ADAPTIVE LINE ENHANCEMENT USING A PARALLEL IIR FILTER WITH A STEP-BY-STEP ALGORITHM

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Abstract A step-by-step algorithm for enhancement of periodic signals that are highly corrupted by additive uncorrelated white gausian noise is proposed. In each adaptation step a new parallel second-order section is added to the previous filters. Every section has only one adjustable parameter, i.e., the center frequency of the self-tuning filter. The bandwidth and the convergence factor of each section is adjusted nonadaptively by a deterministic simple method which results in a stable and accurate regulation of the adaptive parameters. The step-by-step detection of sinusoidal signals prevents the convergence difficulties encountered in adaptive parallel IIR filters. Computer simulation results are presented to show the noise canceling performance of the proposed algorithm. Some comparisons with a new adaptive lattice notch filter for detection of multiple sinusoids are also provided.

Key Words Adaptive Filters, IIR Filters, Line Enhancement, Parallel Structures

چکیده دراین مقاله یک الگوریتم گام به گام برای تشخیص سیگنالهای متناوب که شدیداً توسط نویز گوسی سفید ناهمبستهٔ جمع شونده آلوده شده اند پیشنهاد می شود. در هرگام تطبیق، یک شاخهٔ مرتبهٔ دوم موازی جدید به فیلترهای قبلی افزوده می گردد. هر شاخه، تنها یک پارامتر تنظیم شونده دارد که فرکانس میانی فیلتر خود تنظیم است. پهنای باند و ضریب همگرایی هر شاخه توسط یک روش سادهٔ غیر احتمالاتی به طور غیروفقی تنظیم می شود و نتیجهٔ این کار تنظیم پایدار و صحیح پارامترهای وفقی است. شناسایی و آشکارسازی گام به گام سیگنالهای سینوسی مانع بروز مشکلاتی می شود که در فیلترهای وفقی موازی IIR پیش می آیند. از نتایج شبیه سازی کامپیوتری برای نمایش عملکرد حذف نویز الگوریتم پیشنهادی استفاده می شود. مقایسه ای نیز با یک فیلتر شکافی شبکه ای وفقی جدید برای آشکارسازی امواج سینوسی چندگانه به عمل خواهد آمد.

INTRODUCTION

The adaptive line enhancer (ALE) is a device or algorithm that detects and traces a moving spectral line in broadband noise while enhancing the signal-to-noise ratio (SNR). Adaptive line enhancers have applications in sonar, detection and others areas. Both finite impulse response (FIR) and infinite impulse response (IIR) line enhancers have been proposed by researchers [1,2].

An adaptive IIR filter is preferable to adaptive FIR filters in modeling systems whose outputs contain sinusoid or near sinusoid signals, since an IIR filter can produce high resonant modes [3,4].

The parallel form of an adaptive IIR filter is derived from a partial fraction expansion of the pole-zero filter, resulting second-order sections [5]. The gradient components of this form are easy to compute because the sections are independent. Stability monitoring of an adaptive IIR parallel filter is trivial [6]. A disadvantage of the parallel form is that there are different global minima which can be obtained by reordering the poles among the different sections.

In this paper, an adaptive line enhancer using an IIR filter with a step-by-step algorithm is employed for estimating periodic signals highly contaminated by additive noise. In each step of the algorithm the center frequency of a bandpass filter is the only parameter under adjustment and at the end of the adaptation, if the variance of the error signal does not satisfy the required level, a new section is added in parallel to previous ones, in order to complete the tuning function of the adaptive system. This method have the advantages of parallel form but due to the step-by-step behavior of the proposed algorithm the convergence probelms of adaptive parallel IIR filters mentioned in [5] are not observed.

This paper is organized as follows. In Section I the simple second-order bandpass filter is introduced. In section II the method of step-by-step estimation of center frequencies is presented. Section III explains the adaptive algorithm. Section IV demonstrates some computer simulation results and finally section V provides the conclusions.

