EFFECTS OF SLIP CONDITION ON THE CHARACTERISTIC OF FLOW IN ICE MELTING PROCESS

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Abstract In this paper a laminar flow of water on an ice layer subjected to a slip condition is considered numerically. The paper describes a parametric mathematical model to simulate the coupled heat and mass transfer events occurring in moving boundary problems associated with a quasi steady state steady flow process. The discretization technique of the elliptic governing differential equations of mass, momentum and energy is based on the control volume finite difference approach and enthalpy method. the results illustrate , the distribution of heat transfer coefficient, ice melting thickness, slip velocity at solid moving boundary and boundary layer thickness for some values of slip velocity coefficient , C_u

Keywords Slip velocity; Moving boundary; Melting;

چکیده در این مقاله جریان آب روی سطح یخ با شرط لغزش بصورت عددی بررسی شده است.معادلات انتقال حرارت وجرم به طور همزمان با مرز متغیر و براساس جریان شبه دائم و یکنواخت شبیه سازی شده است. گسسته سازی معادلات بر اساس حجم کنترل و تفاضلات محدود و با استفاده از روش آنتالپی به ازای مقادیر مختلف پارامتر C_u، ضریب سرعت لغزش،انجام شده است . ضخامت یخ ذوب شده، سرعت لغزش روی سطح یخ ، ضریب انتقال حرارت جابجایی و ضخامت لایه مرزی برای C_u های مختلف به دست آمده است.

1. INTRODUCTION

The modeling of moving boundary problems is important field where mathematicians, engineers in different disciplines and physicists have found a common topic interest. Although they have divergent perspectives, they all converge in their passion for solving this class of problems. As a result there is an extensive literature addressing the complexity and solution of moving boundary problems. References [1-4] address these problems from different perspectives.
The moving boundary problems may be found in solidification and melting phenomena commonly encountered in metallurgical processes and dissolving in metal bath [5-7]. Latent heat thermal energy storage [8,9], oceanography and temperature distribution around the polar area [10], effect of frost on the equipment [11], melting, thawing and freezing of food [12] and exothermic heat of mixing [13] are the other applications of this problem.Gebhart, and Shaukatullah [14] were the first to analyze the natural convection flows exprimentally adjacent to the horizontal surface in cold water.
In relation to the numerical side of melting

problems, various solution methods have been considered. Wilson and Lee [15] Presented a numerical study based on the finite difference analysis for the leading portion of a vertical ice sheet as it melted into fresh water. Wang, also, [16] conducted a numerical study into the buoyancy – induced flows next to a vertical wall of ice melting in porous media that was saturated with water. In the paper of Carey, et all.[17] numerical results for laminar, buoyancy induced flow adjacent to a vertical isothermal surface in cold pure and saline water were presented.

In the experimental work of Carey and Gebhart [18], slow patterns were visualized by seeding the water and illuminating the flow field with a sheet of laser light. Wilson and Vyas [19] applied the thymol – blue technique in order to visualize the velocity profile occurring in the natural convection boundary layer. Oosthuizen and Xu [20] presented evidence that the flow around a horizontal melting ice cylinder is three – dimensional in nature. Gebhart and Wang [21] melted short vertical ice cylinders into cold fresh water in order to visualize the melting convective motion. Fukasako and Yamada [22] have presented an extensive summery of the work carried out on water freezing and ice melting problems.

Sparrow, Pattenkar and Ramadhyani [23] used a finite difference scheme to analyze the melt region created by a heated vertical tube embedded in a solid at fusion temperature. Ng et all. [24] used a stream line – upwind / Petrov – Galerkin finite element method to simulate the melting of a phase change material in a cylindrical horizontal annulus heated isothermally from the inside wall. Youngke and lactoix [25] used a stream function vorticity temperature formulation to track the position of the solid – liquid interface for an ice cylinder melting.

