COMPARISON OF EXACT ANALYSIS AND STEPLINES APPROXIMATION FOR EXTERNALLY EXCITED EXPONENTIAL TRANSMISSION LINE

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(Received: April 30, 2004 – Accepted: Feb. 25, 2005)

Abstract In the present paper, the problem of externally excited exponential transmission line has been solved analytically in frequency domains using a simple approach. Then steplines approximation as a first order approximation for the problem of externally excited nonuniform transmission lines in general and exponentially tapered transmission line (ETL) as a special case has been presented. Finally the two approaches are compared and some useful results are obtained to show when the two methods are equivalent.

Key Words Nonuniform, Multiconductor Transmission Lines, Simulation, Coupling

چکیده در این مقاله ابتدا مسئله خطوط انتقال نمایی توسط تابش خارجی با یک روش ساده به طور دقیق حل می شود. سپس با استفاده از تقریب پله ای، همین مسئله مورد بررسی قرار می گیرد. در نهایت نتایج این دو روش با یکدیگر مقایسه می شوند. نتایج نشان دهنده این هستند که تحت چه شرایطی، روش تقریب پله ای، می تواند به نتایج مشابهی، با روش دقیق منجر شود.

1. INTRODUCTION

The effect of the external electromagnetic field to the two wire transmission lines was considered for the first time by Taylor and his group in 1965 [l]. Since that time several research papers have been published in the same field and the method used in [1] has been generalized to several structures including a multiconductor structure [2], and nonuniform two wire structure [3]. The ETL's which are the subject of the present paper are used for some applications such as antenna design [4]. In this paper the coupling of external EM plane waves to nonuniform exponential transmission lines with linear terminations have been considered using two different methods. The first method is analytic and the second method is based on steplines approximations of nonuniform line. Stepline approximations are used to model the nonuniform line by a large number of uniform small steps. Finally, time- and frequency-domain analysis together with the ϕ pattern of the induced terminal voltages presented are compared to show whether and when steplines approximation is a good approximation for coupling problems in nonuniform transmission lines.

2. FIELD COUPLING TO ETL'S EXACT ANALYSIS

Consider an ETL externally excited by a plane wave. It is assumed that the line carries TEM mode as its principal mode of propagation. The differential equations describing this line is given by

$$\frac{d}{dx}\begin{bmatrix} V(x) \\ I(x) \end{bmatrix} + j\omega \begin{bmatrix} 0 & L(x) \\ C(x) & 0 \end{bmatrix} \begin{bmatrix} V(x) \\ I(x) \end{bmatrix} = \begin{bmatrix} F_V(x) \\ F_I(x) \end{bmatrix} \quad (1)$$

$$L(x) = L_0 \exp(2qx) \quad (2)$$

$$C(x) = C_0 \exp(-2qx) \quad (3)$$

$$Z(x) = Z_0 \exp(2qx) = \sqrt{\frac{L_0}{C_0}} \exp(2qx) \quad (4)$$

L_0 and C_0 are constant parameters called per unit length inductance and capacitance at the input terminal of the line. Also q is the taper constant and F_V and F_I are the distributed voltage and current, sources represent the effect of external EM wave. F_V and F_I are given in several references [2-3] as

$$F_V(x) = j\omega\mu_0 \int_0^{h(z)} H_z^t(x, y, 0)dy \quad (5)$$

$$F_I(x) = j\omega\mu_0 \int_0^{h(z)} E_z^t(x, y, 0)dy \quad (6)$$

The superscript *t* represents the total exciting wave. The total field is given by

$$E^t = E^i + E^r \quad (7)$$

in which the superscripts i and r represent the external incident wave and reflected wave from the lower surface (e.g. ground plane) respectively. Now consider two plane waves of parallel (θ) polarization (or TM wave) and of perpendicular (ϕ) polarization (or TE wave). Here, the ϕ angle, is measured from the negative x axis and the θ angle is measured from the positive y axis. For plane wave excitation it is simple to show

$$F_V(x) = -j2E_0 \exp(-jk_x x)\frac{\cos\phi}{\sin\theta}\sin(k_y h(z)) \quad (8)$$

$$F_I(x) = j2E_0 \exp(-jk_x x)\sqrt{\mu_0\varepsilon_0}\,\frac{1}{L(z)}\sin(k_y h(z)) \quad (9)$$

