BOUNDARY LAYERS AND HEAT TRANSFER ON A ROTATING ROUGH DISK

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Abstract The study of flow and heat transfer over rotating circular disks is of great practical importance in understanding the cooling of rotatory machinery such as turbines, electric motors and design and manfacturing of computer disk drives. This paper presents an analysis of the flow and heat transfer over a heated infinite permeable rough disk. Boundary-layer approximation reduces the elliptic Navier-Stokes equations to parabolic equations, where the Keller-Cebeci method of finite-difference solution is used to solve the resulting system of partial-differential equations. The surface roughness is assumed to influence the turbulent boundary layer by adding a roughness parameter height K, while a variable surface temperature induces heat transfer into the flow of fluid over the rotating disk. Blowing and suction is also considered as a means of varying the surface shear distribution. The resulting curve-fit equations to the numerically calculated results of the skin-friction coefficient for three regions of laminar, transition and turbulent flow are shown to be consistent to those obtained for flow over a flat plate or a circular cylinder. To study the influence of surface roughness, calculations for various surface roughness parameters are made and results are presented. Velocity and temperature profiles and the shear stress and heat flux at the surface of the rotating disk are presented for a range of the above parameters.

Key Words Rotating Disk, Boundary Layer Flow and Heat Transfer, Finite Difference Scheme

چکیده مطالعه جریان حرکت سیال و انتقال حرارت بر روی دیسک دوار در ایجاد انتقال حرارت مفید در صنایع استفاده کننده از جمله، توربینها، موتورهای الکتریکی و طراحی و ساخت دیسکهای سخت کامپیوتری دارای اهمیت بسزائی است. در این مقاله به آنالیز جریان و انتقال حرارت بر روی یک دیسک حرارت داده شده زبر نفوذپذیر با ابعاد بی نهایت می پردازیم. تقریب لایه مرزی معادلات بیضوی ناویر استوکس را به معادلات سهموی تبدیل نموده، با استفاده از روش حل تفاضل محدود سیستم معادلات مشتق جزئی به حل می رسند. تاثیر زبری سطح بر روی لایه مرزی متلاطم با افزودن پارامتر ارتفاع زبری تقریب زده شده، در حالیکه دمای سطح متغیر باعث افزودن نرخ انتقال حرارت به جریان سیال بر روی سطح دیسک دوار می شود. دمش و مکش را نیز بعنوان روشی جهت تغییر تنش سطحی در نظر می گیریم. نتایج تولید شده مقادیر تنش برشی برای نواحی لایه ای، گذرا و متلاطم جریان که بصورت منحنی های خورانده شده به نتایج عددی می باشند و تطابق آن با نتایج مربوط به جریان سیال بر روی صفحه مسطح و یا استوانه دلالت بر صحت عملکرد مدل عددی می نماید. برای مطالعه تاثیر زبری سطحی و نتایج مربوط به ارتفاعهای مختلف زبری نشان داده شده است. پروفیلهای سرعت و دما و مقادیر تنش برشی و شار حرارتی ایجاد شده بر روی سطح دیسک دوار برای مقادیر مختلف پارامترهای تاثیر گذار ارائه شده است.

INTRODUCTION

The purpose of this paper is to investigate the problem of a disk rotating in an infinite quiescent fluid with emphasis on the effects of roughness on the convective heat transfer coefficient under the condition of variable wall temperature and transpiration. Depending on the radius, rotational velocity or the kind of fluid (Reynolds number of the flow), laminar, transitional or turbulent may exist. The problem arises in many industrial applications such as disk-drive industry where minimum spacing between head and medium is the objective for increasing the recording density. Investigation of the entire flow requires solution of the elliptic equations of motion, while the flow close to the disk can be approximated by parabolic boundary-layer equations.

Review of the literature indicates that the studies of fluids flow and heat transfer over a rotating disk have been the subject of several works [1-3]. Borisevich and Potanin [1] analyzed the boundary layer flows induced by rotation of a disk with surface inspirations on the basis of an approximate integration of the equations of motion. The solution of laminar convective heat transfer over a rotating disk with sinusoidaly-shaped surface roughness is presented by Palec et. al. [2,3]. On the other hand, there has been no work reported on investigating the phenomenon of convective heat transfer over a rough rotating disk with surface inspirations for laminar and turbulent regions of the flow. This paper, therefore, addresses this issue.

