RELIABILITY OF REPAIRABLE SYSTEM WITH REMOVABLE MULTI-REPAIRMEN

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Abstract In this investigation, the explicit expressions for reliability function and mean time to failure (MTTF) of a repairable system with provision of spares and removable multi-repair facility have been established. In removable repairmen strategy, the repairmen turn on when there are N or more than N failed units and turn-off when system is empty. The failure times of operating/spare units and repair time of failed units are exponentially distributed.

Key Words Reliability, Repairable, Removable Servers, Spare Units, MTTF, Transient Analysis

چکیده در این تحقیق عبارت واضح و روشنی از تابع کارایی و میانگین زمان کارکرد یک سیستم تعمیرپذیر که مجاز باشد از وسایل یدکی و وسایل قابل تعمییر در چندین بار استفاده کند ارائه می شود. استراتژی سیستم تعمیرات بر این مبنا است که اگر سیستم تهی از اجزای معیوب باشد، آگاهی دهنده سیستم تعمیرات خاموش است ولیکن اگر تعداد اجزای معیوب بزرگتر یا مساوی N باشد این آگاهی دهنده روش می شود. زمان طول عمر وسیله های یدکی و زمان تعمیر اجزای معیوب تماماً از توزیع نمایی پیروی می کند.

INTRODUCTION

The provision of spare part support in a repairable system is important in order to increase the reliability of the system. Some interesting results on the reliability of the standby redundant system can be found in Gopalan [1], Kumar and Agarwal [2], Gupta et al. [3]. Goel and Srivastava [4] provided the transient analysis of multi-unit redundant system. Jain and Sharma [5] and Jain [6] established the diffusion equation for a multi-repairmen general machine repair system with spares. Sivazlion and Wong [7] gave cost analysis for an M/M/R system with warm standby spare machines. Cost analysis of a two cold standby system with three modes of failure was discussed by Singh and Singh [8]. Agrafiotis and Singh [9] investigated the stochastic behavior of a two unit standby redundant system with two spare units from cost analysis view point. Agnihotri et al. [10] considered two unit redundant system with n-failure modes and developed fault-detection method. Gopalan and Bhanu [11] gave cost analysis of a two unit repairable system subject to on-line preventive maintenance.

In many real life situations, provision of full-time repairman costs the system which may not be important from the reasonable cost analysis view point. Yadin and Naor [12] first introduced the concept of a removable single-server system with exponential inter-arrival and service time distributions. Bell [13] studied the M/G/1 queueing system with the provision of removable server. Recently Hsieh and Wang [14] tackled a removable single-repairman system with arbitrary spare units. In some situations, there may be multi-repairmen facility in which repairmen turn on only when a threshold number of units are in failed condition. This motivates us to develop the reliability characteristics of a repairable system in which there are removable multi-repairmen and spare units. Such problems may be encountered in machining systems in the production processes.

THE MODEL AND MATHEMATICAL ANALYSIS

We consider a multi-repairmen service facility with the following assumptions:

i) There are M operating and S spare units in the system.

ii) The facility has c repairmen.

iii) System fails if there are less than K operating units i.e. more than L = M+ S-K+1 failed machines in the system.

iv) The operating (spare) units have exponentially distributed failure time with mean rate λ (α) and the repair times of failed units are exponentially distributed with the rate of μ.

v) Repaired units are assumed to be as good as the new ones.

vi) The repairmen turn on when N or more than N units fail and turn off when the system is again empty.

Our main objective is to derive the explicit expressions for reliability function R(t) and mean time to failure (MTTF) by using Laplace transform technique.

We denote

$p_{0,n}(t)$ = Probability that the repairmen are turned off and there are n failed units (n = 0, 1, 2,..., N-1) at time t.

$p_{1,n}(t)$ = Probability that repairmen are turned on and there are n failed units (n = 1,2, ..., L) at time t.