I. SIMPLE BANDPASS FILTER

The transfer function of a single-parameter lowpass filter in z-domain can be considered as follows [7]:

$$H_{lp}(z) = \frac{1-\gamma}{2} \cdot \frac{z+1}{z-\gamma} \tag{1}$$

where $|\gamma|<1$ is the only coefficient used to change the characteristics of the filter. To find the frequency-domain performance of this filter we substitute $z = e^{jw}$ which yields

$$|H_{lp}(e^{jw})| = \frac{1-\gamma}{2}\sqrt{\frac{2+2\cos\omega}{1+\gamma^2-2\gamma\cos\omega}} \tag{2}$$

where we have used $T=1$ as the sampling period of the filter for simplicity. We note that the magnitude of the frequency response (2) is unity and zero at frequencies $\omega = 0$ and $\omega = \pi$, respectively. Figure 1 demonstrates different magnitude curves according to (2) as γ varies.

To calculate the bandwidth of the filter we find the

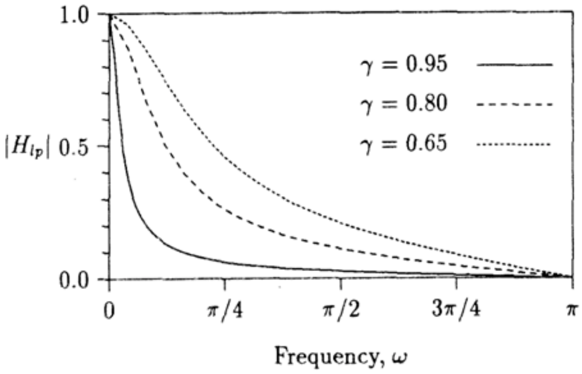

Figure 1. Different magnitude curves of a lowpass filter.

3-dB frequency of the magnitude function (2) as follows:

$$\frac{1}{\sqrt{2}} = \frac{1-\gamma}{2}\sqrt{\frac{2+2\cos\theta_c}{1+\gamma^2-2\gamma\cos\theta_c}} \tag{3}$$

where θ_c denotes the cutoff frequency of the lowpass filter. Solving (3) for θ_c yields

$$\theta_c = \cos^{-1}\frac{2\gamma}{1+\gamma^2} \tag{4}$$

which shows the inverse relationship between γ and the bandwidth of the filter.

Using the transform [8]

$$z^{-1} \rightarrow -\frac{z^{-2}-Az^{-1}+B}{Bz^{-2}-Az^{-1}+1} \tag{5}$$

a lowpass digital filter can be converted to a bandpass one, where

$$A = \frac{2k\cos\omega_c}{k+1} \tag{6}$$

$$B = \frac{k-1}{k+1} \tag{7}$$

and

$$k = cot \frac{\omega_2 - \omega_1}{2} tan \frac{\theta_c}{2} \tag{8}$$

In these equations ω_c is defined as the center frequency of the bandpass filter, and ω_l and ω_2 are the lower and the higher cutoff frequencies, respectively. Substituting (5) into (1) yields

$$H(z) = \frac{(1-\gamma)(1-B)}{2} \cdot \frac{1 - z^{-2}}{(B+\gamma)z^{-2} - A(\gamma+1)z^{-1} + (1+B\gamma)} \tag{9}$$

which has the magnitude

$$\left|H(e^{jw})\right| = \left|\frac{(1-\gamma)(1-B)}{2}\right| \sqrt{\frac{2-2\cos\omega}{D_r^2 + D_i^2}} \tag{10}$$

where

$$D_r = (B+\gamma)\cos 2\omega - A(\gamma+1)\cos\omega + (1+B\gamma)$$
$$D_i = (B+\gamma)\sin 2\omega - A(\gamma+1)\sin\omega$$

We note that as ω_l and ω_2 are determined, the value of ω_c can be computed according to

$$cos\omega_c = cos\frac{\omega_1 + \omega_2}{2} / cos\frac{\omega_1 - \omega_2}{2} \tag{11}$$

