RESEARCH NOTE

$M_1, M_2, ..., M_k/G_1, G_2, \ldots, G_k/l/N$ QUEUE WITH BUFFER DIVISION AND PUSH-OUT SCHEMES FOR ATM NETWORKS

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Abstract In this paper, loss probabilities and steady state probabilities of data packets for an asynchronous transfer mode (ATM) network are investigated under the buffer division and push-out schemes. Data packets are classified in classes k which arrive in Poisson fashion to the service facility and are served with general service rate under buffer division scheme, finite buffer space N is divided into $N_1, N_2, \ldots, N_k$ such that $N = N_1 + N_2 +\ldots+ N_k$. Under push-out scheme if upon arrival of class 1 packet to the system finds that there are less than N_1-1 class 1 packets waiting for service and there is no unoccupied buffer space then one of the packets of other classes is pushed out. The pushed out packet is lost. The packets are served under general service discipline.

Key Words Steady State Probability, ATM, Finite Buffer, Push-Out Scheme

چکیده این مقاله احتمال گم شدن و احتمال حالت پایای بسته داده ها را برای شبکه در وضعیت انتقال ناهمگام به کمک بخش ذخیره و با تلقی اخراج تحت فشار مورد بررسی قرار می‌دهد. بسته داده ها به k کلاس تقسیم می‌شود بطوریکه هر کلاس بر اساس فرایند پواسانی به تجهیزات سرویس دهنده وارد شده و به کمک بخش ذخیره با سرعت عمومی تحت سرویس قرار می‌گیرد. فضای ذخیره N محدود بوده و به زیر بخشهای N_1 ، N_2 ، ... و N_k تقسیم می‌شود بطوریکه $N = N_1 + N_2 +\ldots+ N_k$. اگر بسته داده های کلاس 1 به سیستم وارد شود، با حالتی که کمتر از N_1-1 بسته از کلاس 1 منتظر سرویس هستند و فضای ذخیره ای اشغال نشده باشد، در آن صورت یکی از بسته های غیر از این کلاس به بیرون فرستاده می‌شود. بسته خارج شده، گم شده تلقی می‌شود. سرویس سایر بسته ها طبق نظام عمومی صف انجام می‌شود.

1. INTRODUCTION

In this investigation, $M_1, M_2, \ldots, M_k/G_1, G_2, ..., G_k/l/N$ queuing system with buffer space division and push-out schemes for an asynchronous transfer mode (ATM) network is developed. ATM is a high bandwidth, low-delay switching technique that can switch all types of traffic in a packet format of fixed length. Queuing model under study has its widespread applications to evaluate the performance measures of computer network, ATM network broadband integrated service digital networks (B-ISDN).

Several authors studied various queuing systems, which are well employed in the communication networks. Doshi and Heffes [l] studied overload performance of several processor queuing discipline For M/M/1 queue. Wassal and Hasan [2] proposed architecture for the ATM switches used on board satellites and obtained loss delay in such architecture. Wang and Silvester [3] presented two-state Markov modulated Poisson process bulk (BMMPP) queuing system and studied the performance of ATM multiplexer loaded with various types of traffic. Deng and Chen [4] suggested a multicast bounded-fanout principle to reduce the overflow probability of copy network to an acceptable level. Chaudhry and Gupta [5] obtained the various performance measures for G_l/Geom/l/N queue. Jain and Ghimire [6] analyzed resource sharing finite queue and obtained various performance measures. A channel grouping technique and virtual FIFO architecture was used in ATM switching network in [7].

Breaking the entire geographical barrier ATM switching network has become backbone for public communication networks and very rare literatures can be found in the queuing system in which finite-buffer division and push-out schemes are employed. Satio [8], Sumita and Ozawa [9] have proposed various services and buffer control mechanisms for shared buffer and push-out schemes. Jain and Ghimire [10] presented queuing analysis of cellular radio system under consideration of two types of traffic and obtained loss probability. Timotijevic and Schormans [11] gave the concept for an ATM mechanism in satellite system. Akyildiz and Cheng [12] developed $M_1/M_2/G_1$, $G_2/1/1$ queuing system with buffer division and push-out scheme and studied ATM switching networks under the consideration of two types of traffic with different priority. Kweon and Shin [13] proposed a framework to provide statistical real-time communication services for an ATM networks. Chen et al. [14] developed an efficient cell-scheduling algorithm for multicast ATM switching systems with input queues. Pousada-Carballo et al. [15] presented a new neural scheduler for input-buffered ATM switches for uniform traffic.

