A TWO-THRESHOLD GUARD CHANNEL SCHEME FOR MINIMIZING BLOCKING PROBABILITY IN COMMUNICATION NETWORKS

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Abstract In this paper, we consider the call admission problem in cellular network with two classes of voice users. In the first part of paper, we introduce a two-threshold guard channel policy and study its limiting behavior under the stationary traffic. Then we give an algorithm for finding the optimal number of guard channels. In the second part of this paper, we give an algorithm, which minimizes the number of channels subject to hard constraints on the blocking and dropping probabilities of calls. Finally, we propose an optimal prioritized channel assignment for multi-cells cellular networks with two classes of voice users.

Key Words Call Admission Control, Guard Channels, Two-Threshold Guard Channels, Wireless Networks

چکیده در این مقاله مسئله کنترل پذیرش درخواست ها در شبکه های سیار سلولی با دو سرویس صوتی بررسی می‌شود. در بخش اول مقاله، روش کانال احتیاط دو آستانه ای ارائه و رفتار آن برای ترافیک های ایستا بررسی می‌گردد. سپس الگوریتمی برای پیدا کردن تعداد بهینه کانال های احتیاط ارائه می‌شود. در بخش دوم مقاله، با توجه به محدودیت احتمال قطع رد درخواستها، الگوریتمی برای پیدا کردن تعداد کمینه کانال های مورد نیاز سلول ارائه می‌گردد. در انتها، یک الگوریتم انتساب کانال اولویت دار برای شبکه های سلولی سیار با دو سرویس صوتی ارائه می‌شود.

1. INTRODUCTION

In cellular networks, geographical area covered by mobile network is divided into smaller regions called cells. Each cell has a base station (BS), which is located at its center. A number of base stations are connected to a mobile switching center (MSC), which acts as a gateway of the mobile network to the existing wired-line networks. In order for a mobile user to be able to communicate with other user(s), a connection usually must be established between the users. When a mobile user needs a connection, sends his request to the base station of the cell residing it. Then, the base station determines whether it can meet the requested quality of service (QoS) requirements and, if possible, allocates a channel to the incoming call and establishes a connection. When a call gets a channel, it will keep the channel until its completion, or until the mobile user moves out of the cell, in which case the used channel will be released. When the mobile user moves into a new cell while its call is ongoing, a new channel needs to be acquired in the new cell for further communication. This process is called handoff and must be transparent to the mobile user. During the handoff, if no channel is available in the new cell for the ongoing call, it is forced to terminate (dropped) before its completion. The disconnection in the middle of a call is highly undesirable and one of the goals of the network designer is to keep such disconnections low.

Introduction of micro cellular networks leads to efficient use of channels but increases expected rate of handovers per call. As a consequence, some network performance parameters such as blocking probability of new calls and dropping probability of handoff calls are affected. These two parameters are dependent to each other. For example, accepting more handoff calls increases the blocking probability of new calls and vice versa. As a result, there is a trade-off between these two performance parameters. In order to have these performance parameters at reasonable level, call admission policies are used. The call admission policies determine whether a call should be admitted or blocked. Both blocking probability of new calls and dropping probability of handoff calls are affected by call admission policies. Blocking more new calls generally improves dropping probability of handoff calls and admitting more new calls generally improve blocking probability of new calls. Since dropping of handoff calls is more serious than blocking of new calls, call admission policies give the higher priority to handoff calls. This priority is usually implemented through allocation of more resources (channels) to handoff calls [1]. Many schemes have been proposed to reduce the dropping of voice calls such as guard channel scheme (GC) [2-4], fractional channel scheme (FC) [5], limited fractional channel scheme (LFC) [5], and uniform fractional channel scheme (UFC) [6]. Some schemes allow either handoff calls [7] or new calls [8] to be queued until free channels are obtained in the cell. These schemes may not be used adaptively to deal with changes in such traffic parameters as arrival rates and/or holding time of calls. Therefore, several adaptive call admission schemes have been introduced [9-14].

All of the above mentioned call admission policies consider only one threshold to decide for accepting/rejecting new calls. These policies cannot be used when different classes of users need different level of QoS. In such cases, we need multi-threshold guard channel scheme, which provides different set of guard channels for different classes of users. The only reported multi-threshold guard channel scheme, called dual-threshold reservation (DTR) scheme, is given in [15]. The basic idea behind the DTR scheme is to use two thresholds, one for reserving channels for voice handoff, while the other is used to block data traffic into the network in order to preserve the voice performance in terms of handoff dropping and call blocking probabilities. DTR assumes that the bandwidth requirement of voice and data are the same. The equations for blocking probabilities of DTR are calculated using a two-dimensional Markov chain and the effect of different values for the number of guard channels on dropping and blocking probabilities are plotted, but no algorithm is given to find the optimal number of guard channels.

In this paper, we consider a cellular network with two classes of voice users. In this system, the dropping probability of handoff calls of class 2 is less than the dropping probability of handoff calls of class 1 which is less than the blocking probability of new calls. In order to maintain the predefined level of QoS, we introduce a two-threshold guard channel scheme, which is similar to the idea given in [15]. The proposed scheme minimizes the blocking probability of both types of new calls subject to the hard constraint on the dropping probabilities of handoff calls for both classes. In the proposed call admission scheme, set of channels allocated to the given cell is partitioned into three subsets: ordinary channels, shared guard channels and dedicated guard channels. The ordinary channels are shared among all types of calls while the shared guard channels are shared only among handoff calls of two classes, and the dedicated guard channels are used only for the handoff calls of class 2. The limiting behavior of this scheme is analyzed under stationary traffic. In this paper, we also consider three prioritized channel assignment problems: finding the optimal number of guard channels, minimizing the number of required channels and the optimal channel assignment in multi-cells cellular system subject to the hard constraints on the dropping probabilities of handoff calls of both classes. The call admission scheme proposed in this paper can easily be extended to multi-classes traffic.

The rest of this paper is organized as: Basics of GC policy is given in section 2. Section 3 presents performance parameters of two-threshold guard policy. The properties of performance parameters

```
if (HANDOFF CALL) then
    if c(t) < C then
        accept call
    else
        reject call
    end if
end if
```

```
if (NEW CALL) then
    if c(t) < T then
        accept call
    else
        reject call
    end if
end if
```

Figure 1. Guard channel policy.

are studied in section 4. Section 5 gives an algorithm to find the optimal number of guard channels. An algorithm for finding the minimum number of channels required maintaining the predefined level of QoS is given in Section 6. Section 7, presents an optimal channel assignment algorithm for multi-cells cellular network and section 8 concludes the paper.

2. BASICS IN GUARD CHANNEL POLICY

In this section, we first review guard channel policy and then compute its blocking performance. We assume that the given cell has a limited number of full duplex channels, C, in its channel pool. We define the state of a particular cell at time t to be the number of busy channels in that cell and is represented by c(t). The guard channel policy reserves a subset of channels allocated to a particular cell for handoff calls (say C-T channels) [2]. The description of guard channel policy is given algorithmically in Figure 1. Whenever the channel occupancy exceeds the certain threshold T, the guard channel policy rejects new calls until the channel occupancy goes below the threshold. The guard channel policy accepts handoff calls as long as channels are available.

For computing the blocking performance of GC policy, we consider a homogenous cellular network where all cells have the same number channels C and experience the same new and handoff call arrival rates. In each cell, the arrival of new calls and handoff calls are Poisson distributed with arrival rates λ_n and λ_h, respectively and the channel holding time of new and handoff calls are exponentially distributed with mean μ^{-1}. Let $\lambda = \lambda_n + \lambda_h$, $\alpha = \frac{\lambda_h}{\lambda}$ and $\rho = \frac{\lambda}{\mu}$. This set of assumptions have been found reasonable as long as the number of mobile users in a cell is much greater than the number of channels allocated to that cell. $\{c(t) \mid t \geq 0\}$ is a continuous-time Markov chain (birth-death process) with states 0,1,…,C. The state transition rate diagram of a cell with C full duplex channels and GC call admission scheme is shown in Figure 2.

