MAINTENANCE COST ANALYSIS FOR REPLACEMENT MODEL WITH PERFECT/MINIMAL REPAIR

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Abstract With the evolution of technology, the maintenance of sophisticated systems is of concern for system engineers and system designers. The maintenance cost of the system depends in general on the replacement and repair policies. The system replacement may be in a strictly periodic fashion or on a random basis depending upon the maintenance policy. At failure, the repair of the system may be performed perfectly or minimally associated with some probability. When perfect repair is done, it makes the system as good as the new one. In case of minimal repair, it returns to the working condition of the system at the time of failure. In the present paper, we study the replacement policies for the system wherein minimal or perfect repair is done at the time of failure. The expressions for s-expected cost for the system with replacement and minimal or perfect repair are evaluated. The maintenance costs are discussed for various policies. Numerical simulation is performed to validate the analytical results.

Key Words Maintenance, Reliability, Replacement, Repair, Expected Cost

چکیده با تکامل تکنولوژی، تعمیر و نگهداری سیستمهای پیچیده مورد توجه بیشتر مهندسین و طراحان سیستم قرار می‌گیرد. هزینه تعمیر و نگهداری سیستم در حالت عمومی بستگی به سیاست تعمیر و تعویض پیدا می‌کند. سیاست تعویض ممکن است به صورت دوره‌ای و یا تصادفی باشد و آن بستگی به سیاست تعمیر و نگهداری دارد. در صورت خرابی، تعمیر سیستم ممکن است بطور کامل و یا به میزانی حداقل با یک احتمال مشخص انجام شود. در صورت انجام تعمیر بطور کامل، سیستم مانند نو عمل می کند. در صورت حداقل تعمیر، سیستم به وضعیت کاری خود قبل از خراب شدن می‌رسد. در این مقاله ما سیاست تعمیر و نگهداری را مبنی بر هر دو حالت تعمیر کامل و یا حداقل تعمیر در صورت خرابی در نظر می‌گیریم. منحنی معروف s شکل را برای تعویض سیستم براساس سیاستهای تعمیر کامل و یا حداقل تعمیر محاسبه می‌کنیم. هزینه‌های تعمیر و نگهداری برای سیاستهای مختلف مورد بحث قرار می‌گیرد. به منظور بررسی روایی نتایج تحلیلی، شبیه سازی عددی نیز انجام شده است.

1. INTRODUCTION

With the evolution of technology, the systems are being increasingly sophisticated and require regular maintenance and replacement. Whenever any activity whether a simple repair, regular maintenance or replacement is performed, it is associated with some costs. Therefore, it is essential for system designers and cost analysts to calculate the cost associated with maintenance of the system. The maintenance cost of the system depends, in general, on the replacement and repair policies. The system replacement may be in a strictly periodic fashion or on a random one, depending upon the maintenance policy. The repair may be of various types. At failure, the repair of the system may be performed perfectly or minimally associated with some probability. When perfect repair is done, it makes the system as good as new one. In case of minimal repair it returns to the condition of the system at the time of failure.

The replacement of the part or the device can be made without considering its failure with strict time schedule after a fixed interval of time. Or it can be made more realistically considering the time of replacement i.e., the time of replacement is associated with some random variables. The first one is, in general, coupled with security systems and safety devices in banking, fire fighting, aviation etc., where system designer cannot make any compromise with safety. The second one is the case where compromise on replacement time is possible. In mechanical systems, communication and electronic systems that are not highly sophisticated and have a capacity to work for over scheduled time, replacement policy can be ascertained with concede.

When the working system fails, it goes for repair, which can be minimal or perfect. In other words, when mechanical device fails, it can be repaired by tightening its nuts and bolts (minimal repair) or by replacing the broken parts (perfect repair). For example, in computer systems the repair at failure may be made by tightening the joints or soldering the IC's (minimal repair), or by replacing the cards/shouters (perfect repair).

The maintenance engineers may be interested in the number of failures, replacement or repair whereas for the cost analysts the average long run maintenance cost is of utmost concern. They are more interested in knowing the long run amount to be invested in maintenance and replacement so that they may know the prospect of the working device. One interesting example to be considered here is the success of petrol and diesel automobiles, although they are not environment friendly in comparison to E-automobile. It is because fuel-consuming automobiles, having less wear and tear, squeeze maintenance cost in comparison to E-automobile, which require regular maintenance schedule.