In slip condition at surface of melted ice, water flows at solid boundary with slip velocity less than the free stream velocity [26]. The effects of slip velocity are used for industrial cleansing applications [27] and drag reduction purposes [28-32]. Inspection of velocity on the ice melted is significant from both reducing skin-friction drag and heat and mass transfer aspects. It should be noted that the slip velocity strongly affects the friction and pressure drags because of the delay of the separation point. In Watanabe reports [28-32], the slip velocity at solid boundary was constant, but for the ice melted problem this assumption is not accurate. The reason of this dissimilarity was variation of thickness of boundary layer, and pursuant to it, convection heat transfer coefficients. Thus the slip velocity can be function of thickness of melted ice; free stream velocity and temperature. Note that various approaches have been taken by the preceding authors in their attempts to numerically simulate the melting process. However none of them has so far attempted to solve the forced convection boundary layer with slip velocity at solid boundary. This paper attempts to solve numerically the forced convection boundary layer. The aim of this study is to understand the effects of the process of ice melting on the slip velocity at solid boundary by applying slip velocity coefficient C_u.

2. PHYSICAL CONFIGURATION AND MATHEMATICAL FORMULATION

Figure.1 shows a laminar water flow over an ice plate subjected to a moving boundary due to the ice melting.

In an effort to develop the mathematical formulation the following simplifying assumptions are considered

1. The thermo physical properties are concentration dependent except for density, which varies with temperature.
2. Surface temperature of ice is $0.01\,^\circ C$ and there is no temperature gradient in the ice layer.

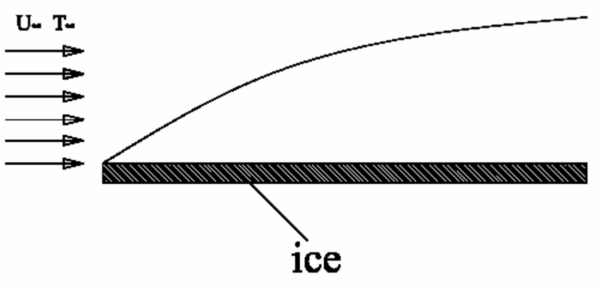


Figure 1. Two dimensional boundary layer water flow over an ice plane

Figure 2 shows the temperature distribution in the ice layer and water. The ice is assumed to be initially at its melting point.

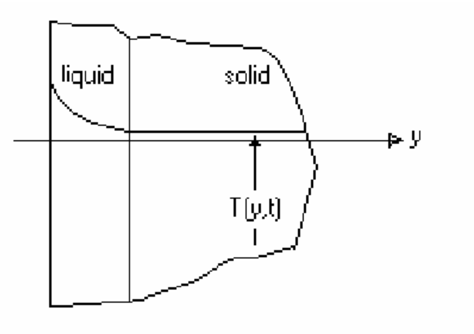


Figure 2. Temperature distribution in ice layer and water boundary layer

The governing differential equations of mass, momentum and energy for the laminar and incompressible flow with no viscous dissipation in a Cartesian coordinate system are represented as follows; respectively.

$$\frac{\partial \rho}{\partial t}+\frac{\partial(\rho u)}{\partial x}+\frac{\partial(\rho v)}{\partial y}=0 \qquad (1)$$

x – Component:

$$\frac{\partial}{\partial t}(\rho u)+u\frac{\partial}{\partial x}(\rho u)+v\frac{\partial}{\partial y}(\rho u) = \left[\frac{\partial}{\partial x}(\mu\frac{\partial u}{\partial x})+\frac{\partial}{\partial y}(\mu\frac{\partial u}{\partial y})\right] \qquad (2)$$

y – Component:

$$\frac{\partial}{\partial t}(\rho v)+u\frac{\partial}{\partial x}(\rho v)+v\frac{\partial}{\partial y}(\rho v) = \left[\frac{\partial}{\partial y}(\mu\frac{\partial v}{\partial x})+\frac{\partial}{\partial y}(\mu\frac{\partial v}{\partial y})\right] \qquad (3)$$

$$\frac{\partial}{\partial t}(\rho h)+u\frac{\partial}{\partial x}(\rho h)+v\frac{\partial}{\partial y}(\rho h) = \frac{\partial}{\partial x}(k\frac{\partial T}{\partial x})+\frac{\partial}{\partial y}(k\frac{\partial T}{\partial y}) \qquad (4)$$