In this work, the boundary-layer equations describing the three-dimensional flow over the rotating disk is solved using an implicit finite difference scheme as described by Cebeci & Smith [4]. The boundary-layer approximation reduces the Navier-Stokes equations to a parabolic set of nonlinear partial differential equations. The resulting system of PDEs is then solved using an efficient implicit finite difference scheme. A nonuniform mesh is used and eddy viscosity concept models the turbulent Reynolds stress and heat flux terms. The model is validated using available experimental data.

METHOD OF SOLUTION

The time-averaged conservation equations for mass, momentum and energy in a stationary cylindrical co-ordinates for the steady incompressible axisymmetric boundary-layer on a rotating disk and in the absence of radial pressure gradient are:

$$\frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$u\frac{\partial u}{\partial r} + v\frac{\partial u}{\partial y} - \frac{w^2}{r} = \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y} - \rho\overline{u'v'}\right) \tag{2}$$

$$u\frac{\partial w}{\partial r} + v\frac{\partial w}{\partial y} + \frac{uw}{r} = \frac{1}{\rho}\frac{\partial}{\partial y}\left(\mu\frac{\partial w}{\partial y} - \rho\overline{v'w'}\right) \tag{3}$$

$$u\frac{\partial T}{\partial r} + v\frac{\partial T}{\partial y} = \frac{1}{\rho c_p}\frac{\partial}{\partial y}\left[-\frac{\partial q}{\partial y} - \mu\phi\right] \tag{4}$$

$$q = -\left(k\frac{\partial T}{\partial y} - \rho c_p\overline{T'v'}\right) \tag{5}$$

$$\mu\phi = \tau_r\frac{\partial u}{\partial r} + \tau_\phi\frac{\partial w}{\partial y} \tag{6}$$

$$\tau_r = \mu\frac{\partial u}{\partial y} - \rho\overline{u'v'} \tag{7}$$

$$\tau_\phi = \mu\frac{\partial w}{\partial y} - \rho\overline{v'w'} \tag{8}$$

Here, u, v, and w are the mean radial, axial, and tangential components of velocity, respectively.

For a given variable disk surface temperature, the appropriate boundary conditions for the above equations considering the effect of surface inspiration denoted by surface normal velocity v_m are:

$$y = 0,\ u = 0,\ v = v_w(r),\ w = \omega r \tag{9}$$

$$y \to \infty,\quad u = 0,\quad w = 0,\quad T = T_\infty \tag{10}$$

EDDY-VISCOSITY FORMULATION

The turblent boundary-layer formulation follows that of Cebeci & Smith [4] in which the boundary-layer is divided into inner and outer layer regions.

In the inner region of the boundary-layer, the turbulent viscosity is given by:

$$\varepsilon_i = L^2\left[\left(\frac{\partial u}{\partial y}\right)^2 + \left(\frac{\partial \overline{w}}{\partial y}\right)^2\right]^{\frac{1}{2}} \qquad 0 \le y \le y_c \qquad (11)$$

with

$$\overline{w} = \omega r - w \quad \text{and } L = \kappa y \left[1 - \exp\left(-\frac{y}{A}\right)\right] \qquad (12)$$

And, the outer region of boundary-layer,

$$\varepsilon_0 = 0.01681 \left| \int_0^\infty \left[\omega r - (u^2 + \overline{w}^2)^{0.5}\right] dy \right| \qquad y_c \le y \le \delta \ (13)$$

here y_c is where $\varepsilon_i = \varepsilon_0$.

