RESEARCH NOTE

APPLICATION OF SVC ON LARGE SYNCHRONOUS MOTORS BASED ON LOC DESIGN

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Abstract   This paper describes the application of static var compensators, (SVC) on an electrical distribution network containing two large synchronous motors, one of which is excited via a three-phase thyristor bridge rectifier. The second machine is excited via a diode bridge rectifier. Based on linear optimization control (LOC), the measurable feedback signals are applied to the control system loops of SVC and the excitation control loop of the first synchronous motor. The phase equations method was used to develop a computer program to model the distribution network. Computer results were obtained to demonstrate the system performance for some abnormal modes of operation. These results show that employing SVC based on the LOC design for electrical distribution networks containing large synchronous motors is beneficial and may be considered a first stage of the system design.

Key Words   SVC Application, Synchronous Motor, LOC

چکیده   این مقاله بکارگیری جبرانسازهای استاتیکی SVC بر یک شبکه توزیع که شامل دو موتور سنکرون بزرگ است را شرح می‌دهد. یکی از این ماشینها توسط یک پل سه فاز تایریستوری و ماشین دوم از طریق یک یکسو کننده پل سه فاز دیودی تحریک می‌گردند. بر پایه روش کنترل بهینه سازی خطی LOC سیگنالهای فیدبک قابل اندازه گیری بر حلقه های کنترلی SVC و سیستم تحریک موتور اول اعمال می‌گردند. بر پایه معادلات فازی، یک برنامه کامپیوتری برای شبیه سازی شبکه مذکور نوشته شده است. نتایج کامپیوتری برای بعضی از شرایط کاری غیر نرمال سیستم بدست می‌آید. این نتایج نشان می‌دهند که بکارگیری SVC بر پایه روش طراحی LOC سودمند بوده و حتی لازم است که در مرحله اول طراحی چنین سیستمهایی مورد توجه قرار گیرد.

1. INTRODUCTION

General idea of applying a voltage regulator is to optimally stabilize the equipment voltage during and immediately after the momentary disturbance of supply voltage, so that a satisfactory functioning of the equipment is composed of a motor load group for which operation continuity is to be maintained when subjected to unavoidable momentary sags of the supply voltage, which is of particular interest to continuous process plants [1,2]. The technique of "riding" through voltage dips by keeping motors connected to their sources during a voltage sag can minimize potential harmful effects to the motors and enhancing operation continuity can be further improved by voltage stabilization at the point of service.

Previous researchers have reported about the applications of SVC in large induction motors [1-4]. During last decade applications of SVC in combination with PSS (Power System Stabilizer) to the power systems were reported, too [5-7]. Although, the SVC systems have been applied to the power systems as well as to the distribution
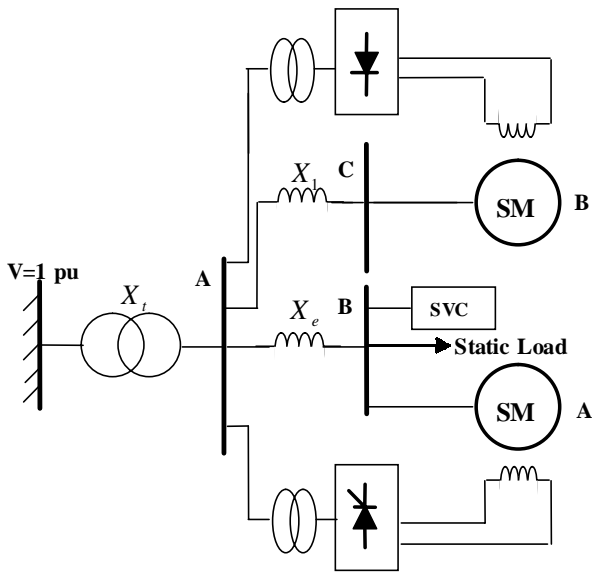
Figure 1. The distribution network.

networks containing large induction motors however, little attention have been paid to applying the SVC compensators on large synchronous motors [8].

The application of synchronous motors in electric drives has the advantage of constant speed operation, and of being capable to operate at a leading power factor. In large sizes, these motors also operate more efficiently than induction motors to maintain operation continuity in the presence of momentary voltage sags [8]. A method has been presented in [8] to determine the minimum capacitive SVC rating necessary for operational continuity of a critical synchronous motor load group. The critical load is driven by a group of synchronous motors for which the dynamic characteristics were justifiably assumed to be co-herent so that they were represented by a single equivalent synchronous motor. In the proposed method, the utility-related voltage disturbances were reflected in the model by abrupt momentary voltage sags at the source end, while the system was operating under steady state rated load conditions. In addition, the well-known equal area criterion was used in order to determine the minimum capacitive SVC rating necessary for the motor to be transiently stable in the first swing. The method described in [8] can only be applied to coherent synchronous motor load groups and not to

the stiffer distribution systems containing large incoherent synchronous motor loads. In addition, the transient and nonlinear nature of the dynamic system was not taken into account. It may be noted that the operational continuity of a critical synchronous motor load group heavily depends not only on its own characteristics but on those of the remaining system as well.

The present paper describes the application of SVC on a typical distribution network containing two large synchronous motors as illustrated in Figure 1. This figure shows that synchronous motor A is excited via a three phase thyristor converter, and motor B via a diode bridge rectifier. A SVC is directly connected to busbar B.

Based on the LOC method, using the reduced order equations of the synchronous machines, the measurable signals are applied to the control system loops of the SVC and excitation control loop of the synchronous motor A. Using the phase equation method, a computer program was written to predict the system dynamic performance, especially during abnormal conditions such as motor starting or shunt or series faults.

Thyristor-switched capacitors (TSCs) are used to survive the immediate effects of the voltage sag (transient stable), although the thyristor-switch reactor (TSRs) may also be added to the TSCs to improve damping of the ensuing system oscillations (oscillatory stable).

In this study, the three single-phase compensators are designed which can be individually used to deal with voltage sags in any of the three phases. It may be noted that three phase voltage sags have the most severe effect on the operation continuity of synchronous motors.