$\overline{P}_{i,s}(s)$ Laplace transform of $p_{i,n}(t)$, i = 0,1.

The mean failure rate and repair rate are respectively given by

$$\lambda_n = \begin{cases} M\lambda + (S-n)\alpha & \text{if } n = 0,1,\ldots,S \\ (M+S-n)\lambda & \text{if } n = S+1,\ldots,L-1 \\ 0 & \text{otherwise} \end{cases}$$

and

$$\mu_n = \begin{cases} 0, & i=0, 0 \le n \le N-1 \\ n\mu, & i=1 \text{ and } 1 \le n \le c-1 \\ c\mu, & i=1 \text{ and } c \le n \le L \end{cases}$$

Using birth death process, we obtain the governing equations (in terms of Lapalce of probabilities) for our model as follows:

$$(s+\lambda_0)\overline{P}_{0,0}(s) - \mu\, p_{1,1}(s) = p_{0,0}(0) \tag{1}$$

$$(s+\lambda_n)\overline{P}_{0,n}(s) - \lambda_{n-1}\overline{P}_{0,n-1}(s) = p_{0,n}(0), \quad 1\le n\le N-1 \tag{1.2}$$

$$(s+\lambda_1+\mu)\overline{P}_{1,1}(s) - \lambda_0\overline{P}_{1,0}(s) - 2\mu\overline{P}_{1,2}(s) = p_{1,1}(0) \tag{1.3}$$

$$(s+\lambda_{n-1}+(n-1)\mu)\overline{P}_{1,n-1}(s) - \lambda_{n-2}\overline{P}_{1,n-2}(s) - n\mu\overline{P}_{1,n}(s) = p_{1,n-1}(0) \qquad 2\le n\le c-1 \tag{1.4}$$

$$(s+\lambda_n+c\mu)\overline{P}_{1,n}(s) - \lambda_{n-1}\overline{P}_{1,n-1}(s) - c\mu\overline{P}_{1,n+1}(s) = p_{1,n}(0) \qquad c\le n\le N-1 \tag{1.5}$$

$$(s+\lambda_N+c\mu)\overline{P}_{1,N}(s) - \lambda_{N-1}\overline{P}_{1,N-1}(s) - \lambda_{N-1}\overline{P}_{0,N-1} - c\mu\overline{P}_{1,N+1}(s) = p_{1,N}(0) \tag{1.6}$$

$$(s+\lambda_n+c\mu)\overline{P}_{1,n}(s) - \lambda_{n-1}\overline{P}_{1,n-1}(s) - c\mu\overline{P}_{1,n+1}(s) = p_{1,n}(0) \qquad N+1\le n\le L-2 \tag{1.7}$$

$$(s+\lambda_{L-1}+c\mu)\overline{P}_{1,L-1}(s) - c\mu\overline{P}_{1,L} - \lambda_{L-2}\overline{P}_{1,L-2}(s) = p_{1,L-1}(0) \tag{1.8}$$

$$s\overline{P}_{1,L}(s) - \lambda_{L-1}\overline{P}_{1,L-1}(s) = p_{1,L-1}(0) \tag{1.9}$$

Also we note that

$p_{0,0}(0) = 1$; $p_{0,n}(0) = 0$ for n = 1, 2, ..., N -1

$p_{1,n}(0) = 0$ for n = 1, 2, ..., L.

The above set of equations (1.1-1.9) can be written in the matrix form as

$$A(s)\,\overline{P}(s) = P(0) \qquad (2)$$

where A is the square matrix of order (N+L) × (N+L) and $\overline{P}(s)$ and P(0) are column vectors of order (N+L)× 1. Matrix A is shown is Figure 1, and the matrices $\overline{P}(s)$ and P(0) are given respectively as

