NUMERICAL SOLUTION FOR HEAVE OF EXPANSIVE SOILS

S. A. Sadrnezhad

Department of Civil Engineering
Khajeh-Nasirodin Toosi University of Technology
Tehran, Iran

Abstract A numerical solution for heave prediction is developed within the context theories for both saturated and unsaturated soil behaviors. Basically, lowering the potential level of compressing on a saturated layer will cause heaving due to water absorption. This water absorption is in an opposite way, similar to water dissipation as what happens during unloading in consolidation process. However, in unsaturated layers any change of the stability of potential energy level will cause the tendency of change in particle interconnection forces. So, any change by either distressing or the variation of moisture ratio will lead to soil heave. In this paper a finite element solution is employed for predicting the heave in saturated soil similar to unloading in consolidation. Also, in the case of unsaturated soil, equivalent soil suction as negative pore water pressures is applied to soil elements as equivalent nodal forces. To show the potential of this mehtod, test results were compared with those obtained from computations. These comparisons show that the presented method is capable of predicting the heave phenomenon quite well.

Key Words Heave, Numerical Solution, Finite Elements, Equilibrium, Continuity, Equivalent Suction, Nodal Force, Swelling Pressure

چکیده بر اساس تئوریهای رفتار خاکهای اشباع و نیمه اشباع تعیین میزان تورم بصورت حل عددی ارائه شده است. اساساً با کاهش سطح پتانسیل فشار بر روی لایه های خاکی اشباع تغییر شکلهایی در اثر جذب آب بصورت تورم حاصل می گردد. روال این جذب آب برعکس روال خروج آب در اثر تحکیم خاک اشباع و مشابه باربرداری از لایه در حال تحکیم در نظر گرفته شده است. این پدیده در هر حال بخاطر تغییر در نیروهای بین ذره ای خاک در اثر تغییر سطح انرژی پتانسیل حاصله در مجموعه می باشد. بر این اساس هر تغییر در اثر کاهش تنشها و یا تغییر در میزان رطوبت خاک باعث تورم خواهد گردید. در این مقاله به کمک روش اجزاء محدود، تورم خاکهای اشباع بصورت مشابه با باربرداری در تحکیم خاک پیش بینی گردیده است. در صورت نیمه اشباع بودن خاک، فشار موجود منفی بصورت بارهای معادل گرهی بر گره های اجزاء، اثر داده شده اند. جهت نمایش آثار روش ارائه شده، نتایج آزمایشات با نتایج محاسباتی مقایسه گردیده اند. این مقایسه حاکی از آنست که روش ارائه شده قادر به پیش بینی تورم خاک می باشد.

INTRODUCTION

In recent years, a number of constructive models of the behavior of soil consolidation subjected to monotonic, cyclic or transient loads have been proposed. A large number of these models follow the mathematical theory of plasticity in fully coupled or unsaturated condition. They attempt to match the experimental behavior of soils under the standard test configurations such as triaxial compression or extension. An investigation of currently proposed models reveals that Darcy law is mostly valid for loading and unloading behavior of saturated soils. However, the contribution of non-Darcian flow can be expected to unsaturated soil. Given the known complexity of clay particle/adsorbed water/free water interaction, it seems unlikely that the present approach to heave modeling would suffice.

The structures founded on unsaturated layers of soil, usually, suffer from sever distresses subsequent to their construction. Changes in the environment around the structure result in changes in the negative pore water pressure, thereby producing volume changes in the soil. The prediction of the heave of cohesive layers of soil, commonly requires a knowledge about the initial in situe of stress, swelling potential and also the final state of stress [1]. Normally, the initial stress state and swelling potential as swelling index are obtained from one dimensional odometer tests. However, the final stress state may be strongly influenced by local experience [1]. Any change in stress state changes the potential level between the initial and the final conditions together with the swelling index which is used to predict the amount of heave. Therefore, the most improtant parameter needed to predict the heave of the soil is the swelling pressure. The swelling pressure represents the initial in situe stress state of the soil. This pressure is generally measured in a one dimensional odometer test using either the method of constant volume or the free swelling procedures[2]. The real swelling pressure represents the in situe state translated to the net normal stress plane that is equal to the overburden pressure plus the matrix suction equivalent [3].

