APPROPRIATE LOADING TECHNIQUES IN FINITE ELEMENT ANALYSIS OF UNDERGROUND STRUCTURES

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Abstract Stability of underground structures is assessed by comparing rock strength with induced stresses resulted from ground stresses. Rock mass surrounding the opening may fail either by fracture or excessive deformation caused. Accurate calculation of induced stresses is therefore fundamental in the stability analysis of an opening. Although numerical methods, particularly finite element method, are very promising methods in finding out induced stresses, special care must be taken at various stages of constructing and analysing such models. This paper describes the significance of loading technique; the way that ground stresses are applied to the finite element model in finite element analysis (FEA) of underground structures. The purpose of this research is to illustrate the results obtained from similar models which were constructed in different ways regarding the loading technique. Key factors for choosing an appropriate loading method whilst considering the in-situ condition of the structure are addressed. To carry out this investigation, the three-dimensional finite element program, NASTRAN, was used. The results of FE models were compared with those obtained from closed solution methods as well as field investigations conducted both during this research and reported by others. Based on the results of this study, appropriate loading techniques are developed and suggested for various conditions. The application of these techniques to the stability analysis of underground structures resulted in encouraging findings.

Key Words Loading (Technique), Stability, Underground Structures, Finite Element Analysis

چکیده این مقاله اهمیت تکنیک بارگذاری در تحلیل سازه های زیرزمینی به روش المانهای محدود را توضیح می دهد. هدف تحقیق ارائه نتایج بدست آمده از مدلهای مشابه است که با تکنیکهای متفاوتی بارگذاری شده اند. عوامل کلیدی در انتخاب روش مناسب بارگذاری به همراه در نظر گرفتن شرایط واقعی بر جای سازه های زیرزمینی ارائه شده اند. برای انجام این تحقیق برنامه سه بعدی المانهای محدود NASTRAN استفاده شد. نتایج حاصل از مدلهای المانهای محدود با نتایج حاصل از تحقیقات صحرایی که هم در طی این تحقیق و هم در تحقیقات مشابه فعلی که توسط محققین دیگر انجام شده بود مقایسه شدند. براساس نتایج حاصل از این مطالعه تحقیقی، روشهای بارگذاری مناسبی ابداع و برای شرایط مختلف پیشنهاد شدند. کاربرد این تکنیکها برای تحلیل پایداری فضاهای زیرزمینی نتایج بسیار رضایتبخشی داد.

INTRODUCTION

General approach in stability analysis of underground structures is to compare rock strength and its stress-strain behaviour with induced loads resulted from ground stresses. An opening has failed if it dose not satisfies its in-service requirements. Rock mass surrounding the opening may fail either by fracture or excessive deformation. In either case, mechanical failure results from some combination of induced stresses. These stresses produce a state stress in each element of rock mass which can be specified by principal stresses; σ_1, σ_2, σ_3, as $\sigma_1 \geq \sigma_2 \geq \sigma_3$. The set of σ_1 , σ_2, σ_3 values for which failure occurs can be presented by a point in σ_1, σ_2, σ_3, space, and the totality of these points describes the surface of the failure called limiting surface f $(\sigma_1, \sigma_2, \sigma_3) = 0$ or in terms of the dependent variables such as σ_1=f (σ_2, σ_3). More details on this subject as well as common failure criteria in rock mechanics are discussed by Hematian and Porter [1].

Therefore, stresses in rock masses are of fundamental concern in the design of underground excavations for civil and mining engineering projects. Only an assessment of stresses in rock will allow the application of strength determination and failure theories to a rational design of openings in rock.

From ground control stand point, there are three conditions of stress state in relation to an underground opening as: in-situ stresses (so called virgin stresses), induced-stresses and support loads. In-situ stresses are the ground pressures associated with every location in the ground before any opening is excavated or any disturbance is made. In-situ stresses derive from several sources. The most important one is the gravity loading of the rocks, and others are: tectonic stresses, local stresses, residual stresses and thermal stresses [2].