for TM case, and

$$F_V(x) = j2E_0 \exp(-jk_x x)\sin\phi\,\sin(k_y h(z)) \quad (10)$$

$$F_I(x) = 0 \quad (11)$$

for TE case. We now define the new variables called incident $V^+(x)$, and reflected $V^-(x)$ traveling waves along the line as

$$\begin{bmatrix} V^+(x) \\ V^-(x) \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1 & Z(x) \\ 1 & -Z(x) \end{bmatrix}\begin{bmatrix} V(x) \\ I(x) \end{bmatrix} = [T(x)]\begin{bmatrix} V(x) \\ I(x) \end{bmatrix} \quad (12)$$

By differentiating 12 one gets to

$$\frac{d}{dx}\begin{bmatrix} V^+(x) \\ V^-(x) \end{bmatrix} - \begin{bmatrix} -j\frac{\omega}{u}+q & -q \\ -q & j\frac{\omega}{u}+q \end{bmatrix}\begin{bmatrix} V^+(x) \\ V^-(x) \end{bmatrix} = [T(x)]\begin{bmatrix} F_V(x) \\ F_I(x) \end{bmatrix} \quad (13)$$

The above differential equation can be solved directly to get to the following solution.

$$\begin{bmatrix} V(d) \\ I(d) \end{bmatrix} = [T(d)]^{-1} \exp([R]d)[T(0)]\begin{bmatrix} V(0) \\ I(0) \end{bmatrix} + [T(d)]^{-1}[F(d)] \quad (14)$$

Thus for the case of perfect ground plane in lower surface of the structure one has

$$E_x^t(x, y, 0) = j2E_0 e_{ox} \sin(\beta_y y)\exp(-j\beta_x x) \quad (15)$$

$$E_y^t(x, y, 0) = 2E_0 e_{oy} \cos(\beta_y y)\exp(-j\beta_x x) \quad (16)$$

where E_0 is the amplitude of the electric field of the incident plane wave, and

$$\beta_x = \beta_0 \cos\phi \sin\theta \quad (17)$$

$$\beta_y = \beta_0 \cos\theta \quad (18)$$

β_0 is the phase constant of the incident wave. For TM wave incident e_{ox} and e_{oy} are defined as

$$e_{ox} = \cos\phi\cos\theta,\ e_{oy} = \sin\theta \quad (19)$$

and for the TE case they are defined as

$$e_{ox} = \sin\theta,\ e_{oy} = 0 \quad (20)$$

$$p = \sqrt{q^2 - \left(\frac{\omega}{u}\right)^2} \quad (21)$$

When $q \neq \frac{\omega}{u}$, and for $q = \frac{\omega}{u}$

$$\lambda_1 = \lambda_2 = q \quad (22)$$

Without loss of generality we suppose $q \neq \frac{\omega}{u}$, so

$$\exp([R]x) = \beta_0 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \beta_1 [R] \quad (23)$$

where

$$\begin{bmatrix} \beta_0 \\ \beta_1 \end{bmatrix} = \begin{bmatrix} 1 & \lambda_1 \\ 1 & \lambda_2 \end{bmatrix}^{-1} \begin{bmatrix} \exp(\lambda_1 x) \\ \exp(\lambda_2 x) \end{bmatrix} \quad (24)$$

Setting 24 in 23 gets to

$$\exp([R]x) = \frac{1}{2p} \begin{bmatrix} b_1 \exp(\lambda_1 x) + b_2 \exp(\lambda_2 x) & q\exp(\lambda_1 x) - q\exp(\lambda_2 x) \\ q\exp(\lambda_1 x) - q\exp(\lambda_2 x) & b_2 \exp(\lambda_1 x) + b_1 \exp(\lambda_2 x) \end{bmatrix} \quad (25)$$

Where

$$b_1 = p + \frac{j\omega}{u} \quad (26)$$

$$b_2 = p - \frac{j\omega}{u} \quad (27)$$

For ETL's one has

$$h(z) = \frac{a}{2}\exp[k\ \exp(2qx)] \quad (28)$$

where a is the radius of the wire and

$$k = \frac{2\pi\varepsilon_0}{C_0} \quad (29)$$

Finally $F(x)$ can be calculated from 15 using numerical integration.