There are now two parameters that can determine the bandwidth of the filter, i.e., γ and Δ, defined as

$$\Delta = \omega_2 - \omega_1 \tag{12}$$

Since we are mostly interested in the minimum number of adjustable parameters, we choose $\gamma = 0.8$, noting that any other values between 0 and 1 could be taken as well. The value of Δ is also considered as a nonadaptive parameter and the center frequency, ω_c, is the unique adaptive coefficient of the filter. As we are mostly concerned with detecting periodic signals, it seems that a proper value for Δ should be very close to zero. On the other hand a very small Δ can destroy the stability of the adaptive filter because the poles of (9) move very close to the unit circle. Another parameter that affects the determination of a fixed Δ is the convergence factor μ which controls the rate of convergence and stability of the adaptive algorithm. An strategy that can solve these problems is: decreasing both Δ and μ nonadaptively. At the start of the adaptation a relatively larger value for Δ and μ can accelerate the convergence without producing unstability because the initial value of Δ is normally chosen so that the stable poles are far from the unit circle. Near the end of the adjustment, as the adaptive coefficient ω_c is in the neighborhood of its correct value, we can use a relatively smaller value for Δ and μ which leads to a more accurate adjustment of ω_c. Substituting $\gamma = 0.8$ into (4), we have

$$\tan\frac{\theta_c}{2} = \tan\left[(\cos^{-1}\frac{2\times 0.8}{1+0.8^2})/2\right] = \frac{1}{9} \tag{13}$$

and using (12) and (13) in (8) gives

$$k = \frac{1}{9}\cot\frac{\Delta}{2} \tag{14}$$

Now, substituting (6), (7) and (14) into (9) yields

$$H(z) = \frac{b_0(1-z^{-2})}{1-a_1z^{-1}-a_2z^{-2}} \tag{15}$$

where we have defined

$$b_0 = \frac{\tan\frac{\Delta}{2}}{\tan\frac{\Delta}{2} + 1} \tag{16}$$

$$a_1 = \frac{2\cos\omega_c}{\tan\frac{\Delta}{2} + 1} \tag{17}$$

$$a_2 = \frac{\tan\frac{\Delta}{2} - 1}{\tan\frac{\Delta}{2} + 1} \tag{18}$$

It should be mentioned again that there are only one adjustable parameters, i.e., the center frequency ω_c which has appeared in (17).

II. STEP-BY-STEP ALE

It should be noted that, only second-order ALE's are considered in our approach because the cascade of second-order ALE's provides better results for detection of multiple sinusoids than that of the higher-order ALE's [9]. Figure 2, shows the adaptive IIR self-tuning filter in the adaptation step I; $I=1,2,\ldots$. The signals $x(n)$ and $v(n)$ are the undistorted signal and noise, respectively. The signal $y(n)$ is the input to all adaptive sections. The output signal of the section i; $i=1,2\ldots,I$; is denoted by $\hat{x}_i(n)$. The transfer function of the i'th section from (15) is