The enthalpy transforming model [2,33] is used to convert the energy equation into only one dependent variable, enthalpy .viz,$dh=C_p dT$.

The water density-temperature relationship has been formulated within 0.01-17$^\circ C$ by Gebhart and Mollendorf [35] according to:

$$\rho=\rho_m[1-w(T-T_m)^q] \qquad (5)$$

Where ρ_m is the maximum density (ρ_m =999.972kg/m^3) and w=9.2793$\times$10^{-6} ($^\circ C$)$^{-4}$, T_m= 4.0293$^\circ C$ and q=1.899816 are constant values proposed by Gebhart and Mollendorf. In order to solve this problem in more general way, the above governing equations are nondimensionlized by the following dimensionless variables.

(6)

$$x^*=\frac{x}{L} \qquad y^*=\frac{y}{L} \qquad u^*=\frac{uL}{\alpha_i}$$

$$V^*=\frac{VL}{\alpha_l} \qquad h^*=\frac{h}{H_f} \qquad \rho^*=\frac{\rho}{\rho_m}$$

$$t^*=\frac{t\alpha_i}{L^2} \qquad \Gamma_\mu=\frac{\mu}{\alpha_i\rho_m} \qquad \Gamma_\alpha=\frac{k}{\alpha_i\rho_m c_p}$$

$$\frac{\partial \rho^*}{\partial t^*}+\frac{\partial u^*}{\partial x^*}+\frac{\partial v^*}{\partial y^*}=0 \qquad (7)$$

$$\frac{\partial u^*}{\partial t^*}+u^*\frac{\partial u^*}{\partial x^*}+V^*\frac{\partial u^*}{\partial y^*} = \left[\frac{\partial^2}{\partial x^{*2}}(\Gamma_\mu u^*)+\frac{\partial^2}{\partial y^{*2}}(\Gamma_\mu u^*)\right] \qquad (8)$$

$$\frac{\partial v^*}{\partial t^*}+u^*\frac{\partial v^*}{\partial x^*}+V^*\frac{\partial v^*}{\partial y^*}$$

$$=\left[\frac{\partial^2}{\partial x^{*2}}(\Gamma_\mu v^*)+\frac{\partial^2}{\partial y^{*2}}(\Gamma_\mu v^*)\right] \quad (9)$$

$$\frac{\partial h^*}{\partial t^*}+u^*\frac{\partial h^*}{\partial x^*}+V^*\frac{\partial h^*}{\partial y^*}$$

$$=\left[\frac{\partial}{\partial x^*}(\Gamma_\alpha\frac{\partial h^*}{\partial x^*})+\frac{\partial}{\partial y^*}(\Gamma_\alpha\frac{\partial h^*}{\partial x^*})\right] \quad (10)$$

The initial velocity for the momentum equation is set equal to Blasius result. The initial temperature of the ice layer is set equal to its melting point.