3. STEPLINE APPROXIMATIONS

The steplines approximation methodcan be applied for all kinds of nonuniform transmission lines (NTL's). Here the main idea is presented for a general case and then used for the case of exponential transmission line. Consider an externally excited lossless NTL as shown in Figure 1.

To find the induced current and voltage over the line, the total length of the line d is subdivided into N equal (without loss of generality) intervals Δx. The inductance and capacitance matrices of NTL over each subinterval are taken to be independent of x. The partial differential equations which describe the system in the: k-th step are given by

$$\frac{d}{dx}\begin{bmatrix} V(x) \\ I(x) \end{bmatrix}_n + [Z]_n \begin{bmatrix} V(x) \\ I(x) \end{bmatrix}_n = 0 \quad (30)$$

In which V(x) and I(x) are the voltage and current along the line, the subscript n. indicates the n-th step (n=1,2,...,N), and

$$[Z]_n = \begin{bmatrix} 0 & j\omega L_n \\ j\omega C_n & 0 \end{bmatrix} \quad (31)$$

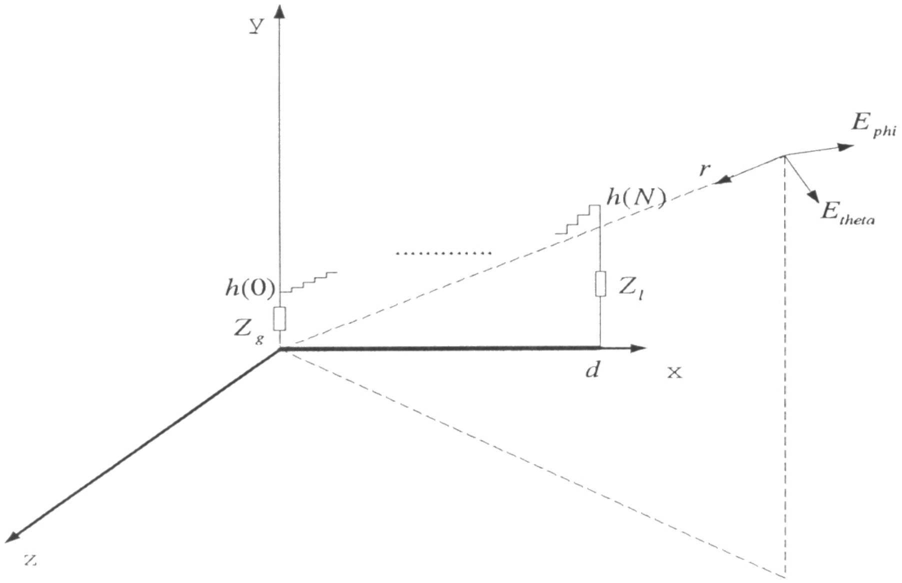

Figure 1. Externally exited lossless nonuniform transmission line (steplines approximation).

L_n And C_n represent per unit length inductance, and capacitance in the n-th step respectively. Using the starting point of the n-th step $((n-1)\Delta x < x < n\Delta x)$ as the reference point for it, 31 has a solution as

$$\begin{bmatrix} V(x) \\ I(x) \end{bmatrix}_n = [A(x)]_n \begin{bmatrix} V(0) \\ I(0) \end{bmatrix}_n - [F(x)]_n \tag{32}$$

This solution is valid for $0 < x < \Delta x$. In 30 we used the following notations

$$[A(x)]_n = \begin{bmatrix} \cos(\beta_{on} x) & jZ_{on}\sin(\beta_{on} x) \\ \dfrac{j}{Z_{on}}\sin(\beta_{on} x) & \cos(\beta_{on} x) \end{bmatrix} \tag{33}$$

$$Z_{on} = \sqrt{L_n / C_n} \tag{34}$$

$$\beta_{on} = \omega\sqrt{L_n C_n} \tag{35}$$

$$[F(x)]_n = \int_0^x [A(x - x')] \begin{bmatrix} F_V(x' + (n-1)\Delta x) \\ F_I(x' + (n-1)\Delta x) \end{bmatrix} dx' \tag{36}$$

$F_V(x)$ and $F_I(x)$, the induced voltage and current sources, are defined in 8 and 11 for both TM and TE cases. Setting $x = \Delta x$ in 32 and applying the boundary conditions

$$\begin{bmatrix} V(\Delta x) \\ I(\Delta x) \end{bmatrix}_n = \begin{bmatrix} V(0) \\ I(0) \end{bmatrix}_{n+1} \tag{37}$$

and terminal conditions

$$V(N\Delta x) = Z_L I(N\Delta x) \tag{38}$$

$$V(0) = -Z_g I(0) \tag{39}$$

TABLE l. Some Parameters and the Error for Steplines Approximations with Different N.