Cleroux et al. [9] studied generalized age replacement policy, where failure was corrected by minimal repair or replacement. Feldman [10] considered a model wherein a device was replaced subject to damage and determined replacement policy. Abdel-Hameed [1] discussed the imperfect maintenance model with block replacement and calculated the long run average cost. Brown et al. [8] analyzed imperfect-repair model in which the failed device went for perfect repair or minimal repair with complimentary probability. Block et al. [6] extended the results and made the perfect repair associated with the age of the device. Lim et al. [13] proposed a Bayes model for imperfect-repair assuming the probability of perfect repair as a random variable. Rangan et al. [15] discussed replacement and repair policy for deteriorating systems.

We study the replacement policies and s-expected total maintenance cost in Section 2. Here the replacements are not made in a strict periodic fashion but are assigned with some probability distribution function. In Section 3, we develop a model wherein replacements are made in strictly periodic fashion. At failure, the repair is done either perfectly or minimally assigned with some distribution. Average s-expected maintenance cost for the systems with replacement and minimal/perfect repair is evaluated in Section 4. Numerical simulation to validate the analytical results is performed in Section 5. In the last section conclusions and further possibilities for the extension of the results are discussed.

2. PLANNED REPLACEMENT WITH IMPERFECT MAINTENANCE

Assume that a device has a planned replacement, with distribution function U(t) and $\{u_k, k > 0\}$ is the process associated with the time between planned replacement. This process does not take any account of any failure. At failure, the device is either restored to its condition prior to failure, by minimal repair with probability q or by perfect repair with probability p=1-q.

Let F(t) be a distribution function of the failure time of a device. If the device failed at time t, and was perfectly repaired with probability p, then $\{v_k, k > 0\}$ (say), the process would describe the time between two perfect repairs with inter-occurrence distribution V(t). If the device is not perfectly repaired, it should to be minimally repaired with probability q=1-p. The process of repair repeats itself after each planned replacement, perfect and minimal repair.

Let us define $\{z_k, k > 0\}$ as the process where $z_k = \min(u_k, v_k)$, then $\{z_k, k > 0\}$ is a process associated with the time interval caused by either planned replacement or perfect repair. The corresponding distribution function is given by

$$Z(t) = 1 - \overline{U}(t)\overline{V}(t) \tag{1}$$

The distribution function of inter-occurrence time between perfect repair in time (0,t] denoted by H(t) is given by

$$H(t) = 1 - \overline{U}(t)\overline{V}(t) + \int_0^t H(t-x)V(t)dU(x) \qquad (2)$$

The distribution function G(T) of the inter-occurrence replacement time for planned replacement in time (0,t] is

$$G(t) = 1 - \overline{U}(t)\overline{V}(t) + \int_0^t G(t-x)\, U(t)\, dV(x) \quad (3)$$

2.1 s-Expected Total Maintenance Cost in (0,t]

To obtain the s-expected total maintenance cost in (0,t] we first calculate the expected number of replacement, perfect repair and minimal repair in the following manner:

The replacement made with every interval of time with inter-occurrence time distribution G(t) in time (0,t] forms a renewal process. The s-expected number of replacement is

$$E\left[N^R(0,t]\right] = \sum_{k=1}^{\infty} G^{(k)} \qquad (4)$$

- The inter-occurrence time distribution H(t) of perfect repair in time (0,t] form a renewal process. The s-expected number of perfect repair is given by

$$E\left[N^P(0,t]\right] = \sum_{k=1}^{\infty} H^{(k)} \qquad (5)$$

- $\{Z_k, k > 0\}$ is a process which deals with the time interval caused by either planned replacement or perfect repair. Therefore, the s-expected number of replacement/perfect repair in (0,t] is

$$E\left[N^{RP}(0,t]\right] = \sum_{k=1}^{\infty} Z^{(k)} \qquad (6)$$

- To know the number of minimal repair we make the following assumptions:

a) $N^{RP}(0,t] = n$

b) n planned replacement or perfect repair occurs at time $t_1, t_2, ..., t_n$

c) n-th planned replacement/perfect repair takes place at time t_n

d) no planned replacement/perfect repair occurs in $(t_n, t]$, all failures in $(t_n, t]$ are minimally repaired.