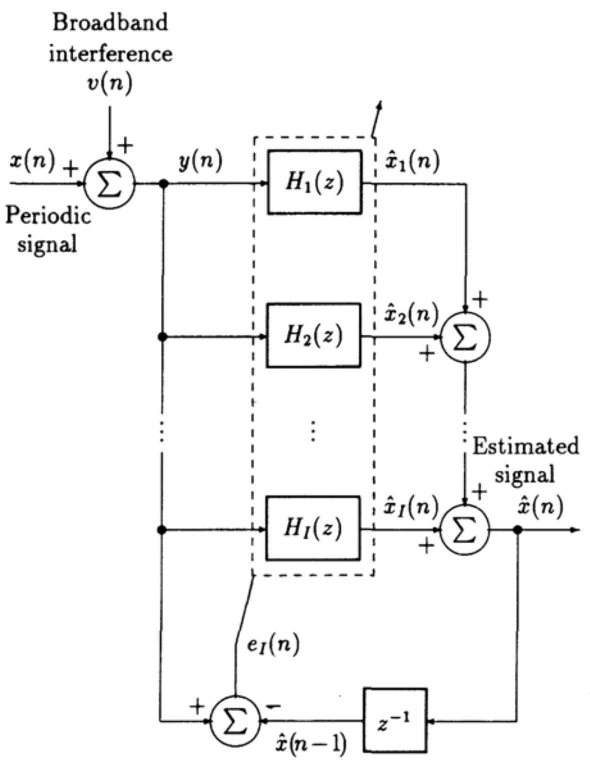

Figure 2. Adaptive IIR self-tuning filter in the adaptation step I.

$$H_i(z) = \frac{b_0(1-z^{-2})}{1 - a_{1i}z^{-1} - a_2 z^{-2}} \tag{19}$$

where b_0 and a_2 are defined in (16) and (18), respectively, and a_{1i} is rewritten from (17) as follows:

$$a_{1i} = \frac{2\cos w_{ci}}{\tan\frac{\Delta}{2} + 1} \tag{20}$$

where the dependence of the section to the center frequency ω_{ci} is reflected in the coefficient a_{1i} of $H_i(z)$. We have

$$\hat{x}_i(n) = H_i(q)\, y(n) = \frac{b_0(1 - q^{-2})}{1 - a_{1i}q^{-1} - a_2 q^{-2}} y(n) \tag{21}$$

where q denotes the shift operator. The output signal of the system, $\hat{x}(n)$, is the sum of the intermediate outputs of the sections, i.e.,

$$\hat{x}(n) = \sum_{i=1}^{I} \hat{x}_i(n) \tag{22}$$

The error signal $e_I(n)$ is defined as

$$e_I(n) = y(n) - \hat{x}(n-1) \tag{23}$$

Here $\hat{x}(n-1)$ is used instead of $\hat{x}(n)$, otherwise the error can be minimized, in fact it can be made zero, by choosing $\hat{x}(n) = y(n)$, i.e., by choosing the signal estimate identical to the measurement, which amounts to making the transfer function $\Sigma H_i(z)$ identical one.

In the first step, $I = 1$, and we have only a single adjustable parameter, ω_{c1}. At the end of this step, ω_{c1}

is near the main frequency of the input signal $x(n)$. This frequency should be normally the lowest frequency of $x(n)$ which reduces the mean-squared-error (MSE) to its minimum value in the first adaptation step. In the second step, we have two adjustable coefficients, i.e., ω_{c1} and ω_{c2}. Normally ω_{c1} will be only fine tuned toward its exact value and ω_{c2} will be determined at the end of this adaptation step. This procedure is continued until a satisfactory MSE is obtained.

It may be interesting for the reader to refer to [10], which also uses a step-by-step algorithm for time-delay estimation and parameter adjustment of adaptive IIR delay filters.

III. ADAPTIVE ALGORITHM

The well-known Least-mean-squares (LMS) algorithm [11] is used to adjust the parameters ω_{ci}. To compute the gradient components we define for $i = 1,2,\ldots,J$

$$\alpha_i(n) = \frac{\partial e_I(n)}{\partial \omega_{ci}} = -\frac{\partial \hat{x}(n-1)}{\partial \omega_{ci}} \tag{24}$$

and because of the independence of parallel sections we have

$$\alpha_i(n) = -\frac{\partial \hat{x}_i(n-1)}{\partial \omega_{ci}} \tag{25}$$

Using (21), we can write

$$\alpha_i(n) = -\frac{\partial}{\partial \omega_{ci}} \left[\frac{b_0(1-q^{-2})q^{-1}}{1-a_{1i}q^{-1} - a_2 q^{-2}} y(n) \right]$$

$$= b_0 q^{-2}(q^{-2}-1) \frac{\frac{\partial a_{1i}}{\partial \omega_{ci}}}{(1-a_{1i}q^{-1} - a_2 q^{-2})^2} y(n) \tag{26}$$