N	$\Delta h_{max}(mm)$	$\Delta Z_{o\,max}(\Omega)$	$e_{max}(\%)$	$h(N)\,(mm)$	$Z_o(N)\,(\Omega)$
100	7.2	4.64	3	102	312.185
50	12.6	9.09	5	95	307.53
25	19.7	17.38	7	82.1	298.44
10	24.7	37.99	17	54.8	272.76
5	18.6	60.8475	20	32	234.76

one gets to

$$\left[\begin{bmatrix}1\\ \frac{1}{Z_L}\end{bmatrix} - [T]\begin{bmatrix}1\\ \frac{-1}{Z_g}\end{bmatrix}\right]\begin{bmatrix}V_L\\ V_g\end{bmatrix} = -[B] \qquad (40)$$

in which

$$[T] = \prod_{n=N}^{1}[A(\Delta x)]_n \qquad (41)$$

$$[B] = \sum_{n=N}^{1}\left(\prod_{i=N}^{n}[A(\Delta x)]_i\right)[F(\Delta x)]_{n-1} + [F(\Delta x)]_N \qquad (42)$$

and Z_L and Z_g are the right- and left-hand loads respectively. From 40 V_L and V_g (voltages at the right and left hand terminals) can be obtained simply for both TM and TE incident.

4. EXAMPLES AND RESULTS

Consider an exponential nonuniform transmission line with the following design parameters.

$$Z(0) = 70.71\Omega;$$
$$Z(d) = 316.6\Omega;\ d = 0.5\,m$$

This design parameters yields to taper' constant $2q = 1.5$ and wire diameter $2a = 3\,mm$. Also here we made the following assumptions:

$Z_L = 50\Omega;\ Z_g = 150\Omega$.

To see the effect of the number (and length) of the steps the total length of the line has been divided into N = 5, 10, 25, 50, 100 steps. Table 1 shows the maximum height step or step in y-direction (Δh_{max}), length step or step in x-direction (Δx), the maximum difference of characteristic impedance of two adjacent steps (ΔZ_{omzx}), the length and characteristic: impedance of the final step ($h(N)$, $Z_o(N)$), and the maximum percent error (e_{max}) in frequency domain for pulse excitation.

i) Time-Domain Analysis The incident electric field is given by a triangular pulse with $\frac{35}{3}$ns rise- and fall-time and $35\,ns$ duration.

The induced terminal voltage for constant $\theta = 0$ and $\phi = \frac{\pi}{3}$, calculated using exact and stepline approximations for N=50, and N=10 are compared in Figures 2-3 respectively.

ii) Frequency-Domain Analysis The error for induced voltages in frequency domain at all terminals, computed using exact and stepline approximation methods, for $\phi = 0$ and $\theta = \frac{\pi}{3}$, are shown in Figures 4-5.

iii) ϕ Pattern Figures 6-7 show the comparison of the exact ϕ pattern and the stepline approximation of the induced terminal voltages for constant $\theta = \frac{\pi}{4}$ and f = 100MHz.

Figure 8 shows the ϕ pattern of the error for N = 100.