$\overline{P}(s)=[p_{0,0}(s), p_{0,1}(s)\ldots, p_{0,N-1}, p_{1,1}(s)\ldots, P_{1,N+1}(s), P_{1,N+1}(s)\ldots, P_{1,L}]^T$
and
$\overline{P}(0)=[p_{0,0}(s), p_{0,1},(0)\ldots, p_{0,N-1}, p_{1,1}(0),\ldots P_{1,N+1}(0), P_{1,N+1}(0)\ldots, P_{1,L}]^T$

Equation 2 can be solved by Cramer's rule for $\overline{P}_{1,L}(s)$ as

$$\overline{P}_{1,L}(s) = \frac{|A_{N+L}(s)|}{|A(s)|} \qquad (3)$$

Before giving the solution for $|A_{N+L}(s)|$, the sequence of tridiagonal matrices should be constructed as follows:

$$A_1 = (a_{11})$$

$$A_2 = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$A_3 = \begin{pmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & a_{23} \\ 0 & a_{32} & a_{33} \end{pmatrix}$$

$$A_{m-1} = \begin{pmatrix} a_{11} & a_{12} & 0 & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & 0 & 0 & 0 \\ 0 & \ddots & \ddots & \ddots & \vdots & \vdots \\ \vdots & \cdots & \ddots & \ddots & a_{m-2,m-2} & a_{m-2,m-1} \\ 0 & \cdots & \cdots & \cdots & a_{m-1,m-2} & a_{m-1,m-1} \end{pmatrix}$$

where

$$a_{kp} = \begin{cases} -\mu_{k+1} & \text{for} \quad p=k+1 \\ s+\lambda_k+\mu_k & \text{for} \quad p=k \\ -\lambda_{k-1} & \text{for} \quad p=k-1 \\ 0 & \text{for} \quad |p-k|\geq 2 \end{cases}$$

Also let

$\Delta_1(s) = |A_1|$

$\Delta_2(s) = |A_2|$
::::::::::::::::::
$\Delta_{m-1}(s) = |A_{m-1}|$

For $\Delta_0(s) = 1$ and using recursive formula, we obtain

$$\Delta_k(s) = a_{kk}\,\Delta_{k-1}(s) - \lambda_{k-1}\,\mu_k\,\Delta_{k-2}(s) \qquad (4)$$

where k = 1, 2, ..., m - 1 (m≥2)

The complete solution for $|A_{N+L}(s)|$ is given by

$$|A_{N+L}(s)| = \left[\prod_{k=0}^{L-1} \lambda_k\right] \Delta_{N-1}(s)$$

$$A = \left|\begin{array}{ccccccccccccccccccccccc}
s+\lambda_0 & 0 & 0 & \cdots & 0 & 0 & -\mu & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
-\lambda_0 & s+\lambda_1 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
0 & -\lambda_1 & s+\lambda_2 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & -\lambda_{N2} & s+\lambda_{N1} & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
0 & 0 & 0 & \cdots & 0 & 0 & s+\lambda_1+\mu & -2\mu & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
0 & 0 & 0 & \cdots & 0 & 0 & -\lambda_1 & s+\lambda_2+2\mu & 3\mu & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & -\lambda_{r-1} & s+\lambda_r+\mu c & -\mu c & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & -\lambda_r & s+\lambda_{r+1}+\mu c & -\mu c & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & 0 & -\lambda_{N1} & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & -\lambda_{N-1} & s+\lambda_N+\mu c & -\mu c & 0 & \cdots & 0 & 0 & 0 \\
0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & -\lambda_N & s+\lambda_{N+1}+\eta c & -\mu c & \cdots & 0 & 0 & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\
0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & \lambda_{L2} & s+\lambda_{L1}+\mu c & \mu c \\
0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 & -\lambda_{L1} & s
\end{array}\right|$$

Figure. 1. Matrix A.