In-situ or virgin stresses, at a certain location in underground are the sum of all possible source of stresses and represent three-dimensional (3-D) quantities which are mathematically described as tensors. In many cases, the principal directions of the virgin stress tensor are parallel at right angles to the earth's surface (Figure 1). Hence, the vertical and horizontal stress components can be treated separately to describe the change of stress magnitudes with depth. A summary of the most important investigations to express virgin stresses as a function of depth is presented by Hematian [3].

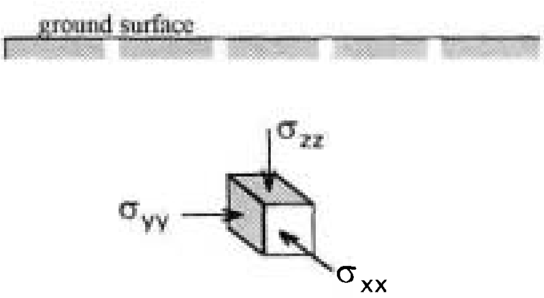
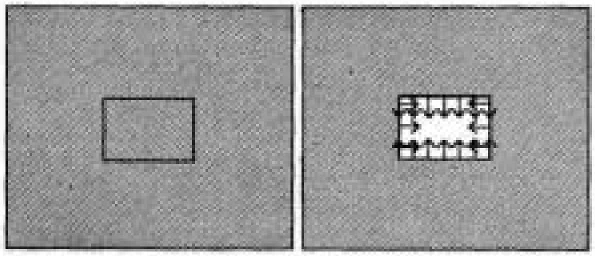


Figure 1. Virgin stresses.



(a) (b)

Figure 2. Stress state at a location (**a**) before constructing the opening, virgin state, (**b**) after constructing the opening, disturbed state.

Induced stresses are mobilised when an opening is made. Consider a cross-section of rock mass where an opening is to be excavated (Figure 2). In its virgin state, the rock mass is in equilibrium everywhere in the cross-section (Figure 2a). Once an opening such as a tunnel is made, the surrounding rocks in the vicinity of the opening are no longer in equilibrium (Figure 2b). The roof has lost support from below, the floor no longer has an applied load from above, and the side-walls are no longer constrained along the sides of the opening. Therefore, the surrounding rock mass tend to deform into the opening. If no artificial supports are erected and rock is not strong enough to withstand the new loading conditions (induced stresses), the opening will soon fail to satisfy the requirements for its in-services.

The closest analogy to express re-distribution of stresses around an opening is the stream line analogy for laminar flow. If an object similar in shape as the cross-section of the opening is placed as an obstruction in the flow lines, a crowding of stream lines (acceleration of flow) at the sides and spreading (slowing of flow) in front and behind the obstacle is observed. State of the induced stresses around openings is a function of many variables of which the important ones are: virgin stresses, shape of the opening, properties of rock mass as well as number of and distance between the openings.

Calculation of induced stresses can be carried out by (1) mathematical theory, (2) numerical modelling, and (3) photo-elastic models. The calculation of the stress distribution around a circular hole in an infinite plate was first solved by the aid of mathematical theory of linear elasticity by Kirsch (1988) quoted in Obert and Duvall [4]. The Kirsch equation gives the radial, tangential and shear stresses at any point in an infinite plate with polar coordinates (Equations 1 to 3 and Figure 3). A comprehensive explanation on the theoretical methods to determine induced stresses around various openings has been given by Obert and Duvall [4].

$$\sigma_r = 0.5\,(\sigma_x+\sigma_y)\,(1-a^2/r^2) + 0.5\,(\sigma_x-\sigma_y)\,(1+3a^4/r^4 - 4a^2/r^2)\,\mathrm{Cos}\,2\theta \qquad (1)$$

$$\sigma_t = 0.5\,(\sigma_x+\sigma_y)\,(1-a^2/r^2) - 0.5\,(\sigma_x-\sigma_y)\,(1+3a^4/r^4)\,\mathrm{Cos}\,2\theta \qquad (2)$$

