, we consider the production scheduling problem in such systems where all parameters (such as demand rates) are deterministic and constant over a given finite planning horizon. The problem has several sub-problems. Machine assignment and sequencing sub-problems (assignment of products to machines at work centers with parallel identical machines, and their sequencing on each machine of each stage) are the combinatorial part of the problem, and lot sizing and scheduling sub-problems (determination of lot sizes and production starting and ending times for each product at each stage) are the continuous part of the problem. The objective is minimizing the average of setup and inventory holding costs per time unit without backlogging.

To solve the problem, we introduce two simplifying and practical assumptions. First, we assume a common cycle for all products. Second, it is required that the planning horizon is an integer multiple of the common cycle length. So, a new mixed zero-one nonlinear program is developed whose optimal solution simultaneously determines the optimal assignment of products to machines at stages with multiple parallel machines, the optimal sequence for each machine at each stage, the optimal lot sizes and the optimal beginning times for each production run.

It is noteworthy that the problem considered here (Problem P), is an extension of well-known economic lot-scheduling problem (ELSP) to flexible flow line systems in finite horizon case. The ELSP deals with lot sizing and scheduling issues for several products with constant demand rates on a machine (or continuous production line) over an infinite horizon, whereas our problem investigates these issues in flow line systems with possibly parallel machines at each stage over a finite horizon. So, we call it the FH-ELSP-FFL problem. Hsu [8] has proven that ELSP is NP-hard; therefore it is obvious that our more general problem is definitely NP-hard.

The problem is common in supply chain environments, where a supplier produces multiple products in a flexible flow line for an assembly facility. In such cases, the product demand rates are deterministic and fairly constant based on contract between supplier and assembler. Moreover, delivery of each finished product to assembler is continuous with fixed rate per time unit. An example for this situation is a large assembly facility such as an automotive assembly plant (customer) and its immediate suppliers. Other applications of concerned problem could be different industries such as the wire & cable industry, food canning, beverage bottling industries and printed circuit board assembly systems.

The most of the contributions reported in the literature dealing with static demand lot sizing and scheduling problems, have focused on particular policies, the *cyclic schedules*, i.e., a schedule that is repeated periodically. In this paper we have adopted the common cycle approach in which at each production cycle; one lot of each product at each stage is produced. This policy allows constructing production schedules that are easy to implement and generally preferred in real-life situations [10].

Moreover, according to contract between supplier and assembler, we assume that planning horizon is finite and fixed by management. It is noted that in the most of previous contributions on economic lot scheduling, planning horizon is assumed to be infinite. There are several reasons for this assumption. First, constructing a mathematical model for infinite case is easier. Further, this assumption makes feasible solution space larger and consequently may lead to better solutions. However, this assumption considerably reduces the usefulness of the proposed contributions, because in practice, planning horizons are always finite and rarely longer than 12 months. Further, in most cases, the schedules obtained by infinite horizon assumption could not be repeated an integer number of times during the finite planning horizon chosen in practice. Thus practitioners usually adjust such schedules to meet this condition, which may lead to a non-negligible increase in the total cost [10].

Literature review in economic lot scheduling problems reveals that the most of contributions are related to infinite planning horizon case at the single stage systems with only one machine [9], the single stage systems with parallel identical or non-identical machines [2, 3, 13] and the flow shop systems [4,5,6,7,11]. Moreover, in the finite horizon case there are only four contributions from Ouenniche et al. [10,12] and Torabi et al. [14,15]. In [10,12], the production scheduling problem in job shops is studied under constant demand rates over a finite planning horizon either using the common cycle approach [10] or the multiple cycle approach [12], to obtain a cyclic schedule. The authors developed an optimal solution method in common cycle case and an efficient heuristic method to obtain a near optimal solution in multiple cycle case. It is noted that these two works are extensions for ELSP problem where demands (deliveries) are continuous and optimization issue is focused on a supplier with a job shop production system, but not on the supply network. Torabi et al [14]. extended the common cycle economic lot scheduling problem (ELSP) to flexible job shops in finite horizon case and developed an optimal enumeration method to obtain optimal solution of this problem. Moreover, they considered the common cycle economic lot and delivery scheduling problem (ELDSP) in flexible flow lines in finite horizon case and developed an efficient hybrid genetic algorithm to obtain optimal or near-optimal solutions for this problem [15]. It is noted that the ELDSP is an extension of ELSP to supply chain environments where a supplier produces multiple products on a single machine, accumulates these products and delivers them directly to an assembly facility (AF). However, to the author's best knowledge, there is no contribution for economic lot-scheduling problem in flexible flow lines under constant demand rates over a finite planning horizon. Thus, in this paper, a new mathematical model and an efficient solution method are developed for this problem.