$$\Delta_{N-1}(S)=\begin{vmatrix} s+\lambda_1+\mu & -2\mu & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ -\lambda_1 & s+\lambda_2+2\mu & -3\mu & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & 0 & 0 \\ 1 & 1 & 1 & \ddots & 1 & 1 & 1 & 1 & 1 & \ddots & 1 & 1 \\ 0 & 0 & 0 & \cdots & -\lambda_{n-2} & s+\lambda_{n-1}+(c-1)\mu & -\mu c & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & -\lambda_{n-1} & s+\lambda_n+\mu c & -\mu c & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & -\lambda_n & s+\lambda_{n+1}+\mu c & -\mu c & \cdots & 0 & 0 \\ 1 & 1 & 1 & \ddots & 1 & 1 & 1 & 1 & 1 & \ddots & -\mu c & 1 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & s+\lambda_{N-2}+\mu c & -\mu c \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & 0 & \cdots & -\lambda_{N-2} & s+\lambda_{N-1}+\mu c \end{vmatrix}$$

where

Now we solve numerator $|A(s)|$. It is clear that s=0 is a root of $|A(s)| = 0$ which is a trivial solution. For a non-trivial one, let $s = -\gamma$, then we have

$$A(-\gamma) = A(0) - \gamma I \qquad (6)$$

where γ is an eigen value, and I is the identity matrix. From Equations 2 and 6, we have

$$A(-\gamma)\,\overline{P}(s) = (A - \gamma I)\overline{P}(s) = p(0)$$

Let $\gamma_1, \gamma_2, \ldots, \gamma_i$ be i distinct non-zero real eigen values and $(\gamma_{i+1}, \overline{\gamma}_{i+1})$, $(\gamma_{i+2}, \overline{\gamma}_{i+2})$, ..., $(\gamma_{i+j}, \overline{\gamma}_{i+j})$ be j pairs of distinct conjugate complex eigen values where i+2j=N+L-1. Hence $|A(s)|$ can be given by

$$|A(s)| = s\left\{\prod_{k=1}^{i}(s+\gamma_k)\right\}\left[\prod_{k=1}^{i}\left[s^2+(\gamma_{i+k}+\overline{\gamma}_{i+k})\,s+\gamma_{i+k}\,\overline{\gamma}_{i+k}\right]\right] \qquad (8)$$

Using Equations 5 and 8, Equation 4 gives

$$\overline{P}_{1,L}(s)=\frac{\left[\prod_{k=0}^{L-1}\lambda_k\right]\Delta_{N-1}(s)}{s\left[\prod_{k=1}^{i}(s+\gamma_k)\right]\left[\prod_{k=1}^{j}\left(s^2+(\gamma_{i+k}+\overline{\gamma}_{i+k})\,s+\gamma_{i+k}\,\overline{\gamma}_{i+k}\right)\right]}=$$

$$\frac{b_0}{s}+\frac{b_1}{s+\gamma_1}+\ldots+\frac{b_i}{s+\gamma_i}+\frac{c_1 s+d_1}{s^2+(s_{i+1}+\overline{\gamma}_{i+1})\,s+\gamma_{i+1}\,\overline{\gamma}_{i+1}}+\ldots$$
$$+\frac{c_j s+d_j}{s^2+(\gamma_{i+j}+\overline{\gamma}_{i+j})\,s+\gamma_{i+j}\,\overline{\gamma}_{i+j}} \qquad (9)$$

where

$$b_0=\frac{\left[\prod_{k=1}^{L-1}\lambda_k\right]\Delta_{N-1}(0)}{\left[\prod_{k=0}^{i}\gamma_k\right]\left[\prod_{k=1}^{i}\gamma_{i+k}\,\overline{\gamma}_{i+k}\right]} \qquad (10)$$

and

$$b_r=\frac{\left[\prod_{k=0}^{L-1}\right]\Delta_{N-1}(-\gamma_r)}{(-\gamma_r)\left[\prod_{k=1}^{i}(\gamma_k-\gamma_r)\right]\left[\prod_{k=1}^{j}\gamma_r^2+(\gamma_{i+k}+\overline{\gamma}_{i+k})(-\gamma_r)+\gamma_{i+k}\,\overline{\gamma}_{i+k}\right]},$$

