RESEARCH NOTE

A PROPOSED METHOD TO EVALUATE ULTIMATE RESISTANCE OF PLATE GIRDERS SUBJECTED TO SHEAR AND PATCH LOADING

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Abstract Experimental investigation of the ultimate resistance of slender, steel-plate girder web panels to combined shear-and-patch loading indicates significant interaction between shear loading and patch loading. However, an existing interaction formula is based on experimental results. Herein, an improved design procedure for slender plate girders subjected to combined shear and patch loading is proposed. A modified formula to evaluate shear resistance of plate girders in presence of patch loading is proposed that shows satisfactory correlation with the available test data and is acceptable for practical purposes.

Key Words Plate Girders, Combined Shear and Patch Loading, Modified Shear Resistance

چکیده بررسی‌های تجربی نشان داده است که نیروهای برشی و موضعی همزمان اندرکنش قابل ملاحظه‌ای در تیر ورقها خواهند داشت. روابط موجود در دسترس عمدتاً بر اساس نتایج حاصل از آزمایشات تخمین زده شده‌اند. در این مقاله، یک روش بهبود یافته طراحی برای تیر ورق‌های تحت اثر بار همزمان برشی و موضعی ارائه شده و رابطه‌ای نیز جهت تخمین مقاومت برشی تیر ورقها پیشنهاد شده است. این رابطه تطابق قابل قبول و مناسبی با نتایج تجربی دارد.

1. INTRODUCTION

Slender steel plates are used in a variety of structural engineering applications owing to their high strength-to-weight ratio and post-buckling reserve of stiffness and strength. Typical structural forms are slender-plate and bog girders, in which the flanges primarily resist bending and the web primarily, resist shear and localized, in-plane, compressive edge loading, often described as patch loading.

Shear loading and localized edge or patch loading of slender plate girders are frequently encountered in practice. Significant shear loading arises from nonuniform bending and support reactions, while examples of patch loading are wheel loads on gantry girders, loads from purlins onto the mainframe members of buildings, and roller loads during the launching of plate girder bridges. Loading conditions that result in pure patch loading, pure shear loading, and combined shear-and-patch loading are illustrated in Figure 1.

During the past three decades, numerous tests have been performed on slender plate girders to provide a better understanding of their modes of failure. In the majority of these tests, shear loading and patch loading have been considered separately [1,2,3]. There are only few experimental studies of the interaction between shear and patch loading [4,5]. While these tests indicated significant interaction between the two forms of loading. To date the only solution, which is available for predicting the resistance of plate girders to combined shear-and-patch loading is based on experimental results, in the form of an interaction formula and not as a rigorous solution [5].

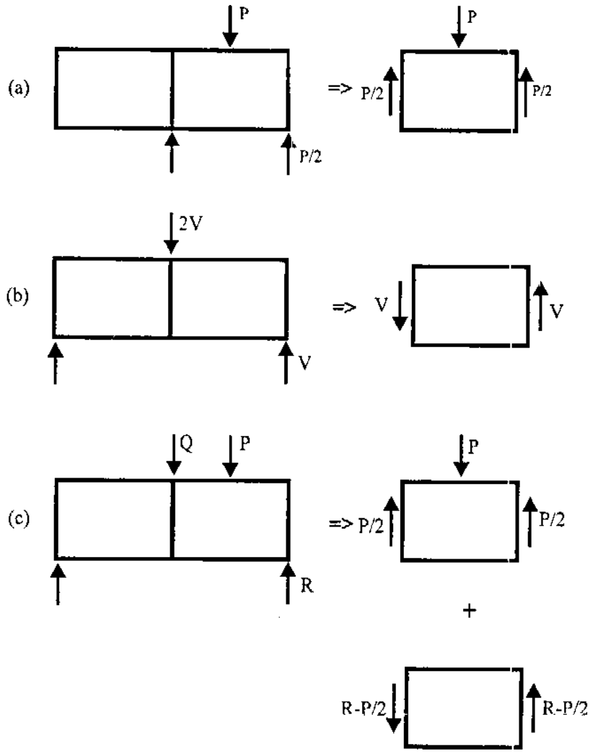

Figure 1. Shear-and patch loading of plate girders: (a) Pure patch loading; (b) Pure shear loading; (c) Combined shear-and-patch loading.

