A ROBUST STRUCTURAL FINGERPRINT RESTORATION

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Abstract Fast and accurate ridge detection in fingerprints is essential to each AFIS (Automatic Fingerprint Identification System). Smudged furrows and cut ridges in the image of a fingerprint are major problems in any AFIS. This paper investigates a new online ridge detection method that reduces the complexity and costs associated with the fingerprint identification procedure. The noise in fingerprint is highly correlated and the statistics of such noises are unknown. In this case, image enhancement techniques based on probabilistic approach may not be suitable. In view of imprecise knowledge about the fingerprint noise, a fuzzy set theoretical approach would be more effective. A new structural algorithm for ridge restoration which is based on unsupervised fuzzy classification technique is described. The accuracy and speed of the proposed method are tested for a large number of fingerprint images with different initial qualities, and are found to be superior to the conventional methods.

Key Words Fingerprint, Ridge Detection, Fuzzy Classifier, Structural Restoration

چکیده اثر انگشت مهمترین خطوط پوستی در انسان می باشد، که از دیرباز مورد توجه متخصصین تشخیص هویت بوده است. این علامت در دوران جنینی انسان شکل گرفته و تا بعد از مرگ و قبل از متلاشی شدن پوست بدون تغییر باقی می ماند. منحصر به فرد بودن خطوط اثر انگشت آدمی باعث شده که به عنوان یک نشانه مطمئن برای شناسایی و تایید افراد بکار گرفته شود، و سیستمهای گوناگون دستی و یا خودکار جهت طبقه بندی، شناسایی و تطابق آن طراحی گردند. اثر انگشت اخذ شده عموما دارای اغتشاشات، لکه ها و بریده گی های نامربوط می باشد، که در مراحل انگشت نگاری و تصویر برداری بوجود می آیند. لذا، قبل از هرگونه عمل گروه بندی و شناسایی، پردازش اثر انگشت امری اجتناب ناپذیر می باشد. این مرحله شامل بازیافت خطوط اثر انگشت است، که در نهایت تصویری با دو ناحیه کاملاً مجزای رگه و شیار حاصل می شود. خصوصیات رگه ها بعنوان ویژگی استخراج گردیده و برای شناسایی اثر انگشت بکار می رود. لذا، کارآیی سیستم شناسایی، مستقیما، به صحت و دقت در استخراج خطوط اثر انگشت بستگی دارد. این مقاله یک روش جدید برای بازیافت و استخراج خطوط اثر انگشت بوسیله کامپیوتر، که هم از نظر دقت در حفظ ویژگی ها مناسب بوده، و هم دارای سرعت پردازش مناسب برای یک سیستم خودکار بیدرنگ باشد، ارائه می کند. با توجه به همبستگی نویز و اثر انگشت یک الگوریتم شناسایی ساختاری مبتنی بر منطق فازی پیشنهاد شده است، که مبتنی بر استفاده از ساختار هندسی و طبیعی، یعنی جهت دار بودن، خطوط اثر انگشت می باشد. مقایسه نتایج حاصل از الگوریتم پیشنهادی با نتایج حاصل از روشهای متداول برتری این الگوریتم را، از نظر سرعت و دقت، نشان می دهد.

INTRODUCTION

Fingerprints have been used as a positive human identifier for more than a century. It is considered one of the most reliable signs for human discrimination. The unchangeability of fingerprints during the human life span and the uniqueness of each individual's fingerprints are the basis for using fingerprints for identification purposes [1]. Fingerprint identification is one of the few scientific aids to investigate crimes, and it is extensively used in day to day crime investigations. The fingerprint matching technique and the matching results are acceptable to the court of law as conclusive proof of identity of a person. A fingerprint could be identified by the shape of ridge lines and their end points (minutiae) in the fingerprint image (Figure 1). The court requires a minimum of 8/16 common minutiae for accepting the identity [2]. There are some other application for AFIS falling into one of four basic

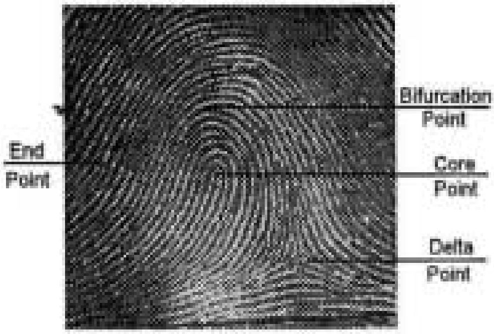

Figure 1. A typical set of common minutiae in a fingerprint.

categories: access control (securing area), transaction control (banking and credit card security), production processing (personality identification), and latent print identification (law enforcement).