For an aerodynamically unsmooth rotating disk, employing the concept of equivalent sand-grain roughness as a means of characterizing the surface roughness of the disk, and assuming that the plate has a uniform average roughness height κ_s, based on a roughness Reynolds number $\kappa^+_s = \kappa_s u_\tau / \nu$, we distinguish three regions of, aerodynamically-smooth wall:

$$\kappa^+_s < 5 \qquad (14)$$

transitional-roughness regime:

$$5 \le \kappa^+_s \le 70 \qquad (15)$$

fully-rough flow:

$$70 < \kappa^+_s < 2000 \qquad (16)$$

To account for the fact that L, and ε_i, cannot go to zero at the wall, i.e. the eddy diffusivity and mixing length must be finite at y=0, we denote L in Equation 11 as:

$$L = \kappa (y + \Delta y) [1 - \exp(-(y + \Delta y)/A)] \qquad (17)$$

with

$$\kappa = 0.4 \qquad (18)$$

and Δy is expressed as a function of equivalent sand-grain-roughness parameter κ^+_s, by defining Δy^+ as;

$$\Delta y^+ \equiv \frac{\Delta y u_\tau}{\nu} \qquad (19)$$

$$\Delta y^+ = 0 \qquad \kappa^+_s < 5 \qquad (20)$$

$$\Delta y^+ = 0.9\left[\sqrt{\kappa^+_s} - \kappa^+_s \exp(-\kappa^+_s/6)\right] \qquad 5 \le \kappa^+_s \le 70 \qquad (21)$$

$$\Delta y^+ = 0.7 (\kappa^+_s)^{0.58} \qquad 70 \le \kappa^+_s \le 2000 \qquad (22)$$

The effect of the surface inspiration is accounted for in the inner layer by the following relation;

$$A = A^+ \frac{\nu}{N} u_\tau^{-1} \qquad (23)$$

where;

$$A^+ = 26, \quad N = \{\exp(11.8 v^+_w)\}^{1/2}, \quad v^+_w = \frac{v_w}{u_\tau} \qquad (24)$$

In case there is no suction and blowing;

$$A = A^+ \ \nu u_\tau^{-1} \qquad (25)$$

Now, a similarity variable;

$$\eta = \left(\frac{\omega}{\nu}\right)^{\frac{1}{2}} y \qquad (26)$$

and a non-dimensional stream function $f(r, \eta)$, defined by:

$$f(r, \eta) = \frac{\psi(r, \eta)}{r^2 (\nu \omega)^{0.5}} \qquad (27)$$

are introduced. Here, the stream function ψ (r, η) is defined as;

$$ur = \frac{\partial \psi}{\partial z} \qquad wr = -\frac{\partial \psi}{\partial r} \tag{28}$$

Using the concept of eddy viscosity, the Reynolds stress terms can be written as:

$$-\rho\, \overline{u'v'} = \rho\, \varepsilon_m \left(\frac{\partial u}{\partial y}\right) \tag{29}$$

$$-\rho\, \overline{v'w'} = \rho\, \varepsilon_m \left(\frac{\partial w}{\partial y}\right) \tag{30}$$

$$-\rho\, c_p \overline{T'v'} = \rho\, c_p \frac{\varepsilon_m}{pr_t} \left(\frac{\partial T}{\partial y}\right) \tag{31}$$

Using the above transformation, the two momentum and energy Equations 2-4 become;

$$(\alpha f'')' + 2ff'' - (f')^2 + (g')^2 = r\left(f'\frac{\partial f'}{\partial r} - f''\frac{\partial f}{\partial r}\right) \tag{32}$$

$$(\alpha g'')' + 2fg'' - 2f'g' = r\left(f'\frac{\partial g'}{\partial r} - g''\frac{\partial f}{\partial r}\right) \tag{33}$$

$$(e_1 S')' + 2fS' + e_4 r^2 ((f'')^2 + (g'')^2) = r\left(f'\frac{\partial S}{\partial r} - S'\frac{\partial f}{\partial r}\right) \tag{34}$$

where

$$f'(r,\eta) = \frac{u}{\omega r}, \quad g'(r,\eta) = \frac{w}{\omega r}, \quad S(r,\eta) = \frac{T}{T_\infty}$$