φ(0,y,0)=[u_∞ , 0 , T_∞]

φ(x,0,0)=[0, 0 ,273$^\circ k$]

At the interface between the ice layer and the water flow, the boundary conditions are :

φ(x,0,t)=[u_s , 0 ,273$^\circ k$]

φ(x, δ ,t)=[u_∞, 0 , T_∞]

Where φ=[u,v,T]^T u_s is the tangential vector of the ice layer and δ is the boundary layer thickness.
The convective heat-transfer coefficient is obtained by the following equation.

$$h(x)=\frac{\dot{q}''(x)}{T_s-T_\infty} \quad (11)$$

The thickness of the ice melting is obtained by:

$$D_m(x)=\frac{\dot{q}''(x)}{\rho_i h_{fg}} \quad (12)$$

Where $\dot{q}''(x)$ the local heat transfer, hif is is the latent heat and ρ_i is the density of ice.
The dimensionless thickness of the ice melting is defined as:

$$D_m^*(x)=\frac{\text{Dm(x)}}{(\text{Dm})_{\text{Max}}} \quad (13)$$

Where $(D_m)_{\text{Max}}$ is the thickness of the ice melted at the leading edge.
The slip velocity at solid boundary is obtained by the slip velocity coefficient C_u as:

$$U_s = C_u.\, u_\infty\, D^* (x). \quad (14)$$

3. METHOD OF SOLUTION

The governing equations are discretized with the control volume implicit finite difference procedure referred to as SIMPLER (Semi-Implicit Method for Pressure Linked Equations Revised) [34]. The computation is done on a domain with 27x$\times$35y nodes. The node is concentrated near the surface of the ice for the purpose of more accuracy. Figure.3 is illustrates a set of nodes in the domain. The convergence for each iteration is assured with the dimensionless time step under 9×10^{-7}.
For this problem a code is developed and solution is iterated for one step of time. Solution is started with no slip condition i.e. Blasius's results [36]. In the next iteration the slip velocity coefficient C_u is entered and the iteration is started with slip velocity condition so that the solution is converged.
It should be noted that the slip velocity coefficient is determined by Raoufpanah et al. and with the change of this parameter the slip velocity, velocity and temperature distributions are changed.

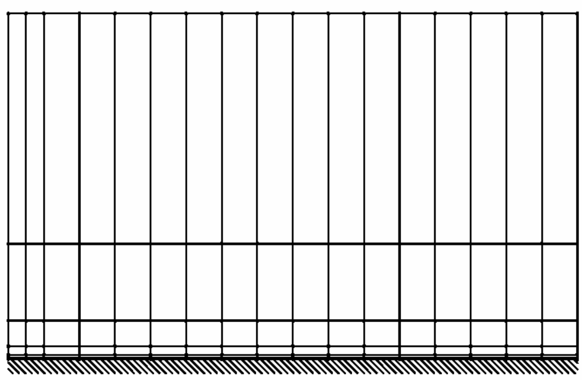


Figure 3. Illustration of grid in the domain

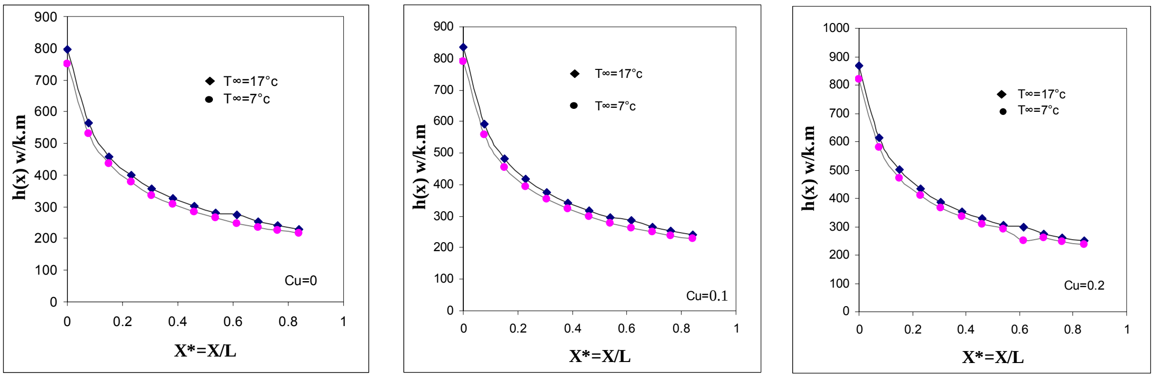
4. RESULTS AND DISCUSSIN

The stability and consistency of the finite difference method depend on the amount of ice melted in each time step. The convergence of each iteration is assured with the dimensionless time step under 9×10^{-7}.