THE CONSTITUTIVE EQUATIONS

The state of stress in an element of soil is represented by effective stress $\sigma' = \sigma'_x, \sigma'_y, \sigma'_z, \tau_{xy}, \tau_{yz}, \tau_{zx}$ and pore water pressure **u**. Thus effective stress is written as:

$$\sigma = \sigma' + mu \tag{1}$$

where

$$m = \{1, 1, 1, 0, 0, 0\}$$

When analyzing the consolidation of either saturated or partly saturated soils, if the drainage is allowed to occur in all directions, the continuity conditions for liquid, solid, and gaseous phases take the following forms respectively:

$$\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} + \frac{\partial V_z}{\partial z} + \frac{\partial n_w}{\partial t} = 0 \tag{2}$$

$$\frac{\partial \overline{V_x}}{\partial x} + \frac{\partial \overline{V_y}}{\partial y} + \frac{\partial \overline{V_z}}{\partial z} + \frac{\partial (1\text{-}n)}{\partial t} = 0 \tag{3}$$

$$\frac{\partial w_x}{\partial x} + \frac{\partial w_y}{\partial y} + \frac{\partial w_z}{\partial z} + \frac{\partial n_a}{\partial t} + \beta \frac{\partial u}{\partial t} = 0 \tag{4}$$

$$\beta = (e_a + C_h e_w)/(u_0 + u) \tag{5}$$

where V stands for the apparent drainage velocity of water, $\overline{V}$ for the apparent velocity of the solid grain movements and w for the air, n for the porosity, C_h for Henry's constant of solubility of air in water, and the suffix w and a for water and air respectively.

The generalized continuity conditions lead to the following general equation of three dimensional consolidation:

$$(1 + e)\left[\frac{\partial}{\partial x}\left(k_x \frac{\partial u}{\gamma_w \partial x}\right) + \frac{\partial}{\partial y}\left(k_y \frac{\partial u}{\gamma_w \partial y}\right) \frac{\partial}{\partial z}\left(k_z \frac{\partial u}{\gamma_w \partial z}\right)\right] = \frac{\partial e}{\partial t} + \beta \frac{\partial u}{\partial t} \tag{6}$$

For saturated soil the coefficient β is zero. In isotropic conditions k_x, k_y, and k_z are equal to k. The use of Equation 6 leads to obtain the following equation:

$$\frac{\partial \sigma'_{oct}}{\partial t} - \frac{k(1+e)}{\gamma_w \frac{\partial e}{\partial \sigma_{oct}}}\left[\left(\frac{\partial u^2}{\partial x^2}\right) + \left(\frac{\partial u^2}{\partial y^2}\right) + \left(\frac{\partial u^2}{\partial z^2}\right)\right] = \left(1 - \frac{\beta}{\frac{\partial e}{\partial \sigma_{oct}}}\right) \frac{\partial e}{\partial t} \tag{7}$$

However, attempting to apply a finite element method solution to saturated condition, one has to embark on achieving the constitutive relationships from the basic laws of continuity and equilibrium.

Any stress-strain relationship may be assumed over limited stress intervals. Consequently, the following expression can be substituted for $\partial e / \partial \sigma'_{oct}$.

$$\frac{\partial e}{\partial \sigma_{oct}} = -\frac{1+e}{K} \tag{8}$$

$$\varepsilon_v = \frac{3(1-2v)'}{E'}\sigma'_{oct} = \varepsilon_x + \varepsilon_y + \varepsilon_z \quad (9)$$

where K denotes the modulus of compressibility. The linear relationship between effective spherical stress and volume strain has been assumed without taking into account the influence of seepage forces. Therefore, Equation 7 can be written in following form:

$$\frac{k}{\gamma_w}\left[\left(\frac{\partial^2 u}{\partial x^2}\right) + \left(\frac{\partial^2 u}{\partial y^2}\right) + \left(\frac{\partial^2 u}{\partial z^2}\right)\right] = -\frac{\partial \varepsilon_v}{\partial t} \quad (10)$$

where

$$\frac{\partial \varepsilon v}{\partial t} = -\frac{\partial u}{\partial t} m + \mathrm{M} \quad (11)$$