2. MATHEMATICAL DESCRIPTION OF THE MODEL

In our queuing model, we allow buffer management, push-out schemes according to which total space N is divided as $N = \sum_{i=1}^{k} N_i$. There cannot be packets of class i more than N_1-l (i = 1,2, ..., k) waiting for services.

We have the following notations for our model:

$\lambda_i =$ Poisson arrival rates of packets of class i (i = 1, 2, ..., k).

$l_i =$ Loss probability of a packet of class i (i =1, 2, ..., k).

$l =$ The loss probability for a packet.

$b_i =$ Probability density function for service time of class i packets.

R = Ratio of packets lost during a long period of time to packets served in the same period of time.

T = Time period when the system reaches the steady state.

$R_1 =$ Average number of class I packets lost per packet served.

$b(x) =$ Probability density Function of time.

$r_i =$ Number of packets of class i (i = 1, 2, ..., k) at the departure time.

$\alpha_i(x) =$ Probability that classes i (i = 1, 2, 3, ..., k) packet is served when there are $r_1, r_2, ..., r_k$ packets of classes 1, 2, ..., k respectively present in the system.

$P(x) =$ Steady state probability that the system is in state r at the departure time of a packet.

$P(r; m) =$ One step transition probability from state to state r to state m

3. LOSS PROBABILITY OF PACKETS

Loss of packets takes place if either there is no waiting space available in the buffer upon its arrival or it is pushed out from buffer while waiting for service. Number of packets lost during a busy period of server is dependent so that.

$$l = \frac{\sum_{i=1}^{k} \lambda_i T(1-l)R}{\sum_{i=1}^{k} \lambda_i T} = 1 - l \tag{1}$$

Equation 1 can be written as

$$l = \frac{R}{1+R} \tag{2}$$

Loss probability of a class 1 packet is given by [1]

$$\frac{\sum_{i=1}^{k} \lambda_i T(1-l)R_1}{\lambda_i T} = \frac{\sum_{i=1}^{k} \lambda_i (1-l)R_1}{\lambda_i} \tag{3a}$$

Similarly the loss probabilities of classes 2, 3, ..., k packets are obtained by

$$l_u = \frac{\left(\sum_{i=1}^{k}\lambda_i\right)\left(1-\left(\sum_{\substack{j=1\\ j\neq i}}^{k}\lambda_j l_j\right)\right)}{\lambda_u}, \quad u = 2,3,\ldots,k \tag{3b}$$

4. STEADY STATE PROBABILITIES

The quantity r = (r_1, r_2, …, r_k) constitutes an imbedded Markov chain where $0 \le r_1 < N_1$, $r_2, r_3, \ldots, r_k \ge 0$ and $r_1 + r_2 + \ldots + r_k \le N-1$. We may define the following probabilities, which may help to facilitate the expressions for P(r) and P(r; m).

1. $I(n, \lambda, b(x))$ is the probability that there are exactly n arrivals with rate λ and is given by

$$I(n,\lambda,b(x)) = \int_0^\infty \frac{(\lambda x)^n}{n!} e^{-\lambda x} b(x)dx$$
$$= \frac{\lambda^n}{n!}\int_0^\infty x^n e^{-\lambda x} b(x)dx \tag{4}$$