The conditional s-expected number of replacements/perfect repairs in (0,t] is

$$E\left[N^m(0,t] / N^{RP}(0,t] = n, T_1 = t_1, T_2 = t_2, ..., T_n = t_n\right]$$

$$= E\left[\sum_{i=1}^{n} N^m(t_i - t_{i-1}) / N^{RP}(0,t] = n, T_1 = t_1, T_2 = t_2, ..., T_n = t_n\right]$$

$$+ E\left[N^m(t - t_{n-1}) \; / N^{RP}(0,t] = n, T_n = t_n\right]$$

$$= qR(w_i) + qR(t - t_i) \qquad (7)$$

where $w_i = t_i - t_{i-1}$ and $R(t) = -\log[\,\overline{Z}(t)\,]$

Taking the s-expectation on the both sides of Equation 7, we have the s-expected number of minimal repairs in (0, t] as

$$E\left[N^m(0,t]\right] =$$

$$qE\left[E\left[\sum_{i=1}^{n} R(w_i) / N^{RP}(0,t] = n\right]\right] +$$

$$qE\left[E\left[R\left(t - T_{N^{RP}(0,p]}\right) / N^{RP}(0,t] = n\right]\right] \qquad (8)$$

Using the results (4) - (8), we obtain s-expected total cost, as follows:

s - expected cost in (0, t] =
(cost per planned replacement).
(s - expected number of planned replacement in (0,1])
+ (cost per perfect repair).
(s - expected number of perfect repair in (0,1]) + (cost per minimal repair). (s - expected number of minimal repair in (0,1])

$$E[C(0,t]] = C_R E[N^R(0,t]] + C_P E[N^P(0,t]]$$

$$+ C_P E[N^m(0,t]]$$

$$= C_R \sum_{k=1}^{\infty} G^{(k)} + C_P \sum_{k=1}^{\infty} H^{(k)}$$

$$+ C_m q \{ E[E[\sum_{i=1}^{n} R(w_i) / N^{RP}(0,t] = n]]$$

$$+ E[E[R(t - T_{N^{RP}}) / N^{RP}(0,t] = n]] \} \quad (9)$$

where C_R, C_P, C_m, denote the cost associated with each planned replacement, perfect repair and minimal repair respectively.

Particular Case We consider the case when the replacement is exponentially distributed. The lifetime of the device is also exponentially distributed. If device is perfectly repaired at failure with probability p then we have

$$U(t) = 1 - \exp(-\lambda_R t)$$
$$F(t) = 1 - \exp(-\lambda t) \quad (10)$$
$$V(t) = 1 - \exp(-p\lambda t)$$

Therefore

$$Z(t) = 1 - \exp(-\lambda_R + p\lambda)\exp\{-(\lambda_R + p\lambda)t\} \quad (11)$$

and $H(t) = U(t), G(t) = V(t)$. (12)

The s- expected number of replacement or perfect repair, replacement and perfect repair in (0,t] are respectively given by

$$E[N^{RP}(0,t]] = (\lambda_R + p\lambda)t \quad (13a)$$

$$E[N^R(0,t]] = \lambda_R t \quad (13b)$$

$$E[N^P(0,t]] = p\lambda t \quad (13c)$$

$$E[N^m(0,t]] = q(\lambda_R + p\lambda)t \quad (13d)$$

The s-expected total maintenance cost is

$$E[C(0,t]] = C_R(\lambda_R t) + C_P(p\lambda t) + C_P q(\lambda_R + p\lambda)t \quad (14)$$