$$e_4 = \frac{\omega^2}{c_p T_\infty}(1 + \varepsilon_m^+) \tag{35}$$

$$e_1 = \frac{1}{pr}\left(1 + \frac{pr}{pr_t}\varepsilon_m^+\right) \tag{36}$$

$$\alpha = (1 + \varepsilon_m^+) \tag{37}$$

Here the primes indicate differentiation with respect to η. In terms of the transformed variables, the boundary conditions, become;

$$\eta = 0,\ f' = 0,\ g' = 1,\ f_w = \frac{-1}{r^2 (\nu\,\omega)^{0.5}} \int_0^r v_w\, r\, dr \tag{38}$$

$$\eta = 0, \left\{ \begin{array}{c} S = S_w^{(r)} = \frac{T_w^{(r)}}{T_\infty} \\ \text{or} \\ \frac{\partial S}{\partial \eta} = -\frac{1}{K T_\infty}\left(\frac{\nu}{\omega}\right)^{1/2} \dot{q}''\, w(r) \end{array} \right\}$$

$$\eta \to \infty,\ f' = 0,\ g' = 0,\ S = 1 \tag{39}$$

NUMERICAL METHOD OF SOLUTION

Equations 32-34 subject to boundary Conditions 38 and 39 are transformed into a system of eight first-order differential equations. The complete system of first order differential equations are then discretized using the semi-implicit Keller-Cebeci scheme[5] where the derivatives are approximated by centered difference quotients and averages centered at the mid-points of net rectangles.

Linerization of the system of nonlinear difference equations is then obtained by Newton's method. The computations are performed using a nonuniform mesh with a geometrically increasing grid system in the axial direction. Therefore, a fine mesh is obtained near the wall and a coarse one away from the solid wall and near the outer edge of the boundary layer. Using the above method a second-order accurate solution is obtained.

In order to quantify the above procedure, the data of Cham and Head [6] is chosen. The measurements were made on a 3 ft. in diameter and 1/2 in. thick steel smooth rotating disk. Traverses of the boundary-layer were made at three rotational speeds of 515, 1000 and 1550 rev/min and at three radii on the disk for each speed. The range of Re_r (=$\omega r^2/\nu$) covered in the Cham & Head data was from 3×10^5 to 2×10^6. Figure 1 compares the experimentally obtained values of the $C_{f\theta}$, the circum ferential skin-friction coefficient, definde as;

$$C_{f\theta} = \frac{2\mu}{\rho\,\omega^2 r^2}\left(\frac{\partial \omega}{\partial y}\right)_{wall} \tag{40}$$

The Figure indicates that there exists a fair agreement between the predictions and the experimentally obtained values.

Based on the calculated values of $C_{f\theta}$ vs. Reynolds number, and in the spirit of curve-fit equations given for a flow inside a cylinder or over a flat plate, the following equations are obtained by passing a smooth curve through the calculated values:

A-Laminar Region

Similar to the skin friction coefficient equation for a laminar flow over a flat plate, the following equation is derived by curve-fitting the results of the numerical model shown in Figure 1;

$$C_{f\theta} = \frac{1.23}{\sqrt{Re_r}} \tag{41}$$

B- Transition Region

For the transition region, curve-fitting the calculated results in a fashion similar to that suggested by Schlichting [7], we have;

$$C_{f\theta} = \frac{0.039}{Re_r^{1/7}} - \frac{1413}{Re_r} \tag{42}$$

C- Turbulent Region

For the turbulent region, an equation similar to that suggested by White [8] for a smooth surface is as follows;

$$C_{f\theta} = \frac{0.448}{\ln^2(0.096\,Re_r)} \tag{43}$$

HEAT TRANSFER CALCULATIONS

The results of the heat transfer calculations of the scheme is shown in two parts. First, simulating the laminar, constant wall temperature conditions of Mccomas & Hartnett [9], yields Figure 2, where there exists an excellent agreement