$$m = \frac{\partial \varepsilon_v}{\partial \sigma'_1} + \frac{\partial \varepsilon_v}{\partial \sigma'_2} + \frac{\partial \varepsilon_v}{\partial \sigma'_3} \quad (12)$$

$$\mathrm{M} = \frac{\partial \varepsilon_v}{\partial \sigma'_1} \times \frac{\partial \sigma_1}{\partial t} + \frac{\partial \varepsilon_v}{\partial \sigma'_2} \times \frac{\partial \sigma_2}{\partial t} + \frac{\partial \varepsilon_v}{\partial \sigma'_3} \times \frac{\partial \sigma_3}{\partial t} \quad (13)$$

In the case of constant surface tractions **M** is equal to zero. Otherwise, volumetric strain can be written as:

$$\varepsilon_v = \mathrm{m}^T \varepsilon \quad (14)$$

In finite element framework $\varepsilon = \mathbf{B}\ \delta$ and also, δ and **u** are written as follows;

$$\delta = \sum_{i=1}^{m} \mathrm{N}_i \delta_r \quad (15a)$$

$$\mathrm{u} = \sum_{i=1}^{m} \mathrm{N}_i \mathrm{p}_r \quad (15b)$$

where N_i is the shape function of the position in terms of thickness, δ_r and p_r are, respectively, the nodal displacements and excess pore water pressure. Substituting these two equations and minimizing the residual obtained by Galerkin's method give:

$$\int_v \mathrm{N}_r \left[\frac{\partial}{\partial x}\left(\frac{k\,\partial u}{\gamma_w \partial x}\right) + \frac{\partial}{\partial t}\left(\frac{\partial \delta}{\partial x}\right) + \frac{\partial}{\partial y}\left(\frac{k\,\partial u}{\gamma_w \partial y}\right) + \frac{\partial}{\partial t}\left(\frac{\partial \delta}{\partial y}\right) + \frac{\partial}{\partial z}\left(\frac{k\,\partial u}{\gamma_w \partial z}\right) + \frac{\partial}{\partial t}\left(\frac{\partial \delta}{\partial z}\right)\right] dv = 0 \quad (16)$$

This can be written more concisely as:

$$[H]\{p\} - [L]\{\mathring{\delta}\} = \{g\} \quad (17)$$

where

$$[H] = \int_v \left[\frac{k\,\partial N_r \partial N_i}{\gamma_w \partial x \partial x} + \frac{k\,\partial N_r \partial N_i}{\gamma_w \partial y \partial y} + \frac{k\,\partial N_r \partial N_i}{\gamma_w \partial z \partial z}\right] dv \quad (18)$$

$$[L] = \int_v N_r\left[\frac{\partial N_i}{\partial x} + \frac{\partial N_i}{\partial y} + \frac{\partial N_i}{\partial z}\right] dv \quad (19)$$

$$\{g\} = N_r\left[\frac{k\partial p}{\gamma_w \partial x} + \frac{k\partial p}{\gamma_w \partial y} + \frac{k\partial p}{\gamma_w \partial z}\right] dv \quad (20)$$

It is noted that both matrices **H** and **g** include the coefficient of permeability and hence their respective values vary with any change in this coefficient, whereas the matrix **L** remains constant.

The continuity equation in finite element framework as substituted form in virtual work equation gives:

$$\int_v [B]^T\{\sigma\}dv - \int_v [B]^T\{m\}[N_p]\{p\}\,dv = \{f\} \quad (21)$$

where $\{f\} = \int_v [N_i]^T\{d\sigma\}dv$.

The concept of linear elasticity for the behavior of soil skeleton gives the stress/strain relationship as follows:

$$\{\sigma\} = [D](\{\varepsilon\} - \{\varepsilon_0\}) \quad (22)$$

Where D is the elasticity matrix, ε and ε_0 total and initial strain vectors. The substitution of Equations 14 and 20 in 21 yields:

$$\int_v [B]^T [D] [B] \{\delta\} dv - \int_v [B]^T \{m\} [N_p] \{p\} dv = \{f\} + \int_v [B]^T [D] \{\varepsilon_0\} dv \quad (23)$$