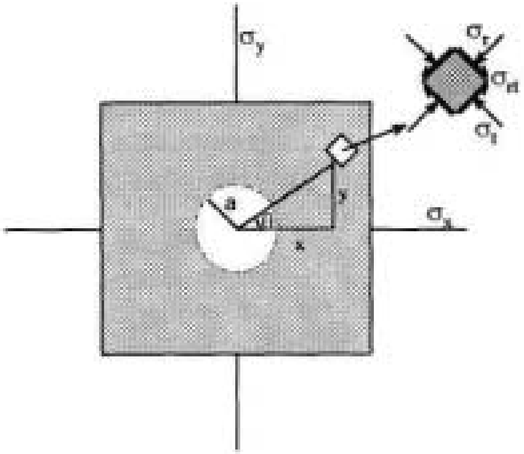


Figure 3. Induced stresses for a circular opening subjected to biaxial stresses.

$$\tau_r = 0.5\,(\sigma_x-\sigma_y)\,(1-3a^4/r^4-2a^2/r^2)\,\mathrm{Sin}\,2\theta \qquad (3)$$

It is important to consider that mathematical solutions (closed solutions) are based on elasticity theory and assume a continuum, homogeneous and isotropic media. If any of these assumptions is not satisfied, it will be almost impossible to use closed solutions to calculate stresses induced around openings, and a numerical method such as finite element method (FEM) should be used. This is why the application of numerical methods such as FEM to stability analysis of underground structures is a common practice.

There is, however, some concern about the accuracy and validity of results obtained from numerical methods. The accuracy and validity of results are highly influenced by the parameters employed in the various stages of the modelling procedure. The analysis will usually produce some results which may or may not be correct or accurate. It has been mentioned in some reports that the magnitudes of deformation obtained from FE analysis were unrealistic compared with field data or those from theoretical solutions [5,6]. It is, therefore, very important to calibrate and optimise these parameters by preparing simple models that can have theoretical or obvious solutions. The results from FE analysis are then compared with the results from theoretical solutions.

Parameters used in a FE modelling procedure can be categorised in three major following groups:

- *Geometrical parameters*: dimensions of the model and mesh density.

- *Structural parameters*: element types and properties, freedom condition of grid points, boundary condition and loading technique.

- *Material parameters*: constant values and/or constitutive equations that explain the behaviour of materials in different conditions.

Of all parameters which may significanly influence the results of FE analysis, loading technique is one of the most fundamental ones. Loading technique may either mean the method that virgin stresses are applied to the FE model or the procedure of solution used to analyse for stress and strain in every elements in the FE model. The former meaning is a modelling concept while the latter is an analytical concept. The aim of this paper is to demonstrate that the method of applying the load to the FE model (the former meaning of loading technique) is very important regarding the accuracy of the results as well as the reduction of the size of the model which in turn saves computer costs.

In this research, 12 models were constructed and analysed utilising NASTRAN code to determine the most appropriate loading techniques for different condition, which will result in the most accurate ouput. The Kirsch equation was used as the bench mark to calculate stresses at any point in an infinite plate. Then the results obtained from different techniques of loading were compared with those from Kirsch solution. In order to apply Kirsch solution, it was inevitable to use 2-D plane strain models and analyse them for stresses and displacements using static linear solutions. However, it should be mentioned that 3-D models of three-way and four-way intersections of tunnels were constructed and analysed with non-linear solutions. The resutls of 3-D models of roadway insections will be presented in independent papers in the near future.

During this research, the 3-D finite element code, NASTRAN, was utilised. This program is a general purpose 3-D FE code for static and dynamic displacement and stress analysis of structures, solids and fluid systems [7]. NASTRAN can be empolyed to perform linear and non-linear analyses. The non-linear solutions consider both geometrical and material non-linearity. It executes the model with specific material properties under increasing load increments.

LOADING TECHNIQUES

There are two major methods to simulate the stress state in the model. The first method is to consider gravitational load throughout the model. The second method is applying an equivalent force or stress at grid points or on the free faces of the model.

