STATE DEPENDENT M/M/C/K/N MACHINING SYSTEM WITH MIXED SPARES AND REMOVABLE REPAIRMEN

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Abstract This study deals with a state dependent machining system having provision of mixed spares. The service facility of the system consists of permanent as well as removable additional repairmen. When all the spares are utilized, the system works in short mode. The steady state solution of the queue size distribution is derived using product type solution. Expressions for some performance measures are established. Some earlier models are deduced as special cases of the model for specific values of the parameters.

Key Words Machine repair, Queue, Mixed Spares, Removable Repairmen, State Dependent Rates, Queue Size

چکیده این مقاله به بحث در باره سامانه ماشینکاری وابسته به شرایطی که از ماشین رزرو به صورت مخلوط استفاده می‌شود، پرداخته است. تسهیلات خدمت گزاری از تعمیر کار دائمی و تعمیر کار اضافی موقت تشکیل شده است. وقتی تمام ماشینهای رزرو مورد استفاده قرار گرفته اند، سامانه در وضعیت کوتاه کار می‌کند. حل حالت ثبات مربوط به توزیع نوبت با استفاده از حل نوع محصول به دست می‌آید. بیانهایی برای سنجش عملکرد ساخته شده است. برای مقادیر مشخصی از متغیر ها، مدل های اولیه تحت عنوان حالات خاص نتیجه گیری شده است.

1. INTRODUCTION

Machines are an indispensable part of any manufacturing or production system. Therefore, the efficiency of machining system is critical to the overall competitiveness. Machine repair modeling is being used to predict the system performance of such systems. This may be fruitful to the system designer to determine the optimal combination of spare units and removable repairmen in order to increase the system efficiency. When the number of failed units exceeds the number of permanent repairmen, the provision of removable repairmen often ensures the desired reliability with a limited number of spares at a reasonable cost.

The machine repair problems have been an area of interest for many researchers. Sivazlian and Wang [1] considered M/M/R machine repair model with warm standby. Wang and Sivazlian [2] presented cost analysis of a M/M/R machine repair model having spares and operating units with variable service rates. The M/G/1 machine repair model with spares was investigated by Gupta and Rao [3].

A transient analysis of the M/M/C machine repair problem with spares was provided by Jain and Dhyani [4]. Jain and Singh [5] studied a finite queuing model with random failures and delayed repairs. Shawky [6] considered M/M/C/K/N machine interference model with balking, reneging and cold spares. An optimal repair/replacement policy for a general repair model was discussed by Jiang, et al. [7] and Grassmann, et al. [8]. Armstrong [9] suggested age-repair policies for machine repair problems addressing the problem of choosing the optimal service rate for a one-server queue with state-dependent Poisson arrivals.

Some works have been reported in literature regarding provision of additional servers. Jain [10] analyzed M/M/R machine repair problem with spares and additional repairmen by using queue size distribution. Jain, et al. [11] extended the work of Shawky [12] by including one additional repairman in case of long queue of failed units. Jain, et al. [13] studied a machine repair problem with discouragement and additional servers. A multi-server queuing system with additional repairmen and discouragement was investigated by Jain and Sharma [14]. Jain and Singh [15] proposed a loss and delay Markovian queuing system having removable additional repairmen.

In this paper, we develop a M/M/C/K/N queuing model for a machining system with mixed spares, additional repairmen and variable failure and repair rates. Using birth-death process, the steady state queue size distribution is obtained to facilitate performance measures. The rest of the paper is organized as follows: Section 2 covers model description and governing equations. The steady state solution for queue size distribution is given in the next section. Some special cases are deduced in section 4. Section 5 establishes some system characteristics and cost function. We conclude our investigations in section 6 by highlighting the scope of the work done.

2. DESCRIPTION OF THE MODEL AND GOVERNING EQUATIONS

The basic assumptions governing the queuing model for multi-components repairable machining system are as follows:

1. The machining system operates in normal mode with N operating units.
2. There is a provision of mixed spares, of which Y are cold and S warm.
3. The repair facility consists of a pool of C permanent and r removable repairmen.
4. When a unit fails, a spare is immediately substituted for it. The switch over time from standby to operating state is negligible.
5. As long as all the spares are not utilized i.e. system works in normal mode with N operating units, the life times of operating units are exponentially distributed with mean rate λ. The repair times of permanent repairmen are identical exponentially distributed random variables.
6. Once all the spares are utilized, the system starts working in short mode as there are less than N operating units but more than or equal to m. In short mode, due to efficiency degradation, failure rate λ increases to λ_d. To cope up with this situation, the permanent repairmen work with faster rate so that the repair rate μ changes to μ_f ($>\mu$).
7. μ_i is the repair rate for i^{th} (i = 1, 2, ..., r) removable repairman which provides repair according to exponential distribution and FIFO discipline.
8. If the number of failed units is more T, the additional removable repairmen turn on one by one with additional load of T failed units so that j^{th} (j = 1, 2, ..., r) additional repairman starts repairing when number of failed units is more than jT but less than or equal to (j+1)T and removes as number of failed units decreases to jT.
9. When a failed unit is repaired, it joins the standby group if system is working in normal mode, otherwise works with other operating units. After repairing, the units are as good as a new one.

The failure rates and repair rates of the units are state dependent and given by

$$\lambda_n = \begin{cases} N\lambda + S\alpha & o \le n < Y \\ N\lambda + (S+Y-n)\ \alpha & Y \le n < Y+S \\ (N+S+Y-n)\ \lambda d & Y+S \le n < N+Y+S-m \end{cases} \tag{1}$$

$$\mu_n = \begin{cases} n\mu & 0 < n \le C \\ C\mu & C < n \le Y + S \\ C\mu f & Y + S + 1 \le n \le T \\ C\mu f + \sum_{i=1}^{j} \mu_i & jT < n \le (j+1)T \\ C\mu f + \sum_{i=1}^{j} \mu_i & rT < n \le N + S + Y - m \end{cases} \tag{2}$$

We consider two cases for formulating the problem. For both cases, Chapman-Kolmogorov equations governing the model are constructed for steady state.

Case I: C ≤ Y

Using state dependent failure and repair rates given in (1) and (2), the steady state difference equations for this case are as follows:

$$-(N\lambda + S\alpha)P_0 + \mu P_1 = 0,\ (n = 0) \tag{3a}$$

$$-(N\lambda + S\alpha + n\mu)P_n + (N\lambda + S\alpha)P_{n-1} + (n + 1)\mu P_{n+1} = 0,(0 < n< C) \tag{3b}$$

$$-(N\lambda + S\alpha + C\mu)P_n) + (N\lambda + S\alpha)P_{n-1} + C\mu P_{n+1} =0, (C \le n \le Y) \tag{3c}$$

$$-[N\lambda + (S+Y-n)\alpha + C\mu]P_n + [N\lambda + (S+Y+1- n)\alpha] P_{n-1} + C\mu P_{n+1} = 0,(Y < n < Y+S) \tag{3d}$$

$$-(N\lambda_d + C\mu)P_{Y+S} + (N\lambda + \alpha]P_{Y+S-1} + C\mu_f P_{Y+S+1} = 0, (n = Y + S) \tag{3e}$$

$$-[(N+S+Y-n)\lambda_d + C\mu_f]P_n + (N+S+Y+1-n)\lambda_d P_{n-1} + C\mu_f P_{n+1}(t),(Y+S < n < T) \tag{3f}$$

$$-[(N+S+Y-T)\lambda_d + C\mu_f]P_T + (N + S + Y + 1 - T) \lambda_d P_{T-1} + (C\mu_f + \mu_1)P_{T+1} = 0\ (n = T) \tag{3g}$$

$$-[(N+S+Y-jT)\lambda_d + C\mu_f + \sum_{i=1}^{j-1}\mu_i]P_{jT} + (N + S + Y + 1 - jT)\lambda_d P_{jT-1} + (C\mu_f + \sum_{i=1}^{j} \mu_i)P_{jT+1} = 0,(n = jT; j=1,2,\ldots,r-1) \tag{3h}$$

$$-[(N+S+Y-n)\lambda_d + C\mu_f + \sum_{i=1}^{j}\mu_i]P_n + (N + S + Y + 1 - n)\lambda_d P_{n-1} + (C\mu_f + \sum_{i=1}^{j}\mu_i)P_{n+1} = 0,(jT < n < (j+1)T) \tag{3i}$$

$$-[(N+S+Y-rT)\lambda_d + C\mu_f + \sum_{i=1}^{r-1}\mu_i]P_{rT} + (N + S + Y + 1 - rT)\lambda_d P_{rT-1} + (C\mu_f + \sum_{i=1}^{r}\mu_i)P_{rT+1} = 0,(n = rT) \tag{3j}$$

$$-[(N+S+Y-n)\lambda_d + C\mu_f + \sum_{i=1}^{r}\mu_i]P_n + (N + S + Y + 1 - n)\lambda_d P_{n-1} + (C\mu_f + \sum_{i=1}^{r}\mu_i)P_{n+1} = 0,(rT < n < N + S + Y - m) \tag{3k}$$

$$-[C\mu_f + \sum_{i=1}^{r}\mu_i]P_{N+S+Y-m} + (m+1)P_{N+S+Y-m-1} = 0\ (n = L) \tag{3l}$$