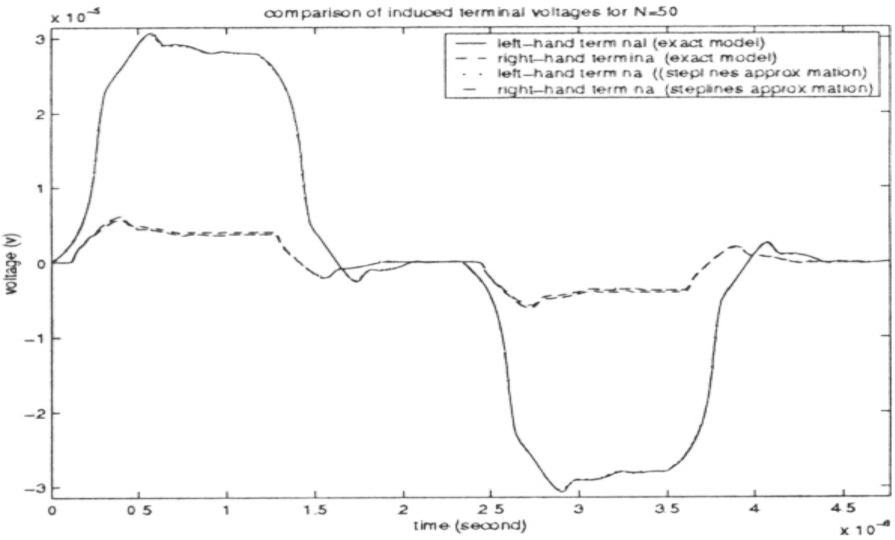

Figure 2. Comparison of time domain induced voltages for exact model and steplines approximations N=50.

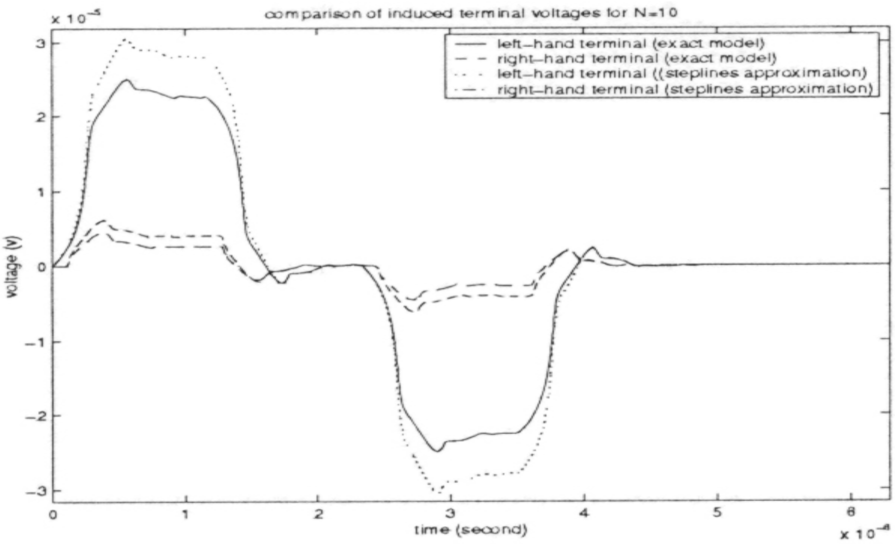

Figure 3. Comparison of time domain induced voltages for exact, model and steplines approximations.

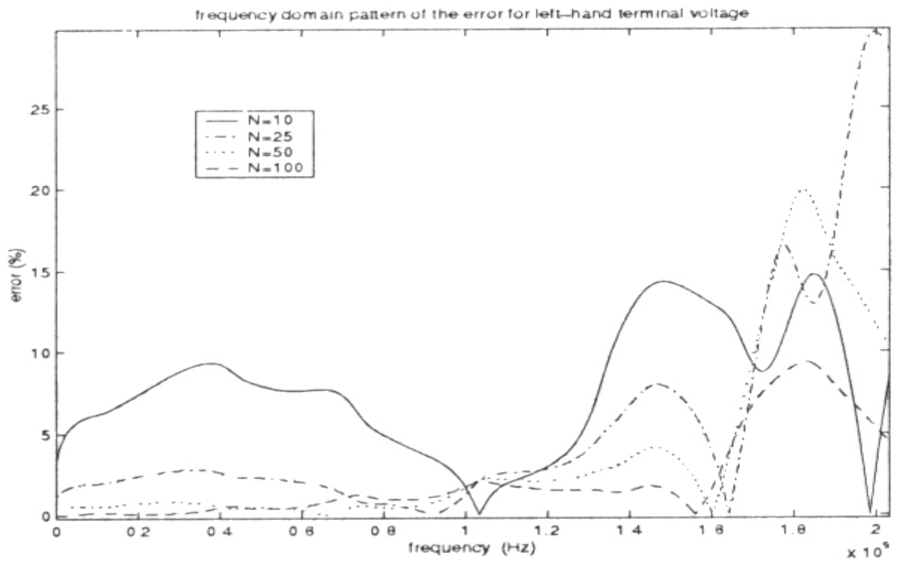

Figure 4. Error for left,-hand terminal induced voltages (in frequency domain) for several numbers of steps.