2. NETWORK SYSTEM WITH TWO MOTORS

Consider motor A in a distribution network as in Figure 1. To design the LOC signals for the motor it is assumed that motor B is disconnected from the busbar C. Then motor A is directly connected from the busbar C. then motor A is directly connected to an infinite busbar via an equivalent reactance X_e as shown in Figure 2.
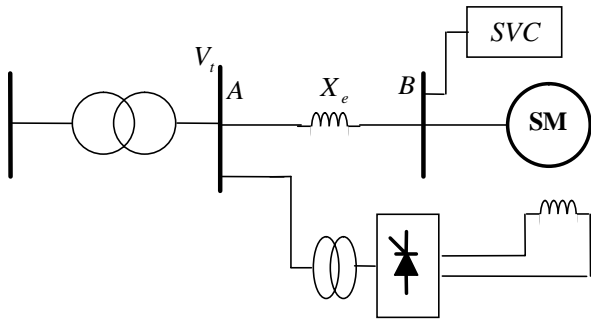
Consider the IEEE SVC model (with an

Figure 2. Power circuit configuration of motor A connected to Busbar A.

integrator block of $\dfrac{k}{s}$ added) and the synchronous machine excitation systems as shown in Figures 3 and 4, neglecting the step-down transformer time constant T. Therefore

$$\dot{E}_{fd} = -\frac{E_{fd}}{T_a} + \frac{k_a}{T_a}(-V_t + V_r + U_E) \qquad (1)$$

$$\Delta \dot{B}_1 = -\frac{\Delta B_1}{T_r} + \frac{k_r}{T_r}(\Delta V_t + U_S), \qquad (2)$$

where E_{fd} = open circuit induced emf of the machine, $\Delta V_t = V_r - V_t$ and B_1 is the SVC capacitive suseptance.

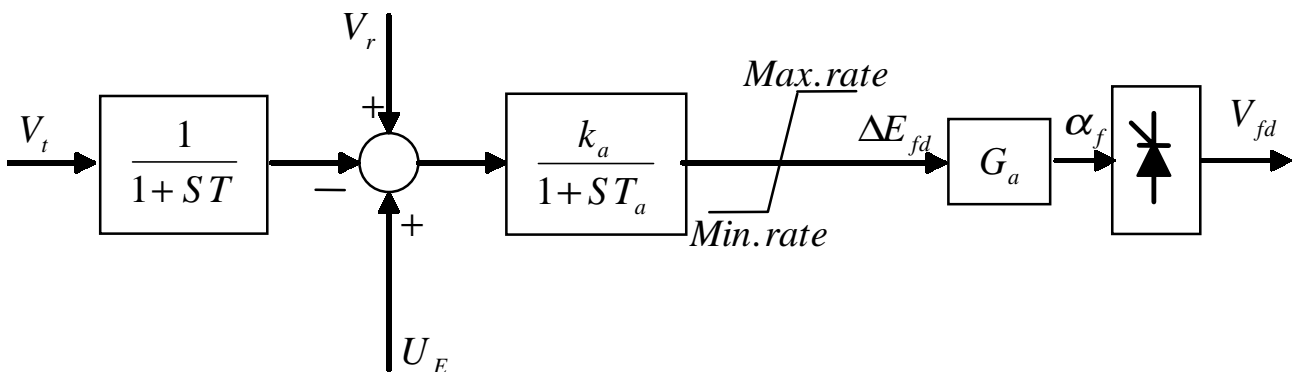   With reference to Appendix I, using the reduced

order equations of the three phase synchronous machines (including torque equation) and Equation 1, the linearized matrix state equations of the system shown in Figure 2, with respect to the initial steady state are

$$\dot{x} = Ax + BU \qquad (3)$$

where A is the state matrix, B is a vector control matrix and

$$x = [\Delta\delta, \Delta\omega, \Delta\psi_{fd}, \Delta\psi_{kd}, \Delta\psi_{kq}, \Delta E_{fd}, \Delta B_1]^T,$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & \dfrac{k_a}{T_a} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \dfrac{k_r}{T_r} \end{bmatrix}^T \qquad (4)$$

Referring to Appendixes II and III, the state variables $\psi_{fd}, \psi_{fd}, \psi_{kq}$ are not measurable and therefore are replaced for measurable variables terminal voltage V_t, developed power P_e, and field current i_{fd}. As a result, the modified matrix state equations is

$$\dot{Z} = FZ + GU \qquad (5)$$

where

$$Z = [\Delta\delta, \Delta\omega, \Delta V_t, \Delta P_e, \Delta i_{fd}, \Delta E_{fd}, \Delta B_1]^T \qquad (6)$$
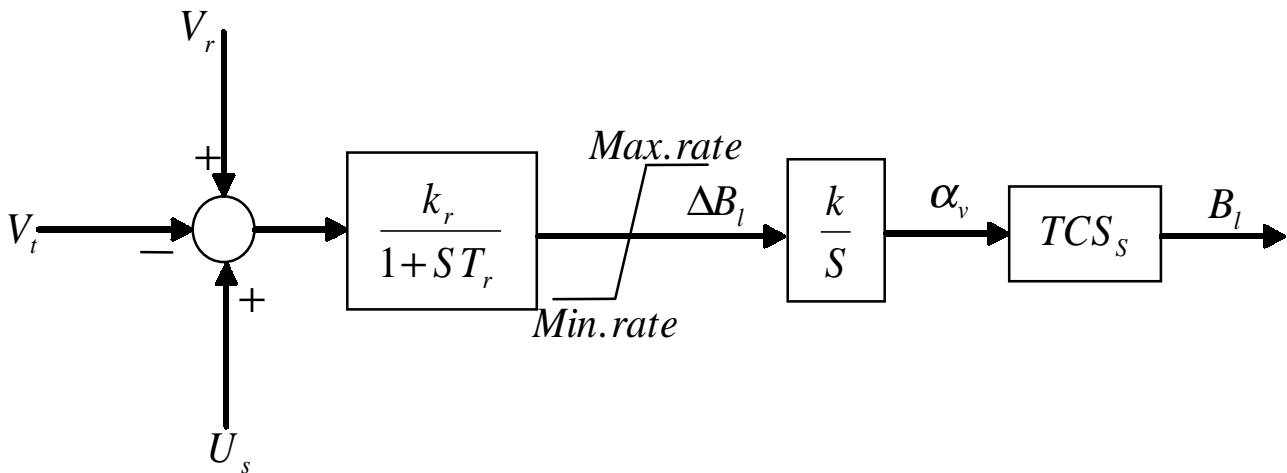

Figure 3. IEEE model of excitation system.