Define the steady state probability

$$P_n = \lim_{t \to \infty} \text{prob}[c(t) = n] \qquad n = 0,1,...,C \qquad (1)$$

From [2], the steady state probability P_n that n channels are busy is given by the following expression.

$$P_n = \begin{cases} \dfrac{\rho^n}{n!} P_0 & 0 \leq n \leq T \\ \dfrac{\rho^n \alpha^{n-T}}{n!} P_0 & T < n \leq C \end{cases} \qquad (2)$$

where P_0 is the probability that all channels are free and is calculated by the following expression.

$$P_0 = \left[\sum_{k=0}^{T-1} \frac{\rho^k}{k!} + \sum_{k=T}^{C} \frac{\rho^k \alpha^{k-T}}{k!} \right]^{-1} \qquad (3)$$

Using above expressions, we can drive an

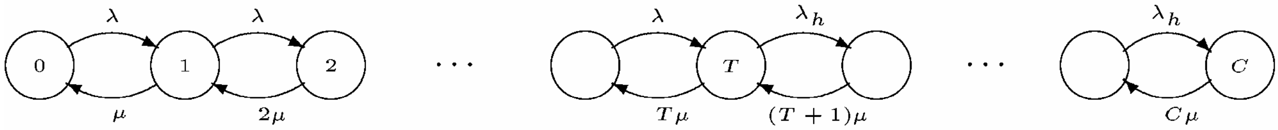

Figure 2. Markov chain model of cell for guard channel scheme.

if (NEW CALL) **then**	**if** (HANDOFF CALL OF CALL 1) **then**	**if** (HANDOFF CALL OF CALL 2) **then**
if ($c(t) < T_1$) **then**	**if** ($c(t) < T_2$) **then**	**if** ($c(t) < C$) **then**
accept call	**accept call**	**accept call**
else	**else**	**else**
reject call	**reject call**	**reject call**
end if	**end if**	**end if**
end if	**end if**	**end if**

Figure 3. Two-threshold guard channel call admission.

expression for dropping probability of handoff calls using C channels and C-T guard channels.

$$B_h(C,T) = P_C = \frac{\rho^C \alpha^{C-T}}{C!} \tag{4}$$

Similarly, the blocking probability of new calls is given by the following expression.

$$B_n(C,T) = \sum_{k=T}^{C} P_k = \sum_{k=T}^{C} \frac{\rho^k \alpha^{k-T}}{k!} \tag{5}$$

It has been shown that there is an optimal threshold T^* in which the blocking probability of new calls is minimized subject to the hard constraint on the dropping probability of handoff calls [5]. Algorithms for finding the optimal number of guard channels are given in [3-5]. If only the dropping probability of handoff calls is considered, the guard channel scheme gives very good performance, but the blocking probability of new calls is degraded to a great extent. In order to have more control on both the dropping probability of handoff calls and the blocking probability of new calls, limited fractional guard channel scheme (LFG) is proposed [5]. The LFG scheme reserves non-integral number of guard channels for handoff calls. The limited fractional guard channel scheme uses an additional parameter π and operates the same as the guard channel scheme except when T channels are occupied in the cell, in which case new calls are accepted with probability π. It has been shown that there is an optimal pair (T^*, π^*), which minimizes the blocking probability of new calls subject to the hard constraint on the dropping probability of handoff calls [5]. An algorithm for finding such optimal parameters is given in [5]. In [6], uniform fractional channel scheme (UFC) is introduced, which accepts new calls with probability of π independent of channel occupancy. It is shown that there is an optimal π^*, which minimizes the blocking probability of new calls with the constraint on the dropping probability of handoff calls. An algorithm for finding such optimal parameter and conditions for which the uniform fractional guard channel performs better than guard channel is given in [6]. It is concluded that, the uniform fractional guard channel scheme performs better than guard channel scheme in low handoff traffic conditions.

3. TWO-THRESHOLD GUARD CHANNEL POLICY

In this section, we first introduce a two-threshold guard channel policy and then compute its blocking performance. We consider a homogenous cellular network where all cells have the same number channels, C, with two classes of voice users and experience the same new and handoff call arrival rates. In each cell, the arrival of new calls and handoff calls for class i (i=1,2) are Poisson distributed with arrival rates λ_{ni} and λ_{hi}, respectively. Thus, the total call arrival rates for classes i are $\lambda_i = \lambda_{ni} + \lambda_{hi}$. In each cell, the channel holding time of new and handoff calls for class i (i=1,2) are exponentially distributed with mean μ_{ni}^{-1} and μ_{hi}^{-1}, respectively. This set of assumptions have been found reasonable as long as the number of mobile users in a cell is much greater than the number of channels allocated to that cell.

Assume that the quality of service (QoS) for handoff calls of class 2 is greater than the QoS for other calls and the QoS for handoff calls of class 1 is greater than the QoS for new calls. In order to maintain such level of QoS, channels allocated to cell are partitioned into three subsets: ordinary channels, shared guard channels and dedicated guard channels. The ordinary channels are shared between all types of calls while the shared guard channels are shared only between handoff calls and dedicated guard channels are used only for handoff calls of class 2. In order to partition the channel sets, we use two thresholds, T_1 and T_2 ($0 < T_1 \le T_2 \le C$). The procedure for accepting calls in the proposed two-threshold guard channel policy, as shown in Figure 3, can be described as follows. When a handoff call of class 2 arrives and an idle channel is available in the channel pool, the call is accepted and a channel is assigned to it; otherwise the call will be dropped. When a handoff call of class 1 arrives, it is accepted provided that the number of busy channels is smaller than T_2; otherwise the call will be dropped. When a new call arrives at the cell, it will be accepted provided that the number of busy channels is smaller than T_1 ($T_1 \le T_2$); otherwise, the incoming call will be blocked. In the above procedure, the highest priority is given to the handoff calls of class 2 and the lowest priority is given to the new calls.

If the cell is in statistical equilibrium, then it can be modeled as a four-dimensional Markov chain with the following state space.

$$S = \{(n_{n1}, n_{h1}, n_{n2}, n_{h2}) \mid n_{n1} \ge 0, n_{h1} \ge 0, n_{n2} \ge 0, \\ n_{h2} \ge 0, n_{n1} + n_{h1} + n_{n2} + n_{h2} \le C\} \tag{6}$$

where n_{n1}, n_{h1}, n_{n2} and n_{h2} denote the number of new calls of class 1, the number of handoff calls of class 1, the number of new calls of class 2 and the number handoff calls of class 2 in the cells, respectively. Let $q(n_{n1}, n_{h1}, n_{n2}, n_{h2}; \overline{n}_{n1}, \overline{n}_{h1}, \overline{n}_{n2}, \overline{n}_{h2})$ be the transition rate from state $(n_{n1}, n_{h1}, n_{n2}, n_{h2})$ to state $(\overline{n}_{n1}, \overline{n}_{h1}, \overline{n}_{n2}, \overline{n}_{h2})$. Then, we have the following.

$$\begin{aligned}
&q(i,j,k,l;i,j,k,l+1) = \lambda_{h2} \\
&\qquad i \ge 0, j \ge 0, k \ge 0, l \ge 0, i+j+k+l < C \\
&q(i,j,k,l+1;i,j,k,l) = (l+1)\,\mu_{h2} \\
&\qquad i \ge 0, j \ge 0, k \ge 0, l > 0, i+j+k+l < C \\
&q(i,j,k,l;i,j,k+1,l) = \lambda_{n2} \\
&\qquad i \ge 0, j \ge 0, k \ge 0, l \ge 0, i+j+k+l < T_1 \\
&q(i,j,k+1,l;i,j,k,l) = (k+1)\,\mu_{n2} \\
&\qquad i \ge 0, j \ge 0, k > 0, l \ge 0, i+j+k+l < C \\
&q(i,j,k,l;i,j+1,k,l) = \lambda_{h1} \\
&\qquad i \ge 0, j \ge 0, k \ge 0, l \ge 0, i+j+k+l < T_2 \\
&q(i,j+1,k,l;i,j,k,l) = (j+1)\,\mu_{h1} \\
&\qquad i \ge 0, j > 0, k \ge 0, l \ge 0, i+j+k+l < C \\
&q(i,j,k,l;i+1,j,k,l) = \lambda_{n1} \\
&\qquad i \ge 0, j \ge 0, k \ge 0, l \ge 0, i+j+k+l < T_1 \\
&q(i+1,j,k,l;i,j,k,l) = \lambda_{n1} \\
&\qquad i > 0, j \ge 0, k \ge 0, l \ge 0, i+j+k+l < C
\end{aligned} \tag{7}$$

Let $p(n_{n1}, n_{h1}, n_{n2}, n_{h2})$ be the steady state probability that there is n_{n1} new calls of class 1, n_{h1} handoff calls of class 1, n_{n2} new calls of class 2, and n_{h2} handoff calls of class 2 in the cells of the network. From Equations 7, we obtain the following balance equations.

$$\begin{aligned}
&(\lambda_{n1} + \lambda_{h1} + \lambda_{n2} + \lambda_{h2})\, p(0,0,0,0) = \\
&\quad \mu_{n1} p(1,0,0,0) + \mu_{h1} p(0,1,0,0) + \mu_{n2} p(0,0,1,0) \\
&\quad + \mu_{h2} p(0,0,0,1) \\
&(\lambda_{n1} + \lambda_{h1} + \lambda_{n2} + \lambda_{h2} + i\mu_{n1} + j\mu_{h1} + k\mu_{n2} + l\mu_{h2}) \\
&\quad p(i,j,k,l) = \\
&\quad (i+1)\mu_{n1} p(i+1,j,k,l) + (j+1)\mu_{h1} p(i,j+1,k,l) + \\
&\quad (k+1)\mu_{n2} p(i,j,k+1,l) + (l+1)\mu_{h2} p(i,j,k,l+1) + \\
&\quad \lambda_{n1} p(i-1,j,k,l) + \lambda_{h1} p(i,j-1,k,l) + \\
&\quad \lambda_{n2} p(i,j,k,l-1) + \lambda_{h2} p(i,j,k,l-1) \\
&\qquad 0 < i+j+k+l < T_1 \\
&(\lambda_{h1} + \lambda_{h2} + i\mu_{n1} + j\mu_{h1} + k\mu_{n2} + l\mu_{h2})\, p(i,j,k,l) = \\
&\quad (i+1)\mu_{n1} p(i+1,j,k,l) + (j+1)\mu_{h1} p(i,j+1,k,l) + \\
&\quad (k+1)\mu_{n2} p(i,j,k+1,l) + (l+1)\mu_{h2} p(i,j,k,l+1) + \\
&\quad \lambda_{h1} p(i,j-1,k,l) + \lambda_{h2} p(i,j,k,l-1) \\
&\qquad T_1 \le i+j+k+l < T_2 \\
&(\lambda_{h2} + i\mu_{n1} + j\mu_{h1} + k\mu_{n2} + l\mu_{h2})\, p(i,j,k,l) = \\
&\quad (i+1)\mu_{n1} p(i+1,j,k,l) + (j+1)\mu_{h1} p(i,j+1,k,l) + \\
&\quad (k+1)\mu_{n2} p(i,j,k+1,l) + (l+1)\mu_{h2} p(i,j,k,l+1) + \\
&\quad + \lambda_{h2} p(i,j,k,l-1) \\
&\qquad T_2 \le i+j+k+l < C
\end{aligned}$$