The outline of this paper is as follows. In section 2, problem formulation as well as necessary conditions to have a feasible solution are presented. In section 3, an enumeration method to obtain optimal solution is developed. In order to validation of the proposed solution method, some numerical experiments are done and the corresponding results are presented in section 4. Two other applicable cases (zero setup costs and lot streaming) as well as required modifications in the model formulation and the solution method are studied in section 5. A numerical example is presented in section 6. Finally, section 7 is devoted to the conclusions and some recommendations for future researches.

2. PROBLEM FORMULATION

The following notations are used for the problem formulation:

Parameters

n : number of products

m : number of work centers (stages)

i , u : products indices

j : stage index

m_j : number of parallel identical machines at stage j

M_{kj} : k-th machine at stage j

d_i : demand rate of product i

p_{ij} : production rate of product i at stage j

t_{ij} : processing time for a lot of product i at stage j ($t_{ij} = d_i . T/p_{ij}$)

s_{ij} : sequence-independent setup time of product i at stage j

A_i : total setup costs of product i over all stages

h_{ij} : inventory holding cost per unit of product i per time unit between stages j and $j+1$

h_i : inventory holding cost per unit of final product i per time unit

H : planning horizon length

M : a large real number

Decision variables

σ_j : production sequence vector at stage j

σ_{kj} : production sequence vector at machine M_{kj}

J_{kj} : the set of products which are assigned to machine M_{kj}

n_{kj} : the number of products which are assigned to machine M_{kj}

T : common production cycle length (time interval between setups)

Q_i : production lot size of product i at different stages ($Q_i = d_i \,.\, T$)

F : the number of production cycles over the planning horizon

b_{ij} : process beginning time of product i at stage j (after setup operations)

$$z_{ilj} = \begin{cases} 1 \;\; \textit{if product i is assigned} \\ \textit{to } l^{th} \;\; \textit{position in } \sigma_j \;\; (j \mid m_j = 1) \\ 0 \;\; \textit{otherwise} \end{cases}$$

$$x_{ilkj} = \begin{cases} 1 \;\; \textit{if product i is assigned} \\ \textit{to } l^{th} \;\; \textit{position in } \sigma_{kj} \;\; (j \mid m_j > 1) \\ 0 \;\; \textit{otherwise} \end{cases}$$

Since after processing each product at each stage, there would be a value added for the product, thus, values of h_{ij} parameters will be non-decreasing, that is:

$h_{ij} < h_i, \;\; h_{i,j-1} \le h_{ij}; \; \forall i = 1, ..., n, \; j = 2, ..., m.$

It is noted that z_{ilj} variables are sequencing sub-problem variables at stages with only one machine and x_{ilkj} variables are both sequencing and machine assignment sub-problem variables at stages with multiple machines.