Figure 4. SVC model.

$$Z = MX \qquad (7)$$

$$F = MAM^{-1} \qquad (8)$$

$$G = MB \qquad (9)$$

U is the vector of control effort.

It may be noted that in our study, the LOC design is based on the linearized seventh order equations given in (5). In order to achieve a zero steady state in B_l as well as to reduce the steady state error of busbar voltage on A, an integrator block is used in Figure 4. However, if the LOC damping signals are designed based on the complete linearized matrix state equations of the distribution network then, in that case no need to include the mentioned block in the SVC model.

3. OPTIMAL CONTROL DESIGN

With reference to [9], the main aim of our LOC design is to minimize a performance index or the performance function of the quadratic form given as

$$J = \frac{1}{2}\int_0^\infty (Z^T Q Y + U^T RU)\,dt \qquad (10)$$

where $Q = dig\begin{bmatrix} q_1 & ... & q_7 \end{bmatrix}$ is the weighting

matrix of the state variables deviations Z and R in that of the control effort.

By introducing a co-state variable vector P, the system dynamics Equation 5 and the Cost Function 10 are appended to form the following Hamiltonian generalized co-energy function [9].

$$H = \frac{1}{2}[Z^T Q Z + U^T R U] + P^T [FZ + GU] \quad (11)$$

Applying Pontryagin's maximum principle [9] to Equation 11 and solving the resultant equations for vector U, it gives

$$[N] = \begin{bmatrix} \dot{Z} \\ \dot{P} \end{bmatrix} = \begin{bmatrix} F & -S \\ -Q & F^T \end{bmatrix} \begin{bmatrix} Z \\ P \end{bmatrix} \qquad (12)$$

which are the combined state and co-state equations for the system.

The co-state variable vector P can be related to the state variable vector Z as

$$P = KZ \qquad (13)$$

Where K is called the Riccati matrix, which is a square matrix. Therefore, the solution of P can be found if k is found.

Combining Equation 13 with Equation 12, the Riccati matrix equation is obtained as:

$$F^T K^T + K^T F - K^T S K^T + Q = 0 \qquad K = \text{constant} \tag{14}$$

where S is a symmetric matrix and is given by

$$S = G R^{-1} G^T \tag{15}$$

The Reccati symmetrical matrix K can be calculated from the eigenvectors of the state and co-state system matrix of 14. The eigenvector equations of N may be written in matrix form [9] as

$$[N][X] = [N][\Lambda] = [N] \begin{bmatrix} \Lambda^+ & 0 \\ 0 & \Lambda^- \end{bmatrix} \tag{16}$$

where $\left[\Lambda^+\right]$ represents the eigenvalues of the right hand side and $\left[\Lambda^-\right]$ those on the left hand side of the complex variable plane respectively.

Here $\left[\Lambda\right]$ is a diagonal eigenvalue matrix with 14 elements, and $\left[X\right]$ is a 14 column eigenvector with 14 elements per column.

Let the eigenvector matrix be partitioned into four 7 by 7 matrices such that

$$[N] = \begin{bmatrix} X_I & X_{III} \\ X_{II} & X_{IV} \end{bmatrix} \tag{17}$$

In [9] has been described that $(X_{II} \, X_I^{-1})$ satisfies the Riccati matrix equation of 14, therefore matrix K is

$$K = X_{II} X_I^{-1} \tag{18}$$

In addition, from Equations 12 and 13, the vector of LOC signals is obtained as

$$U = -(R^{-1} G^T K) x \tag{18}$$

With reference to [8], the incremental deviations of damping and synchronizing torque components of a synchronous machine are given as

$$\Delta T_D = K_D \Delta \omega, \qquad \Delta T_S = K_S \Delta \delta \tag{19}$$

where K_D and K_S are the synchronizing and damping torques coefficients.

From 20, it can be seen that in order to have a nearly high damping torque component, one solution may be to choose a nearly high positive value for second element q_2 in diagonal weighting matrix Q. This value is usually obtained by trial and error method. Also in Reference 9, it has been described that the values of other elements in matrix Q can be very small even equal to zero.

4. SYSTEM SIMULATION

Referring to Figure 1, for simplicity, it was assumed that the neutral points of synchronous motors, SVC and infinite busbar are connected to imaginary neutral points of busbars A and B via very large impedance, (theoretical equal to infinity). Having made this assumption, the a,b,c phase equations of the distribution network can easily be written. Based on applying the LOC signals only to motor A, and using the phase equations for sequence ABC, a step by step computer program was developed to model the system of Figure 1. In this simulation, the commutation overlap was considered for both the SVC and excitation systems of motor A.

Typical parameters of the system may be found with reference to [10], in which parameters of a typical distribution network are given as: f=60Hz, $p=2, \cos(\psi) = 0.9$ (leading), and

$$R_s = 0, X_{ls} = 0.05, X_d = 0.0973,$$
$$X_q = 0.55, X_q'' = 0.133, X_d' = 0.19,$$
$$X_d'' = 0.133, X_{lfd} = 0.2049,$$
$$r_{fd} = 0.000041, X_{lkd} = 0.16, V_{t0} = 1,$$
$$r_{kd} = 0.00041, X_{lkq} = 0.01029, r_{kq} = 0.0136,$$
$$X_e = 0.5, R_e = 0.0$$

all in per unit.

$$T_{d0}' = 7.765 \text{ s}, T_{d0}'' = 0.044 \text{ s},$$
$$T_{q0}'' = 0.094 \text{ s}, H = 4.63 \text{ s}$$

Also, the typical parameters of the SVC and

synchronous machine excitation systems of Figures 2 and 3 are given as

$$k_r = 2500, \; T_r = 0.15 \, s, \; K_a = 50 \; \text{and} \; T_a = 0.05 \, s.$$

From Appendixes I and II, using the above parameters, the numerical values of Matrixes A and M were obtained as