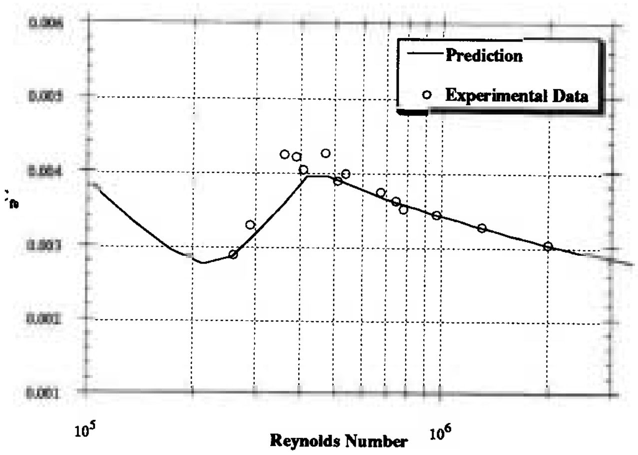

Figure 1. Comparison of calculated circumferential skin-fricition coefficient to experimental values for flow over a smooth disk.

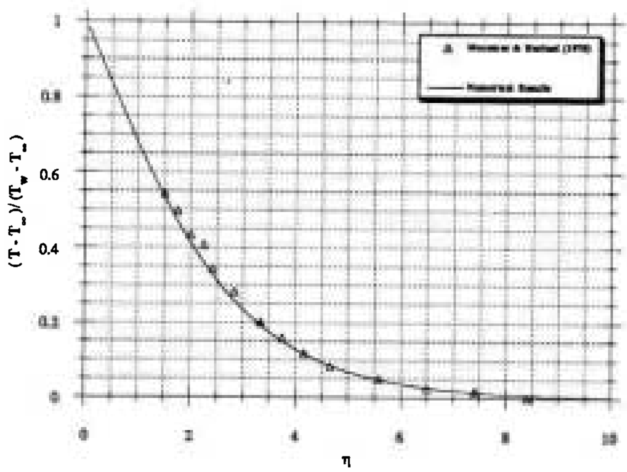

Figure 2. Comparison of dimensionless laminar temperature profiles with experimental values.

between the experimental and the numerically obtained values of the dimensionless laminar temperature profiles. The experimenal data of Mccomas & Hartnett is obtained for the conditions of a smooth rotating disk of 483 mm, where build-in electric heaters were used to maintain a uniform temperature on the disk. In their tests, Mccomas and Hartnett measured average Nusselt numbers for a range of $2\times10^4<Re_r<6\times10^5$.

Next, shown in Figure 3 is the result of the application of the model to the experimental heat transfer data obtained for both laminar and turbulent flow conditions. The solid line compares the results of calculations with the experimental data of Mccomas & Hartnett [9] and Owen & Haynes [10], while the dashed line is a comparison of the numerical results to those of Cobb & Saunders [11], for the respective conditions of constant and variable wall temperatures, given as;

$$T_w(r)/T_\infty = \text{Const.}, \quad T_w(r)/T_\infty = r^2+1 \qquad (44)$$

As for the skin friction results, the proposed model makes a fair prediction of the heat transfer coefficient over a rotating disk.

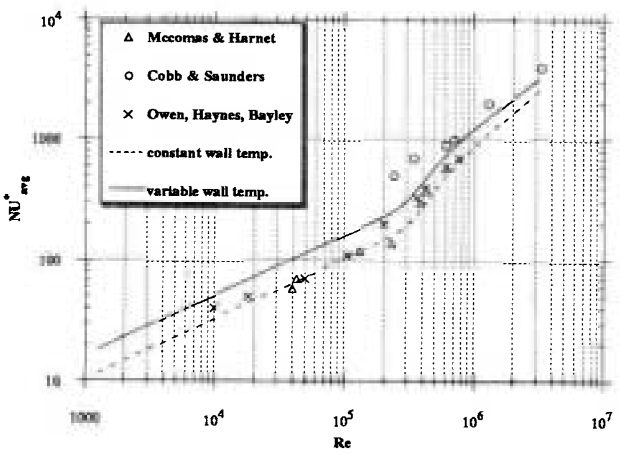

Figure 3. Comparison of calculated values of Nu^*_{avg} vs. Re_r with experimental results for a constant and variable wall temperature.