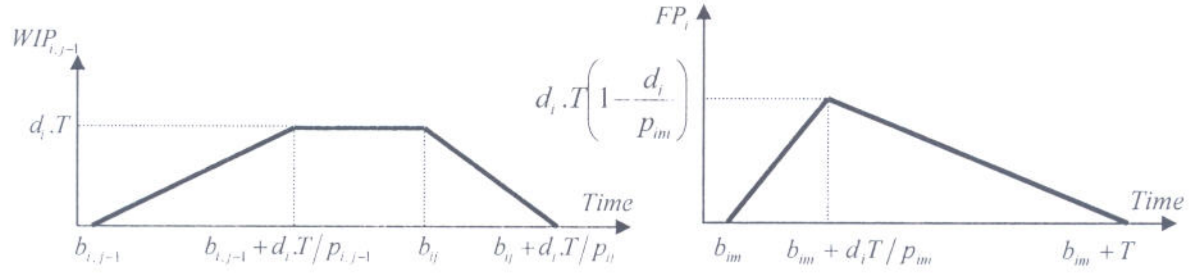
Moreover, in constructing a mathematical model for the problem, the main following assumptions have been considered:

- The production system is a flexible flow line. This system consists of several stages in series, where each stage has one or more parallel machines that are identical in all parameters such as production rates and setup times (costs);
- Machines of different stages are continuously available and each machine can only process one product at a time;
- The common cycle approach is used as production policy i.e. at each production cycle, one lot of each product at each stage is produced;
- The lots of each product have equal sizes at different stages;
- External demands occur only for end products and their delivery is continuous;
- Backlogging is not allowed;
- Setup times and costs are both sequence and lot size independent;
- Inventory holding costs are directly proportional to inventory levels and to holding time;
- Production sequence for each machine at each stage is unique and is determined by mathematical model;
- Preemption is not allowed; that is, at a given stage, once the processing of a lot starts, it must be completed without interruption;
- Lot streaming is not allowed; that is, sub-batches of each product are not transferred to the next stage until the entire lot is processed at the current stage;
- There are unlimited buffers between successive stages, hence in process inventories are allowed, i.e., products may wait for their next operations;
- Total capacity of different stages are sufficient to meet the demands; thus there exists at least one feasible lot and delivery schedule;
- Integer number of cycles *(F)* are repeated until the planning horizon is covered;
- Zero switch rule is used. This means that production of each product at each cycle begins when its inventory level reaches zero.

The problem of simultaneous determination of machine assignment, sequencing, lot sizing and scheduling of n products ($n>1$) manufactured through m stages ($m>1$) in a flexible flow line to minimize average costs per unit time (Problem P), can be formulated as a mixed zero-one nonlinear program. As mentioned earlier, to formulate this problem, we assume a common cycle for all products and choose a cycle time such that the finite horizon H is an integer multiple of T.

The objective of Problem P is to minimize the sum of setup cost, work-in-process and end product inventory holding costs per time unit. The first cost product, the setup costs per time unit is $\sum_i A_i / T$.

Two types of inventory are considered: work-in-process inventory and finished product inventory. From Fig. 1(a), which describes the evolution of work-in-process inventory of product i between two successive stages j-1 and j, we can see that its



(a) WIP inventory between stages *j-1* and *j* (b) Finished product *i* inventory

Figure 1. Inventory evolution curves

average work-in-process inventory is:

$$I_{i,j-1}=\frac{1}{T}\left\{\begin{array}{l}\frac{d_iT}{2}.\frac{d_iT}{p_{i,j-1}}+d_iT\left(b_{ij}-b_{i,j-1}-\frac{d_iT}{p_{i,j-1}}\right)\\+\frac{d_iT}{2}.\frac{d_iT}{p_{ij}}\end{array}\right\}$$

$$=d_i\left(b_{ij}+\frac{d_iT}{2p_{ij}}-b_{i,j-1}-\frac{d_iT}{2p_{i,j-1}}\right) \quad (1)$$

Therefore, the total work-in-process inventory holding cost for all products per time unit is:

$$TC_{WIP}=\sum_{i=1}^{n}\sum_{j=2}^{m}h_{i,j-1}.d_i\left(b_{ij}+\frac{d_iT}{2p_{ij}}-b_{i,j-1}-\frac{d_iT}{2p_{i,j-1}}\right) \quad (2)$$