With the population explosion and the increase in crime rate, there is an urgent need to develop an automatic fingerprint recognition system to improve the efficiency and reliability of personal identification. The aim of all fingerprinting systems is to produce a low-cost procedure that is reliable, fast and accurate. In general, the function of fingerprinting systems can be categorized into several distinct operations which include sensing or reading, image processing, print registration, classification search, comparison, and verification. More powerful and intelligent image processing and pattern analysis techniques are possible by recent developments in computer technology. This technology is being used to automate the fingerprint identification procedure.

In general, the fingerprints are irregular due to imperfections such as: ridge gaps caused by skin fold, touching of ridges caused by spreading of ink by finger pressure or excessive inking or by smearing during rolling of fingers. Moreover the fingerprint size is usually small and ridges are quite close. Therefore, imperfection or noise introduced at any stage of enlarging and digitization makes the ridge detection quite difficult. Enhancement of fingerprint image is necessary where ridges are not very prominent or have non-uniform intensity.

When the quality is not based on an acceptable standard, automatic fingerprint identification or verification becomes extremely difficult. Thus, an AFIS requires special preprocessing algorithms for the image restoration process in order to preserve connectivity of the ridge lines. The purpose of this paper, then, is to investigate ridge detection/restoration methods that reduce the complexity and costs associated with the fingerprint identification procedure.

More specifically, section one reviews the main approaches, and, presents conventional image segmentation techniques on fingerprints. Section two, introduces the new approach to fingerprint segmentation, by ridge detection. The approach is based on the unsupervised fuzzy classification technique. The results of experiments and their evaluation is presented in section three. Experimental results are presented in terms of real fingerprint data to demonstrate the effectiveness of the proposed approach. Finally, the general application of the new method and its possible extensions are considered. It is argued that this approach has the advantage of being simple and having a significant improvement in identification performance.

CONVENTIONAL METHODS

In segmentation of a fingerprint image, a clear ridge area constitutes the foreground, and any other area (including smudged or noisy regions) constitutes the background. Thus, any pixel of the fingerprint image, f(x,y), is classified either to ridges, 1, or to background, ϕ:

$$\psi(f(x,y)) \in \{\phi, 1\} \tag{1}$$

Image segmentation techniques usually aim at an optimal trade off between efficiency and implementation simplicity, according to the user's needs. Basically, two different approaches have been available to segment the fingerprint images: pixel segmentation methods and neighborhood segmentation methods. In the pixel segmentation methods, individual pixel values are manipulated independently of other pixels. In the neighborhood segmentation methods, groups of pixels are operated on either in the space domain or in the spatial-frequency domain. The operations performed can be either linear or nonlinear [6,7]. The simplest pixel segmentation method is to perform a thresholding operation using an appropriate global threshold, TH, [8].

$$\psi(f(x,y)) = \begin{cases} \phi & \text{if} \quad f(x,y) \leq TH \\ 1 & \text{else} \quad f(x,y) > TH \end{cases} \tag{2}$$

Although segmentation by a fixed threshold is very sensitive to smudged and noisy regions, it can be performed very fast. This is a technique that one would normally try first when confronted with high quality fingerprint images. Figure 2 shows the results

Figure 2. Ridges detection by a fixed threshold.

of this technique.

When ridges are not very prominent or have non-uniform intensity, this method can be improved by using adaptive segmentation. A daptive segmentation is performed by manipulating groups of pixels in small moving windows across the image. The pixel value in the segmented image is determined by a function operation on several pixel values in the moving window in the original image. This function can be either linear or nonlinear. One such process is adaptive binarization, where an M by M window is placed over each pixel in the image and the average gray level within the window is computed. Then the center pixel is set to white or black depending on whether the value of the pixel is less than or greater than the local average [9].

$$\psi(f(x,y)) = \begin{cases} \phi & \text{if} \quad f(x,y) \leq TH(x,y) \\ 1 & \text{else} \quad f(x,y) > TH(x,y) \end{cases} \tag{3}$$

where

$$TH(x,y) = \frac{1}{M^2} \sum_{i=-\frac{M}{2}}^{\frac{M}{2}} \sum_{j=-\frac{M}{2}}^{\frac{M}{2}} f(x-i, y-j) \tag{4}$$

Figure 3 shows the result of the adaptive binarization algorithm. This technique suffers from noisy regions.