where $\frac{\partial a_{1i}}{\partial \omega_{ci}}$ can be calculated from (20) which yields

$$\alpha_i(n) = b_0 q^{-2}(q^{-2}-1) \frac{\frac{-2\sin \omega_{ci}}{\tan\frac{\Delta}{2}+1}}{(1-a_{1i}q^{-1} - a_2 q^{-2})^2} y(n)$$

$$= \frac{\frac{2\sin \omega_{ci}}{\tan\frac{\Delta}{2}+1}}{1-a_{1i}q^{-1} - a_2 q^{-2}} \hat{x}_i(n-2) \tag{27}$$

which can be written as

$$\alpha_i(n) = a_{1i}\alpha_i(n-1) + a_2\alpha_i(n-2) + \frac{2\sin\omega_{ci}}{\tan\frac{\Delta}{2}+1}\hat{x}_i(n-2) \tag{28}$$

Since the bandwidth of the filter, i.e., Δ is small throughout the adaptation process, the term $\tan(\Delta/2)$ in the denominator of (28) can be neglected and we will have the simplified gradient as follows:

$$\alpha_i(n) = a_{1i}\alpha_i(n-1) + a_2\alpha_i(n-2) + 2\sin\omega_{ci}\,\hat{x}_i(n-2) \tag{29}$$

Now the LMS algorithm is written as follows:

$$\omega_{ci}(n+1) = \omega_{ci}(n) - 2\mu_{I,i}(n)\,\alpha_i(n)e_I(n) \tag{30}$$

where $\mu_{I,i}(n)$ represents the step size or convergence factor and will be chosen a function of i, I and n.

IV. SIMULATION RESULTS

The periodic signal $x(n)$ used in our computer simulation is

$$x(n) = \frac{1}{2}[\sin(0.05\pi n) + \sin(0.2\pi n) + \sin(0.5\pi n)] \tag{31}$$

and the noise is additive zero-mean white gaussian distributed with a variance of $\sigma_v^2 = 0.375$; thus the signal-to-noise ratio is unity. The convergence factor $\mu_{l,i}(n)$ is calculated as follows:

$$\mu_{l,i}(n) = 3^{1-l}\, i^2 \left(1 - 0.9\frac{n}{n_{max}}\right)\mu_0 \qquad (32)$$

where $i = 1, 2, \ldots, l$; μ_0 is a constant ($\mu_0 = 0.0006$ is used in the simulation); and n_{max} denotes the number of iterations in each adaptation step. This ad hoc relation is derived by trial and error and has the following properties:

- The convergence factor decreases by n in order to compensate for the decrease of Δ and maintain stability.
- The new section has a larger convergence factor because, as mentioned in section I, in each step the center frequencies of previously adapted section do not require a large convergence factor and just need to be trimmed toward their exact values.
- The convergence factor of any parallel section decreases in each step as well, and helps the fine adjustment of previously adapted coefficients.

The value of the bandwidth Δ decreases linearly from 0.1 at $n = 0$ to 0.02 at $n = n_{max}$. The number of iterations in each step is $n_{max} = 10000$.

Figure 3 shows the convergence characteristics of $\omega_{c1}(n)$ in the first step. Figure 4 demonstrates the adaptation of $\omega_{c1}(n)$ and $\omega_{c2}(n)$ in the second step.

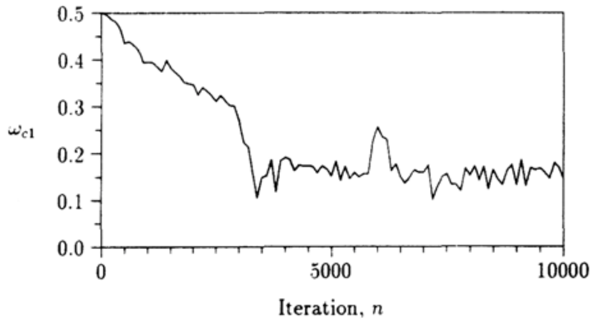

Figure 3. Convergence curved of $\omega_{c1}(n)$ as a function of the iteration in the first step.