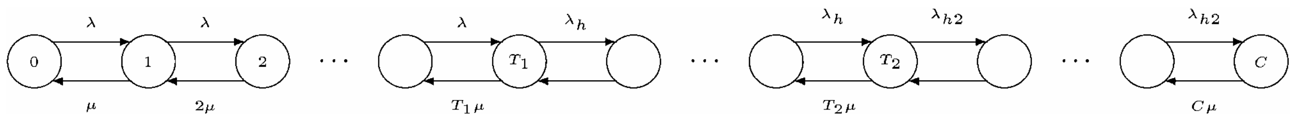

Figure 4. Markov chain model of cell for two-threshold guard channel policy.

$$(i\mu_{n1} + j\mu_{h1} + k\mu_{n2} + l\mu_{h2})p(i,j,k,l) = \lambda_{h2}p(i,j,k,l-1) \quad i+j+k+l=C \tag{8}$$

and $p(i,j,k,l)=0$ for $i<0$ or $j<0$ or $k<0$ or $l<0$. Let $B_{h2}(C,T_1,T_2)$, $B_{h1}(C,T_1,T_2)$, $B_n(C,T_1,T_2)$ be the dropping probability of handoff calls of class 2, the dropping probability of handoff calls of class 1, and the blocking probability of new calls, respectively when using C channels, $C-T_2$ dedicated guard channels and T_2-T_1 shared guard channels. Thus, we obtain

$$B_{h2}(C,T_1,T_2) = \sum_{i+j+k+l=C} p(i,j,k,l) \tag{9}$$

$$B_{h1}(C,T_1,T_2) = \sum_{i+j+k+l\geq T_2}^{i+j+k+l=C} p(i,j,k,l) \tag{10}$$

$$B_n(C,T_1,T_2) = \sum_{i+j+k+l\geq T_1}^{i+j+k+l=C} p(i,j,k,l) \tag{11}$$

In order to calculate the above equations, we need to calculate the steady state probabilities $p(i,j,k,l)$, which can be obtained using a recursive technique first proposed in [16]. This technique is based on typical feature of Chapman-Kolomorogoff system of equations in which there exist a subset of states, called boundary states, for which all other state probabilities can be expressed as a function of state probabilities of the boundary states. The basic idea of this technique is to choose the boundary states first to derive the expressions for all remaining state probabilities and then solve a reduced system of equations for these boundaries. Then all state probabilities can be determined by means of the boundary states. Since, this technique does not lead to suitable closed form equations for all Markov chains including the four-dimensional Markov chain for the proposed model. So, we make some assumptions regarding the traffic parameters in order to reduce the dimensionality of the Markov chain to one. We make the following assumption.

Assumption 1 The channel holding times for all types of calls are exponentially distributed with the same mean μ^{-1}, i.e. $\mu = \mu_{n1} = \mu_{h1} = \mu_{n2} = \mu_{h2}$.

Let c(t) denote the number of occupied channel in the given cell for which $\{c(t) | t \geq 0\}$ is a continuous-time Markov chain (birth-death process) with states 0,1,…,C. The state transition diagram of a particular cell in the network, which has C full duplex channels and uses two-threshold guard channel policy, is shown in Figure 4.

It is apparent that the state dependent arrival rate in the birth-death process, is equal to

$$\lambda(n) = \begin{cases} \lambda & \text{if } n \leq T_1 \\ \lambda_h & \text{if } T_1 \leq n \leq T_2 \\ \lambda_{h2} & \text{if } T_1 \leq n \leq C \end{cases} \tag{12}$$

where $\lambda = \lambda_1 + \lambda_2$ and $\lambda_h = \lambda_{h1} + \lambda_{h2}$. We can easily write down the solution to the steady-state balance equations of the Markov chain. Define the steady state probability

$$P_n = \lim_{t\to\infty} \text{Prob}[c(t)=n] \qquad n=0,1,..C \tag{13}$$

By writing down the equilibrium equations for the steady-state probabilities P_n ($n=0,1,...,C$), we obtain

$$\lambda P_{n-1} = n\mu P_n \qquad \text{if } n \leq T_1$$
$$\lambda_h P_{n-1} = n\mu P_n \qquad \text{if } T_1 < n \leq T_2$$

$$\lambda_{h2}P_{n-1} = n\mu P_n \qquad \text{if } T_2 < n \le C$$

Then, we have the following expression for P_n (n = 0,1, … ,C).

$$P_n = \begin{cases} \dfrac{\rho^n}{n!}P_0 & \text{if } n \le T_1 \\ \alpha^{-T_1}\dfrac{(\rho\alpha)^n}{n!}P_0 & \text{if } T_1 < n \le T_2 \\ \alpha^{-T_1}\left(\dfrac{\alpha}{\alpha_2}\right)^{T_2}\dfrac{(\rho\alpha_2)^n}{n!}P_0 & \text{if } T_2 < n \le C \end{cases} \quad (14)$$

where $\rho = \dfrac{\lambda}{\mu}$, $\alpha = \dfrac{\lambda_h}{\lambda}$, $\alpha_2 = \dfrac{\lambda_{h2}}{\lambda}$ and P_0 is the probability that all channels are free and obtained from equation $\sum_{n=0}^{C} P_n = 1$. Thus, P_0 is equal to the following expression.

$$P_0 = \left[\sum_{n=0}^{T_1-1}\frac{\rho^n}{n!} + \alpha^{-T_1}\sum_{n=T_1}^{T_2-1}\frac{(\rho\alpha)^n}{n!} + \alpha^{-T_1}\left(\frac{\alpha}{\alpha_2}\right)^{T_2}\sum_{n=T_2}^{C}\frac{(\rho\alpha_2)^n}{n!}\right]^{-1}$$

Note that if we set $T_1 = T_2 = C$, then expression (14) reduces to the classical Erlang-B formula [17] and if we set $T_1 = T_2 = T$, then expression (14) reduces to the classical guard channel policy [2]. Now we can write expressions for the dropping probability of handoff calls. The dropping probability of handoff calls of class 2 using C channels, $C - T_2$ dedicated guard channels and $T_2 - T_1$ shared guard channels is equal to

$$B_{h2}(C, T_1, T_2) = P_C = \alpha^{-T_1}\left(\frac{\alpha}{\alpha_2}\right)^{T_2}\frac{(\rho\alpha_2)^C}{C!}P_0 \quad (15)$$

and the dropping probability of handoff calls of class 1 is equal to

$$B_{h1}(C, T_1, T_2) = \sum_{n=T_2}^{C} P_n = \alpha^{-T_1}\left(\frac{\alpha}{\alpha_2}\right)^{T_2}\sum_{n=T_2}^{C}\frac{(\rho\alpha_2)^n}{n!}P_0 \quad (16)$$

Similarly, the blocking probability of new calls is given by the following expression.

$$B_n(C, T_1, T_2) = \sum_{n=T_1}^{C} P_n = \sum_{n=T_1}^{T_2-1}\frac{(\rho\alpha_2)^n}{n!}P_0 + \alpha^{-T_1}\left(\frac{\alpha}{\alpha_2}\right)^{T_2}\sum_{n=T_2}^{C}\frac{(\rho\alpha_2)^n}{n!}P_0 \quad (17)$$

4. PROPERTIES OF DROPPING AND BLOCKING PROBABILITIES

$B_{h2}(C, T_1, T_2)$, $B_{h1}(C, T_1, T_2)$ and $B_n(C, T_1, T_2)$ have interesting properties which will be used in the rest of the paper. In this section, we list some of important properties of $B_{h2}(C, T_1, T_2)$, $B_{h1}(C, T_1, T_2)$ and $B_n(C, T_1, T_2)$ and prove them in the appendix B. From Equation 14 trough 17, it is clear that $B_{h2}(C, T_1, T_2) \le B_{h1}(C, T_1, T_2) \le B_n(C, T_1, T_2)$. For the blocking probability of new calls, $B_n(C, T_1, T_2)$, the following relations hold.