The solution becomes unstable by increasing the free stream temperature and velocity. The solution is also diverged by more increasing T_∞ and u_∞.

The local convection heat transfer coefficient along S vector for two different water temperatures is shown in Figure. 4. The result are restricted between $0.01^\circ c$ to $17^\circ c$ because of the temperature restriction in Eq.(5). The results are obtained for these conditions privately.

The local Nusselt number, Nux=hx/k, is obtained for two different fixed temperature, during the melting process of ice. These results are shown in Figure.5. The thickness of the ice melted for two different free stream temperatures $T_\infty = 7^0 c$ and $17^\circ c$ and free stream velocity u_∞=0.5 m/s are shown in Figure.6. These curves have been after passing of one time step.



(a) (b) (c)

Fig.4 –Local convection heat transfer coefficient for T_∞ =7°C , 17°C and u_∞ =0.5 m/s ; (a) C_u=0 ; (b) C_u=0.1 ; (c) C_u=0.2

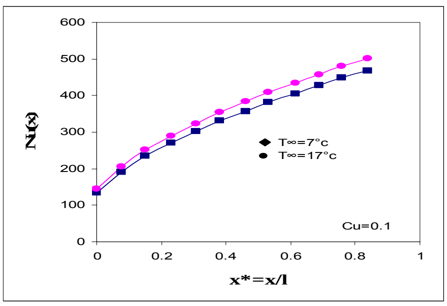
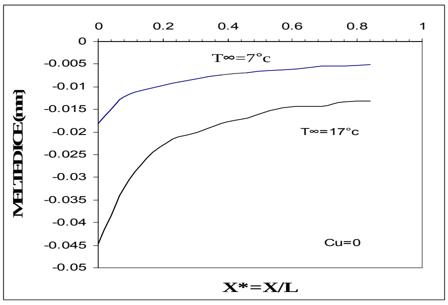
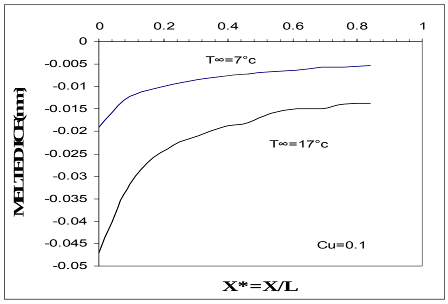


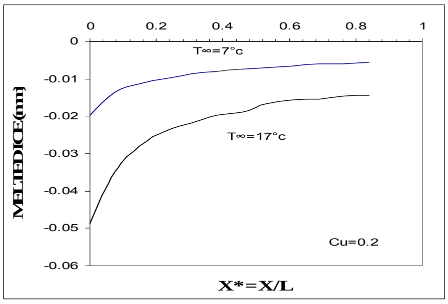
Figure 5. Local Nusselt number for T_∞ =7 °C,17°C , u_∞ =0.5m/s and C_u=0.1



(a)

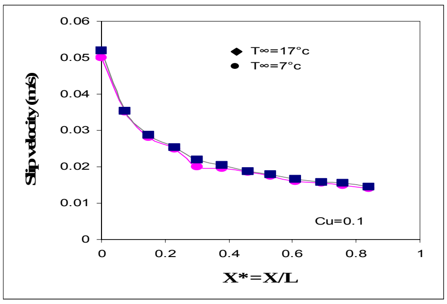


(b)

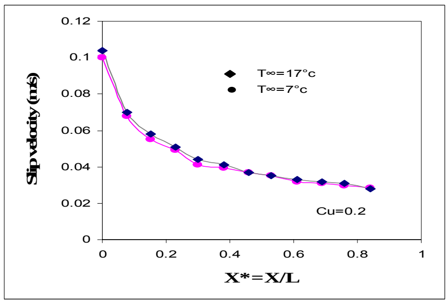


(c)

Figure 6. Thickness of melted ice distribution for T_{∞} = 7 °C and 17°C, u_{∞} =0.5m/s ; (a) Cu=0; (b) C_u=0.1 ; (c) C_u=0.2 after passing of one time step.