2.2 Imperfect Maintenance without Replacement Taking the replacement time of a device to be infinity, we consider the model wherein replacement is not permitted. Let F(t) be the failure time distribution of the device. The perfect repair or minimal repair is performed with probabilities of p and q, respectively. The process of repair repeats itself after each failure. Now s-expected maintenance cost per unit in (0, t] is computed as

$$E[C(0,t]] = C_R E[N^P(0,t]] +$$

$$C_m E[N^m(0,t]] = C_R \sum_{k=1}^{\infty} H^{(k)} +$$

$$C_m \{ E[E[\sum_{i=1}^{n} R(w_i) / N^P(0,t] = n]] \} \quad (15)$$

Particular Case We consider the case where failure time distribution F(t) of a device is distributed according to Weibull distribution. Now

$$F(t)=1-\exp\left(-t^{\alpha}\right) \tag{16a}$$

$$H(t)=1-\exp\left(-pt^{\alpha}\right) \tag{16b}$$

The expected number of perfect repair and minimal repair are

$$E\left[N^{P}(0.t]\right]=\sum_{K=1}^{\infty}H^{(k)} \tag{17a}$$

and

$$E\left[N^{m}(0.T]\right]=q$$

$$\left\{E\left[E\left[\sum_{i=1}^{n}w_i^{\alpha}/N^{P}(0,t]=n\right]\right]\right.$$

$$\left.+\ E\left[E\left[\left(t-T_{N^P}\right)^{\alpha}/N^{P}(0,t]=n\right]\right]\right\} \tag{17b}$$

where w_i are i.i.d. from H(t) and $T_{p(o,t]}=\sum^{N^{P(0,t]}}w_i$

The maintenance cost per unit in (0, t] is

$$E[C(0,t]]=C_R E\left[N^{P}(0,t]\right]+$$

$$C_m\ E\ \left[N^{m}(0,t]\right]=C_R\sum_{k=1}^{\infty}H^{(k)}$$

$$+\ q\left\{E\left[E\left[\sum_{i=1}^{n}W_i^{\alpha}/N^{P}(0,t]=n\right]\right]\right.$$

$$\left.+\ E\left[\left(t-T_{N^P}\right)^{\alpha}/N^{P}(0,t]=n\right]\right\} \tag{18}$$

3. PERIODIC REPLACEMENT WITH IMPERFECT MAINTENANCE

A device is replaced periodically after every T units of time. At failure, the device is either restored to its condition prior to failure by minimal repair or perfect repair as discussed in section 2.

The s-expected number of planned replacement in time (0,t], is given by

$$E\left[N^{P}(0,t]\right]=\lfloor t/T\rfloor \tag{19}$$

where $\lfloor x\rfloor$ denotes the greatest integer value less than or equal to x.

The s-expected number of perfect repair is

$$E\left[N^{P}(0,t]\right]=\sum_{k=1}^{\infty}H^{(k)} \tag{20}$$

The s-expected number of replacements or perfect repairs in (0,t] is

$$E\left[N^{RP}(0,t]\right]=\sum_{k=1}^{\infty}Z^{(k)} \tag{21}$$

s-expected cost in (0, t] Using Equations 19 - 21 for the s-expected number of replacement and repair during (0,t], s-expected total maintenance cost is given by

$$E[C(0,t]]=C_P E\left[N^{P}(0,t]\right]+C_R E\left[N^{R}(0,t]\right]$$

$$+C_m E\left[N^{m}(0,t]\right]=C_p\lfloor t/T\rfloor+C_R\sum_{k=1}^{\infty}H^{(k)}$$

$$+C_m\left\{E\left[E\left[\sum_{i=1}^{n}R(w_i)/\lfloor t/T\rfloor=n\right]\right]\right.$$

$$\left.+E\left[E\left[R\left(t-T_{N^{RP}(0,t]}\right)/N^{RP}(0,t]=n\right]\right]\right\} \tag{22}$$

3.1 Periodic Replacement with Minimal Repair

Now we discuss the case when perfect repair is not taken into consideration. In this case, at failure the device is rehabilitated to its condition prior to failure, i.e. minimal repair. This process of minimal repair repeats itself after each replacement and failure.