Herein, details of a theoretical investigation of the ultimate shear resistance of slender web panels in the presence of patch loading is presented. Theoretical predictions are compared with available experimental results and a modified formula for combined shear-and-patch loading is proposed.

2. THEORETICAL PREDICTION OF ULTIMATE SHEAR RESISTANCE

Theoretical predictions of the ultimate shear resistance of slender web panels can be made in accordance with tension field theory, developed by Porter et al. [6] and Evans [1]. For a web panel having width b_w, depth d_w, thickness t_w, and similar top and bottom flanges (Figure 2), the ultimate shear resistance V_u is given by

$$V_u = \tau_{cr} d_w t_w + \sigma_t^y t_w \sin^2\theta (d_w \cot\theta - b_w) + 4 d_w t_w \sin\theta \sqrt{(\sigma_{yw} M_p^* \sigma_t^y)} \qquad (1)$$

τ_{cr} is the critical shear stress of an assumed simply supported web plate, given by

$$\tau_{cr} = \frac{K_s \pi^2 E t_w^2}{12(1-\nu^2) d_w^2} \qquad (2)$$

where K_s is buckling coefficient. σ_t^y is the web tension field membrane stress, defined by the equation

$$\sigma_t^y = \left\{ \sqrt{\left[1 - \left(\frac{\tau_{cr}}{\tau_{yw}}\right)^2 \left(1 - \frac{3}{4}\sin^2 2\theta\right)\right]} - \frac{\sqrt{3}}{2}\frac{\tau_{cr}}{\tau_{yw}} \sin 2\theta \right\} \sigma_{yw} \qquad (3)$$

where τ_{yw} is shear yield stress of the web. Which is related to the uniaxial yield stress σ_{yw} by the equation

$$\tau_{yw} = \frac{\sigma_{yw}}{\sqrt{3}} \qquad (4)$$

θ is the inclination of the web tension field, assumed to be approximately two-thirds of the inclination of the web panel diagonal, i.e.,

$$\theta = \frac{2}{3} \tan^{-1}\left(\frac{d_w}{b_w}\right) \qquad (5)$$

M_p^* is a nondimensional flange strength parameter defined as

$$M_p^* = \frac{M_{pf}}{d_w^2 t_w \sigma_{yw}} \qquad (6)$$

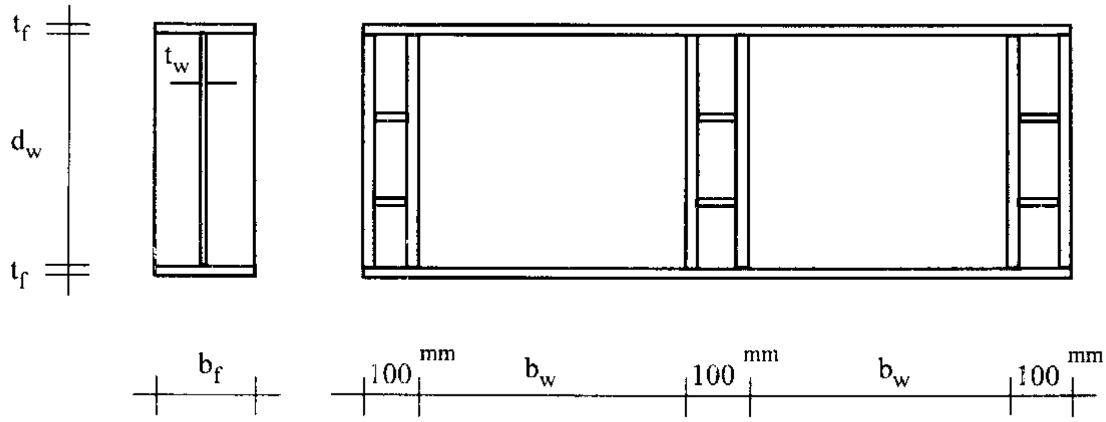

Figure 2. Details of test girders.