Another example of moving window processing in the spatial domain is edge enhancement for highlighting the ridges [10, 11]. These operations can also be performed in the spatial-frequency domain such as directional filtering, with less speed but greater flexibility [12]. One of the more sophisticated spatial-frequency filters is Laplacian filter. Laplacian filter accentuates the high spatial-frequency terms over the low frequency terms. This has the effect of accentuating edges in the original image while suppressing the slowly varying gray level components.

This method is very sensitive to noisy regions and also to the width of the ridges [13].

There are also several segmentation techniques based on edge detection [14, 15, 16]. These techniques suggest that in order to detect the ridges efficiently, one should search for a filter that has two salient characteristics. First and foremost, it should be a differential operator, taking either a first or second spatial derivative of the image. Second, it should be capable of being tuned to act at any desired scale, so that large filters can be used to detect blurry shadow edges, and small ones to detect sharply focused fine details in the image. Marr and Hildreth argued that the most satisfactory operator fulfilling these conditions is the filter ∇^2 **G**, here ∇^2 is the Laplacian operator and **G** stands for the Gaussian distribution which has standard deviation σ.∇^2**G** is a circular symmetric Mexican hat-shaped operator whose distribution in two dimensions may be expressed in terms of the radial distance, **d**, from the origin by the formula:

$$\nabla^2 G\,(d) = \frac{-1}{\pi\sigma^4}\left(1 - \frac{d^2}{2\sigma^2}\right) e^{\frac{-d^2}{2\sigma^2}} \tag{5}$$

where $d^2 = x^2 + y^2$. The standard deviation, σ, could be proportional to the width of the ridges [12]. Then, the segmented image can be computed by:

$$\psi\,(f(x,y)) = \begin{cases} \phi & \text{if} \quad g\,(x,y) \leq TH \\ 1 & \text{else} \quad g\,(x,y) > TH \end{cases} \tag{6}$$

where, g(x,y) is the convolution of input image and ∇^2 G:

$$g\,(x,y) = \nabla^2 G(x,y) * f\,(x,y) \tag{7}$$

Figure 4 illustrates the ridges of a fingerprint which are extracted by using the Marr-Hilderth filter.

Some of the earlier studies cited above [11-19] suffer from sudden changes in fingerprint intensity, while some are highly sensitive to noise. At the same time, most suffer from limitations due to the necessary conditions requiring large amounts of CPU time, and iteration processing. Also some have major shortcomings due to their dependency on manual initiation or supervision for thresholding assignment. In those time consuming approaches where the extraction of ridges is based on directional regions,

Figure 3. Ridges Detection by Marr-Hilderth filter.

Figure 4. Ridges Detection by Marr-Hilderth filter.

even though ridge connectivity may be guaranteed, the extraction of a complex ridge with sudden rotations is not accurate [5].

RIDGE DETECTION BASED ON CONTEXTUAL FEATURE

A good ridge detection method reduces the effect of poor quality data and improves the fingerprint classification performance [3, 4, 5]. This method should not be very sensitive to the contrast in the image, should detect smudged and noisy regions, and should give consistent results for a variety of images expected by the application. In the conventional fingerprint segmentation, a clear ridge area constitutes the foreground and any other area constitutes the background.

It is desirable that all vague regions such as the smudged regions and other such noisy regions be included in the background.

Typically, the vagueness in the detection of the ridges is caused by smudged furrows and cut ridges in the fingerprint image. A ridge could be distinguished on the basis of its intensity feature, using statistical pattern classification techniques. However, the intensity of a ridge often has a weak relation to the finger's dermatoglyphics (it is also related to the finger's pressure on the screen or the stamp inking). One might expect the segmentation accuracy to be higher by using the spectral-contextual feature, rather than using only the measurement made on the pixels' intensity without context [20]. The orientation pattern in the fingerprint which is fuzzy in nature, and its structural information as a grammar rule [21], can be incorporated in a ridge detection process.