Since it is not practical to take into account the total cover of the structure in an FE model, it will be inevitable to use an apparent value for either the density of rocks or the gravity force. In either case, the gradient of vertical stress in the model will be equal to H/h where H and h are depth of overburden and thickness of the model, respectively. This ratio is usually more than 10 when modelling underground structures at depths more than 300 m is considered. Although inertial loading is an accurate method for modelling surface or shallow structures, application of this method for modelling deep structures will no provide a constant uniform stress state around the model (Figure 4).

The second method of simulating the in-situ stress state is to apply the same value of in-situ stress at a certain depth on the free faces of the model or to calculate the equivalent force and apply that at grid points around the model. In both cases, any effect resulting from the high stress gradient along sides of the model will be eliminated. In this way, a constant uniform stress state will be achieved. The key point

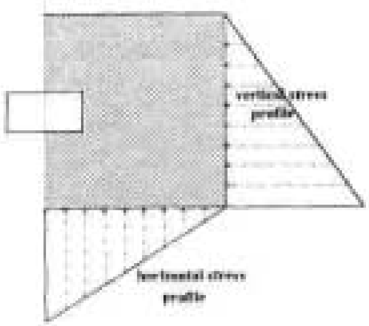
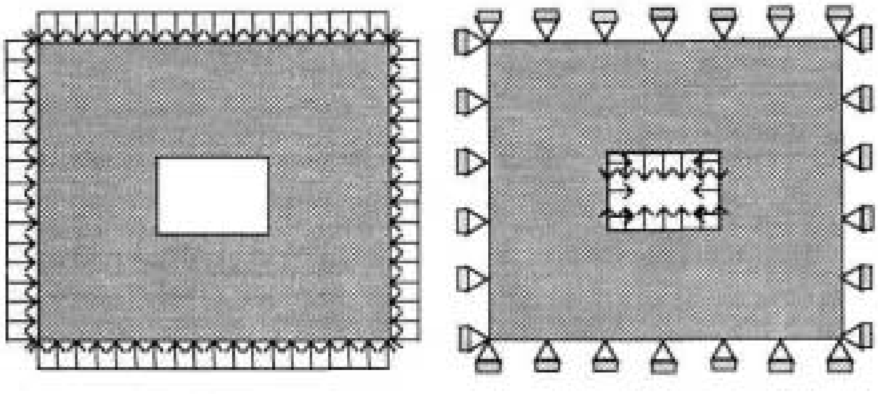


Figure 4. Vertical and horizontal stress profiles around a model using inertial loading.



(a) (b)

Figure 5. Uniform loading of underground structures; **(a)** loading on the external boundary, **(b)** loading on the internal boundary.

in this method is that there are differences whether applying the load on the external or on the internal boundary of the model (Figures 5a and 5b).

Three series of models were constructed and analysed under four different loading conditions (vertical stress S_v= 10 MPa and the horizontal to vertical stress ration, K=1, 2, 3 and 4) to study the differences. The first group included virgin models without any opening where stresses were applied on the external boundary. The second group included models of the structure where stresses were applied on the external boundary. The third group of models were the same as the second group but stresses were applied on the internal boundary. These models are called Null, External and Internal Models, respectively. Figure 6 shows locations at which results were obtained.

Vertical stress at mid-height of the pillar and along the centre-line in the roof as well as vertical displacement on the horizontal line in the roof are given in Figure 7 to 9. Regarding these results, the following conclusions can be made:

(a) The Null Model shows the initial response of the region to the virgin stresses before any stucture was made there. Stress state remains almost constant, but there are some deformations throughout the model.

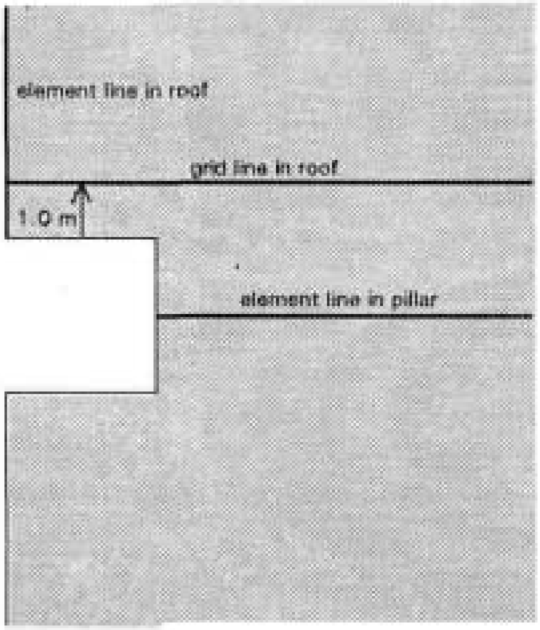


Figure 6. Different locations in roof and pillar for analysing the results.