Property 1 For any given values of $0 < T_1 \le T_2 \le C$, $B_n(C, T_1, T_2)$ is a monotonically decreasing function of T_1 provided that $\left(\dfrac{\lambda_n}{\lambda}\right) < \dfrac{1}{C - T_1}$ and $\rho\alpha < T_1$, i.e. $B_n(C, T_1 + 1, T_2) < B_n(C, T_1, T_2)$. This implies that the blocking probability of new calls decreases whenever the number of ordinary channels is increased.

Property 2 For any given values of $0 < T_1 \le T_2 \le C$, $B_n(C, T_1, T_2)$ is a monotonically increasing function of T_2, i.e. $B_n(C, T_1, T_2 + 1) > B_n(C, T_1, T_2)$, which implies that the blocking probability of new calls is increased when the number of shared guard channels for handoff calls is increased. This is because increasing the number of shared guard channels increases the probability of accepting handoff calls and also the probability of having large number of busy channels. This results in decreasing the probability of accepting new calls and hence increasing the blocking probability of new calls.

Property 3 For any given values of $0 < T_1 \le T_2 \le C$, $B_n(C, T_1, T_2)$ is a monotonically increasing function of C or equivalently $B_n(C+1, T_1, T_2) > B_n(C, T_1, T_2)$.

Property 3 implies that the blocking probability of new calls increases when a new channel is used as the dedicated guard channels. This is because, increasing the number of dedicated guard channels increases the probability of accepting handoff calls of class 2 and also the probability of having large number of busy channels. This results in decreasing the probability of accepting new calls and hence increasing the blocking probability of new calls.

For the dropping probability of handoff calls of class 1, $B_{h1}(C, T_1, T_2)$, the following relations hold.

Property 4 For any given values of $0 < T_1 \le T_2 \le C$, $B_{h1}(C, T_1, T_2)$ is a monotonically increasing function of T_1 ($B_{h1}(C+1, T_1, T_2) > B_{h1}(C, T_1, T_2)$). This implies that the dropping probability of handoff calls of class 1 is increased when the number of shared guard channels is decreased.

Property 5 For any given values of $0 < T_1 \le T_2 \le C$, $B_{h1}(C, T_1, T_2)$ is a monotonically decreasing function of T_2 if $\left(\frac{\lambda_n}{\lambda}\right) < \frac{1}{C - T_2}$ and $\rho\alpha_2 < \frac{1}{T_2 + 1}$, i.e. $B_{h1}(C, T_1, T_2 + 1) < B_{h1}(C, T_1, T_2)$, which implies that the dropping probability of handoff calls of class 1 is decreased when the number of shared guard channels is increased.

Property 6 For any given values of $0 < T_1 \le T_2 \le C$, $B_{h1}(C, T_1, T_2)$ is an increasing function of C, i.e. $B_{h1}(C+1, T_1, T_2) > B_{h1}(C, T_1, T_2)$, which implies that the dropping probability of handoff calls of class 1 is increased when a new channel is added to the set of dedicated guard channels. This is because increasing the number of dedicated guard channels increases the probability of accepting handoff calls of class 2 and hence the probability of having more busy channels. This causes that the dropping probability of handoff calls of class 2 is increased.

For the dropping probability of handoff calls of class 1, $B_{h2}(C, T_1, T_2)$, the following relations hold.

Property 7 For any given values of $0 < T_1 \le T_2 \le C$, $B_{h2}(C, T_1, T_2)$ is a decreasing function of T_1 ($B_{h2}(C, T_1 + 1, T_2) > B_{h2}(C, T_1, T_2)$).

Property 8 For any given values $0 < T_1 \le T_2 \le C$, $B_{h2}(C, T_1, T_2)$ is a decreasing function of T_2. That is $B_{h2}(C, T_1, T_2 + 1) > B_{h2}(C, T_1, T_2)$.

Properties 7 and 8 imply that the dropping probability of handoff calls of class 2 is increased when the number of dedicated guard channels is decreased.

Property 9 For any given values of $0 < T_1 \le T_2 \le C$, $B_{h2}(C, T_1, T_2)$ is a decreasing function of C if $\rho\alpha_2 < (C+1)$, i.e. $B_{h2}(C+1, T_1, T_2) < B_{h2}(C, T_1, T_2)$.

Two graphs of the blocking probabilities $B_n(C, T_1, T_2)$, $B_{h1}(C, T_1, T_2)$ and $B_{h2}(C, T_1, T_2)$ versus T_1 and T_2 are shown in Figures 5 and 6, respectively. The traffic parameters correspond to $\rho = 16$, $\alpha = 0.3$ and $\alpha_2 = 0.2$ and the cell has 12 full duplex channels. These graphs also confirm the properties given in this section.

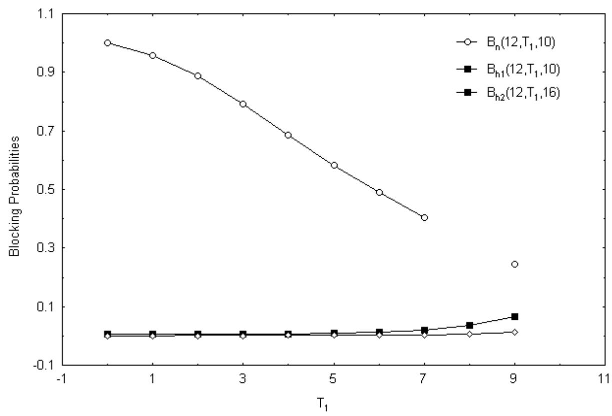

Figure 5. The effect of T_1 on the blocking probabilities.

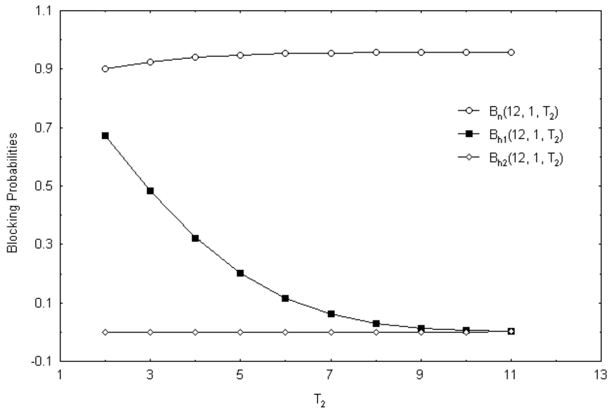

Figure 6. The effect of T_2 on the blocking probabilities.

5. OPTIMAL NUMBER OF GUARD CHANNELS

In this section, we consider the problem of finding the optimal number of guard channels, which can be described as: Given C channels allocated to a cell, the objective is to find T_1^* and T_2^* that minimizes the $B_n(C, T_1^*, T_2^*)$ with constraints $B_{h2}(C, T_1^*, T_2^*) \le p_{h2}$ and $B_{h1}(C, T_1^*, T_2^*) \le p_{h1}$. The values of p_{h1} and p_{h2} are specified by the quality of service provided by the network. Since, cellular networks usually provides the same level of quality of service as that the public switched telephone networks (PSTN), p_{h1} is set equal to the quality of service of PSTN, which may range from one percent to five percent and two percent being the most common used values. Since the handoff calls of class 2 must have the higher quality of service, then p_{h2} must be smaller than the p_{h1}. However, the value of p_{h2} is defined by the network engineers. Thus, we have the following nonlinear integer programming for the prioritized channel assignment problem.

Problem 1 Minimize $B_n(C, T_1, T_2)$ subject to the hard constraint

$$B_{h1}(C, T_1, T_2) \le p_{h1} \tag{18}$$

and

$$B_{h2}(C, T_1, T_2) \le p_{h2} \tag{19}$$

where p_{h2} and p_{h1} ($p_{h1} \ge p_{h2}$) are the levels of QoS to be satisfied for handoff calls of classes 2 and 1, respectively.

We now present an algorithm, which is called MinBlock and shown in Figure 7. The MinBlock algorithm minimizes the blocking probability of new calls with constraints on the dropping probabilities of handoff calls of both classes.

Theorem 1 Algorithm MinBlock minimizes the blocking probability of new calls while satisfying the constraints on the dropping probabilities of handoff calls of class 1 ($B_{h1}(C, T_1, T_2) \le p_{h1}$) and handoff calls of class 2 ($B_{h2}(C, T_1, T_2) \le p_{h2}$).

Proof According to property 1, in order to prove the optimality of algorithm MinBlock, it is

Algorithm *MiniBlock*
if ($B_{h2}(C, C, C) \le p_{h2}$) **then**
  return (C, C)
end if
set $T_2 \leftarrow C$
while ($B_{h2}(C, T_1, T_2) > p_{h2}$) **do**
  set $T_2 \leftarrow T_2 - 1$
end while
set $T_1 \leftarrow T_2$
while ($B_{h1}(C, T_1, T_2) > p_{h1}$) **do**
  if ($B_{h2}(C, T_1, T_2 + 1) > p_{h2}$) **then**
    set $T_2 \leftarrow T_2 + 1$
  else
    set $T_1 \leftarrow T_1 - 1$
  end if
end while
return (T_1, T_2)
end Algorithm

Figure 7. Algorithm for finding the optimal parameters of two-threshold guard channel policy.