Ridges have an oriented pattern thus, each ridge point, r, could be associated with an approximate direction, call it class θ_i:

$$\theta_i = \frac{180°}{5 - i} ; \qquad i = 1, 2, 3, 4. \qquad (8)$$

This " natural association" has a type of imprecision that stems from a grouping of ridge points into classes, θ_i, that do not have sharply defined directions (for example θ_1 is almost 45°). Fuzzy set theory is able to model these classes [22]. Such classes arise whenever we describe ambiguity and vagueness in mathematical models of empirical phenomena. Ever since Zadeh introduced the idea of fuzzy set theory by utilizing the concept of membership grade, many researchers have been concerned with the properties and application of fuzzy sets, as can be observed in the literature [23]. Figure 5 depicts steps for proposed ridge detection.

Let R = {r} denote a space of ridge points. Then a fuzzy set θ in R is a set of ordered pairs:

$$\theta_i = \{r, \mu_{\theta i}(r)\}, \; r \in R \qquad (9)$$

where $\mu_\theta(r)$ is termed "the grade of membership of **r** in θ. " It is assumed that $\mu_\theta(r)$ is a value in the interval [0,1], with the grades 1 and 0 representing full membership and non membership in the fuzzy set, respectively. It is interesting to note that the grade of membership value $\mu_\theta(r)$ of a point **r** can be interpreted as the degree of compatibility of the predicate

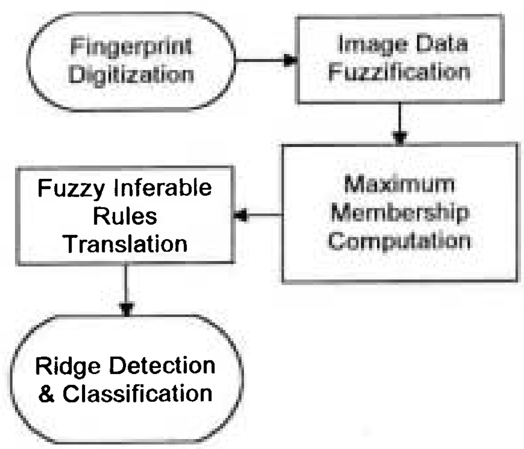

Figure 5. Flow chart of the proposed ridge detection.

associated with direction θ and the ridge **r**. It is also possible to interpret $\mu_\theta(r)$ as the degree of possibility that **r** is the value of parameter fuzzily restricted by θ.

The assignment of the membership function of a fuzzy set is subjective in nature and, in general, reflects the context in which the problem is viewed. The assignment of the membership function of the fuzzy set R is "subjective", and it cannot be assigned arbitrarily. The ambiguity in the ridge detection process can be reduced if the spatial dependencies, which exist among adjacent pixels, are intelligently incorporated into the membership function. Based on this assumption, we now introduce a new membership function by:

$$\mu_{\theta i}(r) = \lambda_r \{(1-\alpha r^2(\theta_i))(e^{-\alpha r^2 \theta i})\}\left(e^{-\frac{\alpha}{2} r^2 (90-\theta_i)}\right) \quad i = 1, 2, 3, 4. \tag{10}$$

where, λ_r is a normalization factor such that the membership function $\mu_{\theta i}(r) \in [0, 1]$.

This membership function has two parts. The first part, $\{(1-\alpha r^2(\theta_i))(e^{-\alpha r^2 \theta i})\}$, is a match filter which has high response to a ridge with the direction of θ_i. The constant α is the spatial-frequency of ridges in the fingers, i.e. α^{-1} approximately equal to the width of a ridge plus a furrow. Also, the first part is very close to the human's retina responses to a ridge with the direction of θ_i[10].

The second part $\{(e^{-\frac{\alpha}{2} r^2 (90-\theta_i)})\}$ is a Gaussian shape spatial-filter in the direction perpendicular to θ_i. The reason the Gaussian shape for membership function was chosen, is that Gaussian distribution has the desirable characteristic of being smooth and localized in both the spatial and spatial-frequency domains. Strictly speaking, it has a unique distribution in spatial and frequency domains [9], which are optimally localized in both domains. Also, the Gaussian part of the membership function, blurs the image, effectively wiping out all structures at scales smaller than minutiae, which is approximately equals to the width of a ridge plus a furrow.