Also, Fig. 1(b), shows the inventory evolution of finished product *i*, and we can see that its average inventory per time unit is:

$$I_{im}=\frac{1}{T}\left\{\begin{array}{l}\frac{d_iT}{2p_{im}}\left(d_iT-\frac{d_i^2T}{p_{im}}\right)+\\\frac{1}{2}\left(T-\frac{d_iT}{p_{im}}\right)\left(d_iT-\frac{d_i^2T}{p_{im}}\right)\end{array}\right\} \quad (3)$$

$$=\left(1-\frac{d_i}{p_{im}}\right).\frac{d_iT}{2}$$

Thus, the total inventory holding cost for all finished products per time unit is:

$$TC_{FPI}=\sum_{i=1}^{n}h_i.\frac{d_i}{2}\left(1-\frac{d_i}{p_{im}}\right).T \quad (4)$$

Therefore, the average costs per time unit (or objective function of Problem p) can be written as follows:

$$TC=\sum_{i=1}^{n}\frac{A_i}{T}+$$
$$\sum_{i=1}^{n}\left[h_i.\frac{d_i}{2}\left(1-\frac{d_i}{p_{im}}\right)+\frac{d_i^2}{2}\sum_{j=2}^{m}h_{i,j-1}\left(\frac{1}{p_{ij}}-\frac{1}{p_{i,j-1}}\right)\right].T$$
$$+\sum_{i=1}^{n}\sum_{j=2}^{m}h_{i,j-1}.d_i\left(b_{ij}-b_{i,j-1}\right) \quad (5)$$

It is noted that since delivery of products is continuous and one delivery of them occurs at each time unit, thus, there will be a fixed delivery cost per time unit that can be deleted from the objective function.

Given the objective function and logical relationships between variables of Problem P (that some of them are extractable from inventory evolution curves), a new mixed nonlinear model is developed to obtain optimal solution of the problem that is presented in Fig. 2.

Problem P has several constraints. Constraints (6) state that, no product can be processed before it is completed at previous stage. Constraints (7) show that, at each stage with one machine ($m_j=1$), no product can be processed before the completion of its predecessor in the related production sequence. In other words, if i and u are two successive products in the sequence vector of stage j (σ_j), then product u must be processed after completion of processing product i at this stage. Constraints (8) are similar to Constraints (7), but they are used for stages with multiple parallel machines ($m_j>1$). Constraints (9) and (10) are assignment Constraints at stages with only one machine, and state that each product has a unique position in the sequence of these stages. Also, Constraints (11) to (13) are applied to stages with multiple parallel machines. Constraints (11) state that each product has a unique position in the sequence of one of the machines at these stages and Constraints (12) show that at each position of each machine at these stages, there is at most one product, because at each machine such as M_{kj}, it may be assigned less than n products to this machine. Constraints (13) stipulate that, one product can be positioned at one position of machine M_{kj}; if another product is to be positioned at previous position of this machine. Constraints (14) imply that at each stage with only one machine, processing the first product in the related sequence cannot start before setting up the corresponding machine. Also, Constraints (15) show that if product i is the first product in the sequence related to one of the machines in stage j ($mj>1$), its processing cannot start before setting up the corresponding machine. Constraints (16) assure that the obtained schedule is cyclic and state that the processing completion time of each product at its final stage is less than or equal to cycle time, and the required time for setting up and processing of all products at each stage is less than or equal to T. Constraint (17) imply that the common cycle is such that the planning horizon H is an integer multiple of T and Constraint (18) shows that F is an integer greater than or equal to one. Finally, Constraints (19) are the non-negativity and the type of other decision variables Constraints.

Problem P (A common cycle model for Problem P)*:*

$$Min\ Z=\frac{\sum_i A_i}{T}+\sum_{i=1}^{n}\left[h_i.\frac{d_i}{2}\left(1-\frac{d_i}{p_{im}}\right)+\frac{d_i^2}{2}\sum_{j=2}^{m}h_{i,j-1}\left(\frac{1}{p_{i,j}}-\frac{1}{p_{i,j-1}}\right)\right].T+\sum_{i=1}^{n}\sum_{j=2}^{m}h_{i,j-1}.d_i\left(b_{i,j}-b_{i,j-1}\right) \quad (5)$$

