DIFFUSION PROCESS FOR MULTI – REPAIRMEN MACHINING SYSTEM WITH SPARES AND BALKING

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Abstract In this paper we describe G/G/R+s multi- repairmen machining system with balking. The system consists of M operating machines, S spare machines, R permanent and s additional repairmen. Assuming the discrete flow of machines by continuous one, the diffusion approximation technique for the machine repair system has developed. The system will be in normal working mode if there is M operating machines. When there are less than M and $\leq$ m, the system is called as short system. The failure rates of operating units in short and normal modes are different. By using the mean and square coefficient of variation of failure and repair time distributions, the queue size distribution has been established. Various performance indices viz. expected number of failed machines, average operating machines etc. have been derived.

Key Words Queue, Machine Repair, Warm Standby, Diffusion Approximation, Additional Repairmen, Multi-Repairmen, Balking

چکیده در این مقاله ما یک سیستم صف G/G/R+s با چندین تعمیر کار را در نظر می‌گیریم که بعضی از مشتریها از گرفتن سرویس مایوس می‌شوند. سیستم شامل M ماشین در حال کار، S ماشین یدکی، R تعمیر کار و s تعمیر کار اضافی است. تکنیک تقریبی پخش برای سیستم تعمیر ماشین به کار گرفته شده است؛ با این فرض که جریان ناپیوسته با جریان پیوسته تقریب زده شود. اگر M ماشین در حال کار باشد، می گوییم سیستم در حال نرمال است. وقتی که تعداد ماشینهای در حال کار کمتر از M باشد میگوییم سیستم کمبود دارد. نرخ خرابی ماشینهای در حال کار، در حال کمبود و در حالت نرمال با هم تفاوت دارد. با استفاده از میانگین و مربع ضریب تغییرات خرابی و زمان تعمیر، به یکی از حالات سیستم صف می رسیم. شاخص های مختلفی از عملکرد، مثل میانگین تعداد ماشینهای خراب شده، میانگین تعداد ماشینهای در حال کار و غیره محاسبه گردیده اند.

INTRODUCTION

In many fields characterized by rapid technological or theoretical development, the practice of queuing theory has logged behind the theoreticians. It is difficult to obtain the exact solution of general queuing model. The diffusion process that is based on continuous time, continuous state process can be used to approximate a discrete stochastic process of a general queuing system. The provision of spares and additional repairmen may improve the running efficiency and operating utilization of the machining system having multi-components. Whenever a machine fails, any spare available in standby group replaces it. The failed machine is sent for repairing to repairman at once. If two or more machines are failed at the same time, only one machine can be checked or repaired at a time. In case of several repairmen if the number of machines in operating group at any time more than the number of repairmen, the more number of machines will have to wait, until repairmen are available. The provision of additional repairmen may be helpful to reduce the backlog in case of longer queue of failed machines.

Several results on the machining system can be found in Gross and Harris [1], Kleinrock [2]. Sivazlian and Wang [3] gave the economic analysis of the M/M/R machine repair problem with warm standby. Agnihothri [4] analyzed multi-server finite queue with general input, general service. He obtained the characteristics of whole system consisting of the set of m machines. Knessl [5] analyzed the transient behavior of the repairmen problem using singular perturbation method to scaled equations for the number of failed machines. Gupta and Rao [6] derived a recursive method to compute the steady state probabilities of the M/G/1/m machine interference model. Steady state solutions of the machine repair problem with no spares are obtained for the M/E_k/1 model by Wang and Kuo [7]. Steady state solution of the single server machine repair problem with balking, reneging and an additional server for longer queues were developed by Shawky [8]. Ke and Wang [9] gave the cost analysis of the M/M/R machine repair problem with balking, reneging and server breakdowns. Shawky [10] developed M/M/C/K/N machine interference model with balking, reneging and spares.