TABLE 1. Prioritized Channel Assignment by Algorithm Given in Figure 7.

Case	ρ	α	α_2	T_1	T_2	B_n	B_{h1}	B_{h2}
1	8	0.2	0.2	10	10	0.0196	0.0196	0.0023
2	6	0.2	0.1	11	11	0.0011	0.0011	0.0011
3	9	0.4	0.2	10	11	0.0594	0.0077	0.0077
4	11	0.5	0.1	9	10	0.1627	0.0160	0.0013
5	13	0.4	0.2	8	11	0.3251	0.0095	0.0095
6	13	0.5	0.1	8	10	0.3648	0.0179	0.0017
7	13	0.5	0.2	6	11	0.6241	0.0099	0.0099
8	14	0.2	0.1	9	10	0.1215	0.0151	0.0015
9	14	0.5	0.1	7	10	0.5359	0.0172	0.0018

sufficient to prove that this algorithm finds the largest value for T_1 such that constraints on B_{h1} and B_{h2} are satisfied. The MinBlock algorithm first (**if-then** 1) checks for maximum allowed value of T_1. When the maximum allowed value of T_1 doesn't satisfy the constraint on B_{h2}, then **while-do** 5 finds the largest value of T_1 satisfying the constraints on B_{h2}. The value of T_1 obtained from **while-do** 5 is a feasible solution in a sense that it finds the largest value of T_1 to satisfy constraint (19). Since the assignment obtained from **while-do** 5 doesn't necessarily satisfy the constraint on the B_{h1}, the **while-do** 9 finds the largest value of T_1 and a value of T_2 such that the constraints (18) and (19) are satisfied.

Example 1 In Table 1, the usefulness of two-threshold guard channel policy is shown. This example assumes that the given cell has 12 full duplex channels (C=12) and constraints are $p_{h1} \leq 0.025$ and $p_{h2} \leq 0.01$. Columns 2 through 4 show the normalized arrival rate, probability that a call is being a handoff call, and probability that a call being a handoff call of class 2, respectively. Columns 5 and 6 show that the optimal number of guard channels obtained by algorithm of Figure 7. Columns 7 through 9 show the blocking probabilities of different types of calls.

The following cases are considered in this example. The first row of Table 1 (Case 1) considers the case when there are no handoff calls of class 1. When there are no handoff calls of class 1, there is no need to have shared guard channels. In this case the probability of a call being a handoff call of class 1 is zero and the proposed algorithm does not reserve any shared guard channels. This result is the same as results found by algorithms given in [3-5]. The second row of Table 1 (case 2) considers the situations that no shared guard channel is needed. The next 7 cases show different traffic patterns, which each of them needs different sets of guard channels.

6. MINIMIZING NUMBER OF CHANNELS WITH HARD CONSTRAINTS

In this section, we consider the problem of finding a call admission policy that minimizes the number of channels while satisfying the hard constraints on the blocking probability of new calls and the dropping probabilities for both handoff calls. Thus, we have the following nonlinear optimization problem.

Problem 2 Minimize C and find the maximum values of T_1 and T_2 such that

$$B_n(C,T_1,T_2) \leq p_n \tag{20}$$

$$B_{h1}(C,T_1,T_2) \leq p_{h1} \tag{21}$$

$$B_{h2}(C,T_1,T_2) \leq p_{h2} \tag{22}$$

with constraints $0 < T_1 \leq T_2 \leq C$ and $p_n \geq p_{h1} \geq p_{h2}$.

Algorithm *MinChannels*

1. **set** C=T_2=$T_1 \leftarrow 0$
2. **while** ($B_n(C,T_1,T_2) > p_n$ **or** $B_{h1}(C,T_1,T_2) > p_{h1}$ **or** $B_{h2}(C,T_1,T_2) > p_{h2}$) **do**
3.   **if** $B_{h2}(C,T_1,T_2) > p_{h2}$ **then**
4.   **set** $C \leftarrow C+1$
5.   **else if** $B_{h1}(C,T_1,T_2) > p_{h1}$ **then**
6.   **set** $T_2 \leftarrow T_2+1$
7.   **else**
8.   **set** $T_1 \leftarrow T_1+1$
9.   **end if**
10. **end while**
11. **while** ($B_n(C,T_1,T_2) \le p_n$ **and** $B_{h1}(C,T_1,T_2) \le p_{h1}$ **and** $B_{h2}(C,T_1,T_2) \le p_{h2}$) **do**
12.   **set** $T_2 \leftarrow T_2+1$
13. **end while**
14. **return** (C,T_1,T_2)

end Algorithm

Figure 8. Algorithm for finding the minimum number of channels of two-threshold guard channel policy required by cell.

Algorithm MinChannels shown in Figure 8 finds minimum value of C such that constraints (20) through (22) are satisfied.

Theorem 2 Algorithm shown in Figure 8 minimizes the number of channels used by the cell and also finds pair of (T_1, T_2), while satisfying the constraints on the blocking probability of new calls ($B_n(C,T_1,T_2) \le p_n$), the dropping probabilities of handoff calls of class 1 ($B_{h1}(C,T_1,T_2) \le p_{h1}$) and handoff calls of class 2 ($B_{h2}(C,T_1,T_2) \le p_{h2}$).

Proof In order to prove the optimality of the given algorithm, from properties 1 through 9 it is sufficient to prove that the algorithm finds minimum value of C and maximum values of T_1 and T_2. The initial assignment obtained by **while-do** 2 is an undominated solution, in the sense that it uses the minimum number of channels to satisfy the constraints defined by problem 2. This assignment results in the minimum value of C subject to the given constraints, because it assigns the channels to the given cell one by one and also finds T_1 and T_2. Since the initial assignment may not maximize T_1 and T_2, then it is not necessarily the optimal solution. In order to find the maximum values of T_1 and T_2, **while-do** 13 and 15 maximizes them. Hence, the given algorithm minimizes C and maximizes T_1 and T_2 subject to the hard constraints defined by problem 2, which results in the optimal solution.

Example 2 In Table 2, the result of algorithm MinChannels is given. This example assumes that the constraints are $p_n \le 0.05$, $p_{h1} \le 0.025$ and $p_{h2} \le 0.01$. Columns 2 through 4 of Table 2 show the normalized arrival rate, probability that a call is being a handoff call, and probability that calls being a handoff call of class 2, respectively. Columns 5 through 7 of this table show the minimum number of channels and the optimal number of guard channels obtained by algorithm MinChannels. Columns 8 through 10 of show the

TABLE 2. Prioritized Channel Assignment by Algorithm Given in Figure 8.

Case	ρ	α	α_2	T_1	T_2	C	B_n	B_{h1}	B_{h2}
1	27	0.6	0.1	34	35	36	0.047	0.015	0.001
2	27	0.6	0.2	34	35	36	0.048	0.016	0.002
3	28	0.6	0.1	35	36	37	0.048	0.016	0.001
4	29	0.2	0.1	35	36	36	0.049	0.006	0.006
5	29	0.9	0.1	38	38	39	0.020	0.020	0.001
6	29	0.9	0.6	37	38	39	0.049	0.024	0.007

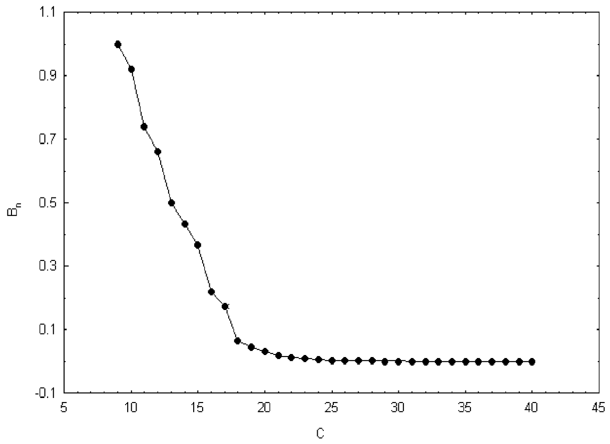

Figure 9. Changes in $B_n(C,T_1,T_2)$ with respect to C.

blocking probabilities of different types of calls.