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ -25.63 & 0 & -25.01 & -20.16 & -5.67 & 0 & 0 \\ 0.06 & 0 & -.45 & 0.32 & 0.01 & 0.15 & 0 \\ -2.78 & 0 & -18.2 & -23.6 & 0.36 & 0 & 0 \\ -1.72 & 0 & -0.45 & -0.36 & -129 & 0 & 0 \\ 75.1 & 0 & -422 & -340 & 231 & 20 & 0 \\ 11.44*10^3 & 0 & 32313*10^3 & 35.577*10^3 & 4.99*10^3 & 0 & -1667 \end{bmatrix}$$

(24)

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0.686 & 0 & 1.94 & -2.315 & 0.2945 & 0 & 0 \\ 2.009 & 0 & 0.4797 & 0.5727 & 0.0489 & 0 & 0 \\ -3.108 & 0 & -74.985 & 11.45 & -.005 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(25)

Also, from Equations 7 and 8, the numerical values of Matrixes G and F were obtained as

$$G = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 10^3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1666.7 \end{bmatrix}^T$$

(26)

$$F = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & | & 0 \\ 10806 & 0 & -8.62 & -64.06 & -.3 & 0 & | & 0 \\ -43.48 & .686 & -20.7 & 30.35 & -.89 & 0.291 & | & 0 \\ 34.08 & 2.01 & 1.60 & -18.23 & .067 & 0.072 & | & 0 \\ 536 & -3.108 & 62 & -30296 & 1.995 & -11.25 & | & 0 \\ -26541 & 0 & 58516 & 12027 & 28.46 & 20 & | & 0 \\ - & - & - & - & - & - & | & - \\ -143990 & 0 & 43012 & 76158 & 167.56 & 0 & | & -16.67 \end{bmatrix}$$

(27)

For Q and R given by

$$R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \; \text{and} \; Q = \text{diag} \begin{bmatrix} 0 & 10 & 0 & 0 & 0 & 0 & 0.00001 \end{bmatrix}$$

(28)

Matrix Equation 14 was solved for the Riccati Matrix K

$$K = \begin{bmatrix} 24.186 & 3.23 & -1.78 & -11.61 & -0.065 & -0.0188 & -0.0009 \\ 3.23 & 1.04 & -0.346 & -2.005 & -0.0107 & -0.0028 & 0.0001 \\ -1.78 & -0.346 & 0.1819 & 1.057 & 0.0061 & 0.0024 & 0 \\ -11.61 & -2.005 & 1.057 & 6.74 & 0.035 & 0.0121 & 0.0004 \\ -0.065 & -0.0107 & 0.0061 & 0.035 & .0002 & 0.0001 & 0 \\ -0.0188 & -0.0028 & 0.0024 & 0.0121 & 0.0001 & 0.0001 & 0 \\ -0.0009 & 0.0001 & 0 & 0.0004 & 0 & 0 & 0 \end{bmatrix}$$

(25)

Consequently, using Equation 18, the vectors of control signals U_E and U_S were obtained as

$$U = \begin{bmatrix} U_E \\ U_S \end{bmatrix} = \begin{bmatrix} -1.3319 & 5.6571 & 0.7393 & -3.0494 & 0.0001 & 0 & 0 \\ 0.0021 & 0.0779 & 0.0039 & -0.403 & -0.0002 & 0 & -0.0002 \end{bmatrix} [Y]$$

(26)

Computer results were obtained for a few system abnormal conditions, such as starting of motor B while A is in steady state initially, the three and single phase faults on busbar A, occurring at zero second and clearing at 0.07 s and 0.015 s respectively. The results from these simulations are shown in Figures 5-7 respectively. Figures 5, 6 and 7 demonstrate that if damping signals are not applied to the control loop of the SVC as well as the excitation system of synchronous motor A, then both motors would be hunting and also the busbar voltage on A would remain in a transient state. In addition, due to the system low frequency oscillations (for a nearly long duration of time), not only a mechanical damage could cause to the motors shafts but also it could effectively change the system dynamic stability conditions [11].

From Figure 7, a slight steady state error (roughly less than five percent) is seen in the busbar voltage on A. That is because of our LOC design, which is based on using the seventh order linearized Equations 5 and not based really on using the complete linearized equations of the system. However, an integrator block has been used in the SVC model in order to achieve a zero steady state error in SVC state variable B_1 and also to reduce the above-mentioned error.