2. $I(\ge n, \lambda, b(x))$ is the probability that there are at least n arrivals with rate λ during service time of which probability density function is b(x) and it is given by

$$I(\ge n,\lambda,b(x)) = 1 - \sum_{r_1=0}^{n-1} I(r_1,\lambda,b(x)) \tag{5}$$

3. $II(n, \Lambda, b(x)$ is the probability that there are exactly r_1, r2, ..., n_k arrivals with rates λ_1, $\lambda_2, \ldots, \lambda_k$ respectively and it may be expressed as

$$II(n,\Lambda) = \int_0^\infty \prod_{i=1}^{K} \frac{(\lambda_i x)^{n_i}}{n_i!} e^{-\lambda_i x} b(x)dx$$

$$= \frac{\prod_{i=1}^{k}\lambda_i^{n_i}\left(\sum_{i=1}^{k} n_i\right)!}{\left(\sum_{i=1}^{k}\lambda_i\right)^{\sum_{i=1}^{k} n_i}\prod_{i=1}^{k} n_i!} I\left(\sum_{i=1}^{k} n_i, \sum_{i=1}^{k}\lambda_i, b(x)\right) \tag{6}$$

4. $II(\ge n, \Lambda, b(x))$ is the probability that there are at least n_1 and exactly n_2, n_3,…, n_k arrivals with rates λ_1, λ_2,…, λ_k respectively. Now

$$II(\ge n,\Lambda,b(x)) = I(n,\Lambda,b(x)) - \sum_{r_1}^{n_1-1} II(r_1, n, \Lambda, b(x)) \tag{7}$$

Also

$$II(n_1, \ge n_2, \ldots, \ge n_k, \Lambda b(x)) =$$
$$II(\ge n_2, \ge n_3, \ge \ldots, \ge n_k, n_1, \lambda_2, \lambda_3, \ldots, \lambda_k, b(x)) \tag{8}$$

$$II(n_1, \ge n_2, \ldots, \ge n_k, \Lambda b(x)) =$$
$$\int_0^\infty \sum_{r_1=n_1}^{\infty}\sum_{r_2=n_2}^{\infty}\cdots\sum_{r_k=n_k}^{\infty}\prod_{i=1}^{k}\frac{(\lambda_i x)^{r_i}}{r_i!}e^{-\lambda_i x}.b(x)dx$$
$$= I(\ge n_2, n_3, \ldots, n_k, \Lambda, b(x)) -$$
$$\sum_{r_1=0}^{n_1-1} II(r_1, \ge n_2, \ge n_3, \ldots, >, n_k, \Lambda b(x)dx) \tag{9}$$

Now For $r_1 = r_2 = \ldots = r_k = 0$, the class of packet which is to be served next depends upon the class form which next packet comes. Since with the probability $\lambda_2 / \sum_{i=1}^{k}\lambda_i$, from class 2 and so on and with the probability $\lambda_k / \sum_{i=1}^{k}\lambda_i$ from class k, so that state transition probability P(o ; m) is given by:

$$p(o;m)=$$

$$\begin{cases} [\beta_1 II(m,\Lambda,b_1(x))+\beta_2 II(m,\Lambda,b_2(x))+\ldots+ \\ \beta_k II(m,\Lambda,b_k(x))] \\ \qquad \text{if } m_i < N_i - 1 \text{ and} \sum_{i=1}^{k} m_i < N-1 \\ [\beta_1 II(m_1,\geq m_2,\ldots,\geq m_k,\Lambda,b_1(x)+ \\ \beta_2 II(m_1,\geq m_2,\ldots,\geq m_k,\Lambda b_2(x))+\ldots+ \\ \beta_k II(m_1 \geq m_2,\ldots,\geq m_k,\Lambda,b_k(x)] \\ \qquad \text{if } m_i < N_i - 1 \text{ and} \sum_{i=1}^{k} m_i < N-1 \\ [\beta_1 II(\geq m_1,m_2,\ldots,m_k,\Lambda,b_1(x)+ \\ \beta_2 II(\geq m_1,m_2,\ldots,m_k,\Lambda,b_2(x)+\ldots+ \end{cases}$$