7. OPTIMAL PRIORITIZED CHANNEL ASSIGNMENT IN MULTI-CELLS CELLULAR NETWORKS

In this section, we extend problem 1 to the multi-cells cellular networks and introduce a prioritized channel assignment algorithm for multi-cells cellular networks. We consider a multi-cells system consisting of several clusters, where a typical cluster m contains N_m cells. Assume that a total of C full duplex channels are allocated to the whole network and hence to each cluster. Under our prioritized channel assignment scheme, the allocated channels will be divided into N_m disjoint channel sets, where each channel set is allocated to one cell in the cluster. Then, the channel set of each cell is divided into three subsets: ordinary channels, shared and dedicated guard channels. By applying the proposed algorithm to each cluster in the system, the prioritized channel assignment is obtained for the whole network. Let $\Lambda_n = \sum_{i=1}^{N_m} \lambda_n^{\ i}$ be the total arrival rate of new calls over all cells in cluster m and $\lambda_n^{\ i}$ is the arrival rate of new calls in cell i of cluster m. Define the overall blocking probability of new calls by

$$B_N = \sum_{i=1}^{N_m} \frac{\lambda_n^{\ i}}{\Lambda_n} B_n^{\ i}\left(C^i, T_1^{\ i}, T_2^{\ i}\right) \tag{23}$$

where $B_n^{\ i}\left(C^i, T_1^{\ i}, T_2^{\ i}\right)$ is the blocking probability of new calls in cell i when C^i channels are allocated to this cell and $T_1^{\ i}$ and $T_2^{\ i}$ are thresholds for this cell. The objective is to find the optimal value for tuple $\left(C^i, T_1^{\ i}, T_2^{\ i}\right)$ (i = 1, 2, …, N_m), which minimizes the overall blocking probability of new calls subject to the hard constraint on the dropping probabilities of both calls. This problem is formulated as the following non-linear optimization problem.

Problem 3 Minimize the overall blocking probability of new calls, B_N, subject to the following hard constraints.

$$B_{hv}^{\ i}\left(C^i, T_1^{\ i}, T_2^{\ i}\right) \le p_{h1} \tag{24}$$

$$B_{ht}^{\ i}\left(C^i, T_1^{\ i}, T_2^{\ i}\right) \le p_{h2} \tag{25}$$

$$\sum_{i=1}^{N_m} C^i = C \tag{26}$$

where $0 < T_1^{\ i} \le T_2^{\ i} \le C^i$ for all i = 1, 2, …, N_m in cluster m.

In what follows, we propose a greedy algorithm for solving problem 3. This algorithm can be described as follows. Initially for each cell i, the smallest number of channels required to satisfy the given QoS is found. To do this, we use the algorithm MinChannels with the constraint $p_n = 1 - \varepsilon$, where ε is a small positive value. Then the remaining channels, if any, are allocated to cells one by one. Let γ_i denotes the amount of decrement in $B_n^{\ i}$ brought by allocation of an additional channel to cell i. Note that the additional channel can be used as an ordinary channel/ shared guard channel/ dedicated guard channel. In order to find the usage of the additional channel, the algorithm used for solving problem 1 is used. The amount of decrement in $B_n^{\ i}$ are computed for all cell i (for i = 1, 2, …, N_m) according to the

Algorithm MultiCells

1. solve problem 2 for cell i (for I=1,2,…,N_m) with constraint $p_n = 1-\varepsilon$, where ε is a small positive value.
2. **set** $S \leftarrow C - \sum_{i=1}^{N_m} C^i$
3. if S = 0 **then** terminate. $\left\{ \left(C^i, T_1{}^i, T_2{}^i\right) \mid i = 1,2,\ldots,N_m \right\}$ is optimal.
4. **if** S < 0 **then** terminate. C channels cannot satisfy the specified QoS.
5. **for** $i \leftarrow 1$ **to** N_m **do**
6. Solve problem 1 for cell with C^i and C^i + 1 channels.
7. **set** $\gamma_i \leftarrow \frac{\lambda_n{}^i}{\lambda_n}\left[B_n{}^i(C^i, T_1{}^{*i}, T_2{}^{*i}) - B_n{}^i(C^i+1, T_1{}^{*i}, T_2{}^{*i})\right]$
8. **end for**
9. **for** $i \leftarrow 1$ **to** S **do**
10. **set** $j \leftarrow \text{argmax}_i\, \gamma_i$.
11. **set** $C^j \leftarrow C^j + 1$.
12. **set** $\gamma_i \leftarrow \frac{\lambda_n{}^i}{\Lambda_n}\left[B_n{}^i(C^i, T_1{}^{*i}, T_2{}^{*i}) - B_n{}^i(C^i+1, T_1{}^{*i}, T_2{}^{*i})\right]$.
13. **end for**
14. $\left\{ \left(C^{*i}, T_1{}^{*i}, T_2{}^{*i}\right) \mid i = 1,2,\ldots,N_m \right\}$ is the optimal solution.

end Algorithm

Figure 10. Multi-cells prioritized channel assignment algorithm.

following equation:

$$\gamma_i = \frac{\lambda_n{}^i}{\Lambda_n}\left[B_n{}^i(C^i, T_1{}^{*i}, T_2{}^{*i}) - B_n{}^i(C^i+1, T_1{}^{*i}, T_2{}^{*i})\right]$$

where T_j^{*i} (for j = 1, 2) are the thresholds obtained by MinBlock algorithm. Note that γ_i is always nonnegative. Then a cell with the largest decrement in B_n is found among all cells in the cluster and an additional channel is assigned to it. This procedure is repeated until all available channels C in the cluster are used. This fact is shown in Figure 9. Algorithm given in Figure 10 summarizes this procedure.

Theorem Algorithm given in Figure 10 finds the optimal solution of problem 3.

Proof The initial assignment is an undominated solution, in the sense that it uses the minimum number of channels to satisfy the constraints (24) and (25). This assignment results in the maximum value of B_N subject to the constraints (24) and (25). Then the algorithm assigns the remaining channels one by one to cells which results in the largest decrement in blocking probability of new calls. This strategy results in the optimal solution. Let j_i be the index of the cell with the largest decrement in B_N at step i (for I =1, 2, …, S). Assume that there is another strategy which is optimal and chooses cell $k_i \neq j_i$ at step i. Thus we have $\gamma_{k_i} = \gamma_{j_i} - \delta_i$ for $\delta_i > 0$. Then interchanging cell j_i with cell k_{i_i} results in assignment $B_N^{k_i} = \sum_{l=1}^{N_m} \frac{\lambda_n^l}{\Lambda_n} B_n^l\left(C^l, T_1{}^l, T_2{}^l\right)$.

Subtracting $B_N^{j_i}$ from $B_N^{k_i}$, we obtain $B_N^{k_i} - B_N^{j_i} = \delta_i$. Repeating this procedure for S steps, we obtain $\sum_{i=1}^{S}\left[B_N^{k_i} - B_N^{j_i}\right] = \sum_{i=1}^{S}\delta_i$, which is positive. Thus, no index other than the index with the largest value of γ_i would results in the optimal solution. Hence, the proposed cell selection mechanism minimizes the value of B_N subject to the hard constraints (24) and (25) and results in the optimal solution.

TABLE 3. The Traffic Parameters of Cellular Network.

Cell	λ_n	λ_h	λ_{h2}
1	4	4	1
2	6	6	1
3	8	6	2
4	7	4	1
5	5	6	2
6	10	8	2
7	4	3	1

Example 3 Consider a cellular system with clusters having 7 cells. Assume that a total of 110

TABLE 4. The Result of Prioritized Channel Assignment for Multi-Cell System.

Case	T_1	T_2	C	B_n	B_{h1}	B_{h2}
1	11	13	13	0.112727	0.008052	0.008052
2	13	16	16	0.233747	0.009029	0.009029
3	14	17	17	0.267828	0.008846	0.008846
4	13	15	15	0.154768	0.008658	0.008658
5	13	15	16	0.178981	0.021289	0.002365
6	19	21	22	0.198081	0.021024	0.001752
7	9	11	11	0.161206	0.009545	0.009545

full duplex channels are available in this system. The upper bounds on the dropping probabilities of handoff calls of classes 1 and 2 are set to 0.025 and 0.01, respectively. The call arrival rates, which are normalized to the call holding time, are given in Table 3. The result of algorithm of Figure 10 is given in Table 4.

From Table 4, it is evident that all constraints are satisfied and the obtained solution is also optimal.

Remark 1 Extension of problem 2 to multi-cell networks has no engineering profit, because the number of channels allocated to the network is fixed and minimizing the number of channels wastes the system resources.

8. CONCLUSIONS

In this paper, we studied the problem of call admission in cellular network. We introduced a two-threshold guard channel policy when there are two classes of voice users and derived blocking probabilities for the network. We also introduced and solved three prioritized channel assignment problems: finding the optimal number of guard channels, minimizing the number of required channels and the optimal channel assignment in multi-cells cellular network subject to the hard constraints on the dropping probabilities of handoff calls of both classes. Through simulations, we showed that for all the proposed algorithms, the numerical results confirm the analytical results.

9. ACKNOWLEDGMENT

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10. APPENDIXES

A. Appendix In this appendix, we first give some notations and then some properties that will be used in next appendix for proofs of properties 1 through 9. We will use the following notations throughout the appendices.

$$\phi(C,T_1,T_2)=\alpha^{-T_1}\left(\frac{\alpha}{\alpha_2}\right)^{T_2}\frac{(\rho\alpha_2)^C}{C!} \tag{A.1}$$

$$D_1(C,T_1,T_2)=\sum_{n=0}^{T_1-1}\frac{(\rho)^n}{n!} \tag{A.2}$$

$$D_2(C,T_1,T_2)=\alpha^{-T_1}\sum_{n=T_1}^{T_2-1}\frac{(\rho\alpha)^n}{n!} \tag{A.3}$$

$$D_3(C,T_1,T_2)=\alpha^{-T_1}\left(\frac{\alpha}{\alpha_2}\right)^{T_2}\sum_{n=T_2}^{C}\frac{(\rho\alpha_2)^n}{n!} \tag{A.4}$$

$$D(C,T_1,T_2)=\sum_{i=1}^{3}D(C,T_1,T_2) \tag{A.5}$$

$D(C,T_1,T_2)$ have the following properties, assuming that all other system parameters are fixed.