$$1 \le r \le i \qquad (11)$$

Similarly from Equation 9, we have

$$c_r(-\gamma_{i+r})+d_r=$$
$$\frac{\left[\prod_{k=0}^{L-1}\lambda_k\right]\Delta_{N-1}(-\gamma_{i+r})}{(-\gamma_{i+r})\left[\prod_{k=1}^{i}(\gamma_k-\gamma_r)\right]\left[\prod_{k=1}^{j}(-\gamma_{i+r})^2+(\gamma_{i+k}+\overline{\gamma}_{i+k})(-\gamma_{i+r})+(\gamma_{i+k}\,\overline{\gamma}_{i+k})\right]} \qquad (12)$$

Inverting Lapalce transform in Equation 9, we get

$$P_{1,L}(t) = b_0 + \sum_{r=1}^{i} b_r e^{-\gamma_r t} +$$

$$\sum_{r=1}^{i}\left[b_r e^{u_r t}\cos(v_r t) + \frac{d_r - c_r u_r e^{u_r t}\sin(v_r t)}{v_r}\right] \qquad (13)$$

where u_r and v_r represent the real and imaginary parts of the complex number γ_{i+r} and b_0, b_r, c_r, and d_r all are real numbers.

Since the system has failed during the infinite period of time, therefore $\lim_{t \to \infty} p_{1,L}(t) = a_0 = 1$. Hence the reliability function is given by,

$$R(t) = 1 - p_{1,1}(t) \qquad (14)$$

The mean time to system failure (MTTF) is given by

$$\text{MTTF} = \int_0^\infty R(t)\,dt = \lim_{t \to 0} \bar{R}(s)$$
$$= -\sum_{k=1}^{i} \frac{b_k}{\gamma_k} - \sum_{k=1}^{i} \frac{d_k}{\gamma_{i+k}\,\gamma_{i+k}} \qquad (15)$$

where b_k and d_k can be determined by Equations 11 and 12 respectively.

NUMERICAL ILLUSTRATION

In order to depict the effect of various parameters on reliability, we consider a numerical example by choosing M=7, μ=1.0. Figures 2-6 give the time variations of reliability by varying different parameters. In Figure 2 reliability R(t) is plotted for different values of α and λ = 0.3, 0.5. It should be noted that R(t) is insensitive with respect to α as curves are almost concident for α = 0.1, 0.2, 0.3. Therefore, for other figures we set α = 0.1. Figure 3 exhibits that R(t) decreases with N. Figures 4 and 5 display the effect of S and K respectively on R(t). Figure 4 shows that R(t) improves significantly by employing more spare parts. We notice in Figure 5 that by increasing K, R(t) decreases drastically. In Figure 6, by varying λ we observe that reliability decreases as λ increases.

