RESEARCH NOTE

ON THE PERFORMANCE OF DIFFERENT EMPIRICAL LOSS EQUATIONS FOR FLOW THROUGH COARSE POROUS MEDIA

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Abstract In this paper, the empirical equations that estimate hydraulic parameters for non-linear flow through coarse porous media are evaluated using a series of independent data collected in the laboratory. In this regard, three different relatively uniform soils ranging in size from 8.5 to 27.6 mm have been selected and three random samples drawn from each material. The physical characteristics such as size distribution, porosity, and shape factor have been measured for the 9 samples. In total, 9 permeameter tests have been conducted on the samples to create a set of reliable hydraulic gradient vs. bulk velocity data. Statistical measures have been used to compare the permeameter data with those predicted by the empirical equations. The study shows that McCorquodale et al. and Stephenson equations, which some subjective parameters related to the surface characters of the material, have been incorporated in their structures, can give good results. Also, equation developed by Adel who considers d_{15} as characteristic size of the media, shows acceptable performance. Other equations either underestimate or overestimate hydraulic gradient based on experimental work and analysis conducted in this study.

Key Words Rockfill, Coarse Porous Media, Non-linear Flow, Non-Darcy Flow, Forchheimer Equation, Missbach Equation

چکیده دراین مقاله، معادلات تجربی، که پارامترهای هیدرولیکی برای جریان غیر خطی در محیط متخلخل درشت دانه را تخمین می‌زنند، با استفاده از یک مجموعه اطلاعات آزمایشگاهی مستقل مورد ارزیابی قرار می‌گیرند. در این رابطه، سه مصالح نسبتا یکنواخت با اندازه های متوسط ۸/۵ تا ۲۷/٦ میلیمتر انتخاب گردیده است و از هریک از مصالح سه نمونه تصادفی برداشت شده است و خصوصیات فیزیکی نمونه ها نظیر توزیع دانه بندی، تخلخل و ضریب شکل برای هر ۹ نمونه اندازه گیری گردیده اند. در مجموع ۹ آزمایش نفوذ پذیری با دستگاه اندازه گیری نفوذ پذیری مصالح درشت دانه انجام شده است تا یک مجموعه اطلاعات گرادیان هیدرولیکی – سرعت قابل اعتماد حاصل شود. نتایج آزمایشهای نفوذ پذیری با نتایج حاصل از اعمال خصوصیات فیزیکی بر معادلات تجربی مقایسه شده اند و در این مقایسه معیارهای آماری به خدمت گرفته شده اند . این مطالعه نشان می‌دهد که معادلات مک کورکادل و همکاران و معادله استیفنسن، که اطلاعات مربوط به زبری و شکل دانه ها در آنها ملحوظ شده است، می‌توانند به نتایج خوبی منجر شوند. همچنین، معادله عادل، که d_{15} مصالح را به عنوان اندازه مشخصه محیط در نظر می‌گیرد، عملکرد قابل قبولی دارد. براساس این تحلیل، معادلات دیگر، گرادیان هیدرولیکی را به صورت بالا دست یا پایین دست تخمین می‌زنند.

1. INTRODUCTION

A commonly used method to obtain hydraulic parameters for non-linear flow through rockfill is the use of empirical relations based on physical properties of the media. Although the research in this area has been extensive and several equations have been developed in this regard, there is no general agreement on one specific equation. The results produced by these equations can be quite different from one equation to another equation when they are applied to a specific medium even under controlled laboratory conditions [1]. Hansen et al. [1] compared these equations and by observing simulated hydraulic gradient-bulk velocity curves against the measured curves stated that the Wilkins equation can act as good as other equations. However, they preferred to use the Wilkins equation in their study because it is traditionally used in Canada for the analysis of flow through waste rock dumps.

In this study nine main equations in the literature that some of them produced reasonable results for Hansen et al.'s study [1] is examined using the independent permeameter data collected by the author. The method of examination is different from that of Hansen et al. The difference is that overall statistical criteria are used here to examine the behavior of the equations in comparison with permeameter results.

2. MATHEMATICAL DESCRIPTION OF NON-LINEAR FLOW

The first theories developed to account for non-linear effects in porous media are models more or less intuitive and empirical in nature. The first equation to account for non-linear effects was proposed by Forchheimer [2,3] who suggested the following one-dimensional forms:

$$i = aV + bV^2 \tag{1}$$

$$i = aV + bV^2 + cV^3 \tag{2}$$

where i is hydraulic gradient, V is bulk velocity and a, b and c are constants. Although Forchheimer used the third term in Equation 2 to make the equation fit to experimental data, Equation 1, in quadratic form, is the generally accepted Forchheimer equation.

Another commonly used non-linear equation, in power form, is the Missbach equation [3]:

$$i = lV^{\lambda} \tag{3}$$

where l and λ are constants which depend on media and fluid properties and λ is a variable between 1 and 2 and changes from case to case. Although Equation 3 has been widely used in the literature, there is no theoretical basis for it. Its advantage is that has only one term and thus better suits the analytical solution of some field problems. Another approach is the concept of introducing a friction factor for the porous media that can be obtained from a friction factor-Reynolds number diagram similar to the Moody diagram for pipe flow. This approach results in an equation similar to Equation 1.