where M_{pf} is fully plastic moment of the flange, which for a rectangular flange having width b_f, thickness t_f, and yield stress σ_{yf} is given by

$$M_{pf} = 0.25 b_f t_f^2 \sigma_{yw} \tag{7}$$

3. THEORETICAL PREDICTION OF ULTIMATE PATCH RESISTANCE

Theoretical predictions of the ultimate patch resistance of slender web panels can be made in accordance with the theory developed by Roberts and Newark [3]. The ultimate patch resistance P_u should be taken as the lesser of the resistance to web-crippling P_{ub}, and web-yielding P_{uy} which are given by

$$P_{ub} = \left[1.1 t_w^2 \left(E \sigma_{yw} \right)^{0.5} \left(\frac{t_f}{t_w} \right)^{0.25} \left(1 + \frac{c_e t_w}{d_w t_f} \right) \right] \frac{1}{F} \tag{8}$$

$$P_{uy} = \left(16 M_{pf} \sigma_{yw} t_w \right)^{0.5} + \sigma_{yw} t_w c_e \tag{9}$$

where F is a numerical factor to be taken as 1.45 for a lower bound 95% confidence limit and 1.0 for a mean prediction and c_e is the effective length of the patch load, which is related to the actual length c by the equation

$$c_e = c + 2 t_f \tag{10}$$

4. INTERACTION BETWEEN SHEAR AND PATCH RESISTACE

When a girder web subjected to patch loading in addition to shear, the determination of the ultimate load capacity becomes more complex. Analysis of test results for combined shear and patch loading conducted by first author indicates significant interaction between the two forms of loading [7]. The following interaction formula for this combined loading is proposed that shows satisfactory correlation with the test data.

$$\left(\frac{V_{dc}}{V_u} \right)^2 + \left(\frac{P_{dc}}{P_u} \right) \leq 1.0 \tag{11}$$

In which V_{dc} and P_{dc} are the design shear and patch loads for combined loading, and V_u and is the ultimate shear resistance in the absence of patch loading, and P_u is the ultimate patch resistance in the absence of shear.

The presence of the patch loading requires two important additional factors to be considered:

(a) The reduction in the shear buckling resistance

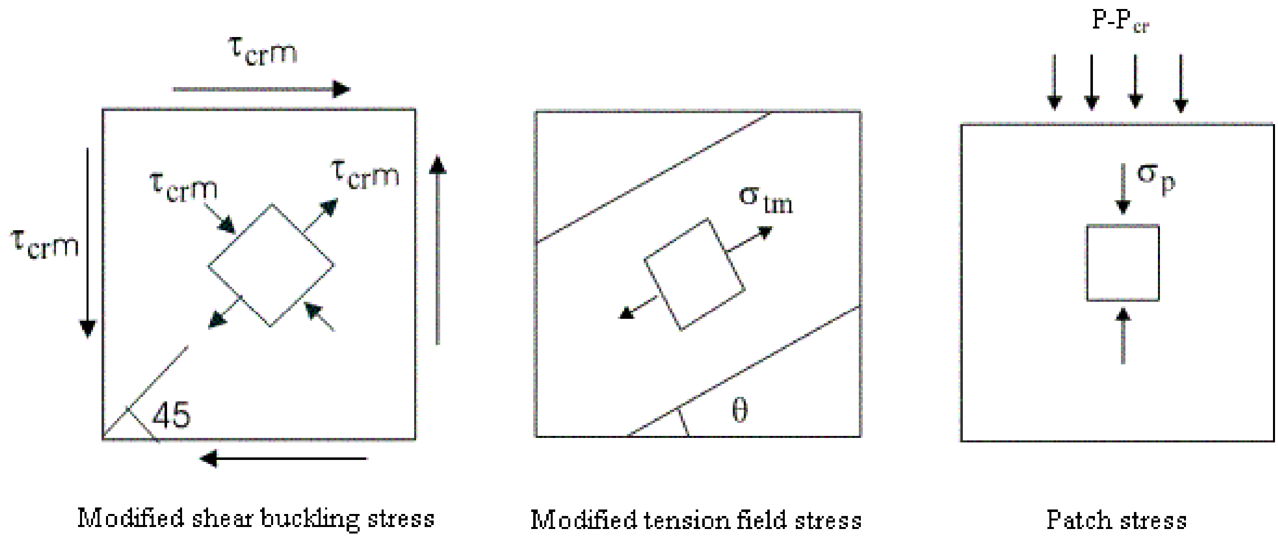

Figure 3. State of stresses in the seb plate girder.

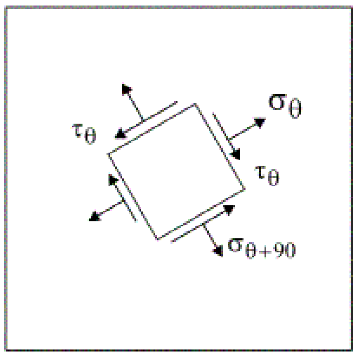

Figure 4. State of combined stresses in the web plate girder.

due to the patch stresses.
(b) The influence of the patch stresses upon the magnitude of the membrane stresses required producing yield in the web.