This equation also is written in matrix form as:

$$[K] \{\delta\} - [L] \{p\} = \{f\} \quad (24a)$$

and in incremental form:

$$[K] \{\overset{\circ}{\delta}\} - [L] \{\overset{\circ}{P}\} = \{\overset{\circ}{f}\} \quad (24b)$$

Where

$$[K] = \int_v [B]^T [D] [B] dv \quad (25a)$$

$$[L] = \int_v [B]^T \{m\} [N_p] dv \quad (25b)$$

$$\{f\} = \int_v [N]^T \{b\} dv + \int_A [N]^T \{dt\} dA + \int_v [B]^T [D] \{\varepsilon_0\} dv \quad (25c)$$

$$\{\overset{\circ}{f}\} = \int_v [N]^T \{db\} dv + \int_A [N]^T \{dt\} dA + \int_v [B]^T [D] \{\varepsilon_0/\partial t\} dv \quad (25d)$$

To obtain a general solution, the combination of Equations 17 and 24b can be stated in matrix form as:

$$\begin{bmatrix} K & -L \\ -L^T & 0 \end{bmatrix} \begin{Bmatrix} \dot{\delta} \\ \dot{p} \end{Bmatrix} - \begin{bmatrix} 0 & 0 \\ 0 & H \end{bmatrix} \begin{Bmatrix} \delta \\ p \end{Bmatrix} = \begin{Bmatrix} \dot{\delta} \\ -p \end{Bmatrix} \quad (26)$$

The solution of these equations represents the values of the two unknowns; pore water pressures and deformations.

THREE LEVEL FINITE DIFFERENCE TIME SCHEME

The dependence of the variables, deformations and excess pore water presures, on time are overcome by employing a finite difference three level scheme. This three level system is a typical central difference form as follows:

$$\phi^{(t)} = \frac{\phi^{(t-\Delta t)} + \phi^{(t)} + \phi^{(t+\Delta t)}}{3} \quad (27)$$

As a three level method requires two initially known values of the function ϕ, it is inappropriate to employ this at the start of the process for as much as only half the requirement can be met. Applying this method in Equation 18 gives:

$$\begin{bmatrix} K & -L \\ -L^T & -\frac{2\Delta t}{3} H^T \end{bmatrix} \begin{Bmatrix} \delta^{t+\Delta t} \\ p^{t+\Delta t} \end{Bmatrix} = \begin{bmatrix} K & -L \\ -L^T & +\frac{2\Delta t}{3} H^T \end{bmatrix} \begin{Bmatrix} \delta^{t-\Delta t} \\ p^{t-\Delta t} \end{Bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & +\frac{2\Delta t}{3} H^T \end{bmatrix} \begin{Bmatrix} \delta^{t} \\ p^{t} \end{Bmatrix} + \begin{Bmatrix} 2\Delta t f \\ -2\Delta t g \end{Bmatrix} \quad (28)$$

$$[R]^{(t)} \{\phi\}^{(t+\Delta t)} = [R]^{(t)} \{\phi\}^{(t-\Delta t)} + [s]^{(t)} \{\phi\}^{(t)} + [T]^{(t)} \quad (29)$$

PARTLY SATURATED SOIL

Normally, there are negative pore water pressures in unloaded unsaturated soil. The term negative pore water pressure is taken here to mean any pressure deficiency which occurs in situe while subjected to some form of externally applied stress system. However, the suction of any element of soil, s, in the unloaded state, is modified by the effect of the overburden. This modified suction is, of course, the final pore water pressure, either positive or negative, that the element has reached. The equivalent hydrostatic pressure stands for suction can be obtained from either Cronley & Coleman [4] or Clisby [5].

EQUIVALENT NODAL FORCES TO SUCTION

In the finite element solution of the most complex boundary value problems, different effects are considered as equivalent nodal forces. In the problems of unsaturated soils simply the existence of suction can be changed by equivalent nodal forces according to energy equalization. Accordingly, the solution procedure is the same as the consolidation of saturated soil but the equivalent nodal forces are added to force vector at each time level. Consequently, upon the use of the same computer program both saturated and unsaturated consolidations and heavings of the soil are computed. For a single node having three degree of freedom in **x**, **y**, and **z** directions the equivalent nodal forces can be written as follows:

$$\begin{Bmatrix} f_x \\ f_y \\ f_z \end{Bmatrix} = \int_v B_i^T . \Delta p . dv \qquad (30)$$

where , Δp is equivalent hydrostatic pressure to suction which must be added to effective stresses.