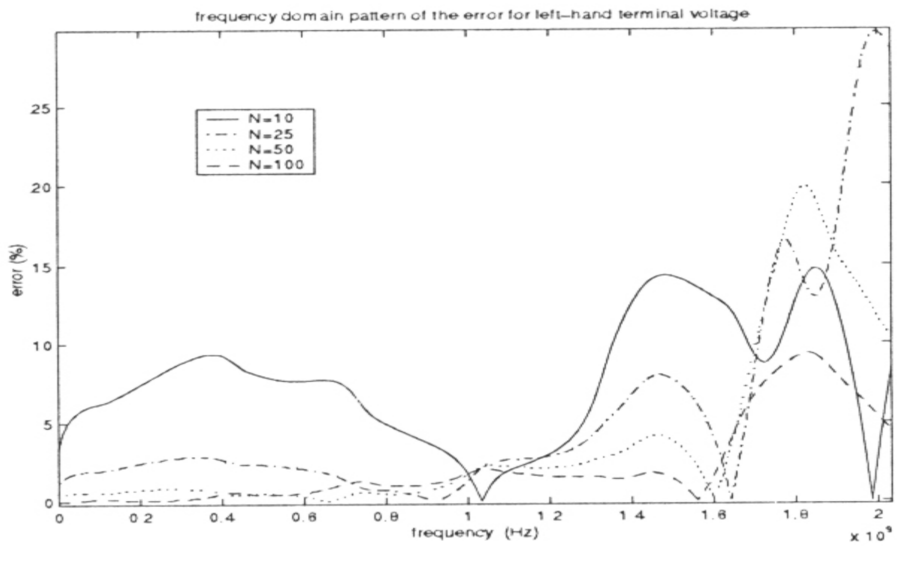

Figure 5. Error for right-hand terminal induced voltages (in frequency domain) for several numbers of steps.

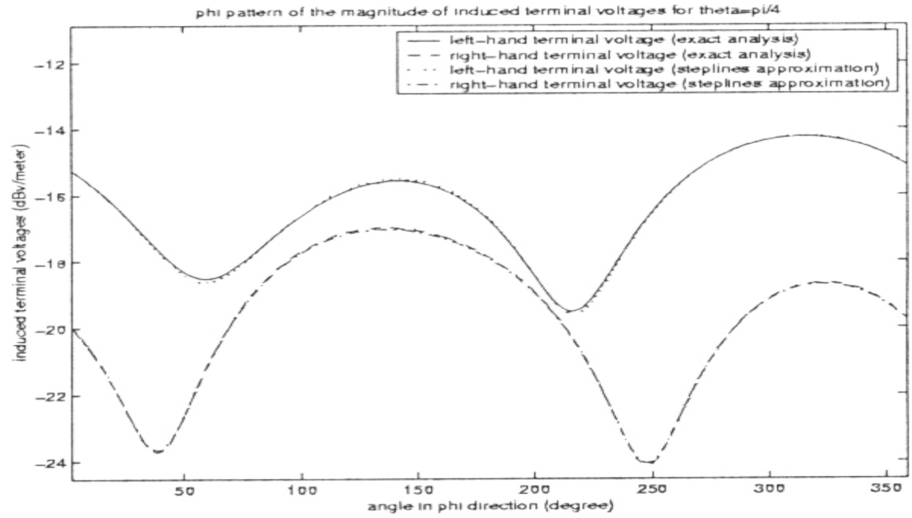

Figure 6. Comparison of magnitude of the exact ϕ pattern and steplines approximation in constant $\theta = \frac{\pi}{4}$ for f = 100MHz and N = 100.