The inexact statement "a ridge has an approximate direction θ_i" is expressed by $r_i \in R$. It is assumed that an exact comparison is possible for the truths of any two inexact statements: "$r_i \in R$" and "$r_k \in R$", and that the exact relation so obtained satisfies the minimal consistency requirements of transitivity and reflexivity. The $\mu_{\theta i}(r) > \mu_{\theta k}(r)$ mean "r_i is at least as true as r_k" and $\mu_{\theta i}(r) < \mu_{\theta k}(r)$ denoting "r_i is not so true as r_k".

The support of θ is the set of points in R at which $\mu_{\theta k}(r)$ is positive. Based on the above assumptions, the membership directions of each pixel is computed. Intuitively, this point belongs to a ridge-class that admits the highest membership in it, $\mu_{\theta i}(r)$:

$$\mu_{\theta i}(r) = \sup_{k=1}^{4} \mu_{\theta k}(r) \tag{11}$$

The crossover point of θ is the point in R whose grade of membership is 0.5. θ is normal if its height is 1; otherwise if $0.5 < \mu_{\theta i}(r) < 1$ it is subnormal. For discrimination of a ridge from a furrow, the unity relation that must exist among the pixels of a ridge is defined. The unity relation is constructed with regard to the adjacency relation, spectral-feature and spatial feature characteristics in a ridge.

When the spectral and spatial features in the membership function are considered:

if $\mu_{\theta i}(r) < 0.5$	then:	$\psi\{r_i\} = \phi$
else:	check the adjacency relation	

And when , the natural structure of fingerprint pattern, as a grammar rule, has been used for adjacency relationship test, the consistency of any ridge is checked by comparing the $\mu_{\theta i}(r)$ with the perpendicular direction membership, $\mu_{\theta i+2}(r)$:

Figure 6. Ridge detection by the proposed technique.

Figure 7. Direction of ridges.

if $\mu_{\theta i+2}(r) < 0.5$	then:	$\psi\{r_i\} = 1$
else:		$\psi\{r_i\} = \phi$

It should be noted that: $\mu_{\theta i}(r) \geq \mu_{\theta k}(r)$ for all k, and $\mu_{\theta i}(r) > 0.5$, thus if $\mu_{\theta i+2}(r) > 0.5$, then the ridge is in favor of both classes θ_i and θ_{i+2}[23]. That means: $\mu_{\theta i+1}(r)$ should be the supremum, not the $\mu_{\theta i}(r)$, thus it is a violated case.

In the case of $\psi\{r_i\} = \phi$, this pixel belongs to background valleys, in the other case, $\psi\{r_i\} = 1$, it belongs to a ridge. Figure 6 depicts the result of ridge detection by the above procedure.

The $\mu_{\theta i}(r)$ has additional information about the ridges direction which can be used as a new feature for fingerprint identification. If $\mu_{\theta i+1}(r) > 0.5$ then the ridge is in favor of both classes θ_i and θ_{i+1}, that means: this ridge is in favor of both directions θ_i and $\theta_i + 45°$, which is $\theta_i + 22.5°$.

With no cost time, more precise direction of this ridge point, θ_r, can be computed by the following simple algortihm:

if $\mu_{\theta i}(r) > 0.5$ then:

if $\mu_{\theta i+1}(r) > 0.5$ then: $\theta_r = \theta_i + 22.5°$.

else if $\mu_{\theta i-1}(r) > 0.5$ then: $\theta_r = \theta_i - 22.5°$

else $\theta_r = \theta_i$

Figure 7 depicts the result of ridges classification.

RESULTS

A program, which implements the procedures described in this paper, was written in C and run on an IBM compatible PC with 80486 DX4 processor. The CPU time for each fingerprint is less than 0.5 second. Experimental results are presented in terms of real fingerprint data to demonstrate the effectiveness of the proposed approach, (the image sampling rate was 300 ppi, with 256 gray levels). Example results show good ridge enhancement and noise reduction (Figure 8).