$$\begin{cases} \beta_k II(\geq m_1,m_2,\ldots,m_k,\Lambda,b_2(x))] \\ \qquad \text{if } m_i = N_i - 1 \text{and} \sum_{i=1}^{k} m_i < N-1 \\ [\beta_1 II(\geq m_1,\geq m_2,\ldots,\geq m_k,\Lambda,b_1(x))+ \\ \beta_2 II(\geq m_1,\geq m_2,\ldots,\geq m_k,\Lambda,b_2(x))+\ldots+ \\ \beta_k II(\geq m_1,\geq m_2,\ldots,\geq m_k,\Lambda,b_k(x))] \\ \qquad \text{if } m_i = N_i - 1 \end{cases}$$

where $i = 1,2,\ldots,k$

(10)

where $\beta_1 = \frac{\lambda_1}{\sum_{i=1}^{K}\lambda_i}; \beta_2 = \frac{\lambda_2}{\sum_{i=1}^{K}\lambda_i}; \ldots; \beta_k = \frac{\lambda_k}{\sum_{i=1}^{K}\lambda_i}$

For $r_1 = r_2 = \ldots = r_{k-1} = 0$ and $t > 0$, in this case packet of class k- is served. Let $\Delta_1 = m_1$, $\Delta_2 = m_2$, …, $\Delta_k = m_k - (r_1 - 1)$. Then we have

$$P(r_i e_i;m)=\begin{cases} 0 & ;\Delta_i < 0 \text{ and } \sum_{i=1}^{k} m_i < N-1 \\ II(\Delta,\Lambda,b_i(x)) & ;m_i < 0 \text{ and } \sum_{i=1}^{k} m_i < N-1 \\ II(\Delta_i,\Delta_{i+1},\ldots,\Delta_{i+j} \geq \Delta_k,\Lambda,b_i(x)) & \\ & ;m_i < N_{i-1} \text{ and} \sum_{i=1}^{k} m_i < N-1 \\ II(\geq \Delta_i,\geq \Delta_{i+1} \geq \Delta_{i+1},\ldots,\Delta_k,\Lambda,b_i(x)) & \\ & ;m_i < N_{i-1} \text{ and} \sum_{i=1}^{k} m_i < N-1 \\ (\geq \Delta_i,\geq \Delta_{i+1} \geq \Delta_{i+1},\ldots,\Delta_k,\Lambda,b_i(x)) & \\ & ;m_i < N_{i-1} \text{ and} \sum_{i=1}^{k} m_i < N-1 \\ (i = 1,2,\ldots,k) & \end{cases} \quad (11)$$

where e: is the vector having 1 at the ith position and zero at the other positions.

For $r_i < 0$; a packet of class i (i = 1, 2, ..., k), will be served with probability $\alpha_i(\gamma)$ and with the assumption that $\Delta_1^1 = m_1 - (r_1 - 1)$ and $\Delta_1^2 = (m_1 - r_1)$, $\Delta_{i+1}^2 = m_{i+1} - (r_{i+1} - 1)$ $(i = 1,2,\ldots,k)$. Here Δ_i^1 indicates the number of changes of classes i packets in a class 1 packet service time, Δ_i^2 denotes the number of changes of class i (i = 1, 2, ..., k) packets in class 2 packet service time and so on. We notice that Δ_i^1, Δ_i^2, Δ_i^3 cannot be negative.

We define

$$\delta(\Delta)=\begin{cases} 0 & \text{If } \Delta_i < 0 \text{ and} \sum_{i=1}^{k} m_i < N-1 \\ 1 & \text{otherwise} \end{cases}$$

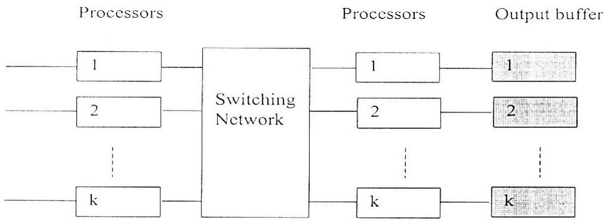

Figure 1. An ATM switching network.