Equations 1 and 3, with unspecified parameters, are the equations most commonly used in the literature to relate hydraulic gradient and bulk velocity under non-linear flow conditions. Parallel to research on theoretical explanation of non-linear flow [4,5], extensive research has also been done to relate the coefficients in these equations to fluid and porous media properties. Bear [2] and Hansen et al. [1] give good review of different non-linear equations presented to the literature. The equations considered in this study are: Ergun equation [6], McCorquodale et al. equation [7], Stephenson equation [8], Adel equation [9], Gent equation [10], Wilkins equation [11], Martins equation [12] and two new equations developed by Li et al. [10]. The equations below (Equations 4, 5 and 6) illustrate the coefficient determinations in three of them taken as examples. The first two equations have a quadratic form and follow a Forchheimer-type constitutive relationship, while the third equation has a power form. A detailed review of the type and structure of the other equations can be found in reference [13] and other related references [1,6-12].

Gent (1991):

$$i = \frac{1207.06\nu(1-n)^2}{gn^3d^2}V + \frac{1.209(1-n)}{gn^3d}V^2 \tag{4}$$

Li et al. [10]:

$$i = \frac{441\nu r_e^2(1-n)^2}{gn^3d^2}V + \frac{2.3r_e(1-n)}{gn^3d}V^2 \tag{5}$$

Li et al. [10]:

For low Reynolds numbers, i.e. $Re = \frac{Vm}{\nu n} \le 200$

$$i = \frac{8.9\nu^{0.17}(1-n)^{1.17}r_e^{1.17}}{d^{1.17}gn^3}V^{1.83} \tag{6a}$$

For high Reynolds number and fully turbulent flow, i.e. $Re = \frac{Vm}{\nu n} > 200$

$$i = \frac{2.36(1-n)r_e}{gn^3 d} V^2 \qquad (6b)$$

In Equations 4 to 6, i is hydraulic gradient, V is bulk velocity, n is porosity, ν is cinematic viscosity, g is gravitational acceleration, d is harmonic mean particle size, and r_e = relative surface area efficiency, a coefficient that accounts for the deviation from a smooth spherical shape [11] (= 1 for sphere, ≈ 1.6 for crushed limestone, up to 2 for crushed rock); m = mean hydraulic radius which is defined as: $m = ed/6r_e$ where e is void ratio, d is particle diameter which can be calculated as harmonic mean according to hydraulic radius theory [14].

Although for brevity the structure of all equations are not given here, from the foregoing presentation, it can be concluded that investigators have included different parameters in their equations and that there is no unique, acceptable, non-linear equation in the literature, which can be applied to all field conditions. Each equation is only representative of a set of data obtained from experiments conducted on some materials under some specific circumstances. Therefore, this paper examines the accuracy of the equations using an independent data set.

3. EXPERIMENTAL WORK

In the following, the key physical properties of different materials, the apparatus used, and the experimental procedures applied to measure these properties are briefly introduced. Complete description of the experimental work is found in reference [15].

Three types of materials selected in this study were different in size ranging from 8.5 mm to 27.6 mm. Materials were obtained from a sand and gravel quarry. Although each material had been mechanically sorted in the quarry, it was washed and completely mixed in the laboratory to produce a media as uniform as possible. Three samples were randomly drawn from each material. Each sample was large enough to fill the permeameter (about 25 kg). For each sample, the size distribution, particle density, porosity and shape factor were determined. The porosity considered for each of the three samples was the in-situ porosity measured in the permeameter. To estimate the shape factor, three major axes were measured for the particles using a digital calliper and the average axes lengths calculated for each sample. Shape factors (SF) were estimated using the relationship $SF = c^* / \sqrt{a^* b^*}$ where a^* is length in longest direction and b^*, c^* are lengths measured in mutually perpendicular medium and short directions, respectively. Table 1 summarises the properties of the three samples randomly drawn from each of the three materials.

TABLE 1. Material Properties.

Material	d_{50} (mm)	d_{15} (mm)	Coef. of Uniformity (-)	Coef. of Concavity (-)	Particle Density (g/cm^3)	Porosity (-)	Shape Factor (-)
Small							
(1)	8.7	6.1	1.63	1.06	2.76	0.477	0.49
(2)	8.5	6.0	1.61	1.03	2.76	0.483	0.49
(3)	8.5	6.0	1.61	1.06	2.74	0.489	0.42
Medium							
(1)	21.2	16.7	1.44	1.14	2.75	0.456	0.52
(2)	21.0	16.2	1.46	1.13	2.69	0.458	0.54
(3)	21.1	16.3	1.46	1.13	2.70	0.459	0.54
Large							
(1)	27.4	21.7	1.38	1.04	2.62	0.443	0.47
(2)	25.6	20.7	1.36	0.96	2.63	0.443	0.56
(3)	27.6	21.6	1.41	1.05	2.54	0.443	0.48

Most parameters related to the equations discussed in Section 2 are reported Table 1. However, some of the empirical equations include parameters that are mainly selected by engineering judgment considering the recommendations made by the developers of the equations. For example, r_e in Li et al. equations was selected = 1.4 for medium and large material and = 2.0 for small material based on shape and surface characters of the materials.