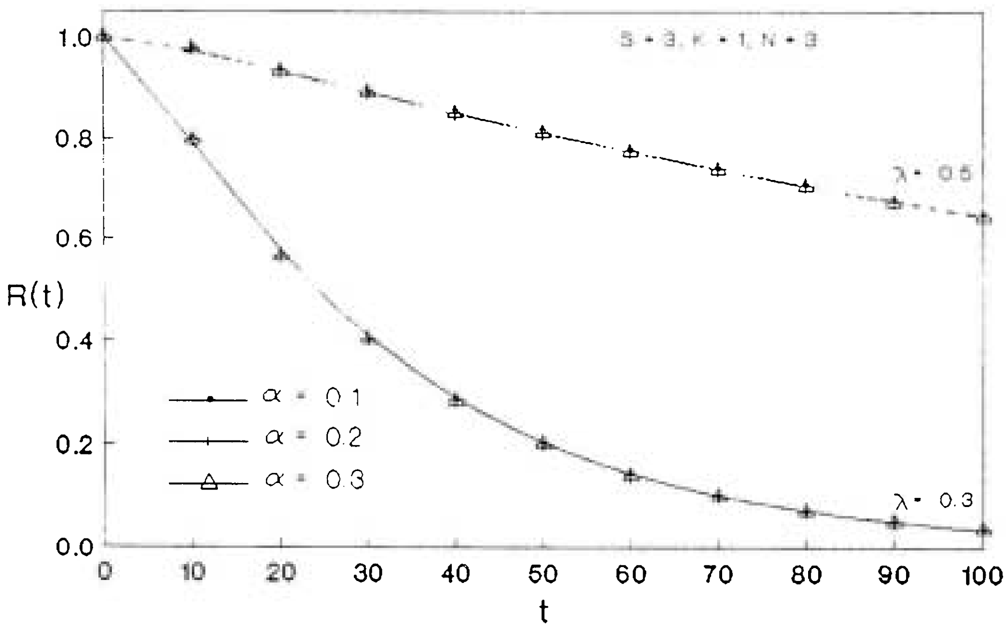


Figure 2. Reliability vs.

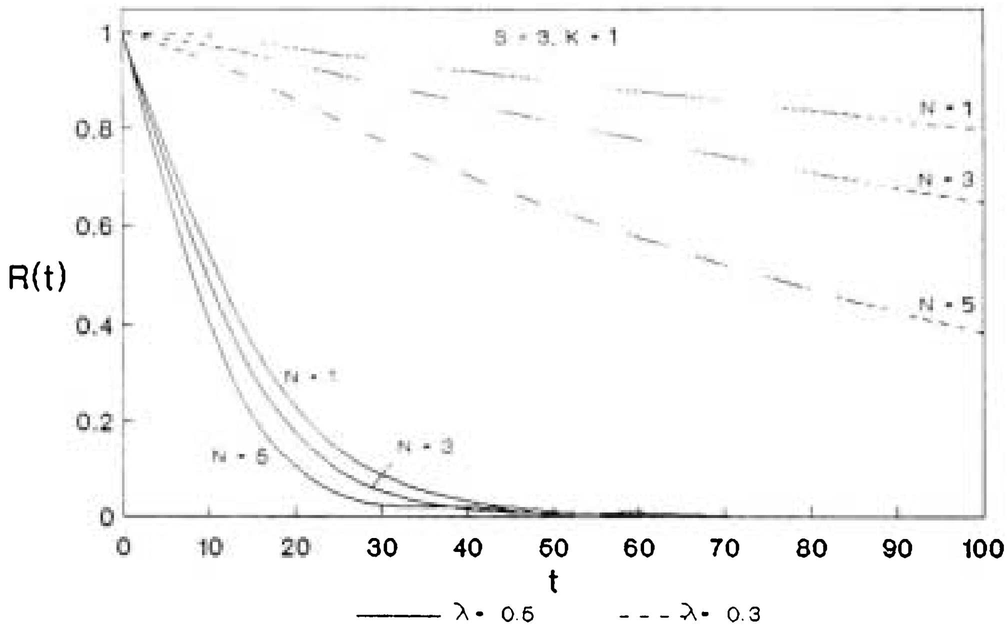


Figure 3. Reliability vs. N.

In Tables 1 and 2, mean times to failure (MTTF) are summerized by taking different set of (N,S) and varying λ. It should be noted that by providing more spares, the mean time to failure increases. Also MTTF decreases significantly as failure rate λ increases.