Subject to :

$$b_{i,j-1}+\frac{d_i.T}{p_{i,j-1}}\le b_{i,j};\ i=1,...,n,\ j=2,...,m. \quad (6)$$

$$b_{ij}+\frac{d_i.T}{p_{ij}}+s_{uj}-b_{uj}\le M\left(2-Z_{i\ell j}-Z_{u,\ell+1,j}\right);\quad j\mid m_j=1,\ i=1,...,n,\ u\ne i,\ \ell<n. \quad (7)$$

$$b_{ij}+\frac{d_i.T}{p_{ij}}+s_{uj}-b_{uj}\le M\left(2-x_{i\ell kj}-x_{u,\ell+1,kj}\right);\quad \forall k,\ j\mid m_j>1,\ i=1,...,n,u\ne i,\ \ell<n. \quad (8)$$

$$\sum_{\ell=1}^{n}z_{i\ell j}=1;\ j\mid m_j=1,\ i=1,...,n. \quad (9)$$

$$\sum_{\ell=1}^{n}z_{i\ell j}=1;\ j\mid m_j=1,\ i=1,...,n. \quad (10)$$

$$\sum_{k=1}^{m_j}\sum_{\ell=1}^{n}x_{i\ell kj}=1;\ j\mid m_j>1,\ i=1,...,n \quad (11)$$

$$\sum_{i}x_{i\ell kj}\le 1;\quad j\mid m_j>1,\ k=1,...,m_j,\ \ell=1,....,n \quad (12)$$

$$\sum_{i}x_{i,\ell+1,kj}\le\sum_{i}x_{i\ell kj};\quad j\mid m_j>1,\ k=1,...,m_j,\ \ell<n \quad (13)$$

$$b_{ij} \geq s_{ij} . z_{i1j} ; \ j \mid m_j = 1, \ i = 1, \ldots, n \tag{14}$$

$$b_{ij} \geq s_{ij} . \sum_{k=1}^{m_j} x_{i1kj} ; \ j \mid m_j > 1, \ i = 1, \ldots, n \tag{15}$$

$$b_{i,m} + \frac{d_i . T}{p_{i,m}} \leq T ; \ i = 1, \ldots, n \tag{16}$$

$$F.T = H \tag{17}$$

$$F \geq 1 \ and \ integer \tag{18}$$

$$T \geq 0, \ b_{ij} \geq 0; \forall i, j, \ z_{i\ell j}, x_{i\ell kj} \in \{0,1\}; \forall i, \ell, k, j. \tag{19}$$

Moreover, since some time must be left for setups at each stage, the necessary conditions to have feasible solutions for the problem can be written as follows:

$$\sum_{i=1}^{n} \left(\frac{d_i}{p_{ij}} + \frac{s_{ij}}{T} \right) \leq m_j ; \ \forall j = 1, \ldots, m \tag{20}$$

But the value of variable T is not determined so far, thus we can redefine necessary conditions as follows:

At each stage j ($j=1, \ldots, m$), the products are sorted in a non-increasing order of d_i / p_{ij} values. The term d_i / p_{ij} represents the fraction of one machine at stage j required by product i. Then according to this order, each product is assigned to the first available machine. At the end, if the following conditions are satisfied, then there would be at least one feasible schedule.

$$Min_k \left(1 - \sum_{i \in J_{kj}} \frac{d_i}{p_{ij}} \right) > 0 ; \ \forall j = 1, \ldots, m \tag{21}$$

This procedure attempts to minimize the makespan for independent jobs on m_j parallel identical machines at each stage [1].

3. SOLUTION PROCEDURE

Problem P is a mixed zero-one nonlinear program. Nonlinearity of this model is due to nonlinear term $\sum_i A_i / T$ in objective function (with respect to T) and also nonlinear constraint (17) in constraints sets. Since it will be difficult to solve this mixed nonlinear model directly, we propose an enumeration method with an iterative process for solving it to optimality.