5. CONCLUSIONS
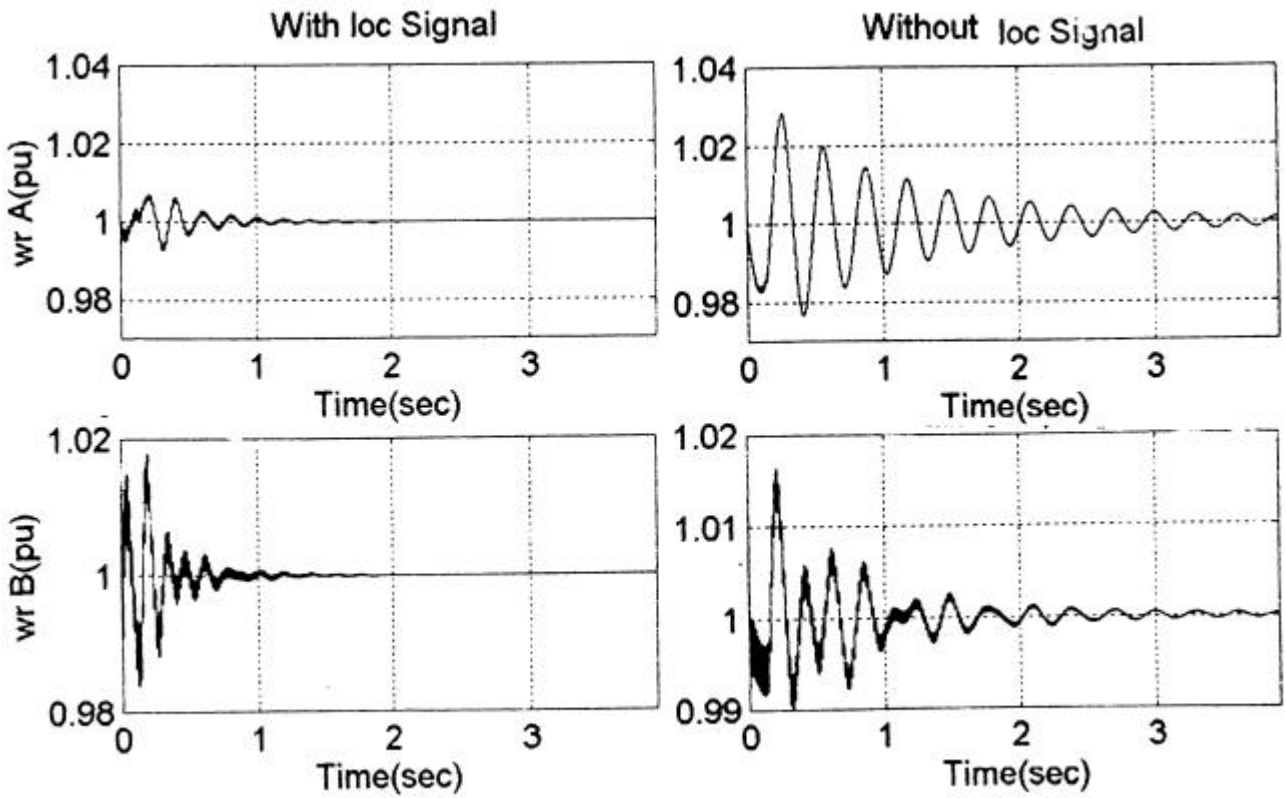
Based on the LOC design, a SVC is applied to a typical

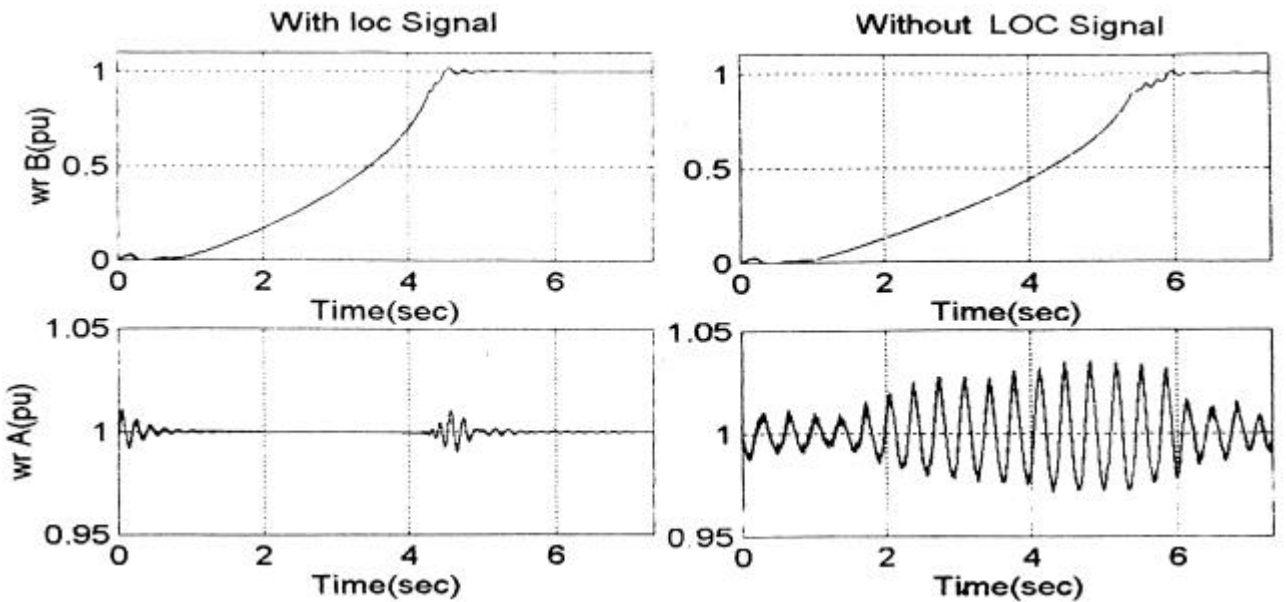**Figure 5**. Phase to earth fault on Busbar A.

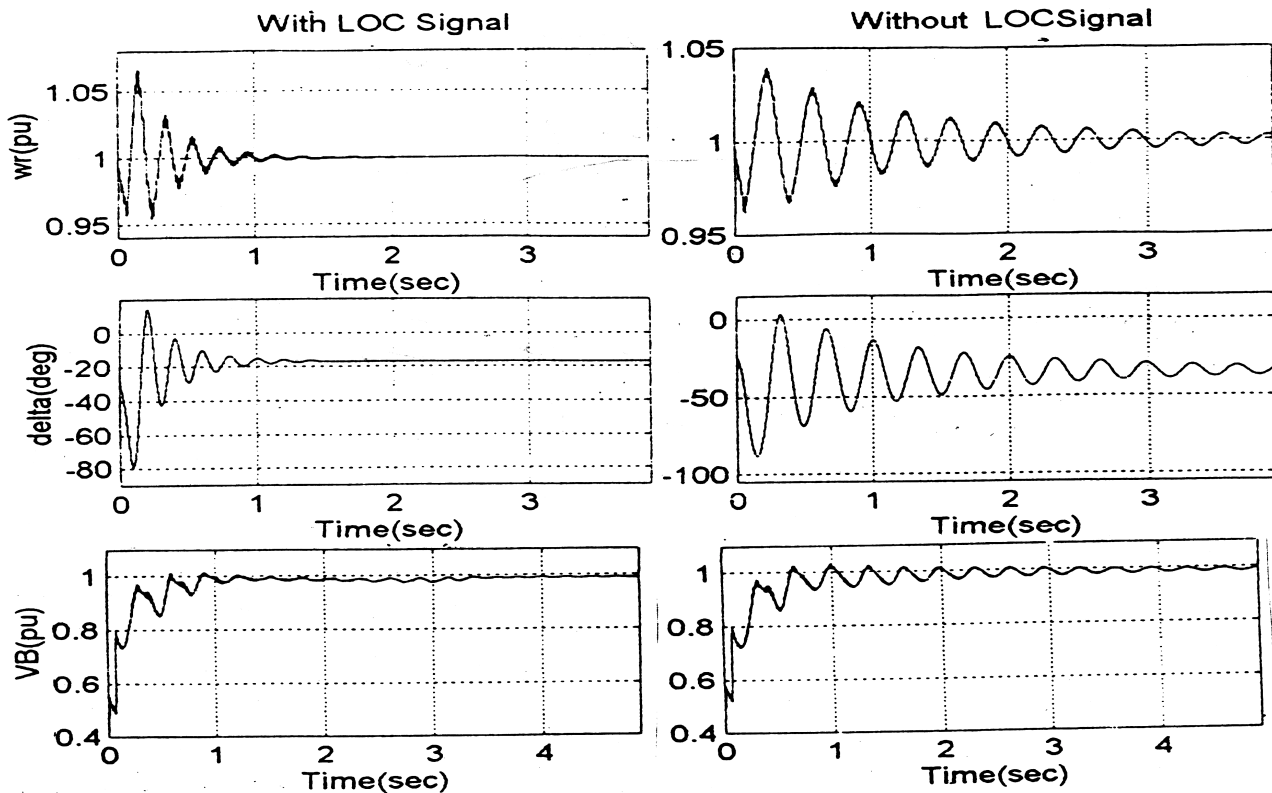**Figure 6**. Starting motor B with motor A in steady state.