DISCUSSION

Spare provisioning plays a crucial role in electronic, computer and communication systems as well as in defence. As huge amount of money is involved to

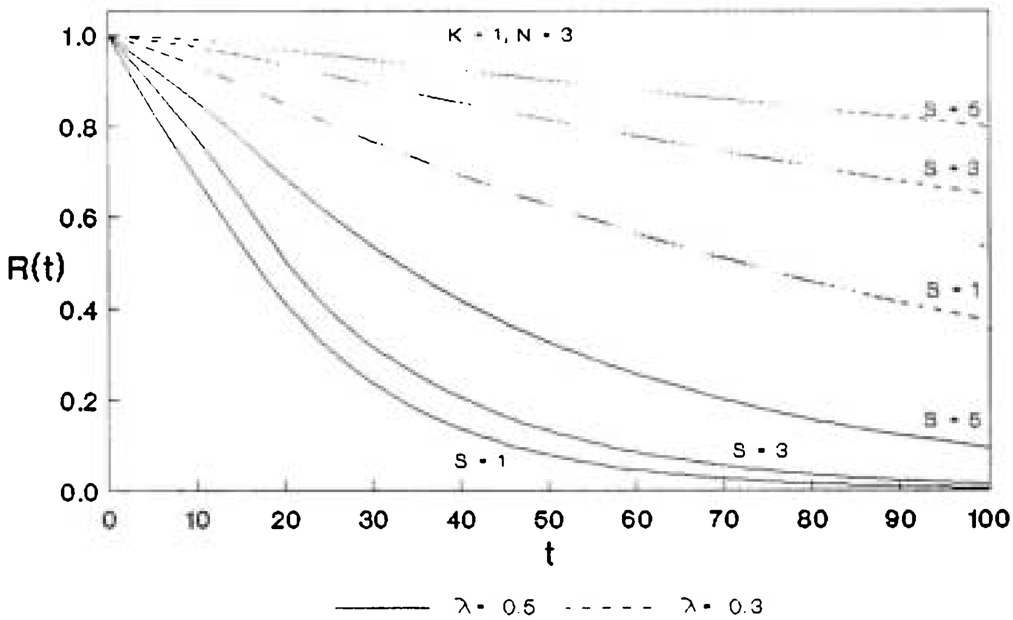
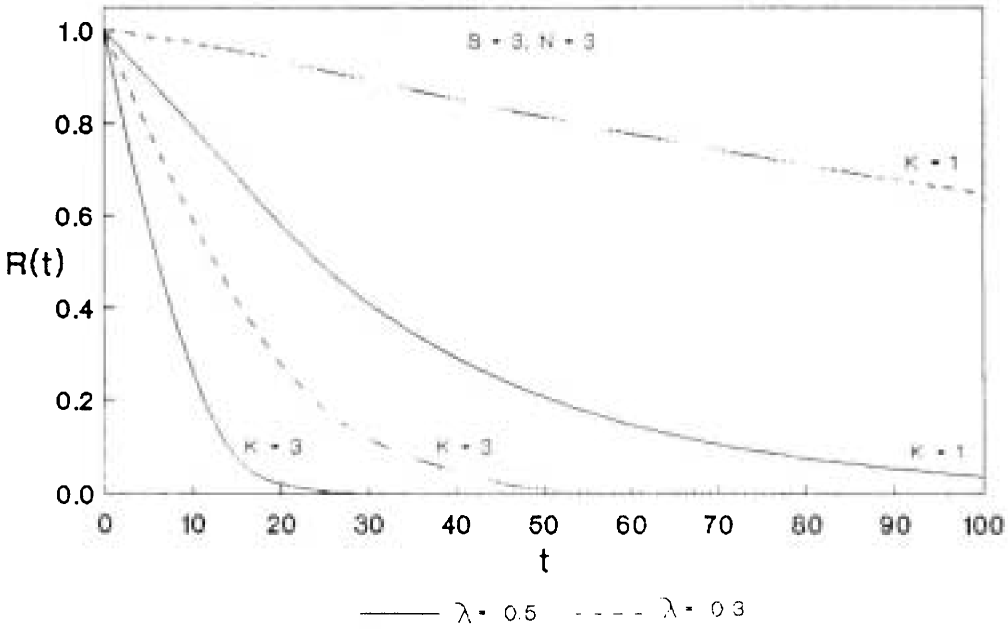


Figure 4. Reliability vs. S.