Case II: C > Y

In this case, we also use (1) and (2) to obtain the steady state difference equations and obtain

$$-(N\lambda + S\alpha)P_0 + \mu P_1 = 0,(n = 0) \tag{4a}$$

$$-(N\lambda + S\alpha + n\mu)P_n + (N\lambda + S\alpha)P_{n-1} + (n + 1)\mu P_{n+1} = 0,(0 < n \le Y) \tag{4b}$$

$$-[N\lambda + (S+Y-n)\alpha + n\mu]P_n + [N\lambda + (S+Y+1- n)\alpha] P_{n-1} + (n+1)\mu P_{n+1} = 0,(Y < n < C) \tag{4c}$$

$$-[N\lambda + (S+Y-n)\alpha + C\mu]P_n + [N\lambda + (S+Y+1- n)\alpha] P_{n-1} + C\mu P_{n+1} = 0, (C \le n < Y+S) \tag{4d}$$

The rest of the equations for this case are identical to equations 3(e) – 3 (l) as obtained in the previous case.

$$
P_n = \begin{cases}
\dfrac{(N\lambda+S\alpha)^n}{n!\mu} P_0 & 0 \le n \le C \\[2ex]
\left(\dfrac{N\lambda+S\alpha}{\mu}\right)^n \dfrac{1}{C!(C)^{n-C}} P_0, & C \le n \le Y \\[2ex]
\left(\prod_{i=Y=2}^{n} S_i\right) \dfrac{\beta}{(C)^{n-C}} P_0, & Y+1 \le n \le Y+S \\[2ex]
\left(\prod_{j=0}^{n-(Y+S+1)} \delta_j\right)\left(\prod_{i=Y=2}^{Y+S} S_i\right) \dfrac{\beta}{(C)^{n-C}} P_0, & Y+S \le n \le T \\[2ex]
\dfrac{\left(\prod_{k=T}^{n-1} (N+S+Y-k)\lambda_d\right)\left(\prod_{j=0}^{T-(Y+S+1)} \delta_j\right)\left(\prod_{i=Y=2}^{Y+S} S_i\right)}{\left(\prod_{l=1}^{j-1}\left(C\mu_f+\sum_{i=1}^{l}\mu_i\right)^T\right)\left(C\mu_f+\sum_{i=1}^{j}\mu_i\right)^{n-jT}} \cdot \dfrac{\beta}{(C)^{T-C}} P_0 & jT < n \le (j+1)T,\ 1 \le j < r \\[2ex]
\dfrac{\left(\prod_{k=T}^{n-1} (N+S+Y-k)\lambda_d\right)\left(\prod_{j=0}^{T-(Y+S+1)} \delta_j\right)\left(\prod_{i=Y=2}^{Y+S} S_i\right)}{\left(\prod_{l=1}^{r-1}\left(C\mu_f+\sum_{i=1}^{l}\mu_i\right)^T\right)\left(C\mu_f+\sum_{i=1}^{r}\mu_i\right)^{n-rT}} \cdot \dfrac{\beta}{C)^{T-C}} P_0 & rT < n \le N+Y+S-m
\end{cases}
\tag{5}
$$

where

$$
S_i = \frac{N\lambda+(Y+S+1-i)\alpha}{\mu}, \beta = \frac{1}{C!}\left(\frac{N\lambda+S\alpha}{\mu}\right)^{Y+1}
$$

$$
\delta_j = \frac{(N-j)\lambda_d}{\mu_f}
$$

We determine P_0, using the normalizing condition

$$
\sum_{n=0}^{N+S+Y-m} P_n = 1
$$

which gives

$$
P_0^{-1} = \sum_{n=0}^{C} \frac{(N\lambda+S\alpha)^n}{n!\mu^n} + \sum_{n=C+1}^{Y} \frac{(N\lambda+S\alpha)^n}{n!\mu^n (C)^{n-c}} + \beta \sum_{N=y+1}^{y+s} \left[\frac{\left(\prod_{i=Y=2}^{N} S_i\right)}{(C)^{n-C}}\right]
$$