For each material, the three random samples were tested in the permeameter. The permeameter was made of a vertical PVC pipe 1.0 m long and 152 mm inside diameter containing the media and head losses were determined using two piezometer taps 717 mm apart. Discharge was measured using a 38-mm diameter orifice in the supply line. At least 17 discharges were tested for each sample and totally 248 hydraulic gradient-bulk velocity data were collected for all samples [15].

4. DATA ANALYSIS

In this section, the physical properties associated with each empirical equation, reported in Section 3, are applied to the equations to find the hydraulic gradients for all velocity values corresponding to the permeameter tests. The simulated hydraulic gradients resulting from different equations are then compared with the corresponding observed values to evaluate the performance of the equations.

Figure 1 shows i (simulated) vs. i (observed) data for McCorquodale et al. equation as one of the equations which their results are in agreement with experimental data. Full line in the graph shows the best-fit line which is obtained by applying the least square method to the data. This line shows the overall prediction trend of the equation. In an ideal situation the slope of this line should be one while the intercept is zero. In other words, it should correspond to the line of perfect agreement shown by dashed line in the Figure. The data points and line are for nine samples resulted from three different materials.

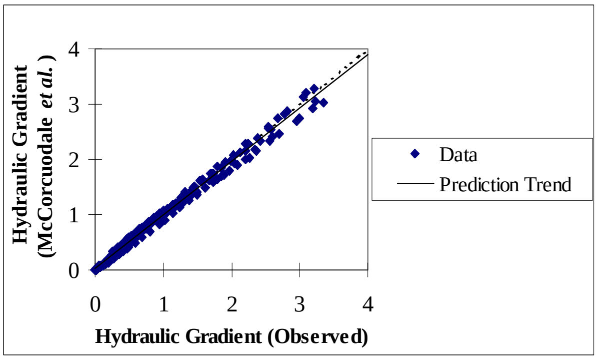

Figure 1. Simulated vs. Observed Hydraulic Gradients for McCorquodale et al. equation.

In addition to this method of analysis, the mean absolute percentage error was also calculated for all equations. Table 2 shows the summary of the results for all equations.

TABLE 2. Examination of Different Non-linear Loss Equations.

Equation	Slope of Prediction Line	Mean Absolute Percentage Error (%)	Overall Prediction Trend
Ergun equation	0.635	32	Underestimate
McCorquodale *et al.* equation	0.978	11	Underestimate
Stephenson equation	1.035	9	Overestimate
Adel equation	0.948	12	Underestimate
Gent equation	0.565	34	Underestimate
Li *et al.* equation (Equation 5)	1.531	53	Overestimate
Wilkins equation	1.218	28	Overestimate
Martins equation	0.530	48	Underestimate
Li *et al.* equation (Equation 6)	1.524	45	Overestimate

5. CONCLUSIONS

From Table 2 and Figure 1, the following conclusions can be made:

1. McCorquodale et al. and Stephenson equations which some subjective parameters related to the surface characters of the material have been incorporated in their structures, can give good results. Following the recommendations made by the developers of the equations can make a reasonable estimation of these subjective parameters. McCorquodale et al. equation is more computationally intensive and needs adjustment of a and b based on Reynolds number. This equation is especially recommended for experimental work in the laboratory.

2. In Adel equation, d_{15} is considered as the characteristic size of the domain and no effect of shape has been incorporated in its structure. However, its result is quite comparable with those of McCorquodale et al. and Stephenson equations. Equations such as this are useful in the presence of the size distribution and absence of any idea about the surface characters of the materials.

3. Ergun, Gent, and Martins equations underestimate the hydraulic gradient based on the data analysis conducted in this study. Some parts of this conclusion agree with the study conducted by Hansen et al. [1].

4. Wilkins equation and the equations developed by Li et al. tend to overestimate hydraulic gradient. Li et al. consider their equations as general equations and include large amount of data in their analysis. They exclude situations where the contact surface between the particles is high. However, they do not give any criterion in this regard. The materials selected in this study were not flaky in shape, and other reasons should be found for poor performance of these equations.

Although some agreement is found between the conclusions made in this study with those made by Hansen et al. [1], these studies do not include all situations and therefore should not be considered absolute, as absolute knowledge is not attainable for open systems. Therefore, the modeller should be aware of the possible reasons for this discrepancy and the consequences of using a specific equation. Different model structures, different error values in experimental work and different methodologies for statistical analysis of data (selection of dependent and independent variables) may explain the reasons for such a discrepancy in the results obtained from different equations.

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