(b) The external Model gives the total values of stress and displacement, including the initial response of the region to the virgin stresses (before constructing the opening) plus the disturbances resulted from making the structure.

(c) The Internal Model gives the relative changes of stresses and displacements around the structure from the initial condition (virgin state) to

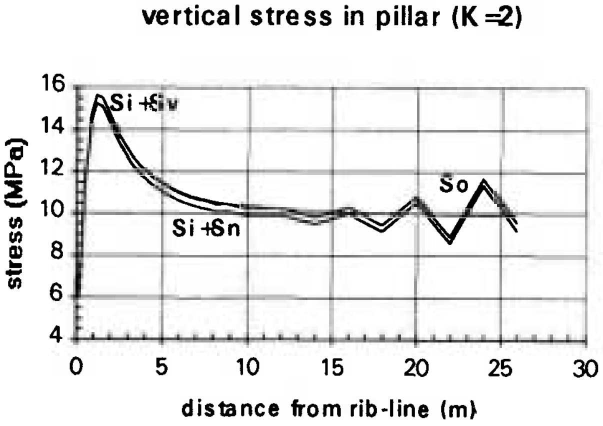


Figure 7. Vertical stress at mid-height of the pillar.

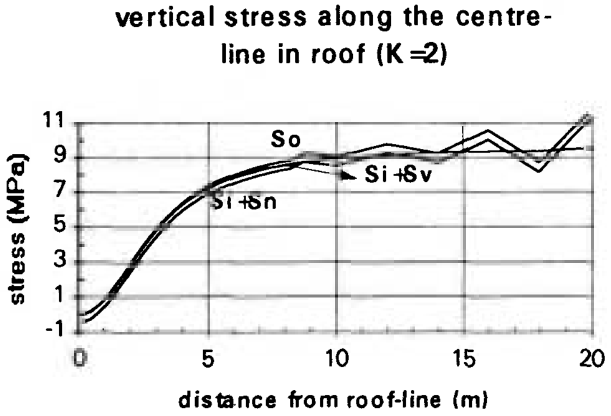


Figure 8. Vertical stress along centre-line in the roof.

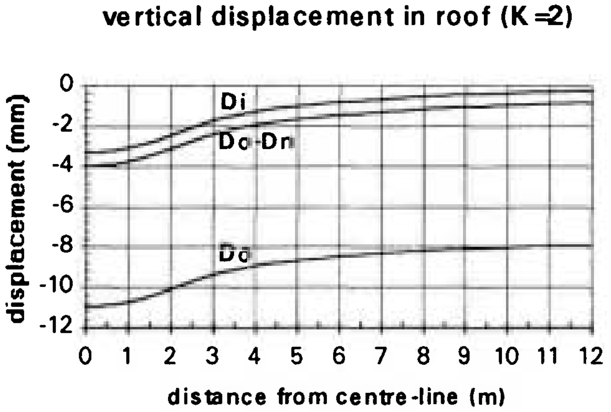


Figure 9. Vertical displacement on horizontal line in the roof.

the final conditon (disturbed state). In other words, the initial conditions of stress and displacement are taken as zero state; hence the results show the induced stress and displacement resulted only from the excavation of the opening.

The above conclusions were exactly the same for all four loading conditions, and in summary it is suggested that:

i- The absolute values of stress around the structure can be obtained from either:

(a) External Model stress (S_0), or

(b) Internal Model stress + Virgin stress, (S_i+S_v).

ii- The relative displacement around the structure can be obtained from either:

(a) Internal Model displacement (D_i), or

(b) External Model displacement-Null Model displacement, (D_o-D_n).