$$
+\beta\left(\prod_{i=Y=2}^{Y+S} S_i\right) \sum_{N=y+s=1}^{T} \left(\prod_{J=0}^{n-(Y+S+1)} \delta_J\right) \frac{1}{(C)^{n-C}} + \beta\left(\prod_{i=Y+2}^{y+s} S_i\right)
$$

$$
\left(\prod_{J=0}^{T-(y+s+1)} \delta_J\right) \frac{1}{(C)^{t-C}} \sum_{J=1}^{r-1} \sum_{n=jT+1}^{(j+1)T} \frac{\left(\prod_{k=T}^{n-1}(N+S+Y-k)\lambda_d\right)}{\left(\prod_{l=1}^{j-1}\left(C\mu_f+\sum_{i=1}^{l}\mu_i\right)^T\right)\left(C\mu_f+\sum_{i=1}^{j}\mu_i\right)^{n-jT}}
$$

$$
+\beta\left(\prod_{i=Y=2}^{Y+S} S_i\right)\left(\prod_{J=0}^{T-(Y+S+1)} \delta_J\right)
$$

$$
\frac{1}{(C)^{t-C}} \sum_{n=rT+1}^{N+S+Y-m} \frac{\left(\prod_{k=T}^{n-1}(N+S+Y-k)\lambda_d\right)}{\left(\prod_{l=1}^{r-1}\left(C\mu_f+\sum_{i=1}^{l}\mu_i\right)^T\right)\left(C\mu_f+\sum_{i=1}^{r}\mu_i\right)^{n-rT}}
\tag{6}
$$

The expected number of failed units in the system is

$$E(n) = \sum_{n=1}^{N+S+Y-m} n.P_n \tag{7}$$

The machine availability i.e. rate of production per machine is obtained as

$$M.A. = 1 - \frac{E(N)}{N+Y+S} \tag{8}$$

Case II: C > Y

In this case, we also obtain product type solution for the steady state queue size distribution using equations 4(a) – 4(d) and 3(e) – 3(l) as
Here, also we employ the normalizing condition

$\sum_{n=0}^{N+S+Y-m} P_n = 1$ to yield

$$P_n = \begin{cases} \left(\dfrac{N\lambda+S\alpha}{\mu}\right)^n \dfrac{1}{n!} P_0, & 0 \le n \le Y \\[2ex] \left(\prod_{i=Y=2}^{n} S_i\right) \dfrac{\beta C!}{n!} P_0, & Y+1 \le n \le C \\[2ex] \left(\prod_{i=Y=211}^{Y+S} S_i\right) \dfrac{\beta}{(C)^{n-C}} P_0, & C \le n \le Y+S \\[2ex] \left(\prod_{j=0}^{n-(Y+S+1)} \delta_j\right)\left(\prod_{i=Y=2}^{Y+S} S_i\right) \dfrac{\beta}{(C)^{n-C}} P_0, & Y+S < n \le T \\[2ex] \dfrac{\left(\prod_{k=T}^{n-1} (N+S+Y-k)\lambda_d\right)\left(\prod_{j=0}^{T-(Y+S+1)} \delta_j\right)\left(\prod_{i=Y=2}^{Y+S} S_i\right)}{\left(\prod_{l=1}^{j-1}\left(C\mu_f + \sum_{i=1}^{l}\mu_i\right)^T\right)\left(C\mu_f + \sum_{i=1}^{j}\mu_i\right)^{n-jT}} . \dfrac{\beta}{(C)^{T-C}} P_0 & jT < n \le (j+1)T,\ 1 \le j < r \\[2ex] \dfrac{\left(\prod_{k=T}^{n-1} (N+S+Y-k)\lambda_d\right)\left(\prod_{j=0}^{T-(Y+S+1)} \delta_j\right)\left(\prod_{i=Y=2}^{Y+S} S_i\right)}{\left(\prod_{l=1}^{r-1}\left(C\mu_f + \sum_{i=1}^{l}\mu_i\right)^T\right)\left(C\mu_f + \sum_{i=1}^{r}\mu_i\right)^{n-rT}} . \dfrac{\beta}{(C)^{T-C}} P_0 & rT < n \le N+Y+S-m \end{cases} \tag{9}$$