Each of these factors will now be considered individually.

(a) Evaluation of the Shear Buckling Resistance When the web panel is subjected to combined patch loading and shear, the buckling coefficients are determined by the author from the following interaction formula [8]

$$\left(\frac{K_{sp}}{K_s}\right)^{\alpha} + \left(\frac{K_{ps}}{K_p}\right)^{\alpha} = 1.0 \tag{12}$$

where K_{sp} and K_s are the buckling shear coefficients in the presence and absence of patch loading, respectively, and K_{ps} and K_p are the

TABLE 1. Girder Dimensions and Material Yield Stresses [5].

Plate Girder	b_w (mm)	D_w (mm)	t_w (mm)	b_f (mm)	t_f (mm)	σ_{yw} (N/mm^2)	σ_{yf} (N/mm^2)
PG1-2	600	600	4.1	200	12.3	339	250
PG1-3	600	600	4.1	200	12.5	338	251
PG2-2	900	900	3.1	300	10.2	284	256
PG2-3	900	900	3.1	300	10.2	282	253
PG3-2	900	600	3.2	200	10.2	273	263
PG3-3	900	600	3.2	200	10.2	275	258
PG4-2	1000	500	1.9	200	9.8	247	313
PG4-3	1000	500	1.9	200	10.0	236	294

Note: For all patch tests c =50 mm

buckling patch coefficients in the presence and absence of shear, respectively. α is given by

$$\alpha = 2.1 - 0.77\beta + 0.77\beta^2 - 0.05\beta^3 \tag{13}$$

in which

$$\beta = \frac{b_w}{d_w} \tag{14}$$

In each case, the buckling load P_{cr} can be expressed as

$$P_{cr} = K_p \frac{\pi^2 E t_w^3}{12(1-\nu^2) d_w} \tag{15}$$

Thus, for any patch load P, by substituting $\frac{K_{ps}}{K_p} = \frac{P}{P_{cr}}$ in Equation 12, the value of the modified shear buckling stress τ_{crm} is determined.

$$\tau_{crm} = \tau_{cr} \frac{K_{sp}}{K_s} \tag{16}$$

(b) Evaluation of the Membrane Stresses

The state of stress in the web plate for combined shear and patch loading is shown in Figure 3. The total state of stress in the web plate may be obtained by superimposing all the stresses as shown in Figure 4 and may be defined as follows

$$\begin{aligned}
\sigma_\theta &= \tau_{crm} \sin 2\theta - \sigma_{tm} - \sigma_p \sin^2 \theta \\
\sigma_{\theta+90} &= -\tau_{crm} \sin 2\theta - \sigma_p \cos^2 \theta \\
\tau_\theta &= \tau_{crm} \cos 2\theta - \frac{\sigma_p}{2} \sin 2\theta
\end{aligned} \tag{17}$$

Upon further increase of the applied loading, the tensile membrane stress (σ_{tm}) developed in the web increases. Eventually, the membrane stress reaches such a value that, when combined with the buckling stress as in Equations 17, the resulting stress (σ_θ) reaches the yield value (σ_{yw}) for the web material. This value of the membrane stress will be denoted as σ_{tm}^y and it may be determined by applying the Von Mises-Hencky yield criterion:

$$\sigma_\theta^2 + \sigma_{\theta+90}^2 - \sigma_\theta \sigma_{\theta+90} + 3\tau_\theta^2 = \sigma_{yw}^2 \tag{18}$$

By substituting the stresses from Equations 17, the following modified expression for the membrane stress is obtained.