Figure 7. Three phase fault on Busbar A.

distribution network containing two incoherent large synchronous motors.

The abc-s phase equations method has been used to model the distribution network and therefore, both types of series and shunt faults can be studied. A transformation matrix has been derived which can be used to transform the machine state variables directly in terms of machine measurable quantities. As a result, the measurable LOC damping feedback signals can be designed for applying to the SVC and synchronous machine excitation systems. The computer results obtained show that for operational continuity of a critical synchronous motor load group during system abnormal condition, applying the damping signals to the SVC and motor excitation systems is beneficial and therefore this should be considered a first stage of the system design. It was also found that based on the LOC design, the minimum capacitive rating required could easily amount to the same order of the critical load. If the feedback damping signals are not applied to the distribution

system, this rating is considerably increased and may provide a cost effective solution or even none feasible technical solution. In addition, because of the low frequency oscillations existing in the system nearly for a long time, therefore, both the motors would be hunting and as a result a mechanical damage could cause to the motors' shafts. Furthermore, these oscillations could also change the system dynamic stability conditions. We also applied a SVC and a SVC in combination with a PSS to the distribution network of Figure 1. Comparing to the method described in this paper, the computer results obtained by these methods has effectively shown the less damping effects on the system low frequency oscillations.

APPENDIX I

From Reference 12, the reduced order equations of the three phase salient pole synchronous machines (including torque equation) are

$$v_{ds} = -\psi_{qs} + R_s\,i_{ds}, \quad v_{qs} = \psi_{ds} + R_s\,i_{qs},$$

$$v_{fd} = \frac{1}{\omega_b}\dot{\psi}_{fd} + R_{fd}\,i_{fd},$$

$$0 = \frac{1}{\omega_b}\dot{\psi}_{kd} + R_{kd}\,i_{kd}, \quad 0 = \frac{1}{\omega_b}\dot{\psi}_{kd} + R_{kd}\,i_{kd}$$

$$\tag{27}$$

$$\dot{\delta} = \omega_r - \omega_0,$$

$$\dot{\omega}_r = \frac{\omega_b}{2H}(T_L - T_e),$$

$$T_e = \psi_{qs}\,i_{ds} - \psi_{ds}\,i_{qs}$$

$$\tag{28}$$

$$\psi_{ds} = X_{ls}\,i_{qs} + \psi_{mq},$$

$$\psi_{fd} = X_{lfd}\,i_{fd} + \psi_{md},$$

$$\psi_{kd} = X_{lkd}\,i_{kd} + \psi_{md},$$

$$\psi_{md} = X_{md}\,(i_{ds} + i_{fd} + i_{kd})$$

$$\tag{29}$$

$$\psi_{kq} = X_{lkq}\,i_{kq} + \psi_{mq},$$

$$\psi_{qs} = X_{ls}\,i_{ds} + \psi_{md}, \quad \psi_{mq} = X_{mq}\,(i_{qs} + i_{kq})$$

$$\tag{30}$$

where

v_{ds}, i_{ds} = direct axis stator voltage and current

v_{qs} = quadrature axis stator voltage and current

ψ_{fd} = field winding flux linkage

ψ_{kd} = direct axis dumper winding flux linkage

ψ_{kq} = quadrature axis damper winding flux linkage

i_{fd}, i_{kd}, i_{kq} = rotor winding currents

δ = torque angle

ω = rotational angular speed

ω_b = synchronous base angular speed

ω_0 = synchronous angular speed

R_s = stator resistance

X_{ls} = stator leakage inductance

X_{md} = direct axis magnetization inductance

E_{fd} = transformed field voltage

Derivation of X-Model of a Three-Phase Synchronous Motor Connected to an Infinite Busbar Linearization of Equations 1, 2, 3 and 29 gives

$$\dot{x} = \tilde{A}x + \tilde{B}\,\Delta I + BU \tag{31}$$

where

$$X = [\Delta\delta, \Delta\omega, \Delta\psi_{fd}, \Delta\psi_{kd}, \Delta\psi_{kq}, \Delta E_{fd}]$$

$$\Delta I = \begin{bmatrix} \Delta i_{ds} \\ \Delta i_{qs} \end{bmatrix} \tag{32}$$