$$\begin{aligned} P_0^{-1} = {} & \sum_{n=0}^{Y} \frac{(N\lambda+S\alpha)^n}{n!\mu^n} \\ & + \beta.C! \sum_{n=Y+1}^{C} \left(\prod_{i=Y=2}^{n} S_i\right)\frac{1}{n!} \\ & + \beta. \sum_{n=c+1}^{y+s} \left(\prod_{i=Y=2}^{n} S_i\right)\frac{1}{(C)^{n-c}} \\ & + \beta.\left(\prod_{i=Y=2}^{Y+S} S_i\right) \sum_{N=Y+S+1}^{T} \left(\prod_{J=0}^{n-(Y+S+1)} \delta_J\right)\frac{1}{(C)^{t-C}} \\ & + \beta.\left(\prod_{i=Y=2}^{Y+S} S_i\right)\left(\prod_{J=0}^{T-(Y+S+1)} \delta_J\right)\frac{1}{(C)^{T-C}} \\ & \sum_{J=1}^{r-1} \sum_{n=jT+1}^{(j+1)T} \frac{\left(\prod_{k=T}^{n-1} (N+S+Y-k)\lambda_d\right)}{\left(\prod_{l=1}^{j-1}\left(C\mu_f + \sum_{i=1}^{l}\mu_i\right)^T\right)\left(C\mu_f + \sum_{i=1}^{j}\mu_i\right)^{n-jT}} \\ & + \beta.\left(\prod_{i=Y+2}^{Y+S} S_i\right)\left(\prod_{J=0}^{T-(Y+S+1)} \delta_J\right)\frac{1}{(C)^{t-C}} \\ & \sum_{n=rT+1}^{N+S+Y-m} \frac{\left(\prod_{k=T}^{n-1} (N+S+Y-k)\lambda_d\right)}{\left(\prod_{l=1}^{r-1}\left(C\mu_f + \sum_{i=1}^{l}\mu_i\right)^T\right)\left(C\mu_f + \sum_{i=1}^{r}\mu_i\right)^{n-rT}} \end{aligned} \tag{10}$$

We can find the expected number of failed units in this machining system using Equation 7. Here also, the machine availability is computed by using Equation 8.

3. SPECIAL CASES

3.1 M/M/C/K/N Model with Additional Repairmen and Cold Spares If $\lambda = \lambda_d$, $\mu = \mu_f$ and $S = 0$ then our model reduces to M/M/C/K/N with cold spares only and having additional repairmen. Now we obtain queue size distribution as indicated in Equation 11.

$P_n =$

$$P_n = \begin{cases} \dfrac{(N\rho)^n}{n!} P_0, & 0 \le n \le C \\ \dfrac{(N0\rho)^n}{C!(C)^{n-C}} P_0, & C \le n \le Y \\ \left(\prod_{j=0}^{n-(Y+1)} \delta_j\right) \dfrac{(N\rho)^{Y+1}}{C!(C)^{n-C}} P_0, & Y \le n \le T \\ \dfrac{\left(\prod_{k=T}^{n-1} (N+Y-k)\lambda\right)\left(\prod_{j=0}^{T-(Y+1)} \delta_j\right)(N\rho)^{Y+1}}{\left(\prod_{l=1}^{j-1}\left(C\mu + \sum_{i=1}^{l} \mu_i\right)^T\right)\left(C\mu + \sum_{i=1}^{j} \mu_i\right)^{n-jT} C!(C)^{T-C}} P_0, & jT < n \le (j+1)T, 1 \le j < r \\ \dfrac{\left(\prod_{k=T}^{n-1} (N+Y-k)\lambda\right)\left(\prod_{j=0}^{T-(Y+1)} \delta_j\right)(N\rho)^{Y+1}}{\left(\prod_{l=1}^{r-1}\left(C\mu + \sum_{i=1}^{l} \mu_i\right)^T\right)\left(C\mu + \sum_{i=1}^{r} \mu_i\right)^{n-rT} C!(C)^{T-C}} P_0, & rT < n \le N+Y-m \end{cases} \quad (11)$$

where

$$\rho = \lambda/\mu, \ \delta_j = \frac{(N-j)\lambda}{\mu}$$

3.2 M/M/C/K/N Model with Mixed Spares

Setting $r = 0$, $\lambda = \lambda_d$ and $\mu = \mu_f$, our model reduces to the usual M/M/C/K/N model with mixed spares.