Let T^* and Z^* denote the optimal common cycle and the corresponding total cost per time unit, respectively. Moreover, let Z_F denotes the objective function value of Problem P for a given value of F. Then this Problem can be solved using the following iterative procedure:

Initialization step. Let $F=1$, and solve the resulting mixed zero-one linear program. Set: $Z^* = Z_1$, $T^* = H$

Iterative step. Increase F by 1 and solve the corresponding mixed zero-one linear program for this new value of F. If this model has no feasible solution, stop; else, if $Z_F < Z^*$ then set $Z^* = Z_F$ and $T^* = H/F$ and go to the next iteration.

Basically, this procedure enumerates all feasible values of F and for each value of F, it solves a mixed linear model to optimality. Thus, this procedure produces the optimal solution of Problem P.

To solve these mixed linear models, we can use one of the large-scale mixed integer optimization tools such as *CPLEX* and *LINGO*. However, within our computational study, we used the LINGO 6.0 solver from LINDO systems, Inc.

4. NUMERICAL EXPERIMENTS

In this section, in order to evaluate the performance of the proposed solution method, we indicate how the computational time increases as the size of the test problems increase. The test problems are randomly generated so that all parameters are drawn from discrete uniform distributions that are presented in Table 1. Moreover, for each problem instance, the necessary conditions are checked in order to make sure that these test problems are suitable for our experiments.

Four sets of test problems with different sizes have been considered (see Table 2), and five problems for each set are randomly generated.

Table 1. Uniform distributions used for the parameters

Parameter	d_i	p_{ij}	s_{ij}	h_{ij}	A_i
Corresponding Uniform Distribution	*U (100, 1000)*	*U (1000, 10000)*	*U (0.01, 0.25)*	*U (1, 20)*	*U (100, 4000)*

Table 2. Structure of the test problems

Problem set number	Number of products	Number of stages	Number of machines at each stage	Problem size		
				Number of integer var.	Number of continuous var.	Number of constraints
1	*5*	*2*	*1,2*	*76*	*11*	*295*
2	*5*	*5*	*1,2,1,2,1*	*176*	*26*	*688*
3	*5*	*10*	*1,2,1,2,1,2,1,2,1,2*	*376*	*51*	*1467*
4	*8*	*5*	*1,2,1,2,1*	*449*	*41*	*2950*

For each set of test problems, a LINGO model has been generated using LINGO 6.0 modeling language, and all of the test problems are solved on a personal computer with an Intel Pentium 4 processor running at 3.2 GHz.

Table 3 represents the average CPU time required to obtain an optimal solution for each set of test problems. It is noted that for 8×5 problems, it was not possible to find an optimal solution within a reasonable CPU time.

Table 3. Average CPU times (in minutes) for the test problems

Problem set number	Number of test problems	Average CPU time
1	*5*	*72*
2	*5*	*146*
3	*5*	*238*
4	*5*	*N. A.*

N. A.: Not available.

Computational results indicate that the proposed solution method can obtain an optimal solution for small-sized and moderately medium-sized problems within a reasonable time. But it can not obtain an optimal solution for medium and large size problems within a reasonable time because solution time grows exponentially with the size of the problem.

Therefore, a more efficient heuristic method should be developed to obtain a near-optimal schedule for medium and large size problems within a reasonable CPU time.

5. PRACTICAL CASES OF PROBLEM *P*

In this section, we consider two practical cases of our problem (zero setup costs and Lot streaming), and present required modifications in the basic model and the solution procedure.

5.1. Zero setup costs

This case ($A_i = 0$, *for all* i) is a special case of the problem and may occur in the following situations: When machine operators perform setup operations as a part of their normal working time and thus the setup costs are negligible.

In such cases that the main emphasis is on inventories reduction and the management's goal is inventory holding cost reduction (that always is the main part of the total cost).

Moreover, the introduction of setup costs leads to longer cycle durations, and consequently more idle time in the schedule. This is the main reason why we ignore setup costs. Also, such an assumption is reasonable in a JIT environment [4].