Property 10 For any given values of $0 < T_1 \le T_2 \le C$, $D(C, T_1, T_2)$ satisfies the following condition.

$$D(C, T_1, T_2) > \frac{\alpha_2}{\alpha} D(C, T_1, T_2 + 1)$$

Proof In order to prove the inequality $D(C, T_1, T_2) > \frac{\alpha_2}{\alpha} D(C, T_1, T_2 + 1)$, we show that $D(C, T_1, T_2) - \frac{\alpha_2}{\alpha} D(C, T_1, T_2 + 1)$ is greater than zero. From (A.5) and by some algebraic simplification, we have

$$D(C, T_1, T_2) - \frac{\alpha_2}{\alpha} D(C, T_1, T_2 + 1) = \left(1 - \frac{\alpha_2}{\alpha}\right) [D_1(C, T_1, T_2) + D_2(C, T_1, T_2 + 1)] \quad \text{(A.6)}$$

Since $\alpha > \alpha_2$, then

$D(C, T_1, T_2) > \frac{\alpha_2}{\alpha} D(C, T_1, T_2 + 1)$ for all values of $0 < T_1 \le T_2 \le C$.

Property 11 For any given values of $0 < T_1 \le T_2 \le C$, $D(C, T_1, T_2)$ satisfies the following condition.

$$D(C, T_1, T_2) > \alpha D(C, T_1 + 1, T_2)$$

Proof In order to prove the inequality $D(C, T_1, T_2) > \alpha D(C, T_1 + 1, T_2)$, we show that $D(C, T_1, T_2) - \alpha D(C, T_1 + 1, T_2)$ is greater than zero. From expression (A.5) and by some algebraic simplification, we have

$$D(C, T_1, T_2) - \alpha D(C, T_1 + 1, T_2) = (1 - \alpha) D_1(C, T_1 + 1, T_2) \quad \text{(A.7)}$$

Since α is less than 1, then $D(C, T_1, T_2)$ is greater than $\alpha D(C, T_1 + 1, T_2)$ for all values of $0 < T_1 \le T_2 \le C$.

Property 12 For any given values of $0 < T_1 \le T_2 \le C$, $D(C, T_1, T_2)$ is a monotonically increasing function of C, that is

$$D(C + 1, T_1, T_2) > D(C, T_1, T_2)$$

Proof From (A.5) and (A.1), we obtain

$$D(C + 1, T_1, T_2) = D(C, T_1, T_2) + \phi(C + 1, T_1, T_2) > D(C, T_1, T_2) \quad \text{(A.8)}$$

which holds for all values of $0 < T_1 \le T_2 \le C$.

Property 13 For any given values of $0 < T_1 \le T_2 \le C$, $D(C, T_1, T_2)$ is a monotonically increasing function of T_2. That is

$$D(C, T_1, T_2 + 1) > D(C, T_1, T_2)$$

Proof In order to prove that $D(C, T_1, T_2)$ is an increasing function of T_2, we show that $D(C, T_1, T_2 + 1) - D(C, T_1, T_2)$ is greater than zero. Let $X^{T_2} = \alpha^{-T_1} \left(\frac{\alpha}{\alpha_2}\right)^{T_2} \frac{(\rho\alpha_2)^{T_2}}{T_2!}$. From (A.5) and by some algebraic simplification, we obtain

$$D(C, T_1, T_2 + 1) - D(C, T_1, T_2) = [D_3(C, T_1, T_2) - X_{T_2}] \left(\frac{\alpha}{\alpha_2} - 1\right) > 0 \quad \text{(A.9)}$$

Since α is less than 1, then $D(C, T_1, T_2)$ is greater than $\alpha D(C, T_1 + 1, T_2)$ for all values of $0 < T_1 \le T_2 \le C$.

B. Appendix In this appendix, we give the proofs of properties 1 through 9.

B.1 Proof of Property 1 In order to prove that $B_n(C, T_1, T_2)$ is a decreasing function of T_1, we show that $B_n(C, T_1 + 1, T_2) - B_n(C, T_1, T_2)$ is negative, that is,

$$B_n(C, T_1 + 1, T_2) - B_n(C, T_1, T_2) = \frac{D_2(C, T_1 + 1, T_2) + D_3(C, T_1 + 1, T_2)}{D(C, T_1 + 1, T_2)}$$

$$- \frac{D_2(C,T_1,T_2)+D_3(C,T_1,T_2)}{D(C,T_1,T_2)}, \tag{B.10}$$

$$< \frac{D_2(C,T_1+1,T_2)+D_3(C,T_1+1,T_2)}{D(C,T_1,T_2)} - \frac{D_2(C,T_1,T_2)+D_3(C,T_1,T_2)}{D(C,T_1,T_2)}, \tag{B.11}$$

$$= \frac{\alpha^{-1}\left[D_2(C,T_1,T_2)+D_3(C,T_1,T_2)-\frac{\rho^{T_1}}{T_1!}\right]}{D(C,T_1,T_2)} - \frac{D_2(C,T_1,T_2)+D_3(C,T_1,T_2)}{D(C,T_1,T_2)}, \tag{B.12}$$

$$= \frac{[D_2(C,T_1,T_2)+D_3(C,T_1,T_2)][1-a]-\frac{\rho^{T_1}}{T_1!}}{\alpha D(C,T_1,T_2)} \tag{B.13}$$

Since $\lambda_n/\lambda < 1/(C-T_1)$ and $\rho\alpha/T_1 < 1$, we have

$$(1-a) = \frac{\lambda_n}{\lambda} < \frac{1}{C-T_1} < \frac{1}{1+\ldots+\left(\frac{\rho\alpha}{T_1}\right)^{C-T_1}} < \frac{1}{1+\ldots+\frac{(\rho\alpha)^{C-T_1}}{(T_1+1)\times\ldots\times C}} \tag{B.14}$$

$$< \frac{\frac{(\rho\alpha)^{T_1}}{T_1!}}{\frac{(\rho\alpha)^{T_1}}{T_1!}} \times \frac{1}{1+\ldots+\frac{(\rho\alpha)^{C-T_1}}{(T_1+1)\times\ldots\times C}} \tag{B.15}$$

$$= \frac{\frac{(\rho\alpha)^{T_1}}{T_1!}}{\sum_{n=T_1}^{T_2-1}\frac{(\rho\alpha)^n}{n!}+\sum_{n=T_2}^{C}\frac{(\rho\alpha)^n}{n!}} \tag{B.16}$$

Since $k > T_2$, we have $\left(\frac{\alpha}{\alpha_2}\right)^k > \left(\frac{\alpha}{\alpha_2}\right)^{T_2}$, the above inequality becomes

$$(1-a) < \frac{\frac{(\rho\alpha)^{T_1}}{T_1!}}{\sum_{n=T_1}^{T_2-1}\frac{(\rho\alpha)^n}{n!}+\left(\frac{\alpha}{\alpha_2}\right)^{T_2}\sum_{n=T_2}^{C}\frac{(\rho\alpha_2)^n}{n!}} \tag{B.17}$$

$$= \frac{\alpha^{-T_1}\frac{(\rho\alpha)^{T_1}}{T_1!}}{\alpha^{-T_1}\sum_{n=T_1}^{T_2-1}\frac{(\rho\alpha)^n}{n!}+\alpha^{-T_1}\left(\frac{\alpha}{\alpha_2}\right)^{T_2}\sum_{n=T_2}^{C}\frac{(\rho\alpha_2)^n}{n!}} \tag{B.18}$$

$$= \frac{\frac{\rho^{T_1}}{T_1!}}{D_2(C,T_1,T_2)+D_3(C,T_1,T_2)} \tag{B.19}$$

From the above inequality and inequality (B.13) the proof will be completed.

B.2 Proof of Property 2 In order to prove that $B_n(C,T_1,T_2)$ is an increasing function of T_2, we show that $B_n(C,T_1,T_2+1)-B_n(C,T_1,T_2)$ is greater than zero. Let $X_{T_2} = \alpha^{-T_1}\left(\frac{\alpha}{\alpha_2}\right)^{T_2}\frac{(\rho\alpha_2)^{T_2}}{T_2!}$. From (17) and (A.5), and by some algebraic simplification, we obtain

$$B_n(C,T_1,T_2+1)-B_n(C,T_1,T_2) = \frac{D_2(C,T_1,T_2+1)+D_3(C,T_1,T_2+1)}{D(C,T_1,T_2+1)} - \frac{D_2(C,T_1,T_2)+D_3(C,T_1,T_2)}{D(C,T_1,T_2)}$$

$$= \frac{D_2(C,T_1,T_2)+\frac{\alpha}{\alpha_2}D_3(C,T_1,T_2)-\left(\frac{\alpha}{\alpha_2}-1\right)X_{T_2}}{D_1(C,T_1,T_2)+D_2(C,T_1,T_2)+\frac{\alpha}{\alpha_2}D_3(C,T_1,T_2)-\left(\frac{\alpha}{\alpha_2}-1\right)X_{T_2}} - \frac{D_2(C,T_1,T_2)+D_3(C,T_1,T_2)}{D_1(C,T_1,T_2)+D_2(C,T_1,T_2)+D_3(C,T_1,T_2)}$$

$$= \left(\frac{\alpha}{\alpha_2}-1\right)\frac{D_1(C,T_1,T_2)\,[D_3(C,T_1,T_2)-X_{T_2}]}{D(C,T_1,T_2)D(C,T_1,T_2+1)} \tag{B.20}$$

which is greater than zero for all values of $0<T_1\le T_2\le C$.