Considering only the case $C \le Y$ we have

$$P_n = \begin{cases} \left(\dfrac{N\lambda + S\alpha}{\mu}\right)^n \dfrac{1}{n!} P_0, & 0 \le n \le C \\ \left(\dfrac{N\lambda + S\alpha}{\mu}\right)^n \dfrac{1}{C!\,(C)^{n-C}} P_0, & C \le n \le S \\ \left(\prod_{i=Y=2}^{n} S_i\right) \dfrac{\beta}{(C)^{n-C}} P_0, & Y+1 \le n \le Y+S \\ \left(\prod_{k=0}^{n-(Y+S+1)} \delta_k\right)\left(\prod_{i=Y+2}^{Y+S} S_i\right) \dfrac{\beta}{(C)^{n-C}} P_0, & Y+S+1 \le n \le Y+S+N-m \end{cases} \quad (12)$$

where

$$S_i = \frac{N\lambda + (Y+S+1-i)\alpha}{\mu}$$

$$\beta = \frac{1}{C!}\left(\frac{N\lambda + S\alpha}{\mu}\right)^{Y+1}$$

$$\delta_k = \frac{(N-k)\lambda}{\mu}$$

If $Y = 0$ and $S = 0$, the present special case provides results for classical M/M/C/K/N machine repair problem as discussed in Kleinrock (1985).

4. COST ANALYSIS

Before presenting a cost analysis, we establish some system characteristics as follows

Expected number of unused cold spare units in the system

$$E(UCS) = \sum_{n=0}^{Y} (Y-n)P_n \quad (13)$$

Expected number of unused warm standby units in the system

$$E(UYS) = Y\sum_{n=0}^{S} P_n + \sum_{n=Y+1}^{Y+S} (Y+S-n)P_n \quad (14)$$

Expected number of operating units in the system

$$E(O) = N - \sum_{n=Y+S+1}^{Y+S+K} [n-(Y+S)]P_n \quad (15)$$

Expected number of idle permanent repairmen

$$E(I) = \sum_{n=0}^{C-1} (C-n)P_n \quad (16)$$

Expected number of busy permanent repairmen

$$E(B) = C - E(I) \tag{17}$$

Expected number of busy removable repairmen

$$E(BR) = \sum_{j=1}^{r-1} \sum_{n=jT+1}^{(j+1)T} jP_n + r \sum_{n=rT+1}^{N+Y+S-m} P_n \tag{18}$$

Operating efficiency is obtained by

$$O.E. = \frac{E(B)}{R + r} \tag{19}$$

5. COST ANALYSIS

We define various cost factors associated with different states as:

$C_N =$ Cost per unit time of an operating unit when system works in normal mode

$C_{SH} =$ Cost per unit time of an operating unit when system works in short mode

$C_{SC} =$ Cost per unit time for providing a cold spare unit

$C_{SW} =$ Cost per unit time for providing a warm spare unit

$C_I =$ Cost per unit time per idle permanent repairman

$C_B =$ Cost per unit time per permanent repairman when he is busy in providing repair

$C_{BR_i} =$ Cost per unit time of i^{th} (i=1,2,…,r) removable repairman when he is busy in providing repair

We formulate the cost function as the expected total cost per unit time given below:

$$E(C) = C_N \sum_{n=0}^{Y+S} NP_n + C_{SH} \sum_{n=Y+S+1}^{N+Y+S-m} [N + Y + S - n)]P_n +$$

$$C_{SC}E(UCS) + C_{SW}E(UYS) + C_I E(I) + C_B E(B)$$

$$+ \sum_{j=1}^{r-1} C_{BR_j} \sum_{n=jT+1}^{(j+1)T} j.P_n + r.C_{BR_r} \sum_{n=rT+1}^{N+Y+S-m} P_n \tag{20}$$

6. CONCLUSION

In this paper we have obtained the product type solution for the state-dependent M/M/C/K/N machine repair model with mixed spares and removable repairmen. The space or cost constraints often restrict the number of spare units as well as the number of permanent repairmen in a machining system.

For such cases, the provision of mixed spares and removable repairmen may be invaluable for the system designer so as to reduce the backlog of failed units to ensure the smooth functioning of the machining system. The cost analysis might give an insight to determine the optimal combination of cold and warm standby units as well as number of permanent/ additional repairmen, which needs further investigation.

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