The results is expressed in Park's parameters which are more familiar to engineers [10]

$$\tilde{A} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \tilde{a}_{23} & \tilde{a}_{24} & \tilde{a}_{25} & 0 & 0 \\ 0 & 0 & \tilde{a}_{33} & \tilde{a}_{34} & \tilde{a}_{35} & \tilde{a}_{36} & \tilde{a}_{37} \\ 0 & 0 & \tilde{a}_{43} & \tilde{a}_{44} & 0 & 0 & 0 \\ 0 & 0 & \tilde{a}_{53} & \tilde{a}_{54} & 0 & 0 & 0 \\ 0 & 0 & \tilde{a}_{63} & \tilde{a}_{64} & \tilde{a}_{65} & \tilde{a}_{66} & \tilde{a}_{67} \\ 0 & 0 & \tilde{a}_{73} & \tilde{a}_{74} & \tilde{a}_{75} & \tilde{a}_{76} & \tilde{a}_{77} \end{bmatrix} \tag{33}$$

$$S_1 = \frac{\omega_0}{2H}, \qquad S_2 = \frac{(X_d - X_{ls})(X_d'' - X_{ls})}{(X_d - X_{ls})(X_d' - X_{ls})},$$

$$S_3 = \frac{(X_d' - X_d'')}{(X_d' - X_{ls})}, \quad S_4 = \frac{(X_q - X_q'')}{(X_q - X_{ls})}$$

$$\tilde{a}_{23} = S_1 S_2 I_{q0}, \qquad \tilde{a}_{24} = S_1 S_3 I_{q0}, \qquad \tilde{a}_{25} = S_1 S_4 I_{d0}$$

$$\tag{34}$$

$$\tilde{a}_{33} = -\frac{1}{T_{d0}'}[1 + \frac{S_3(X_d - X_d')}{X_d' - X_{ls}}],$$

$$\tilde{a}_{34} = -\frac{S_3(X_d - X_{ls})}{T_{d0}'(X_d' - X_{ls})}, \tag{35}$$

$$\tilde{a}_{43} = -\frac{1}{T_{d0}''}$$

$$\tilde{a}_{55} = -\frac{1}{T''_{q0}}, \qquad \tilde{a}_{63} = -\frac{k_a S_2 V_{q0}}{V_{t0}.T_a},$$

$$\tilde{a}_{73} = \tilde{a}_{63} K_T, \qquad \tilde{a}_{64} = \frac{k_a S_3 V_{q0}}{V_{t0}.T_a}, \qquad \tilde{a}_{74} = \tilde{a}_{64} K_T$$

$$\tilde{a}_{65} = k_a S_4 V_{d0} /(V_{t0}.T_a), \qquad \tilde{a}_{75} = \tilde{a}_{65} K_T,$$

$$\tilde{a}_{66} = -\frac{1}{T_a}, \qquad \tilde{a}_{76} = 0, \qquad \tilde{a}_{77} = -\frac{1}{T_r}$$

$$(36)$$

where

$$K_T = \frac{T_a}{K_a} \frac{K_r}{T_r} \qquad (37)$$

Also

$$\tilde{B} = \begin{bmatrix} 0 & \tilde{b}_{21} & \tilde{b}_{31} & \tilde{b}_{41} & 0 & \tilde{b}_{61} & \tilde{b}_{71} \\ 0 & \tilde{b}_{22} & 0 & 0 & \tilde{b}_{52} & \tilde{b}_{62} & \tilde{b}_{72} \end{bmatrix}^T$$

$$(38)$$

where

$$\tilde{b}_{21} = S_1[(X''_q - X''_d)I_{q0} - S_4 I_{q0} X_{mq},$$

$$\tilde{b}_{22} = S_1[(X''_q - X''_d)I_{d0} + S_2 X_{md}(I_{d0} + I_{q0})$$
$$\qquad\qquad + X_{md} I_{d0} S_3],$$

$$\tilde{b}_{31} = (X_d - X_{ls})\frac{(X''_d - X_{ls})}{T''_{d0}},$$

$$\tilde{b}_{41} = \frac{(X''_d - X_{ls})}{T''_{d0}}, \qquad \tilde{b}_{52} = \frac{(X_q - X_{ls})}{T''_{q0}},$$

$$\tilde{b}_{61} = k_a \frac{[R_s V_{d0} + X''_q V_{q0}]}{V_{t0} T_a}, \qquad \tilde{b}_{71} = \tilde{b}_{61} K_T$$

$$\tilde{b}_{62} = \tilde{b}_{72} = -k_a \frac{[-R_s V_{q0} + X''_q V_{d0}]}{V_{t0} T_a},$$

$$\tilde{b}_{72} = \tilde{b}_{62} K_T$$

$$(39)$$

In above Equations, (X'_d, X''_d, X''_q) and $(T'_{d0}, T''_{d0}, T''_{q0})$ are the standard d axis and q axis
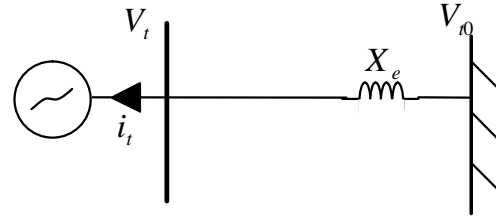

Figure 8. A three phase synchronous motor connected to an infinite power system.

transient and subtransient reactances and time constants of a three-phase salient pole synchronous machine [12].

Following procedure may be followed from Park's equations [6], in order to eliminate the current vector ΔI in Equation 28.

$$\Delta V_{ds} - R_s \Delta i_{ds} + X''_q \Delta i_{qs} = -S_4 \Delta\varphi_{kq}$$
$$\Delta V_{qs} - R_s \Delta i_{qs} - X''_d \Delta i_{ds} = -S_3 \Delta\varphi_{kq} + S_2 \varphi_{fd}$$

$$(40)$$

From Figure 8, the steady state Equations for the external system are

$$\Delta V_{ds} = -X_c \Delta i_{qs} - V_0 \sin(\delta_0)\Delta\delta$$
$$\Delta V_{qs} = -X_c \Delta i_{ds} + V_0 \cos(\delta_0)\Delta\delta$$

$$(41)$$

Also in Equations 33-38, the parameters with subscript zero denote the motor A initial steady state condition.