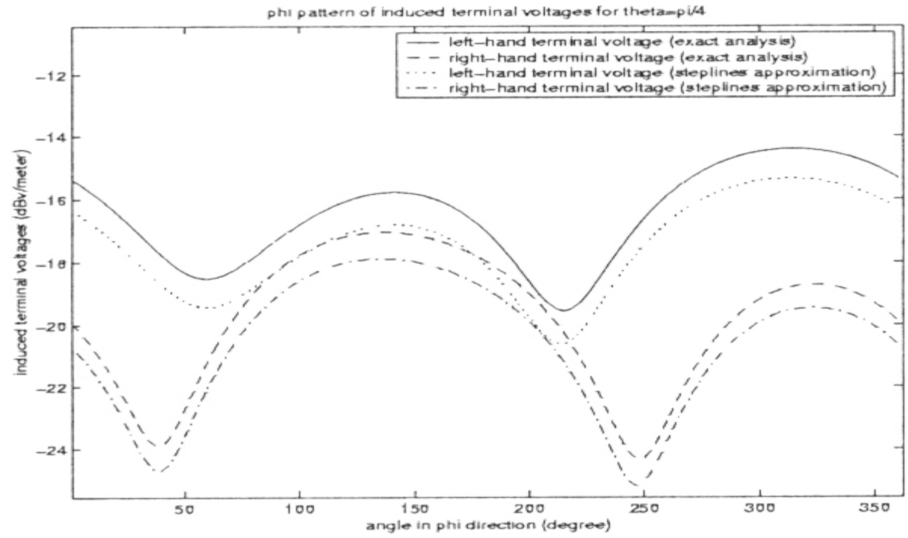

Figure 7. Comparison of magnitude of the exact ϕ pattern and steplines approximation in constant $\theta = \frac{\pi}{4}$ for f = 100MHz and N = 100.

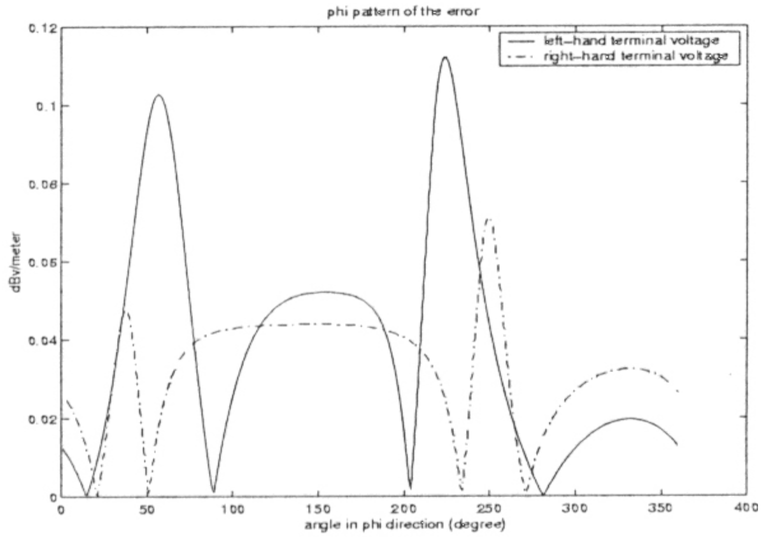

Figure 8. ϕ pattern of the error in constant $\theta = \frac{\pi}{4}$ for f = 100MHz and N = 100.

5. CONCLUSION

Stepline approximation has been compared with the exact method for field coupling to ETL's both in time- and frequency-domain. In the stepline approximation method, choosing constant Δx leads to different Δz and ΔZ_o, for each step. Also choosing constant Δh or ΔZ_o leads to different steps in the other two parameters. The natural question that arises here is that, which choice is better (leads to less error) in stepline approximation method? As it is appears the error for each choice depends on the nature of the problem under consideration, but generally the error is less for the case of constant Δx for the same N. For example choosing N = 50 for constant steps in y-axis leads to, constant $\Delta h = 2.015 mm$, $\Delta Z_{omax} = 3549$ and $\Delta x_{max} = 89$ mm. Also choosing N = 50 for constant steps in Z_o leads to $\Delta h_{max} = 8.4$ mm, constant $\Delta Z_o = 4.92$ and $\Delta x_{max} = 22.4$ mm (compare with the second row of Table 1). The maximum error, in frequency domain, for both two cases is obtained a little more than the case of constant Δx considered in the paper. It may be because of the nature of the induced distributed voltage and current sources $F_V(x)$ and $F_I(x)$ which are functions of height and characteristic: impedance of the steps. It seems that the results obtained in the paper can be generalized for all other cases of nonuniform transmission lines.

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