Where S_0 and S_i = stresses resulted from External and Internal Models, respectively.

D_i, D_n and D_o = displacements resulted from Interna, Null and External Models, respectively.

S_V = virgin stress

These results also indicate that the model size could be reduced by 50% using the internal loading technique without affecting the results, because there are no boundary effect on the results in this model. Although for 2-D problems, this may not appear significant, in 3-D models the size reduction would significantly reduce the amount of required memory and computer running time.

VERIFICATION OF THE LOADING TECHNIQUES

In order to verify the results obtained from the Internal and External loading techniques, it was decided to apply both techniques to the classical problem of a semi-infinite plate having a circular hole inside (Figure 10). The results from the internally and externally loaded models were compared with the results calculated using Krisch solution [4]. The comparison of the stress against distance from the centre of the

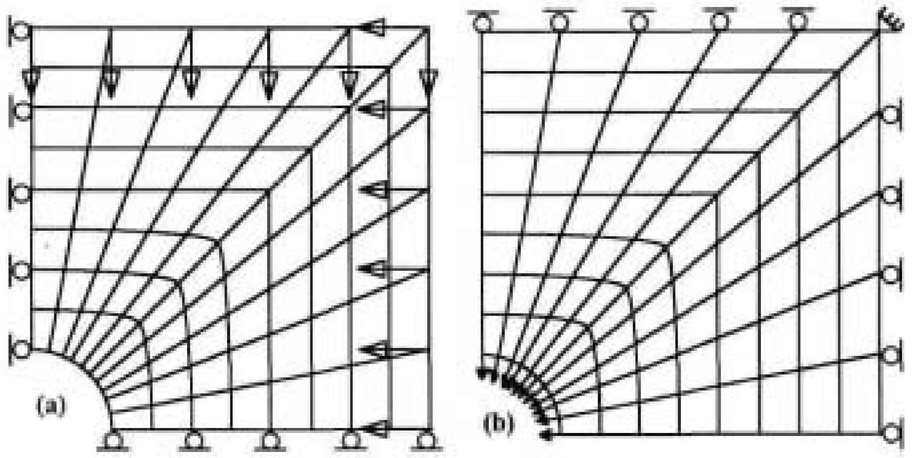


Figure 10. A semi-infinite plate having a circular hole inside, **(a)** External loading and **(b)** Internal loading models.

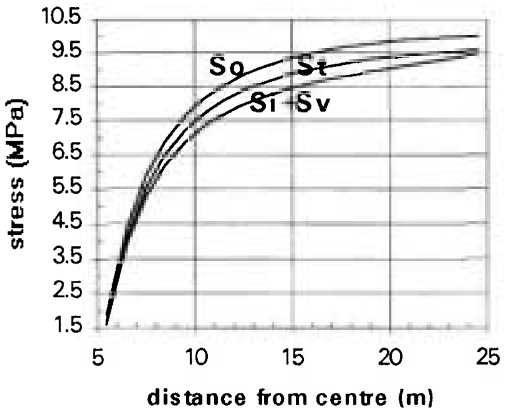


Figure 11. The comparison of the stress against distance from centre of the opening, S_o: External result; S_t: theoretical result; S_i: Internal result; S_v: Virgin stress.

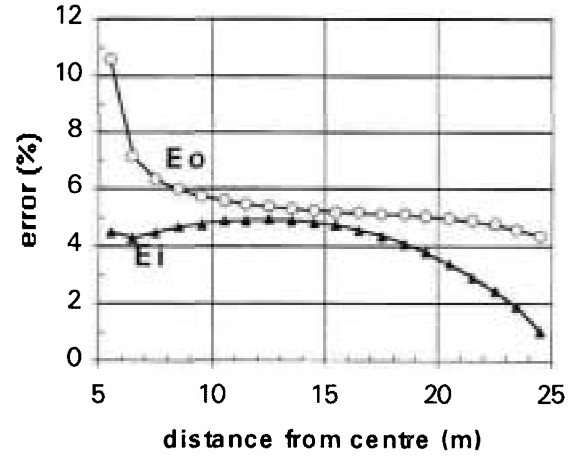


Figure 12. Accuracy of the External and Internal results against theoretical solution, E_0: Error of External loading; E_i: Error of Internal loading.