Several researchers studied queuing models in general frame-works by using diffusion approximation technique. By using diffusion approximation technique, one can derive the steady state queue size distribution for the expected number of failed units in the system. The boundary behaviors of a diffusion process to approximate queuing model are classified into three categories: the reflecting boundary, the elementary return boundary and the instantaneous return boundary. Gaver and Lehoezky [11] considered repairman problem in which the failures may be of two types and the operations may occur simultaneously. The problem was solved by diffusion approximation technique. Haryona and Sivazlian [12] gave the analysis of machine repair problem by using diffusion approach. Yao and Buzacott [13] suggested a diffusion approximation for G/G/r flexible machine interference problem using elementary return boundaries.

Krichagina [14] studied diffusion approximation for a queue in a multi- server system with multi- stage service. Jain [15] developed the (m, M) machine repair problem with spares and state dependent rates by using diffusion approximation technique with reflecting boundaries. Sharma and Jain [16] established refined diffusion approximation for G/G/m/N queue and derived formulae for the mean number of customers in the system, delay probability and mean queue length.

The purpose of this paper is to investigate the general machine repair problem with spares, additional repairmen and balking by using diffusion approximation technique based on reflecting boundaries. The spares are considered to be warm standby that may fail with a rate less than that of operating units. The repair rates of permanent and additional repairmen are different. The steady state queue size distribution is obtained by using mean and variance of the diffusion process. Various performance indices are derived in terms of queue size distribution.

THE MODEL

We consider G/G/R machine repair system with N identical machines. Where N= M (operating) + S (spare) machines that are subject to breakdowns and are maintained by R permanent and s additional repairmen. Each operating machine fails randomly at failure rate λ. The mean failure rate of spare machines is $\alpha (0 \leq \alpha \leq \lambda)$. In case when at least one permanent repairman is free and all spare machines are used, the machine fails in operating group with rate $\lambda_1 (\lambda_1 \leq \lambda)$. However when all repairmen are busy, the failed machines may balk with probability $\beta (0 < \beta \leq 1)$. Let the service rate for idle repairmen be μ_0. When all permanent repairmen are busy, the failed machines are repaired with repair rate $\mu (\mu_0 \geq \mu)$. If there are greater than or equal to lK and less than $(l+1)K$ failed machines, where l = 1, 2, 3, …, s-1, then additional repairmen are available for service with rate μ_1. In this model we use the one dimensional diffusion process with mean a(x, t) and variance b (x, t) at time t.

DIFFUSION PROCESS

The continuous probability density function p (x, t) is taken corresponding to the discrete probability $p_n(t)$, where $p_n(t)$ denotes the probability that there are n machines in the system at time t. The p (x, t) satisfies the Forward Kolmogorov (Fokker- Plank) equation given by (see Cox and Miller, [17])

$$\frac{\partial p(x,t)}{\partial t} = \frac{1}{2}\frac{\partial^2}{\partial x^2}\{p(x,t)b(x,t)\} - \frac{\partial}{\partial x}\{p(x,t)a(x,t)\} \tag{1}$$

where a(x ,t) and b(x ,t) are the mean and variance of the diffusion process respectively. Using reflecting boundaries at x=0 and x= M+S-m, the steady state solution of Equation 1 is given by

$$p(x) = \frac{\text{constant}}{b(x)} \exp\left\{2\int_0^x \frac{a(x)}{b(x)}dx\right\} \tag{2}$$

where a (x) and b (x) are respectively, the mean and variance under steady state. The square coefficients of variation of inter-failure and repair time distributions are represented by C_a^2 and C_s^2 respectively. Here we treat the two cases, first of them is S<R and other is $R \le S$.