$$\begin{aligned}
&\sigma_{tm}^y = \\
&-\frac{3}{2}\tau_{crm}\sin 2\theta + \sigma_p \sin^2\theta - \frac{1}{2}\sigma_p \cos^2\theta \\
&+\frac{1}{2}(-3\tau_{crm}^2 \sin^2 2\theta - 6\tau_{crm}\sin 2\theta\sigma_p \cos^2\theta \\
&\quad -3\sigma_p^2\cos^4\theta - 12\tau_{crm}^2\cos^2 2\theta \\
&\quad +12\tau_{crm}\cos 2\theta\sigma_p \sin 2\theta - 3\sigma_p^2 \sin^2 2\theta + 4\sigma_{yw}^2)^{0.5}
\end{aligned} \tag{19}$$

where σ_p is the value of the patch stress is given by

$$\sigma_p = \frac{\gamma(P - P_{cr})}{P_u}\sigma_{yw}$$

$$\begin{aligned}
&\gamma = 0 && P \le P_{cr} \\
&\gamma = 1 && P_{cr} \le P \le 0.9P_u \\
&\gamma = \frac{P}{P - P_{cr}} && 0.9P_u \le P \le P_u
\end{aligned} \tag{20}$$

5. PROPOSED FORMULA TO EVALUATE THE MODIFIED SHEAR RESISTANCE

The modified shear resistance of the plate girder may be obtained approximately from the following formula

$$\begin{aligned}
V_{um} &= \tau_{crm} d_w t_w + \sigma_{tm}^y t_w \sin^2\theta(d_w \cot\theta - b_w) \\
&\quad + 4 d_w t_w \sin\theta\sqrt{(\sigma_{yw} M_p^* \sigma_{tm}^y)}
\end{aligned} \tag{21}$$

Comparison of the ultimate shear resistance and modified shear resistance in accordance with Equations 1 and 21, shows the values of τ_{cr} and σ_t^y have been replaced with the τ_{crm} and σ_{tm}^y, respectively.

6. COMPARISON OF THE PROPOSED METHOD WITH EXPERIMENTAL AND THEORETICAL RESULTS

A total of 32 specimens were constructed and tests (two tests per girder) have been conducted on slender-plate girders, details of which are shown in Figure 2. The dimensions of the test girders denoted PG2-1 to PG4-3, are given in Table 1.

The modified shear resistance (V_{um}) of the plate girders determined in accordance with Equation 21 is presented in Table 2. To evaluate P_u used in Table 2, the numerical factor F in Equation 8 was assumed equal to 1.0 to provide a mean prediction.

The theoretical ultimate shear resistances (V_u) in the absence of patch loading, determined in accordance with Equation 1, are compared with experimental results (V_{exc}) and the proposed formula (V_{um}) in Table 3 and in Figure 5.

Also, as presented in Table 3 and Figure 6, values of λ are determined as follows

$$\lambda = \frac{V_{exc}}{V_{um}} \ge 1 \tag{22}$$

All the values of λ are greater than unity, indicating a safe and conservative design procedure.

7. DISCUSSION AND CONCLUTIONS

Experimental investigation of the ultimate resistance of slender, steel-plate girder web panels to combined shear-and-patch loading indicate significant interaction between shear loading and patch loading. To date the only solution that is available for predicting the resistance of plate girders to combined shear-and-patch loading is in the form of an interaction formula and not as a

TABLE 2. Modified Shear Resistance in Accordance with the Proposed Formula.

Girder/ Test reference	P_{cr} [Eq. (15)] (kN)	P_u [Eq. (8)] (kN)	P_{exc} [reference 5] (kN)	τ_{crm} [Eq. (16)] (N/mm^2)	σ_p [Eq.(20)] (N/mm^2)	σ_{tm}^y [Eq. (19)] (N/mm^2)	V_{um} [Eq. (21)] (N/mm^2)
PG1-2SP1	74	213	205	0	326	61	71
PG1-2SP2	74	213	208	0	331	43	56
PG1-3SP1	74	213	175	0	168	273	215
PG1-3SP2	74	213	186	0	186	260	207
PG2-2SP1	21	112	51	0	76	263	215
PG2-2SP2	21	112	25	0	10	282	228
PG2-3SP1	21	112	74	0	133	232	194
PG2-3SP2	21	112	16	11	0	268	249
PG3-2SP1	28	119	36	0	18	267	122
PG3-2SP2	28	119	110	0	252	57	39
PG3-3SP1	28	119	60	0	74	245	114
PG3-3SP2	28	119	106	0	180	163	83
PG4-2SP1	6	44	38	0	152	149	40
PG4-2SP2	6	44	20	0	67	215	52
PG4-3SP1	6	44	32	0	118	166	42
PG4-3SP2	6	44	44	0	172	112	32

rigorous solution.

Herein, the shear resistance of plate girders in the presence of patch loading has been investigated, theoretically. A modified formula for this combined loading is proposed that shows satisfactory correlation with the available test results. The solution presented is acceptable for practical purposes.