opening is given in Figure 11. It is shown that all three methods have similar results. If, however, the Krisch solution is accepted as the true values, it can be seen from Figure 12 that the internally loaded model consistently gives more accurate results. This is particularly true for points colse to the opening-which would be the critical region in the case of underground openings. These results suggest that it is desirable to use internal loading when modelling underground structures. This applies to both accuracy of the results and computer costs.

The next point relating to the loading technique is the method of applying horizontal stress on the model. Since each stratum may have a different stiffness, it may seem that horizontal stress would be divided among the strata according to their stiffness. On the other hand, available in-situ measurements have shown a constant uniform distribution of horizontal stress for a limited range of depth. Threfore, two series of models were constructed and analysed to check this point. In the first series, a uniform horizontal

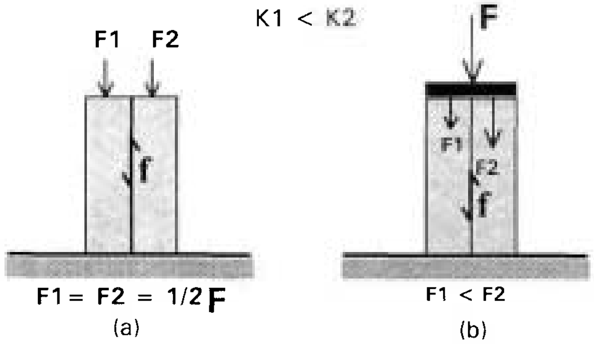


Figure 13. Uniform and Stiffness loading Models (K is stiffness)

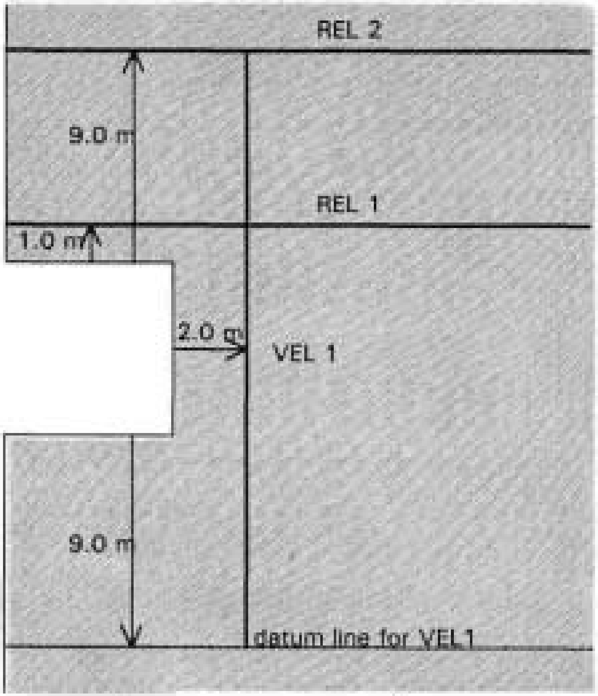


Figure 14. Different locations in the model for analysing the results relating to horizontal loading techniques.

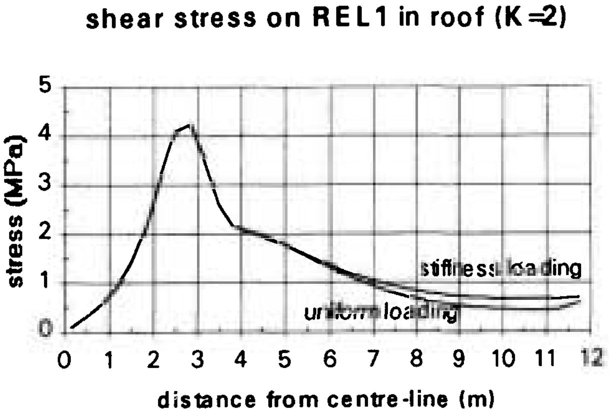


Figure 15. Shear stress along REL1 line in the roof.