Case A: S<R The values of state dependent mean failure rate λ_n and mean repair rate μ_n are as follows:

$$\lambda_n = \begin{cases} M\lambda + (S-n)\alpha & 0 \le n \le S \\ (M+S-n)\lambda_1 & S < n < R \\ (M+S-n)\lambda\beta & R \le n \le M+S-m \end{cases} \tag{3}$$

$$\mu_n = \begin{cases} n\mu_0 & 1 \le n < R \\ R\mu & R \le n < K \\ R\mu + l\mu_1 & lK \le n < (l+1)K, 1 \le l \le s-1 \\ R\mu + s\mu_1 & sK \le n \le M+S-m \end{cases} \tag{4}$$

Let the steady state mean and variance be a (x) and b (x) respectively. The values of a (x) and b (x) for this case are as follows:

1. For $0 \le x \le S$

a (x) = $M\lambda + (S-x)\alpha - x\mu_0$

b (x) = $[M\lambda + (S-x)\alpha] C_a^2 + x\mu_0 C_s^2$

2. For $S < x < R$

a (x) = $(M+S-x)\lambda_1 - x\mu_0$

b (x) = $(M+S-x)\lambda_1 C_a^2 + x\mu_0 C_s^2$

3. For $R \le x < K$

a (x) = $(M+S-x)\lambda\beta - R\mu$

b (x) = $(M+S-x)\lambda\beta C_a^2 + R\mu C_s^2$

4. For $lK \le x < (l+1)K, l = 1,2,...,s-1$

a (x) = $(M+S-x)\lambda\beta - (R\mu + l\mu_1)$

b (x) = $(M+S-x)\lambda\beta C_a^2 + (R\mu + l\mu_1) C_s^2$

5. For $sK \le x \le (M+S-m)$

a (x) = $(M+S-x)\lambda\beta - (R\mu + s\mu_1)$

b (x) = $(M+S-x)\lambda\beta C_a^2 + (R\mu + s\mu_1) C_s^2$

THE QUEUE SIZE DISTRIBUTION

The steady state queue size distribution p (x) is obtained by using the values of a (x) and b (x) in

Equation 2 for various intervals as follows

$$p(x)=\begin{cases} Cg_1(x)\equiv p_1(x), & 0\le x\le S \\ C\dfrac{g_1(S)}{g_2(S)}g_2(x)\equiv p_2(x), & S<x<R \\ C\dfrac{g_1(S)g_2(R)}{g_2(S)g_3(R)}g_3(x)\equiv p_3(x), & R\le x<K \\ C\dfrac{g_1(S)g_2(R)}{g_2(S)g_3(R)}\left(\prod_{i=1}^{1}\dfrac{g_{2+i}(iK)}{g_{3+i}(iK)}\right)g_{3+l}(x)\equiv p_{3+l}(x), & lK\le x<(l+1)K, 1\le l<s \\ C\dfrac{g_1(S)g_2(R)}{g_2(S)g_3(R)}\left(\prod_{l=1}^{s-1}\dfrac{g_{2+l}(lK)}{g_{3+l}(lK)}\right)\dfrac{g_{2+s}(sK)}{g_{3+s}(sK)}g_{3+s}(x)\equiv p_{3+s}(x), & sK\le x\le M+S-m \end{cases} \tag{5}$$

where C is a constant and

$$g_1(x)=\left(x-\frac{\phi}{\alpha-\mu_0\eta}\right)^{\left[2\phi\mu_0(1+\eta)/(\mu_0\eta-\alpha)^2C_a^2\right]-1}\times \exp\left\{\frac{-2(\alpha+\mu_0)x}{(\mu_0\eta-\alpha)C_a^2}\right\} \tag{6}$$

$$g_2(x)=\left(x-\frac{\gamma}{\lambda_1-\mu_0\eta}\right)^{\frac{2\gamma\mu_0(1+\eta)}{(\mu_0\eta-\lambda_1)^2C_a^2}-1}\exp\left\{\frac{-2(\lambda_1+\mu_0)x}{(\mu_0\eta-\lambda_1)C_a^2}\right\} \tag{7}$$

$$g_3(x)=\left(x-\frac{\zeta+R\mu\eta}{\lambda\beta}\right)^{\frac{2R\mu(1+\eta)}{\lambda\beta C_a^2}-1}\exp\left\{\frac{2x}{C_a^2}\right\} \tag{8}$$