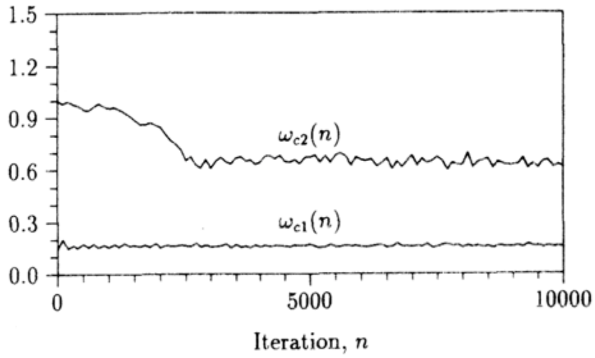

Figure 4. Convergence curved of $\omega_{c1}(n)$ and $\omega_{c2}(n)$ as a function of the iteration in the second step.

Figure 5 shows the convergence of $\omega_{c1}(n)$, $\omega_{c2}(n)$ and $\omega_{c3}(n)$ in the third step. It is obvious from these figures that the main adjustment of every center frequency $\omega_{ci}(n)$ is done during its first adaption step. Figure 6 shows the MSE defined as

$$\text{MSE} = \text{E}\,[e_l^2\,(\text{n})] \qquad (33)$$

as a function of iteration, n. We note that the minimum attainable MSE is the noise variance, i.e., 0.375. Figure 7 illustrates the disturbed signal $y(n)$ along with the estimated signal $\hat{x}(n)$ in the last step and shows the considerable noise cancellation properties of the proposed method. Finally Figure 8 demonstrates the frequency response of $\Sigma H_i(z)$ at the end of the third adaptation step. These magnitude curves show

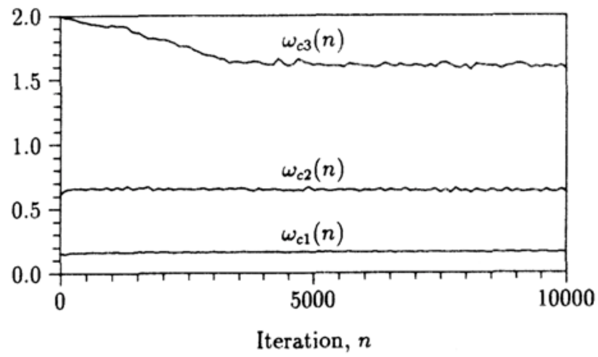

Figure 5. Convergence curved of $\omega_{c1}(n)$, $\omega_{c2}(n)$ and $\omega_{c3}(n)$ as a function of the iteration in the third step.

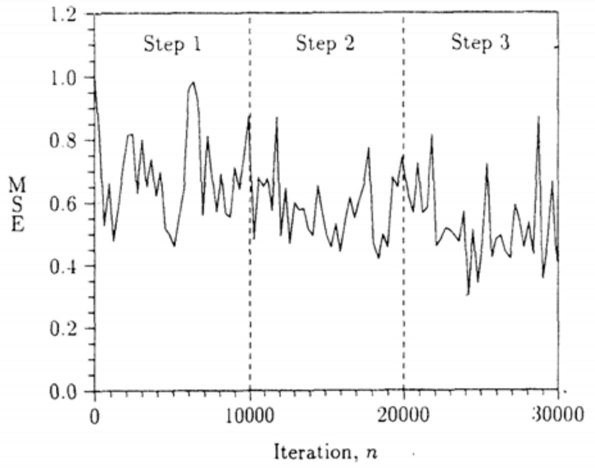

Figure 6. Mean-square-error curve vs. iteration during three steps.

three resonances at frequencies $\omega_{c1}(n)$=0.158, $\omega_{c2}(n)$= 0.635 and $\omega_{c3}(n)$= 1.597, which are very close to the ideal values 0.05π, 0.2π and 0.5π, respectively.