8. NOMENCLATURE

b_f	Width of flange
b_w	Clear width of web plate between stiffeners
c	Length of Patch load
c_e	Effective length of patch load
d_w	depth of web panel
E	Young's modulus

TABLE 3. Comparison of Proposed Method with Test Results and Theoretical Predictions.

Girder/Test reference	V_u [Eq. (1)] (kN)	V_{exc} [Reference 5] (kN)	V_{um} [Eq. (21)] (kN)	$\lambda = \frac{V_{exc}}{V_{um}}$ [Eq. (22)]
PG1-2SP1	386	158	71	2.33
PG1-2SP2	386	97	56	1.73
PG1-3SP1	386	266	215	1.24
PG1-3SP2	386	215	207	1.04
PG2-2SP1	268	241	215	1.12
PG2-2SP2	268	247	228	1.08
PG2-3SP1	268	230	194	1.19
PG2-3SP2	268	260	249	1.04
PG3-2SP1	182	169	122	1.39
PG3-2SP2	182	52	39	1.33
PG3-3SP1	182	161	114	1.41
PG3-3SP2	182	107	83	1.29
PG4-2SP1	70	61	40	1.53
PG4-2SP2	70	72	52	1.38
PG4-3SP1	70	62	42	1.48
PG4-3SP2	70	43	32	1.34

F	Numerical factor
K_p	Buckling coefficient for patch loading
K_{ps}	Buckling coefficient for combined patch loading and shear
K_s	Buckling coefficient for shear
K_{sp}	Buckling coefficient for combined shear and patch loading
M_p^*	Flange strength parameter
M_{pf}	Plastic moment of flange
P	Patch load
P_{cr}	Buckling patch load
P_{dc}	Design patch load for combined loading
P_{exc}	Experimental ultimate patch resistance to combined shear-and-patch loading
P_u	Theoretical ultimate patch resistance
P_{ub}	Theoretical ultimate patch resistance to

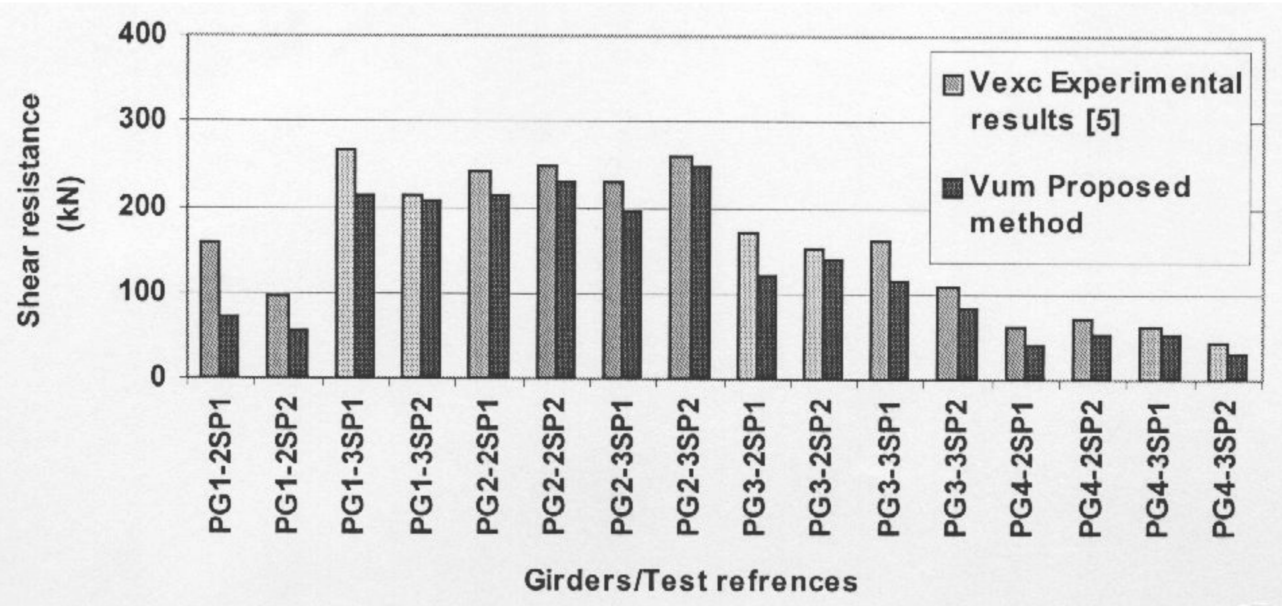

Figure 5. Comparison of proposed method with test results.