RESULTS

The model abilities to predict correct *settlement/time* curve is predominatly the justification for the basic theory and assumptions, hence comparisons between experimental results and model predictions have been carried out in this form. To predict the swelling of a clay sample from Iranshahr, a western town in Iran, the clay in the odometer test considered as eight triangle plane strain 15 nodded elements. This sample with initial 10.68% moisture ratio under constant vertical surcharge of 0.25 kg/cm^2 is merged in water. The maximum swelling obtained after 24 hours and 27.7% moisture ratio. According to stress path shown in Figure 1, and employing Clisby method, equivalent swelling pressure was 4.5 kg/cm^2.

The employed parameters are presented in Table 1.

TABLE 1.The Employed Parameters.

Parameter	Values
x	0.038
λ	0.1082
M	0.94
$e_{cs} = \Gamma - 1$	0.621
ν'	0.25
K_o	0.6
$K_x = K_y$	1.2×10^{-6} cm/min

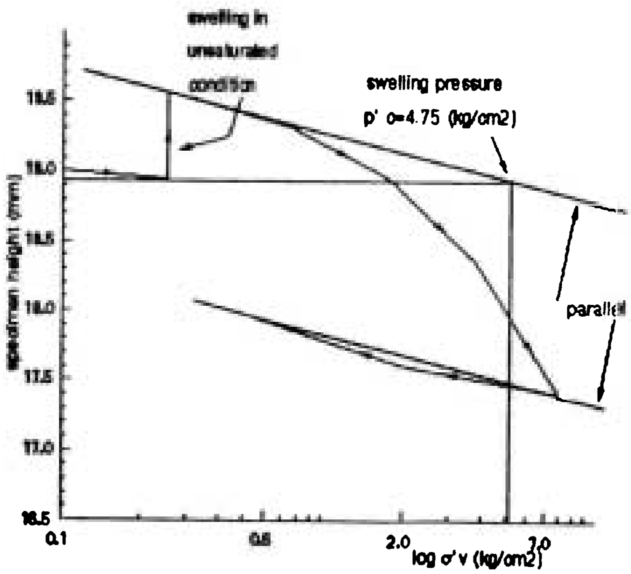

Figure 1. Stress path and procedure for finding swelling pressure.

Figure 2 shows the comparison of experimental and model results as *settlement/time* curves. This comparison shows that the model has been able to predict the swelling behavior of the introduced clay well.

Increasing the moisture ratio in the sample under water, the suction of mid-bottom node gradually vanishes. Figure 3 shows decreasing of soil suction at the stated node versus time. Also, at this point, decreasing the suction the effective vertical stress is decreased. The variation of vertical stress versus time is shown in Figure 4.

The effect of variation of permeability coefficient on the model results as *settlement/time* curves are also shown in Figure 5. However, there is no change in initial and final results but some changes in the mid-range. Therefore, this was intended as a measure of the model abilities.

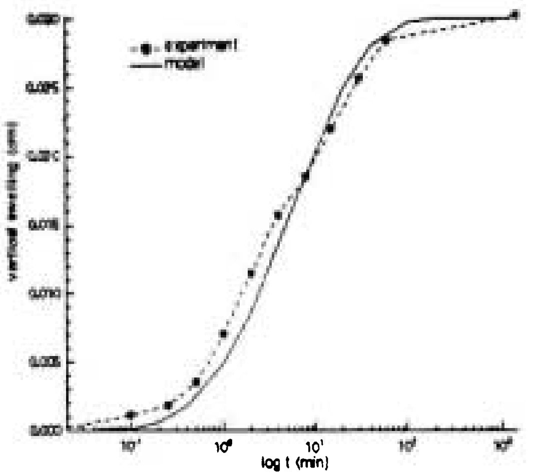

Figure 2. The comparison of experiemntal and model result in variation of settlement/time curves.