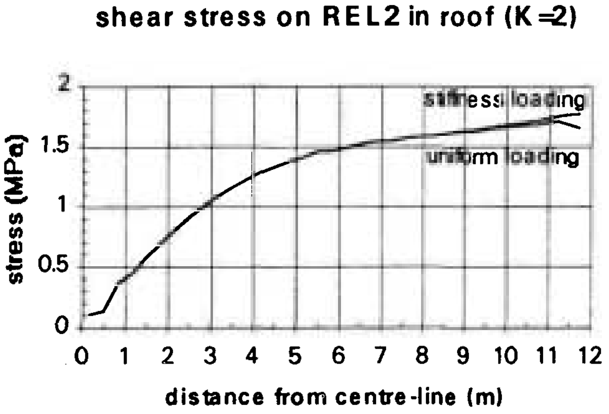


Figure 16. Shear stress along REL2 line in the roof.

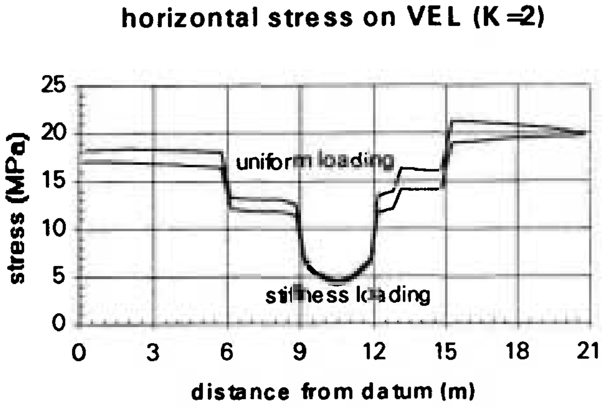


Figure 17. Shear stress along VEL1 line corssing strata vertically.

stress was applied on all strata (Uniform Model) while in the second series the horizontal stress was divided among the strata according to their stiffness (Stiffness Model). These two alternatives are structurally shown in Figure 13.

Theoretical analysis shows that as long as the two parts in the model are bound together, there will be no difference in the average frictional force, f, calculated from either model. This concept was checked by FE analysis. The results from FE models were in good agreement with the theoretical solutions. Figures 15, 16 and 17 show shear stress along three different lines in the model (Figure14). Small discrepancy between the results from Uniform and Stiffness Models around the boundary of the model is due to the fact that the distribution of frictional force along the contact planes is not exactly the same, but not important for the locations close to the structures, about 4 m from the rib-line.

CONCLUSIONS

Based on the findings of this investigation, it can be claimed that the loading technique, the way that virgin stresses are applied to the FE model, has a great influence on the results of any finite element analysis. Gravitational loading method is an appropriate method for modelling surface or shallow structures where it is possible to model the total cover (overburden), or the gradient of the stress along the sides of the model is very small. The uniform loading method is suitable for modelling deep structures but it must be noticed that if the load is applied on the external boundary of the model, results are absolute magnitudes of stresses and displacements. For finding realistic displacements around the structure (relative values), the initial response of the model to the virgin stresses must be taken from the absolute values. On the other hand, if the load is applied on the internal boundary of the model, results are the relative values of the stresses and displacements, and for finding the actual value of stresses, the virgin stresses must be added to the relative values. However, the Internal Loading Technique has advantages in that its results are more accurate, it is easy to add virgin stresses to model results to get the absolute magnitude of the stresses, it is possible to reduce the model size because there is no disturbance around the otuer boundary of the model. There is not significant difference between applying the horizontal stress uniformly or considering transitional zone.

And finally the Internal Loading Technique is recommend for deep underground structures. It will help to choose smaller model in order to save computer time significantly as well as to obtain more accurate results. These benefits will be very appreciable when constructing and analysing 3-D models. Further investigations on the stability of three-and four-way interesections of tunnels (3-D models) are under way of which the results will be presented in separate papers.

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