As a comparison with other new schemes of adaptive detection of multiple sinusoids, we can refer to [12], in which a cascade of second-order adaptive notch filters in three different approaches is proposed.

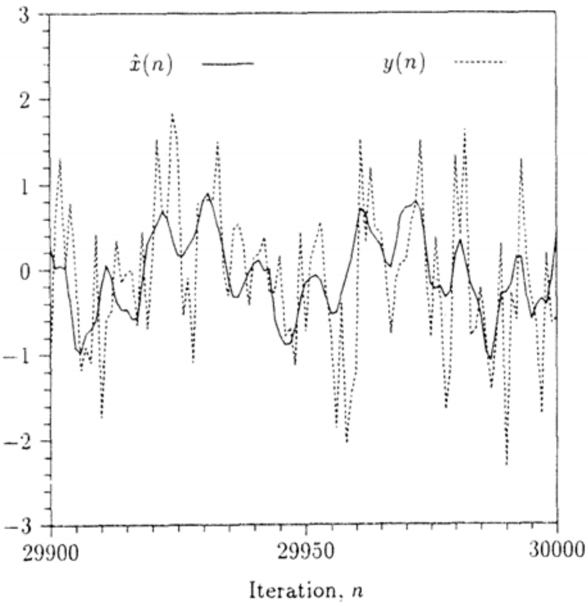

Figure 7. Disturbed signal $y(n)$ and estimated signal $\hat{x}(n)$ at the end of the third step.

The first approach employs the linear cascade structure whereas the second and the third algorithms are based on the triangular cascade structures [13]. Comparing these algorithms with our proposed one, there are some differences and important points which can be summarized as follows:

- The formulations of [12] are based on cascade structures, whereas here, we use a parallel IIR structure, which according to [6] has better convergence properties and less gradient complexity.
- The adaptive algorithm used in [12] is the stochastic Gauss-Newton algorithm [14], which employs the gradient vector for updating the coefficients. This algorithm requires matrix multiplications and the improved behavior is not comparable with the simplicity of the LMS algorithm which is used in this paper.
- In every stage of the proposed algorithm, the previously adjusted coefficients are trimmed toward their best values whereas this property is not seen in [12].
- To improve the performance of the LMS algorithm in this particular structure, we have used a variable step size which depends on iteration and step indices. By running the stochastic Gauss-Newton algorithm we have found that the convergence properties of the variable step-size algorithm is comparable to the

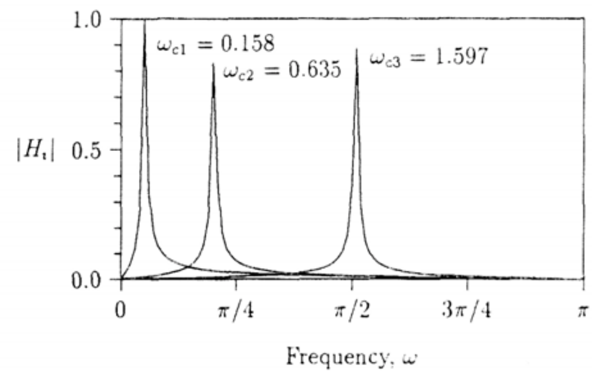

Figure 8. Frequency response of the parallel sections at the end of the adaptation.

stochastic Gauss-Newton algorithm, however, the computations are much less than this algorithm.

● It should also be mentioned that the frequency bias of [12] is better than our algorithm. This can be further investigated in future.

V. CONCLUSIONS

In this paper we proposed a step-by-step algorithm for detection of periodic signals that are highly contaminated by additive noise. Each section of the adaptive configuration employs a bandpass second-order parallel IIR filter which has merely a single adjustable coefficient. In each step of the proposed algorithm the center frequency of the last section is estimated and the center frequencies of the previous sections are fine tuned toward their exact values. The bandwidth and convergence factor are adjusted nonadaptively based on a deterministic relation. The step-by-step procedure prevents the convergence problems of adaptive parallel IIR filters. The convergence properties and complexity of the proposed algorithm is superior to the cascade structure but needs further investigations regarding to the bias in frequency detection.