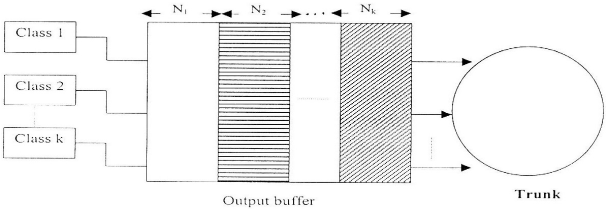

Figure 2. A queuing model.

$$P(r;m) = \begin{cases} \alpha_i(r)\delta(\Delta^1)\Pi(\Delta^1,\Lambda,b_i(x)) + \\ \alpha_i(r)\delta(\Delta^2)\Pi(\Delta^2,\Lambda,b_i(x)) + \\ \alpha_i(r)\delta(\Delta^3)\Pi(\Delta^3,\Lambda,b_i(x)) + \\ \quad \text{for } m_i < N_i - 1 \text{ and } \sum_{i=1}^{k} m_i = N_i - 1 \\ \alpha_i(r)\delta(\Delta^1)\Pi(\Delta_i^1, \geq \Delta_{i+1}^1, \geq \Delta_{i+2}^1, \ldots, \geq \Delta_k^1, \Lambda, b_i(x)) + \\ \alpha_i(r)\delta(\Delta^2)\Pi(\Delta_i^2, \geq \Delta_{i+1}^2, \geq \Delta_{i+2}^2, \ldots, \geq \Delta_k^2, \Lambda, b_i(x)) + \\ \alpha_i(r)\delta(\Delta^3)\Pi(\Delta_i^3, \geq \Delta_{i+1}^3, \geq \Delta_{i+2}^3, \ldots, \geq \Delta_k^3, \Lambda, b_i(x)) + \\ \quad \text{for } m_i < N_i - 1 \text{ and } \sum_{i=1}^{k} m_i = N - 1 \\ \alpha_i(r)\delta(\Delta^1)\Pi(\Delta_i^1, \Delta_{i+1}^1, \ldots, \Delta_k^1, \Lambda, b_i(x)) + \\ \alpha_i(r)\delta(\Delta^2)\Pi(\Delta_i^2, \Delta_{i+1}^2, \ldots, \Delta_k^2, \Lambda, b_i(x)) + \\ \alpha_i(r)\delta(\Delta^3)\Pi(\Delta_i^3, \Delta_{i+1}^3, \ldots, \geq \Delta_k^3, \Lambda, b_i(x)) + \\ \quad \text{for } m_i < N_i - 1 \text{ and } \sum_{i=1}^{k} m_i = N - 1 \\ \alpha_i(r)\delta(\Delta^1)\Pi(\Delta_i^1, \Delta_{i+1}^1, \ldots, \Delta_k^1, \Lambda, b_i(x)) + \\ \alpha_i(r)\delta(\Delta^2)\Pi(\Delta_i^2, \Delta_{i+1}^2, \ldots, \Delta_k^2, \Lambda, b_i(x)) + \\ \alpha_i(r)\delta(\Delta^3)\Pi(\Delta_i^3, \Delta_{i+1}^3, \ldots, \geq \Delta_k^3, \Lambda, b_i(x)) + \\ \quad \text{for } m_i < N_i - 1 \text{ and } \sum_{i=1}^{k} m_i = N - 1 \\ \text{where } i = 1,2,\ldots,k \end{cases} \tag{12}$$

Equation 12 gives complete computation for P(r;m) the steady state probabilities, P (r)obey the law of conservation [12]. So:

$$P(r) = \sum_{\text{all } m_i} P(m).P(r;m);$$

$$r_1 \leq N_1, r_{l+1} \geq 0 \text{ and } \sum_{i=1}^{k} r_i \leq N - 1 \tag{13}$$

and

$$\sum_{i=1}^{k} P(r) = 1 \tag{14}$$

Equation 13 and 14 give the complete computation of P(r).

5. APPLICATIONS OF THE MODEL

Recently: asynchronous transfer mode (ATM) has been increasingly accepted as the basic technology for high-speed packet switching. The model under study has its widespread applications in many areas of computer communication system in modeling and designing process. Loss probabilities of traffic obtained in this research enable queuing theorists as well as practitioners to evaluate performance measures of B-ISDN, ATM, local area network (LAN), wide area network (WAN) in telephony and computer communication networks.

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