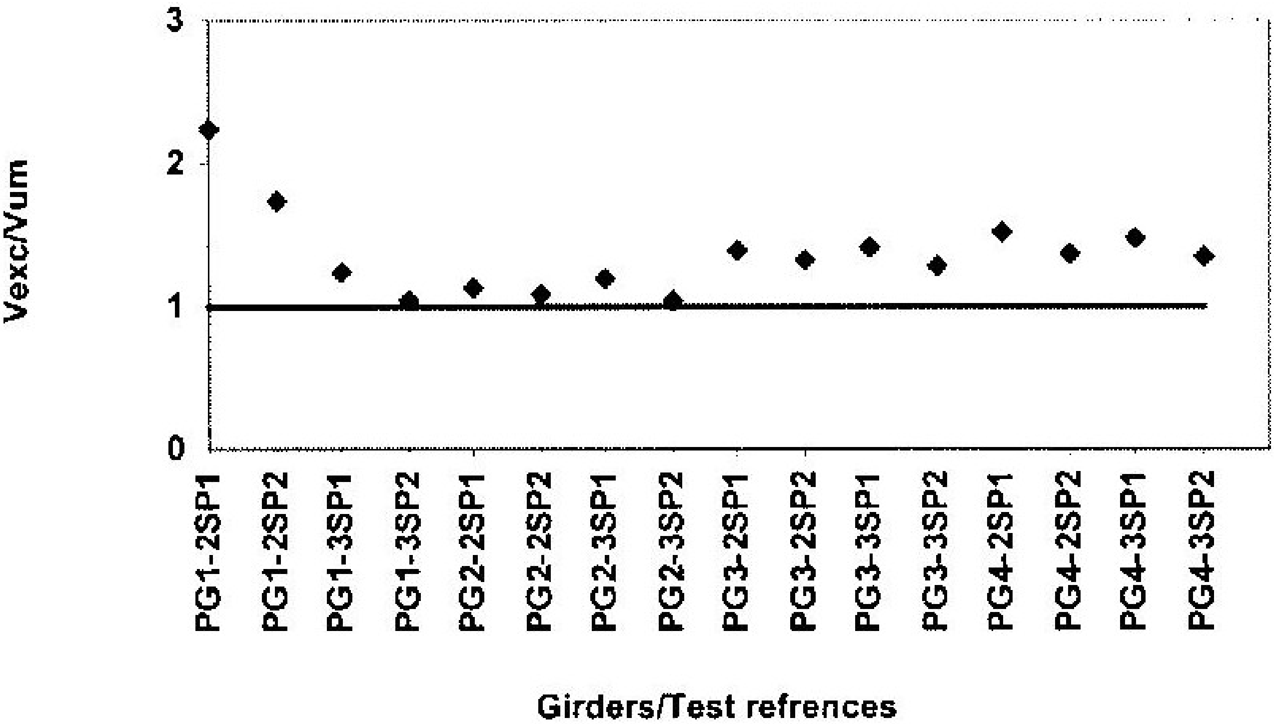

Figure 6. Comparison of modified shear resistance with test data.

	web crippling
P_{uy}	Theoretical ultimate patch resistance to web yielding
R	Reaction at support

t_f	Thickness of flange
t_w	Thickness of web
V	Shear load
V_{dc}	Design shear load for combined loading
V_{exc}	Experimental ultimate shear resistance to combined shear–and-patch loading
V_u	Theoretical ultimate shear resistance
V_{um}	Modified ultimate shear resistance to combined shear-and-patch loading

Greek Symbols

α	Function of β
β	Aspect ratio (b_w/d_w)
γ	Numerical factor for patch stress
θ	Inclination of web tension field
λ	Design factor (defined by equation 21)
ν	Poisson's ratio
σ_t^y	Tension field membrane stress at yield
σ_{tm}^y	Modified tension field membrane stress at yield
σ_{yf}	Yield stress of flange
σ_{yw}	Yield stress of web
σ_θ	Normal stress
τ_{cr}	Buckling shear stress
τ_{crm}	Modified buckling shear stress
τ_{yw}	Yield shear stress of web
τ_θ	Shear stress

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