SURFACE ROUGHNESS CONSIDERATIONS

The existence of surface roughness has no effect on the laminar and transition regions of the flow. In this model, it's influence is seen only in the inner region of the turbulent flow. Figures 4-6 show the results of calculations for the radial and circumferential components of the flow, as well as the skin friction coefficient for various values of surface roughness heights. Figures 4 and 5 which show the two components of velocity plotted versus the non-

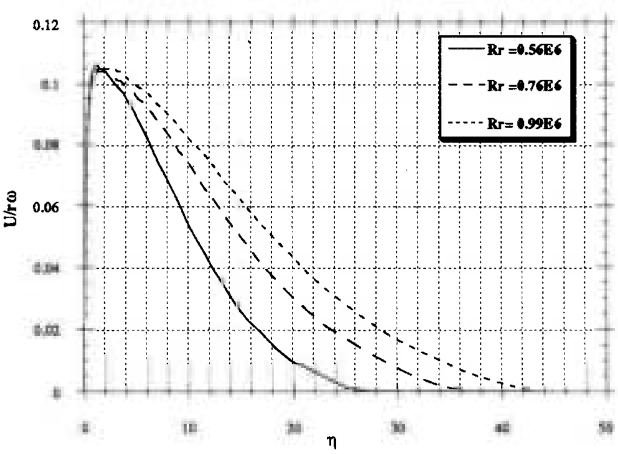

Figure 4. Calculated values of radial component of velocity for surface roughness ($k=1\times10^{-5}$ m) and Re_r.

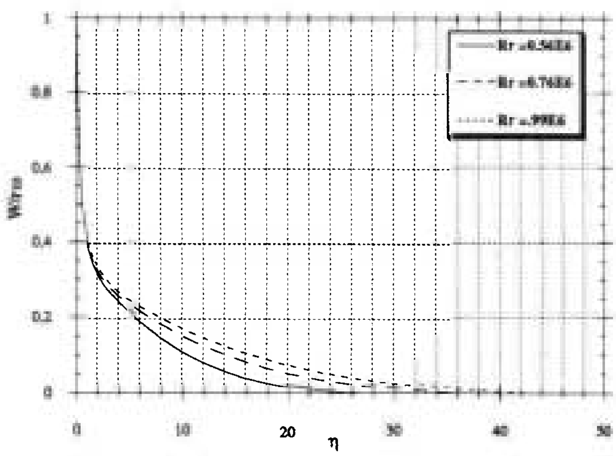

Figure 5. Calculated values of circumferential component of velocity for surface roughness ($k = 1\times10^{-6}$m) and Re_r.

dimensional similarity parameter η, indicate thickening of the boundary-layer for increased surface roughness. Figure 6 indicates that the wall friction dramatically increases with even a small roughness on the disk. The figure also indicates that the effect of surface roughness is similar to that of a flow inside a pipe or a flat-plate with rough walls. Figure 7 shows heat transfer results in terms of Nusselt number plotted versus Reynolds number for various values of wall roughness parameters. As in the case of a flat plate, at a given value of Re_r, the value of Nu increases with increased wall roughness.

THE EFFECT OF SURFACE INSPIRATION

Figures 8 and 9 show circumferential surface friction and heat transfer coefficients on the surface of a rotating disk in the presence of surface suction and blowing. The results indicate that there exists an increase of about 20% in the magnitute of skin friction coefficient and 5% in the heat transfer

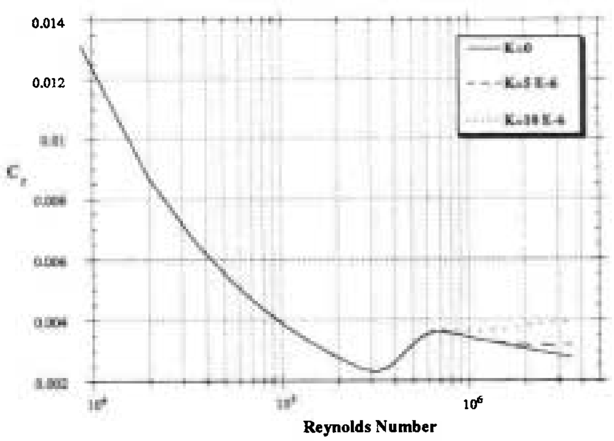

Figure 6. Comparison of circumferential skin-friction coefficient for various values of surface roughness.