In this case, the nonlinear term $\sum_i A_i/T$ in objective function is omitted and only the nonlinear constraint (17) remains. Therefore, in this case Problem P again can be solved with the enumeration method discussed in section 3.

5.2. Lot streaming

In this section, we consider lot streaming case that is a generalization of Problem P. Lot streaming is the process of splitting a lot into a number of portions, often called transfer sublots (or batches) so that successive operations can be overlapped in a multi-stage production system. A major benefit of lot streaming is the reduction in the manufacturing lead time (MLT) and thereby provides an opportunity to the considerably reduction in work-in-process inventories (WIP) and corresponding holding costs. The required modifications in this case, can be examined in the following two sub- cases:

5.2.1. $p_{i,j-1} \geq p_{ij}$

In this sub-case, the production rate for product i at stage j-1 is greater than at stage j. Therefore, the processing start time of this product at stage j must be at the time that the first batch is transferred from stage j-1 to stage j. Then Constraints (6) must be substituted with the following constraints:

$$b_{ij} \geq b_{i,j-1} + \frac{a_{i,j-1}}{p_{i,j-1}} + \tau_{i,j-1}; \; i = 1,\ldots,n, \; j = 2,\ldots,m \tag{22}$$

Where

$a_{i,j-1}$ = transfer batch size of product i from stage j-1 to stage j (determined based on existing unit load)

$\tau_{i,j-1}$ = transfer time for one batch of product i from stage j-1 to stage j.

The evolution of inventory of product i between two successive stages j-1 and j in this sub-case is shown in Fig. 3(a). Therefore, we will have:

$$I_{i,j-1} = d_i \left(b_{ij} - b_{i,j-1}\right)\left(2 - \frac{p_{ij}}{p_{i,j-1}}\right) + \frac{1}{2} d_i^2 \left(\frac{1}{p_{ij}} - \frac{1}{p_{i,j-1}}\right).T \tag{23}$$

5.2.2. $p_{ij} \geq p_{i,j-1}$

In this sub-case, the production rate of product i at stage j is greater than at stage j-1. Therefore, the processing start time on last batch of this product at stage j must be at the time that the processing of entire lot of product i at stage j-1 is completed and last batch of this product is transferred from stage j-1 to stage j. Then, instead of Constraints (6) we will have the following constraints:

$$b_{ij} + \frac{d_i.T}{p_{ij}} - \frac{a_{i,j-1}}{p_{ij}} \geq b_{i,j-1} + \frac{d_i.T}{p_{i,j-1}} + \tau_{i,j-1} \quad \forall i, \; j = 2,\ldots,m \tag{24}$$

The inventory evolution of product i between two successive stages j-1 and j in this sub-case is shown in Fig. 3(b). In this sub-case, $I_{i,j-1}$ will be similar to (23) and thus, the sum of work-in-process inventory holding cost per unit time when lot streaming is allowed, will be:

$$TC_{WIP} = \sum_{i=1}^{n}\sum_{j=2}^{m} h_{i,j-1}.d_i\left[\left(b_{ij} - b_{i,j-1}\right)\left(2 - \frac{p_{ij}}{p_{i,j-1}}\right) + \frac{1}{2} d_i \left(\frac{1}{p_{ij}} - \frac{1}{p_{i,j-1}}\right).T\right] \tag{25}$$