$$g_{3+l}(x)=\left(x-\frac{\zeta+(R\mu+l\mu_1)\eta}{\lambda\beta}\right)^{\frac{2(R\mu+l\mu_1)(1+\eta)}{\lambda\beta C_a^2}-1}\exp\left\{\frac{2x}{C_a^2}\right\} \tag{9}$$

$$g_{3+s}(x)=\left(x-\frac{\zeta+(R\mu+s\mu_1)\eta}{\lambda\beta}\right)^{\frac{2(R\mu+s\mu_1)(1+\eta)}{\lambda\beta C_a^2}-1}\exp\left\{\frac{2x}{C_a^2}\right\} \tag{10}$$

where

$$\phi=M\lambda+S\alpha,\ \eta=\frac{C_s^2}{C_a^2},\ \gamma=(M+S)\lambda_1,\ \zeta=(M+S)\lambda\beta \tag{11}$$

We obtain the value of C by using the following normalizing condition

$$\int_0^{M+S-m}p(x)dx=1$$

i.e.

$$\int_0^S p_1(x)dx+\int_S^R p_2(x)dx+\int_R^K p_3(x)dx+\sum_{l=1}^{n-1}\int_{lk}^{(l+1)K}p_{3+l}(x)dx+\int_{sK}^{M+S-m}p_{3+s}(x)dx=1 \tag{12}$$

The approximate value of p_n is determined as follows:

$$\hat{p}_n=\int_{n-1/2}^{n+1/2}p(x)dx \qquad 1\le n\le M+S-m \tag{13}$$

SOME PERFORMANCE INDICES

Let E (F_A) be the expected number of failed machines in the system, then

$$\begin{aligned} E(F_A)=&\sum_{n=1}^{M+S-m}n\hat{p}_n \\ =&\sum_{n=1}^{S-1}n\int_{n-1/2}^{n+1/2}p_1(x)dx+S\left[\int_{S-1/2}^{S}p_1(x)dx+\int_S^{S+1/2}p_2(x)dx\right]+ \\ &\sum_{n=S+1}^{R-1}n\int_{n-1/2}^{n+1/2}p_2(x)dx+R\left[\int_{R-1/2}^{R}p_2(x)dx+\int_R^{R+1/2}p_3(x)dx\right]+ \\ &\sum_{n=R+1}^{K-1}n\int_{n-1/2}^{n+1/2}p_3(x)dx+K\left[\int_{K-1/2}^{K}p_3(x)dx+\int_K^{K+1/2}p_4(x)dx\right]+ \\ &\sum_{l=1}^{s-1}\sum_{lK+1}^{(l+1)K-1}n\int_{n-1/2}^{n+1/2}p_{3+l}(x)dx+\sum_{l=1}^{s-1}(l+1)K\Big[\int_{(l+1)K-1/2}^{(l+1)K}p_{3+l}(x)dx+ \\ &\int_{(l+1)K}^{(l+1)K+1/2}p_{4+l}(x)dx\Big]+\sum_{sK+1}^{M+S-m-1}n\int_{n-1/2}^{n+1/2}p_{3+s}(x)dx \\ &+(M+S-m)\int_{M+S-m-1/2}^{M+S-m}p_{3+s}(x)dx \end{aligned} \tag{14}$$