Fgiure 5. Reliability vs. K.

provide spares, the provision of removable repair facility may be helpful to reduce the burden to some extent, specially for large systems like aircraft and nuclear submarine, which have hundreds of components. The combination of spare units and removable repairmen employed in the present investigation may also be of great importance in various complex situations of machining system in the production processes of new technology world to minimize the espected cost function. We have derived

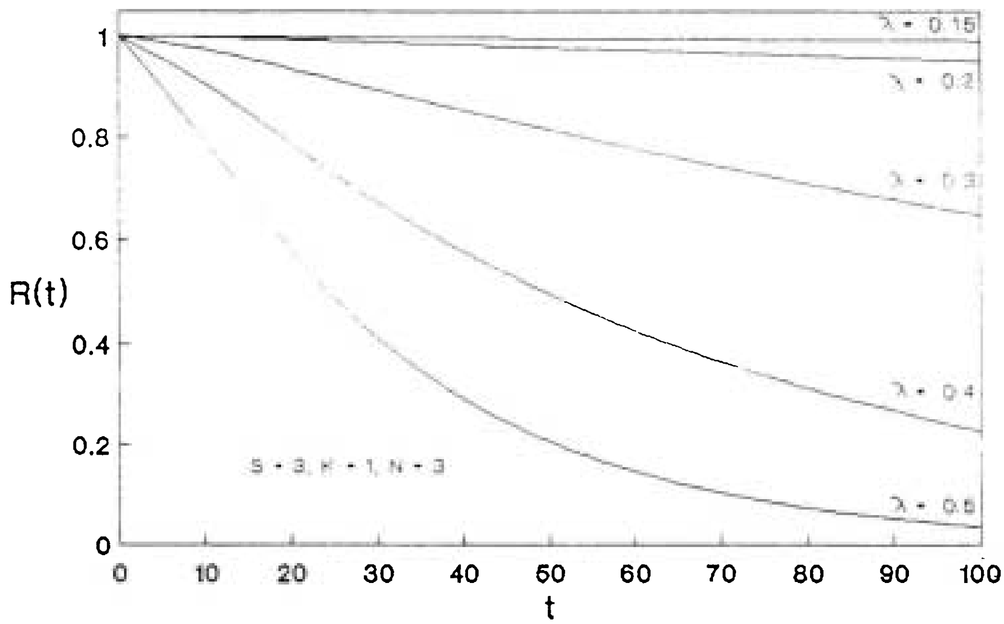


Figure 6. Reliability vs. λ.

TABLE 1. Mean Time to Failure (MTTF).

λ	(N,S)		
	(1,1)	(2,2)	(3,3)
0.2	185.4961	191.7070	196.8141
0.3	42.44509	43.20683	43.83783
0.4	19.79727	20.20192	20.56186
0.5	12.24690	12.54593	12.82229
0.6	8.706705	8.949082	9.177359
0.7	6.704012	6.909126	7.104577
0.8	5.431003	5.609324	5.780722
0.9	4.555735	4.713715	4.866381
1.0	3.919238	4.061087	4.198872

TABLE 2. Mean Time to Failure (MTTF).

λ	(N,S)		
	(3,5)	(5.3)	(5,5)
0.2	207. 4671	187.7974	204.0853
0.3	45.42527	42.92724	44.81965
0.4	21.51797	20.16357	21.20558
0.5	13.52829	12.58714	13.33570
0.6	9.740590	9.021921	9.610224
0.7	7.573660	6.994266	7.479671
0.8	6.182906	5.698308	6.111980
0.9	5.218416	4.802582	5.163006
1.0	4.512025	4.148036	4.467678

expressions for reliability and mean time to failure which can be computed easily as shown through a numerical illustration. These system characteristics may be helpful for system designer to determine the optimal number of repairmen and spare units.

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