The s-expected total maintenance cost in (0,1] is

$$E[C(0,t]]=C_R E[N^R(0,t]]+$$

$$C_m E[N^m(0,t]]=C_R\sum_{k=1}^{\infty}H^{(k)}$$

$$+C_m\{E[E[\sum_{i=1}^{n}R(w_i)/\lfloor t/T\rfloor=n]]$$

$$+E[E[R(t-T_{N^{RP}(0,t]})/N^{RP}(0,t]=n]]\} \quad (23)$$

Particular Case

(a) **Exponential Distribution**: If the life time distribution F(t) of device is exponential and the replacement duration is T, then we have $F(t)=1-\exp(-\lambda t)$, $t\geq 0$, $\lambda>0$ and

$$E[C(0,t]]=C_R\lfloor t/T\rfloor+C_m E[N^m(0,t]]$$

$$=C_R\lfloor t/T\rfloor+C_m\{E[nT/N^R(0,t]=n] \quad (24)$$

$$+E[E[\lambda(t-nT)/N^R(0,t]=n]]\}$$

(b) **Weihull Distribution**: If the life time distribution F(t) of device is distributed according to Weibull distribution, then

$$F(t)=1-\exp(-\lambda.t^{\alpha}),\quad t\geq 0,\ \lambda>0 \quad (25)$$

and

$$E[C(0,t]]=C_R\lfloor t/T\rfloor+C_m E[N^m(0,t]]$$

$$=C_R\lfloor t/T\rfloor+C_m\{E[n\lambda T^{\alpha}/N^R(0,t]=n]$$

$$+E[E[\lambda(t-nT)^{\alpha}/N^R(0,t]=n]]\} \quad (26)$$

4. AVERAGE S-EXPECTED MAINTENANCE COST RATE OVER AN INFINITE TIME SPAN

Now we obtain the average long run maintenance cost for the policies and models discussed in earlier sections.

The average s-expected maintenance cost rate over an infinite time span C_A is given by

$$C_A=\lim_{t\to\infty}\left[\frac{\text{s-expected cost in }(0,t]}{t}\right]$$

$$=\lim_{t\to\infty}\left[\frac{C_R E[N^R(0,t]]+C_P E[N^P(0,t]]+C_P E[N^m(0,t]]}{t}\right]$$

$$=\frac{C_R}{\mu_G}+\frac{C_P}{\mu_H}+\lim_{t\to\infty}\left\{\frac{C_P E[N^m(0,t]]}{t}\right\} \quad (27)$$

where $\mu_I=\int_{t=0}^{\infty}I(t)dt$ is the mean of I(t). Here I stands for G,H,Z.

In general, we cannot get the third term of Equation 27 in an explicit form. Therefore, bounds can be established as follows:

To calculate the s-expected number of minimal repair over an infinite time span, we note that inter-arrival time between t_n and the next perfect repair w_{n+1}, satisfies

$$w_{n+1}\geq t-t_n \quad (28)$$

which gives

$$E\left[R\left(t-T_{N^P(0,t]}\right)\right]\leq E\left[R\left(T_{N^P(0,t]+1}-T_{N^P(0,t]}\right)\right] \quad (29)$$

Adding $\sum_{i=1}^{n}R(w_i)$ on both sides of the inequality and taking expectations, we have

$$E[N^m(0,t]]\leq qE\left[\sum_{i=1}^{N^P(0,t]+1}R(w_i)\right]$$

$$\leq qE[N^P(0,t]+1]\,E[R(w_1)] \quad (30)$$

TABLE 1. The Planned Replacement with Imperfect Maintenance System ($\lambda_R = 12, \lambda = 30, \ C_R = 15, \ \ C_P = 11, \ C_m = 4, \ \ t = 25$).

p	$N^R(0,t)$	$N^P(0,t)$	$N^m(0,t)$	$C(0,t)$
0.00	300.0	0.00	300.00	5700.00
0.10	300.0	75.00	337.50	6675.00
0.20	300.0	150.00	420.00	7830.00
0.30	300.0	225.00	457.50	8805.00
0.40	300.0	300.00	480.00	9720.00
0.50	300.0	375.00	337.50	9975.00
0.60	300.0	450.00	300.00	10650.00
0.70	300.0	525.00	247.50	11265.00
0.80	300.0	600.00	180.00	11820.00
0.90	300.0	675.00	97.50	12315.00
1.00	300.0	750.00	0.00	12750.00

In limiting case, we find

$$\lim_{t\to\infty}\left(\frac{E\left[N^m(0,t)\right]}{t}\right) \le q\,\frac{E\left[R(w_i)\right]}{\mu_z} \qquad (31)$$

Inequality 31 provides the upper bound for s-expected number of minimal repairs over an infinite time span.