From Equations 39 and 40

$$[\Delta I] = \begin{bmatrix} \Delta i_{ds} \\ \Delta i_{qs} \end{bmatrix} = [C][X] \qquad (42)$$

where

$$C = \begin{bmatrix} k_3 k_2 + k_4 k_2 & 0 & k_4 s_2 & k_4 s_3 & -k_5 s_4 & 0 \\ k_5 k_1 + k_3 k_2 & 0 & k_3 s_2 & k_3 s_3 & -k_5 s_4 & 0 \end{bmatrix}$$

$$(43)$$

$$k_1 = V_0 \cos(\delta_0), \quad k_2 = V_0 \sin(\delta_0),$$
$$k_3 = \frac{R_s}{D}, k_4 = \frac{X''_q + X_e}{D}, k_5 = -(X_e + X''_d)/D$$

$$D = R_s^2 - (X_e + X_d'')(X_e + X_q'') \tag{44}$$

Combining Equation 27 with Equation 40, it gives

$$\dot{X} = AX + BU \tag{45}$$

where

$$A = \tilde{A} + \tilde{B}C \tag{46}$$

APPENDIX III

Derivation of Transformation Matrix M   The vector X can be expressed in terms of directly measurable machine terminal equations as

Terminal Voltage

$$\Delta V_t = \frac{V_{d0}}{V_{t0}} \Delta V_{ds} + \frac{V_{q0}}{V_{t0}} \Delta V_{qs} \tag{47}$$

From Equations 39 and 40

$$\Delta V_t = m_{31}\Delta\delta + m_{33}\Delta\varphi_{fd} + m_{34}\Delta\varphi_{kd} + m_{35}\Delta\varphi_{kq} \tag{48}$$

where

$$m_{31} = \frac{[c_{21}(V_{d0}X_q'' - V_{q0}R_s) - c_{11}(V_{d0}R_s + V_{q0}X_d'')]}{V_{t0}}$$

$$m_{33} = \frac{[c_{23}(V_{d0}X_q'' - V_{q0}R_s) - c_{11}(V_{d0}R_s + V_{q0}X_d'')]}{V_{t0}}$$

$$m_{34} = [c_{24}(V_{d0}X_q'' - V_{q0}R_s)$$
$$+ \frac{V_{q0}}{K_3} - c_{14}(V_{d0}R_s + V_{q0}X_d'')/V_{t0}]$$

$$m_{35} = [c_{25}(V_{d0}X_q'' - V_{q0}R_s)$$
$$- c_{15}(V_{d0}R_s + V_{q0}X_d'' - \frac{V_{d0}}{K_3})/V_{t0}]$$
$$\tag{49}$$

Electrical Power   From Figure 8, the incremental

deviation of electric power ΔP_e is

$$\Delta P_e = V_{d0}\Delta i_{ds} + V_{q0}\Delta i_{qs} + I_{d0}\Delta V_{ds} + I_{q0}\Delta V_{qs} \tag{50}$$

Again from Equations 40 and 41

$$\Delta P_e = m_{41}\Delta\delta + m_{43}\Delta\varphi_{fd} + m_{44}\Delta\varphi_{kd} + m_{45}\Delta\varphi_{kq} \tag{51}$$

where

$$m_{41} = S_6 V_{d0} + S_5 V_{q0} - I_{d0}(S_5 X'' - S_6 R_s)$$
$$+ I_{q0}(S_6 X_d'' + S_5 R_s)$$
$$m_{43} = S_2[K_4 V_{d0} + K_3 V_{q0} + I_{d0}(K_3 X_q'' - K_3 R_s)$$
$$+ I_{q0}(1 - K_3 X_{d0}' - K_3 R_s)]$$
$$m_{44} = m_{43}\frac{S_3}{S_2},$$
$$m_{45} = S_4[-I_{q0}(K_3 X_d'' + K_5 R_s)$$
$$+ I_{d0}(1 + K_5 X_q'' - K_3 R_s) - (K_3 V_{d0} + K_5 V_{q0})]$$
$$\tag{52}$$

$$S_5 = K_1 K_5 + K_2 K_3, \quad S_6 = K_1 K_3 + K_2 K_4 \tag{53}$$

Field Current   Consider the linearized form of Equation 25

$$\Delta\varphi_{fd} = X_{lfd}\Delta i_{fd} + X_{md}(\Delta i_{ds} + \Delta i_{kd} + \Delta i_{fd}) \tag{54}$$

$$\Delta\varphi_{kd} = X_{lkd}\Delta i_{kd} + X_{md}(\Delta i_{ds} + \Delta i_{kd} + \Delta i_{fd}) \tag{55}$$

Solving Equations 53 and 54 for Δi_{fd} and Δi_{kd} in terms of $\Delta\varphi_{fd}$, $\Delta\varphi_{kd}$ and Δi_{ds}, it gives

$$\Delta i_{fd} = \frac{X_{kd}}{(X_{fd}X_{kd} - X_{md}^2)}\Delta\varphi_{fd} - \frac{X_{md}}{X_{fd}X_{kd}X_{md}^2}\Delta\varphi_{kd}$$
$$+ \frac{X_{md}(X_{md} - X_{kd})}{X_{fd}X_{xd} - X_{md}^2}\Delta i_{ds}$$
$$\tag{56}$$

Substituting for Δi_{ds} from Equation 40 into Equation 54, it follows that

$$\Delta i_{fd} = m_{51}\,\Delta\delta + m_{53}\Delta\varphi_{fd} + m_{54}\,\Delta\varphi_{kd} + m_{55}\,\Delta\varphi_{kq}$$
$$(57)$$

where

$$m_{51} = c_{11}\,S_2\,X_{md},\ m_{55} = c_{15}\,S_2\,X_{md}$$
$$m_{53} = c_{13}\,S_3\,X_{md}$$
$$+ \frac{(X_d - X_d')}{X_{md}}[1 + \frac{(X_d - X_d')(X_d' - X_d'')}{(X_d' - X_{ls})}]$$
$$m_{54} = c_{14}\,S_2\,X_{md} + \frac{(X_d - X_d')(X_d' - X_d'')}{(X_d' - X_{ls})^2}$$
$$m_{55} = c_{15}S_2(X_d - X_{ls})$$
$$(58)$$

As a result, the transformation matrix M is

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ m_{31} & 0 & m_{33} & m_{34} & m_{35} & 0 & 0 \\ m_{41} & 0 & m_{43} & m_{44} & m_{45} & 0 & 0 \\ m_{51} & 0 & m_{53} & m_{54} & m_{55} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad (59)$$

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