(a)

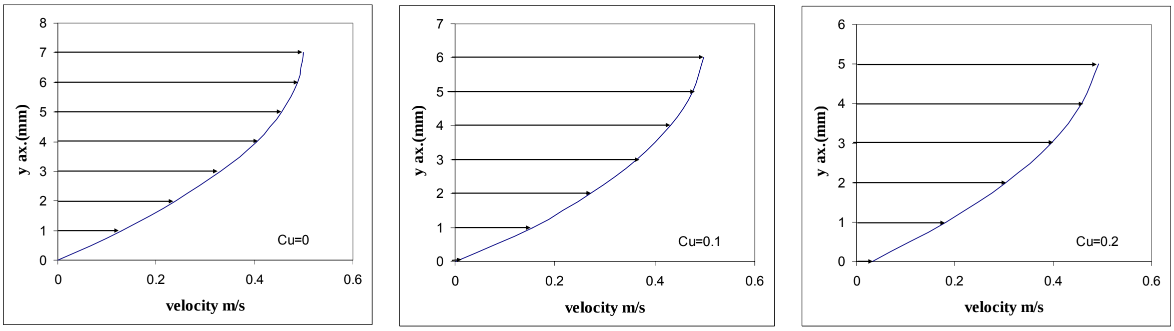


(b)

Figure 7. Slip velocity distribution for T_{∞} =7 °C and 17 °C, u_{∞} =0.5 m/s; (a): C_u= 0.1 ; (b): C_u=0.2

The slip velocity distribution along the ice surface is shown in Figure.7- for different C_u.

The velocity profiles for Re=2.5×10^5 and three different fixed ship velocity coefficients are shown in Figure 8.



(a) **(b)** **(c)**

Figure 8. Velocity profiles for $T_\infty = 7^0 c$, $u_\infty = 0.5\dfrac{m}{S}$ and Re = 2.5×10^5;

(a) c_u = 0; (b) c_u = 0.1; (c) c_u = 0.2

5. CONCLUSIONS

A Parametric numerical investigation for forced convection ice melting under slip boundary condition has been carried out. The following conclusions are reached.

1. The results show that the complex transient phenomenon can be qualitatively captured using a computational fluid dynamic.
2. The convection heat transfer h(s) increase by increasing slip velocity coefficient parameter C_u
3. The thickness of melted ice increase by increasing slip velocity coefficient parameter Cu
4. The no – slip or slip condition and velocity of slip is important boundary conditions in ice melting problem. and results varies effectively by varying this boundary condition.
5. The maximum of slip velocity occurs at the leading edge of the surface and slip velocity decrease by increasing Reynolds number.
6. Nusselt number in this study is different with Churchill's correlation [36] about 5% to 10%.

6.NOMENCLATURE

C_u slip velocity coefficient
D_m melted ice thickness
h enthalpy , heat transfer coefficient
k thermal conductivity
L ice layer length
Nu Nusselt number
$\dot{q}''$ local heat flux
Re Reynolds number
t time
T temperature
u velocity component in x-direction
v velocity component in y- direction
x coordinate system
y coordinate system

Greek letters

α thermal diffusivity
Γ diffusivity dimensionless
δ boundary layer thickness
Δ difference
μ viscosity
ρ density
φ variable vector

subscript

i ice
if fusion heat
m melting
max maximum
s slip
a thermal
μ viscous
* dimensionless
. rate

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