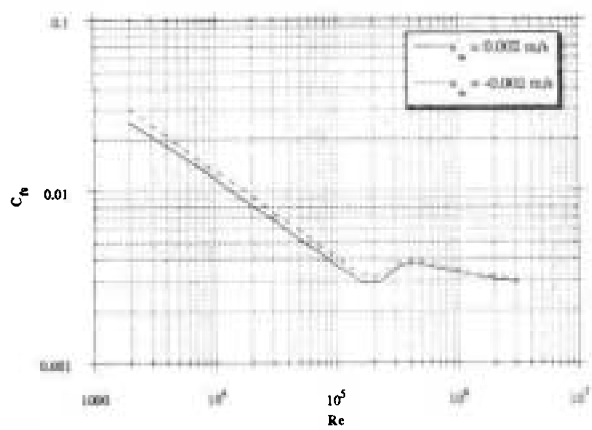

Figure 8. Calculated values of circumferential skin friction coefficient for conditions of surface suction and blowing.

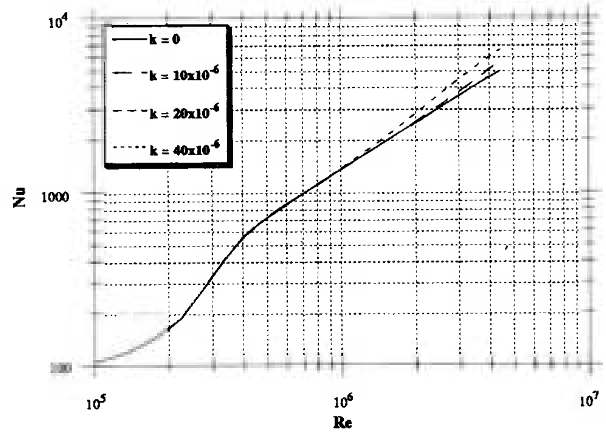

Figure 7. Calculated values of Nusselt versus Reynolds number for specified surface roughness values.

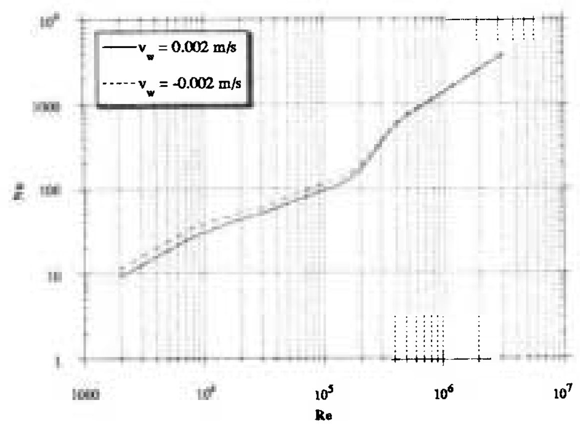

Figure 9. Calculated values of Nusselt number for conditions of surface suction and blowing.

coefficient when surface blowing velocity of 0.002 m/s is reduced to -0.002 m/s of surface suction at $Re_r = 2000$. The figure indicates an expected decrease in the values of $C_{f\theta}$ and NU when comparing the conditions of surface suction and blowing with decreased Re_r.

CONCLUSIONS

The issue of modelling of flow over a rotating disk with surface roughness has not been considered extensively in the past, with the exception of the works of Palec [2, 3]. Though, in his work, Palec models surface roughness assuming a periodical profile with sinusoids, it is merely the definition of surface waviness and not roughness.

In the present work, the problem of fluids flow over a rotating disk with a specified surface roughness is solved using the boundary-layer approximations. The equations of motion coupled with the energy equation are solved using an efficient second order implicit finite-difference scheme. Uniform surface inspiration is included as a boundary condition to the problem. The influence of the surface roughness is contained in the inner-layer of the turbulent flow. The numerical solution of the three dimensional boundary-layer flow and heat transfer is shown to be in good agreement with the available experimental data.

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