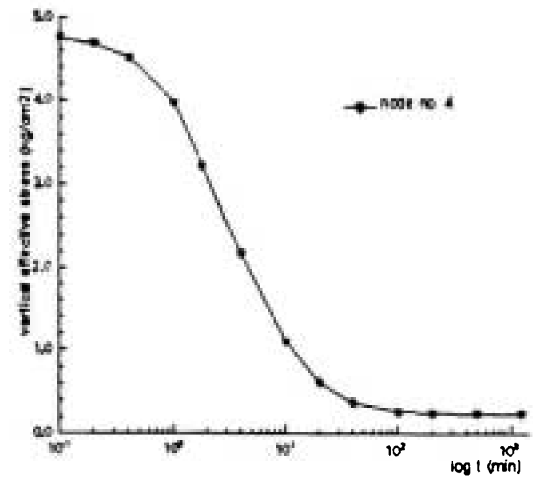

Figure 4. Reduction of vertical effective stress at mid-bottom point in the test sample versus time

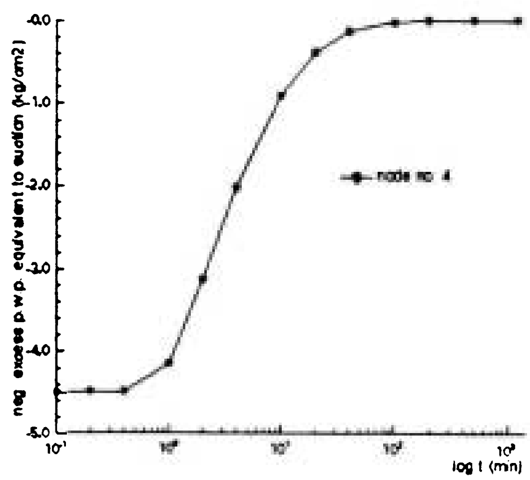

Figure 3. Reduction of soil suction at mid-bottom point in the test sample versus time.

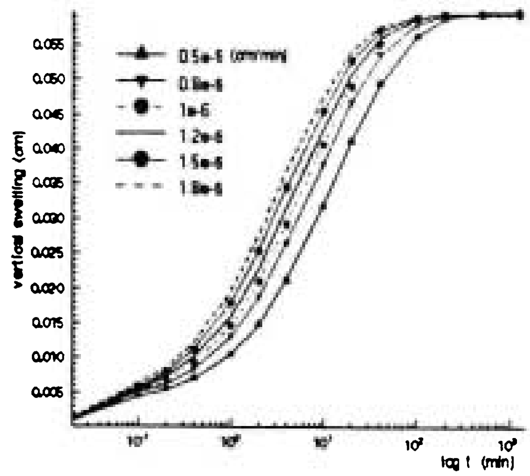

Figure 5. Effect of various values of permeability coefficients on model results.

CONCLUSION

The first objective of this research was the development of a model that could adequately and simultaneously describe the behavior of saturated and unsaturated swelling of clays. This had been achieved by a simple unloading in saturated soil and applying equivalent nodal forces to suction in unsaturated soil while employing the same procedure. Subsequent to the above achievement the validity of the proposed model was brought under scrutiny in a collection of the experimental results with the predictions of the model and this capabilities of the model have been demonstrated.

REFERENCES

1. D. G. Fredlund, "Prediction of Ground Movements in Swelling Clays, Annual Soil Mechanics and Foundation Engineering Conference, Minneapolis, Minn (1983).
2. D. G. Fredlund, "Consolidometer Test Procedural Factors Affecting Swll Properties. Proc., Second International Conference on Expansive Clay Soils Texas A & M University, College Station, Tex., 435-456.
3. R.T. Yoshida, D. G. Fredlund, and J. J. Hamilton, "The Prediction of Total Heave of a Slab-on-grade Floor on Regina Clay," *Can. Geotech. J.* 20 (1), 69 -81
4. D. Croney, and J. D. Colamn , "Soil Moisture Suction Properties and their Bearing on the Moisture Distribution in Soils. "Proc. 3rd. Int. Conf. Soil Mechanics and Foundation Engineering (1953).
5. K. Behnia and A. M. Tabaatabaaii, "Soil Mechanics", Tehran University Publications, (1988).
6. H. R. Thomas and S. A. Sadrnezhad, "Modeling One-dimensionsl Primary and Secondary Consolidation Using a non- Darcian Flow Approach", unpublished paper.