Particular Cases (i) **Planned Replacement with Imperfect Maintenance**: For exponential distributed planned replacement and failure, we have

$$C_A = C_R(\lambda_R t) + C_P(p\lambda t) + C_P q(\lambda_R + p\lambda)t \qquad (32)$$

(ii) **Imperfect Maintenance without Replacement**: For an exponentially distributed failure, we have

$$C_A \le \frac{C_P\left(p^{\frac{1}{\alpha}}\right) + C_m q\left(p^{\frac{1}{\alpha}}\right)E\left[w_1^{\alpha}\right]}{\Gamma\left(1+\alpha^{-1}\right)} = \frac{C_P\left(p^{\frac{1}{\alpha}}\right) + C_m q\left(p^{\frac{1}{\alpha}}\right)\left(p^{\frac{1}{\alpha}-1}\right)}{\Gamma\left(1+\alpha^{-1}\right)} \qquad (33)$$

(iii) **Periodic Replacement with Minimal Repair**: In the case of exponential distribution, we get

$$C_A = C_R/T + C_m q\lambda T \qquad (34)$$

For Weibul distribution

$$C_A = C_R/T + C_m q\lambda T^{\alpha} \qquad (35)$$

5. NUMERICAL ILLUSTRATION

We have performed numerical simulation to compute the expected total maintenance cost, number of replacement and number of perfect and minimal repair. The effect of parameters on the maintenance cost is displayed in Tables 1-3.

In Table 1, the maintenance cost for planned replacement with imperfect maintenance is shown. The replacement and failure time are taken to be exponentially distributed. We fix the parameters as $\lambda_R = 12, \ \lambda = 30, \ C_R = 15, \ C_P = 11, \ C_m = 4, \ t = 25$, and vary p from 0 to 1. The s-expected number of replacements/repair, minimal repair and perfect repair are also given.

In Table 2, we depict the results for the imperfect maintenance without replacement having Weibull failure distribution. By choosing $\alpha = 0.5$, $C_R = 15$, $C_m = 4$, $t = 10$ and varying p from 0 to 1, the s-expected number of perfect repair, minimal repair and s-expected total maintenance cost are tabulated.

We summarize the results for periodic replacement with imperfect maintenance for Weibull distribution with parameter $\alpha = 2$ in Table 3. The s-expected

TABLE 2. The Imperfect Maintenance without Replacement ($\alpha = 0.5,\ C_R = 15,\ C_m = 4,\ t = 10$).

p	$N^P(0,t]$	$N^m(0,t]$	$C(0,t]$
0.00	0.00	3.16	12.64
0.10	0.36	3.12	17.88
0.20	0.77	3.02	23.63
0.30	1.24	2.87	30.08
0.40	1.77	2.66	37.19
0.50	2.38	2.38	45.22
0.60	3.06	2.04	54.06
0.70	3.82	1.64	63.86
0.80	4.68	1.17	74.88
0.90	5.61	0.62	86.63
1.00	6.48	0.00	97.20

cost of total maintenance and number of minimal repair and replacement are given by varying periodic replacement time T for time period t = 100. Here the cost of replacement and minimal repair per unit are 55 and 12 respectively.

6. CONCLUSION

We have analyzed the maintenance policies where the replacement is made either periodically or at random time intervals. Knowing the real world requirements, we have considered two types of repair: minimal and perfect. For cost analysis, the bounds for average maintenance are also provided for the cases in which exact results are difficult to obtain. Numerical illustrations are made to validate the results for various policies. The performance measures for maintenance cost may provide an insight to the system engineers to know the long run maintenance cost.

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TABLE 3. The Periodic Replacement with Imperfect Maintenance ($\alpha = 2,\ C_R = 55,\ C_m = 12,\ t = 100$).

T	$N^R(0,t]$	$N^m(0,t]$	$C(0,t]$
1.00	100.00	600.00	12700.00
5.00	20.00	48000.00	577100.00
7.00	14.00	4140.00	50450.00
21.00	4.00	12120.00	145660.00
25.00	4.00	15000.00	180220.00

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