B.3 Proof of Property 3 In order to prove that $B_n(C,T_1,T_2)$ is an increasing function of C, we show that $B_n(C+1,T_1,T_2)-B_n(C,T_1,T_2)$ is greater than zero. From (17) and (A.5), and by some algebraic simplification, we obtain

$$B_n(C+1,T_1,T_2)-B_n(C,T_1,T_2) = \frac{D_2(C+1,T_1,T_2)+D_3(C+1,T_1,T_2)}{D(C+1,T_1,T_2)}$$

$$- \frac{D_2(C,T_1,T_2) + D_3(C,T_1,T_2)}{D(C,T_1,T_2)} = \frac{D_1(C,T_1,T_2)\phi(C+1,T_1,T_2)}{D(C+1,T_1,T_2)D(C,T_1,T_2)} \quad \text{(B.21)}$$

which is greater than zero for all values of $0 < T_1 \le T_2 \le C$.

B.4 Proof of Property 4 In order to prove that $B_{h1}(C,T_1,T_2)$ is an increasing function of T_1, we show that $B_{h1}(C,T_1+1,T_2)/B_{h1}(C,T_1,T_2)$ is greater than one. By some algebraic simplification, we have

$$\frac{B_{h1}(C,T_1+1,T_2)}{B_{h1}(C,T_1,T_2)} = \frac{D(C,T_1,T_2)D_3(C,T_1+1,T_2)}{D(C,T_1+1,T_2)D_3(C,T_1,T_2)} = \frac{D(C,T_1,T_2)}{\alpha D(C,T_1+1,T_2)} \quad \text{(B.22)}$$

From property 11, it is apparent that the above equation is greater than one for all values of $0 < T_1 \le T_2 \le C$.

B.5 Proof of Property 5 In order to prove that $B_{h1}(C,T_1,T_2)$ is an decreasing function of T_2, we show that $B_{h1}(C,T_1,T_2+1) - B_{h1}(C,T_1,T_2)$ is negative, that is

$$B_{h1}(C,T_1,T_2+1) - B_{h1}(C,T_1,T_2) = \frac{D_3(C,T_1,T_2+1)}{D(C,T_1,T_2+1)} - \frac{D_3(C,T_1,T_2)}{D(C,T_1,T_2)} \quad \text{(B.23)}$$

Using property 13, we obtain

$$B_{h1}(C,T_1,T_2+1) - B_{h1}(C,T_1,T_2) < \frac{D_3(C,T_1,T_2+1) - D_3(C,T_1,T_2)}{D(C,T_1,T_2)} \quad \text{(B.24)}$$

$$= \frac{\left(\frac{\alpha}{\alpha_2}\right)[D_3(C,T_1,T_2) - X_{T_2}] - D_3(C,T_1,T_2)}{D(C,T_1,T_2)} \quad \text{(B.25)}$$

$$= \frac{\left(\frac{\alpha}{\alpha_2} - 1\right) D_3(C,T_1,T_2) - \frac{\alpha}{\alpha_2} X_{T_2}}{D(C,T_1,T_2)} \quad \text{(B.26)}$$

$$= \frac{(\alpha - \alpha_2)\, D_3(C,T_1,T_2) - \alpha X_{T_2}}{\alpha_2 D(C,T_1,T_2)} \quad \text{(B.27)}$$

where $X_{T_2} = \alpha^{-T_1}\left(\frac{\alpha}{\alpha_2}\right)^{T_2} \frac{(\rho\alpha_2)^{T_2}}{T_2!}$. Since $\frac{\lambda_{h1}}{\lambda_{h2}} < 1/(C - T_2)$ and $\rho\alpha_2/T_2 + 1 < 1$, we have

$$\frac{\lambda_{h1}}{\lambda_{h2}} = \frac{\alpha - \alpha_2}{\alpha} < \frac{1}{T_2} < \frac{1}{1 + \ldots + \left(\frac{\rho\alpha_2}{T_2+1}\right)^{C-T_2}}$$

$$< \frac{1}{1 + \ldots + \frac{(\rho\alpha_2)^{C-T_2}}{(T_2+1)\times\ldots\times C}} \quad \text{(B.28)}$$

$$< \frac{X_{T_2}}{X_{T_2}} \times \frac{1}{1 + \ldots + \frac{(\rho\alpha_2)^{C-T_2}}{(T_2+1)\times\ldots\times C}} \quad \text{(B.29)}$$

$$= \frac{X_{T_2}}{D_3(C,T_1,T_2)} \quad \text{(B.30)}$$

From the above inequality and inequality (B.27), the proof will be completed.

B.6 Proof of Property 6 In order to prove that $B_{h1}(C,T_1,T_2)$ is an increasing function of C, we show that $B_{h1}(C+1,T_1,T_2) - B_{h1}(C,T_1,T_2)$ is positive. Thus, we have

$$B_{h1}(C+1,T_1,T_2) - B_{h1}(C,T_1,T_2)$$
$$= \frac{D_3(C+1,T_1,T_2)}{D(C+1,T_1,T_2)} - \frac{D_3(C,T_1,T_2)}{D(C,T_1,T_2)}$$
$$= \frac{D_3(C,T_1,T_2) + \phi(C+1,T_1,T_2)}{D(C+1,T_1,T_2)} - \frac{D_3(C,T_1,T_2)}{D(C,T_1,T_2)}$$
$$= \frac{\phi(C+1,T_1,T_2)\,[D(C,T_1,T_2) - D_3(C,T_1,T_2)]}{D(C+1,T_1,T_2)D(C,T_1,T_2)}$$
$$= \frac{\phi(C+1,T_1,T_2)\,[D_1(C,T_1,T_2) + D_2(C,T_1,T_2)]}{D(C+1,T_1,T_2)D(C,T_1,T_2)} \quad \text{(B.31)}$$

which is positive for all values of $0 < T_1 \le T_2 \le C$.

B.7 Proof of Property 7 In order to prove that $B_{h2}(C,T_1,T_2)$ is an increasing function of T_1, we show that $B_{h2}(C,T_1+1,T_2)/B_{h2}(C,T_1,T_2)$ is greater than one. Thus, we have

$$\frac{B_{h2}(C,T_1+1,T_2)}{B_{h2}(C,T_1,T_2)} = \frac{D(C,T_1,T_2)\phi(C,T_1+1,T_2)}{D(C,T_1+1,T_2)\phi(C,T_1,T_2)} = \frac{D(C,T_1,T_2)}{\alpha D(C,T_1+1,T_2)} \quad \text{(B.32)}$$

From property 11, it is apparent that the above equation is greater than one for all values of $0 < T_1 \le T_2 \le C$.

B.8 Proof of Property 8 In order to prove that

$B_{h2}(C,T_1,T_2)$ is an increasing function of T_2, we show that $B_{h2}(C,T_1,T_2+1)/B_{h2}(C,T_1,T_2)$ is greater than one. Thus, we have

$$\frac{B_{h2}(C,T_1,T_2+1)}{B_{h2}(C,T_1,T_2)} = \frac{D(C,T_1,T_2)\phi(C,T_1,T_2+1)}{D(C,T_1,T_2+1)\phi(C,T_1,T_2)} = \frac{\alpha D(C,T_1,T_2)}{\alpha_2 D(C,T_1,T_2+1)} \quad (B.32)$$

From property 10, it is apparent that the right hand side of the above equation is greater than one for all values of $0<T_1 \le T_2 \le C$.

B.9 Proof of Property 9 In order to prove that $B_{h2}(C,T_1,T_2)$ is a decreasing function of C, we show that $B_{h2}(C+1,T_1,T_2)-B_{h2}(C,T_1,T_2)$ is less than zero. Using property 12 and some algebraic simplification, we obtain

$$\begin{aligned} B_{h2}(C+1,T_1,T_2) - B_{h2}(C,T_1,T_2) = & \\ \frac{\phi(C+1,T_1,T_2)}{D(C+1,T_1,T_2)} - \frac{\phi(C,T_1,T_2)}{D(C,T_1,T_2)} & \\ < \frac{\phi(C,T_1,T_2)}{D(C,T_1,T_2)}\left[\frac{\rho\alpha_2}{C+1} - 1\right] & \quad (B.34) \end{aligned}$$

which is less than zero for all values of $0<T_1 \le T_2 \le C$ if $\rho < C$.

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