The expected number of operating machines in the system E (O_A) is given by

$$E(O_A) = M\sum_{n=0}^{S}\hat{p}_n + \sum_{n=S+!}^{M+S-m}(M+S-n)\hat{p}_n$$

$$= M\left[\sum_{n=0}^{S-1}\int_{n-1/2}^{n+1/2}p_1(x)\,dx + \int_{S-1/2}^{S}p_1(x)\,dx + \right.$$

$$\left.\int_{S}^{S+1/2}p_2(x)\,dx\right] + \sum_{n=S+1}^{R-1}(M+S-n)\int_{n-1/2}^{n+1/2}p_2(x)\,dx +$$

$$(M+S-R)\left[\int_{R-1/2}^{R}p_2(x)\,dx + \int_{R}^{R+1/2}p_3(x)\,dx\right] +$$

$$\sum_{n=R+1}^{K-1}(M+S-n)\int_{n-1/2}^{n+1/2}p_3(x)\,dx + (M+S-K)$$

$$\left[\int_{K-1/2}^{K}p_3(x)\,dx + \int_{K}^{K+1/2}p_4(x)\,dx\right] +$$

$$\sum_{l=1}^{s-1}\sum_{n=lK+1}^{(l+1)K-1}(M+S-n)\int_{n-\frac{1}{2}}^{n+\frac{1}{2}}p_{3+l}(x)\,dx +$$

$$\sum_{l=1}^{s-1}[M+S-(l+1)K]\left[\int_{(l+1)K-1/2}^{(l+1)K}p_{3+l}(x)\,dx + \right.$$

$$\left.\int_{(l+1)K}^{(l+1)K+1/2}p_{4+l}(x)\,dx\right] + \sum_{n=sK+1}^{M+S-m-1}(M+S-n)\int_{n-1/2}^{n+1/2}p_{3+s}(x)\,dx$$

$$+ m\int_{M+S-m-1/2}^{M+S-m}p_{3+s}(x)\,dx \tag{15}$$

where p_{M+S-m} is approximated by

$$\hat{p}_{M+S-m} = \int_{M+S-m-1/2}^{M+S-m}p_{3+s}(x)dx \tag{16}$$

Case B: R≤S In this case the state dependent failure and repair rates are as follows:

$$\lambda_n = \begin{cases} M\lambda + (S-n)\alpha, & 0 \le n \le S \\ (M+S-n)\lambda\beta, & S < n \le M+S-m \end{cases} \tag{17}$$

$$\mu_n = \begin{cases} n\mu_0, & o < n < R \\ R\mu, & R \le n < K \\ R\mu + l\mu_1, & lK \le n < (l+1)K, l = 1,2...,s-1 \\ R\mu + s\mu_1, & sK \le n \le M+S-m \end{cases} \tag{18}$$

The value of a (x) and b (x) for different intervals are given below:

1. For $0 \le x < R$

a (x) = $M\lambda + (S-x)\alpha - x\mu_0$

b (x) = $[M\lambda + (S-x)\alpha]C_a^2 + x\mu_0 C_s^2$

2. For $R \le x \le S$

a (x) = $M\lambda + (S-x)\alpha - R\mu$

b (x) = $[M\lambda + (S-x)\alpha]C_a^2 + R\mu C_s^2$

3. For $S < x < K$

a (x) = $(M+S-x)\lambda\beta - R\mu$

b (x) = $(M+S-x)\lambda\beta C_a^2 + R\mu C_s^2$

4. For $lK \le x < (l+1)K$, l=1, 2, …, s-1

a (x) = $(M+S-x)\lambda\beta - (R\mu + l\mu_1)$

b(x) = $(M+S-x)\lambda\beta C_a^2 + (R\mu + l\mu_1)C_s^2$

5. For $sK \le x \le M+S-m$

a (x) = $(M+S-x)\lambda\beta - (R\mu + s\mu_1)$

b (x) = $(M+S-x)\lambda\beta C_a^2 + (R\mu + s\mu_1)C_s^2$

For this case the queue size distribution under steady state, is determined by using the values of a (x) and b (x) in Equation 2 for different intervals as follows:

$$
p(x)=\begin{cases}
C' g_1(x)\equiv p_1(x), 0\le x<R \\
C'\dfrac{g_1(R)}{g_2'(R)}g_2'(x)\equiv p_2'(x), R\le x\le S \\
C'\dfrac{g_1(R)g_2'(S)}{g_2'(R)g_3'(S)}g_3'(x)\equiv p_3'(x), S<x<K \\
C'\dfrac{g_1(R)g_2'(S)}{g_2'(R)g_3'(S)}\left(\prod_{i=1}^{l}\dfrac{g_{2+i}'(iK)}{g_{3+i}'(iK)}\right)g_{3+l}'(x)\equiv \\
\qquad p_{3+l}'(x), lK\le x<(l+1)K, 1\le l<s \\
C'\dfrac{g_1(R)g_2'(S)}{g_2'(R)g_3'(S)}\left(\prod_{l=1}^{s-1}\dfrac{g_{2+l}'(lK)}{g_{3+l}'(lK)}\right)\dfrac{g_{3+s-1}'(sK)}{g_{3+s}'(sK)}\times \\
\qquad g_{3+s}'(x)\equiv p_{3+s}'(x), sK\le x\le M+S-m
\end{cases}
\tag{19}
$$

where C' is an arbitrary constant and

$$
g_2'(x)=\left(x-\frac{\phi+R\mu\eta}{\alpha}\right)^{\left[\frac{2R\mu(1+\eta)}{\alpha C_a^2}\right]-1}\exp\left\{\frac{2x}{C_a^2}\right\}
\tag{20}
$$

$$
g_3'(x)=\left(x-\frac{\zeta+R\mu\eta}{\lambda\beta}\right)^{\left[\frac{2R\mu(1+\eta)}{\lambda\beta C_a^2}\right]-1}\exp\left\{\frac{2x}{C_a^2}\right\}
\tag{21}
$$

$$
g_{3+l}'(x)=\left(x-\frac{\zeta+(R\mu+l\mu_1)\eta}{\lambda\beta}\right)^{\left[\frac{2(R\mu+l\mu_1)(1+\eta)}{\lambda\beta C_a^2}\right]-1}\times \exp\left\{\frac{2x}{C_a^2}\right\}
\tag{22}
$$

where $1\le l<s$

$$
g_{3+s}'(x)=\left(x-\frac{\zeta+(R\mu+s\mu_1)\eta}{\lambda\beta}\right)^{\left[\frac{2(R\mu+s\mu_1)(1+\eta}{}\right]-1}\times \exp\left\{\frac{2x}{C_a^2}\right\}
\tag{23}
$$

To obtain the value of C', we use the normalizing condition

$$
\int_0^{M+S-m}p(x)dx=1
$$

i.e.

$$
\int_0^R p_1(x)\,dx+\int_R^S p_2'(x)\,dx+\int_S^K p_3'(x)\,dx+
$$

$$
\sum_{l=1}^{s-1}\int_{lK}^{(l+1)K}p_{3+l}'(x)\,dx+\int_{sK}^{M+S-m}p_{3+s}'(x)\,dx=1
\tag{24}
$$

If expected number of failed machines in E (FB) denotes the system, then

$$
E(F_B)=\sum_{n=1}^{M+S-m}n\hat{p}_n
$$

$$
=\sum_{n=1}^{R-1}n\int_{n-1/2}^{n+1/2}p_1(x)dx+R\left[\int_{R-1/2}^{R}p_1(x)dx+\right.
$$

$$
\left.\int_R^{R+1/2}p_2'(x)dx\right]+\sum_{n=R+1}^{S-1}n\int_{n-1/2}^{n+1/2}p_2'(x)\,dx+
$$

$$
S\left[\int_{S-1/2}^{S}p_2'(x)\,dx+\int_S^{S+1/2}p_3'(x)\,dx\right]+
$$

$$
\sum_{n=S+1}^{K-1}n\int_{n-1/2}^{n+1/2}p_3'(x)\,dx+K\left[\int_{K-1/2}^{K}p_3'(x)\;dx+\right.
$$

$$
\left.\int_K^{K+1/2}p_4'(x)\,dx\right]+\sum_{l=1}^{s-1}\sum_{lK+1}^{(l+1)K-1}n\int_{n-1/2}^{n+1/2}p_{3+l}'(x)\,dx+
$$