NOMENCLATURE

ALE	adaptive line enhancer
FIR	finite impulse response
IIR	infinite impulse response
LMS	least-mean-square
MSE	mean squared error
A, b, k	constants
a_1, a_2	feedback coefficient of the bandpass filter
b_o	forward coefficient of the bandpass filter
$H(z)$	transfer function of the bandpass filter
$H_{lp}(z)$	transfer function of the lowpass filter
n	discrete time and iteration index
$v(n)$	noise source
$x(n)$	input signal
$\hat{x}(n)$	estimated signal
$y(n)$	input signal of the adaptive filters
x	variable of the z-transform
α_i, β_i	gradient components
γ	parameter of the lowpass filter
Δ	bandwidth of the bandpass filter
μ	convergence factor
σ^2	noise variance
ω	frequency
ω_1, ω_2	cutoff frequencies of the bandpass filter
ω_c	center frequency of the bandpass filter
θ_c	cutoff frequency of the lowpass filter

REFERENCES

1. B. Widrow and S. D. Stearns, "Adaptive Signal Procssing," prentice-Hall, Englewood Cliffs, N. J., (1985).
2. D. R. Hush et al., "An Adaptive IIR Structure for Sinusoidal Enhancement, Frequency Estimation, and Detection," *IEEE Trans. Acoust, Speech, Sig. Proc.*, vol. ASSP-34, No. 6, (Dec. 1986), 1380-1389.
3. C. R. Johnson, Jr. *Adaptive IIR filtering,* "Current results and open issues," IEEE Trans. Information theory" vol. IT-30, No. 2, Mar. (1984) 237-250.
4. J. J. Shynk *"Adaptive IIR filtering," IEEE ASSP Magazine*, vol. 6, (Apr 1989), 4-21.
5. J. J. Shynk "Adaptive IIR filtering using Parallel form Realizations, . "IEEE Trans. Acoust., Speech, Sig. Proc, vol. ASSP-37, (Apr. 1989), 519-533.
6. J. J. Shynk "Performance of Alternative Adaptive IIR Filter Realizations, "in Proc. 21st Asilomar Conf. Signals, Systems, Computers, Pacific Grove, California, (Nov. 1987), 144-150.
7. A. Zalnieriunas, "Simple Adaptive Recursive Filter for Estimation of Trajectories in Noise", presented in *ISSSE* 92, Paris, (Sep. 1992) 450-453.
8. A. V. Oppenheim and R. W. Schafer, Digital Signal Processing. Englewood Cliffs, N. J: Prentice-Hall,

(1975).

9. D. V. Bhaskar Rao and S. Y. Kung, "Adaptive Notch Filtering for the Retrieval of Sinusoids in Noise", IEEE Trans. Acoust., Speech, Sig. Proc, vol. ASSP-32, (Aug. 1984) 791-802.
10. M. Ghavami, "A New Adaptive Recursive Filter," *IEEE Trans. Circuits Systems*, vol. CAS-40 (Jun. 1993), 426-429.
11. B. Widrow, J. M. McCool, M. G. Larimore and C. R. Johnson, Jr. *Stationary and Nonstationary Learning Characteristics of the LMS Adaptive Filter*. Proc. IEEE, vol. 64, (Aug. 1976), 1151-1162.
12. N. I. Cho and S. U. Lee: "On the Adaptive Lattice Notch Filter for the Detection of Sinusoids," *IEEE Trans. Circuits Systems*, vol. CAS-40, (July 1993), 405-416.
13. T. Kwan and K. Martin: "Adaptive Detection and Enhancement of Multiple Sinusoids using a Cascade IIR Filter," *IEEE Trans. Circuits System*, vol. CAS-36, (July 1989), 937-947.
14. L. Ljung and T. Soderstrom: "Theory and Practice of Recursive Identification," Cambridge, MA., MIT press, (1983).