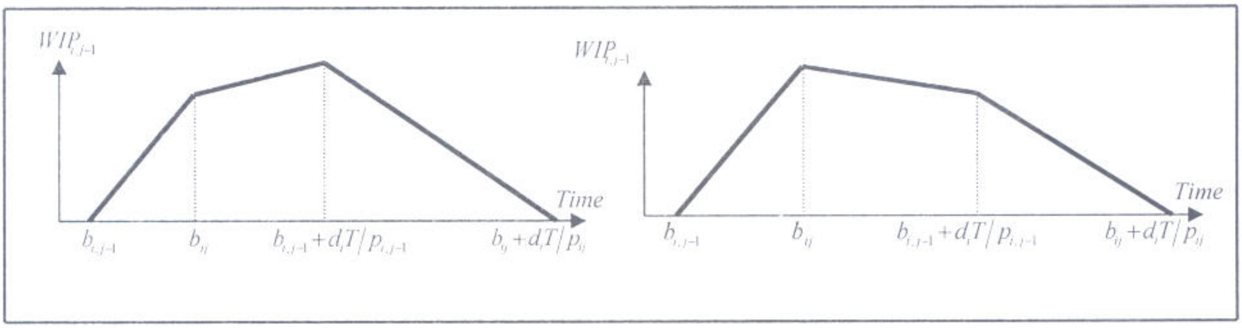


Figure2. WIP inventory curves for the Lot streaming case

Therefore, the objective function in this case can be written as follows:

$$TC = \sum_{i=1}^{n} \frac{A_i}{T} + \sum_{i=1}^{n} \left[h_i \cdot \frac{d_i}{2}\left(1 - \frac{d_i}{p_{im}}\right) + \frac{d_i^2}{2} \sum_{j=2}^{m} h_{i,j-1}\left(\frac{1}{p_{ij}} - \frac{1}{p_{i,j-1}}\right)\right].T + \sum_{i=1}^{n}\sum_{j=2}^{m} h_{i,j-1}.d_i\left(b_{ij} - b_{i,j-1}\right)\left(2 - \frac{p_{ij}}{p_{i,j-1}}\right) \qquad (26)$$

In this case, we again deal with a mixed nonlinear model and we can apply the enumeration method for solving this model to optimality.

6. Numerical example

In this section, a numerical example is presented to illustrate applicability of mathematical model and its solution method for scheduling a Printed Circuit Board (PCB) assembly system that is a typical flexible flow line. Five different types of boards are assembled at two stages of the system. In the first stage there is an axial insertion machine ($m_1=1$) that inserts axial elements such as resistors or transistors on each board, and the second stage has two parallel identical radial machines ($m_2=2$) that insert radial elements such as capacitors on each board. Also, the time unit is assumed one week and planning horizon length is equal to one year or 52 weeks ($H=52$). Table 4 presents other required data for this example.

Table 4. Required data for the example

i	j	d_i	p_{ij}	s_{ij}	A_i	h_{ij}
1	1	550	4500	0.002	1200	1
	2		2000	0.003		3
2	1	400	3000	0.008	3700	2
	2		1200	0.002		5
3	1	750	5000	0.001	350	1
	2		2600	0.008		3
4	1	500	3000	0.006	1000	1
	2		2000	0.005		2
5	1	650	4500	0.004	4000	3
	2		2500	0.006		7

The corresponding mathematical model of this example (at no allowing lot streaming case) is solved using enumeration method and optimal solution is computed as follows:

$$Z = 14437.43, \quad F = 37, \quad T = 1.41,$$
$$\sigma_1 = (3,4,1,2,5),\ \sigma_{12} = (3,1,5),\ \sigma_{22} = (4,2).$$

Therefore, the above optimal schedule should be repeated 37 times over the planning horizon.

7. CONCLUSION REMARKS

In this paper, we have considered the common cycle approach to solve the economic lot sizing and scheduling problem in deterministic flexible flow lines. First, we developed a new mixed zero-one nonlinear model to solve the problem to optimality. Then, to avoid solving this complex mixed nonlinear program directly, we have suggested an efficient enumeration method to determine an optimal solution. Two other applicable cases of Problem P (zero setup costs and lot streaming) are also presented and required modifications in the model formulation and the solution procedure are described. Moreover, through a numerical example, applicability of this formulation and its solution method is shown.

However, based on our experimental results, applying the proposed solution method to determine optimal solution in medium and large size problems requires solving several large-scale mixed zero-one programs that need high computational efforts. Therefore, further research to develop more efficient heuristic methods is on our research line.

Moreover, there are other different directions for future studies. Among them, the following topics are recommended:

- Modeling Problem P using the basic period (multi-cyclic) or time varying lot size approaches,
- Considering non-identical parallel machines at each stage,
- Allowing the backlogging.

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