$$
\sum_{l=1}^{s-1}(l+1)\,K\left[\int_{(l+1)K-1/2}^{(l+1)K}p_{3+l}'(x)\,dx+\int_{(l+1)K}^{(l+1)K+1/2}p_{4+l}'(x)\,dx\right]+
$$

$$
\sum_{sK+1}^{M+S-m-1}n\int_{n-1/2}^{n+1/2}p_{3+s}'(x)\,dx+
$$

$$
(M+S-m)\int_{M+S-m-1/2}^{M+S-m}p_{3+s}'(x)\,dx
\tag{25}
$$

The expected number of operating units in the system is

$$E(O_B) = M\sum_{n=0}^{S}\hat{p}_n + \sum_{n=S+1}^{M+S-m}(M+S-n)\hat{p}_n$$

$$= M\Bigg[\sum_{n=0}^{R-1}\int_{n-1/2}^{n+1/2}p_1(x)\,dx + \int_{R-1/2}^{R}p_1(x)\,dx +$$

$$\int_{R}^{R+1/2}p_2'(x)\,dx + \sum_{n=R+1}^{S-1}\int_{n-1/2}^{n+1/2}p_2'(x)\,dx +$$

$$\int_{S-1/2}^{S}p_2'(x)\,dx + \int_{S}^{S+1/2}p_3'(x)\,dx\Bigg] +$$

$$\sum_{n=S+1}^{K-1}(M+S-n)\int_{n-1/2}^{n+1/2}p_3'(x)\,dx + (M+S-K)$$

$$\left[\int_{K-1/2}^{K}p_3'(x)\,dx + \int_{K}^{K+1/2}p_4'(x)\,dx\right] + \sum_{l=1}^{s-1}\sum_{lK+1}^{(l+1)K-1}$$

$$(M+S-n)\int_{n-1/2}^{n+1/2}p_{3+l}'(x)\,dx + \sum_{l=1}^{s-1}\left[M+S-(l+1)K\right]$$

$$\left[\int_{(l+1)K-1/2}^{(l+1)K}p_{3+l}'(x)\,dx + \int_{(l+1)K}^{(l+1)K+1/2}p_{4+l}'(x)\,dx\right] +$$

$$\sum_{n=sK+1}^{M+S-m-1}(M+S-n)\int_{n-1/2}^{n+1/2}p_{3+s}'(x)\,dx +$$

$$m\int_{M+S-m-1/2}^{M+S-m}p_{3+s}'(x)\,dx \tag{26}$$

Similarly we can get some more results by using the following formulae. The probability of short system is

$$\hat{p}_s = \sum_{i=S+1}^{M+S-m}\hat{p}_i \tag{27}$$

The probability of system failure is

$$\hat{p}_F = \hat{p}_{M+S-m} = \int_{M+S-m-1/2}^{M+S-m}p_{3+s}(x)dx, for S < R \tag{28}$$

$$\hat{p}_F' = \hat{p}_{M+S-m}' = \int_{M+S-m-1/2}^{M+S-m}p_{3+s}'(x)dx,\ \text{for}\, S \geq R \tag{29}$$

The expected number of spare machines in the system as standby is

$$E(S) = \sum_{n=0}^{S}(S-n)\hat{p}_n \tag{30}$$

The expected number of idle repairmen is

$$E(I) = \sum_{n=0}^{R-1}(R-n)\hat{p}_n \tag{31}$$

DISCUSSION

In this investigation, one dimensional diffusion equation is used to obtain the approximate value of queue size distribution under steady state. Since the diffusion equation depends upon the mean and variance of the process, this process is easily applicable to the problem having general independent failure and general repair process. For solution purpose, we have imposed the reflecting boundaries at x = 0 and x = M+S-m. Some performance measures obtained can be easily computed by using Gauss formula for numerical integration.

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