The accuracy and the speed of this method were tested for a large number of fingerprint images with different initial qualities and were found to be quite superior to conventional methods [8, 11, 12, 14, 17, 19]. The resulting images are enhanced, noise reduced, and binary.

The segmented image has additional information about the ridge's direction which can be used as a new feature for fingerprint identification system. Using the ridge's direction, an extremely fast, on-line, and accurate algorithm for thinning the fingerprint images is developed [25]. the thinning process is based on erosion of the ridges from both sides (perpendicular to the ridges direction) towards the ridges' center, with holding the ridges connectivity constrain. The result of this algorithm is depicted in Figure 9. The results have been used for an on line fingerprint verification system. The system has acquired signification improvements in speed and performance [25].

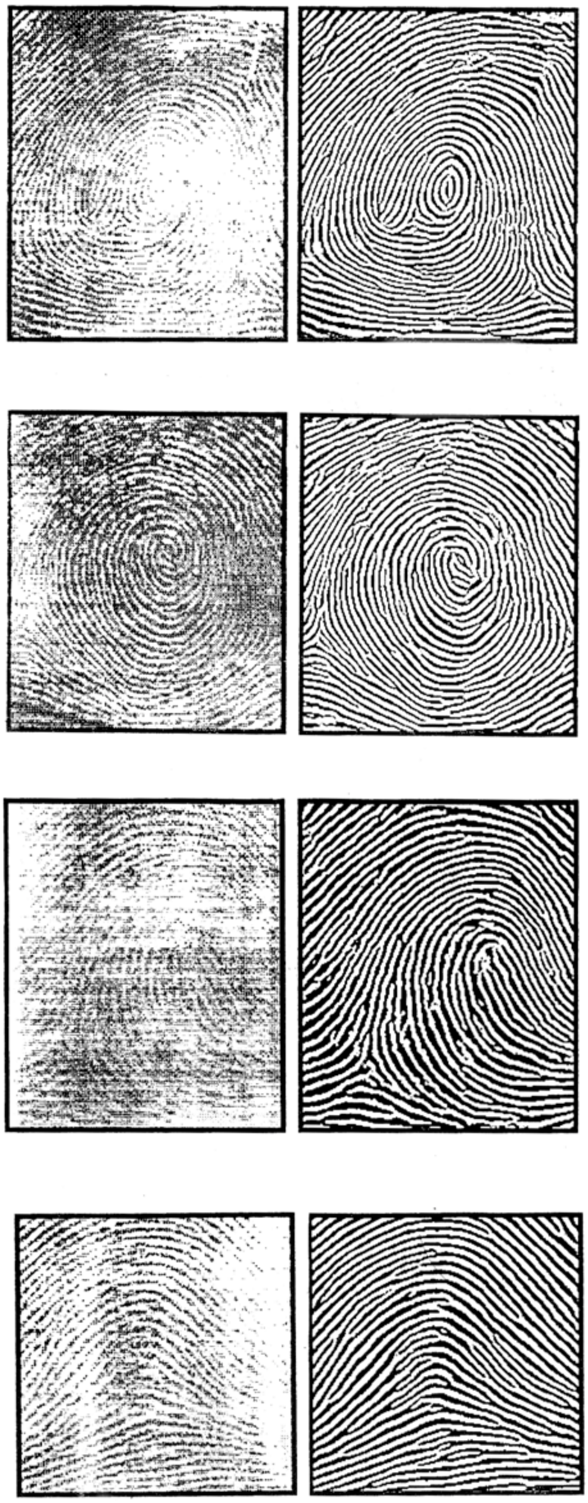

Figure 8. Left images are original fingerprints and the right are results by the proposed method.

SUMMARY AND CONCLUSION

The reliability in the application of fingerprints for individual identification in a security system is explained. The main sources of miss recognition in an automatic fingerprint identification system is also explained. To realize an AFIS the first problem to be

Figure 9. Example result of thinning algorithm.

solved is the acquisition of a high quality segmented fingerprint. Noise elimination and restoration of ridges, which should be incorporated in the preprocessing operation of an automatic fingerprint identification system are discussed. In other words, a robust ridge detection technique is the first step towards further processing, leading to efficient encoding, storage, and recognition of a fingerprint.

An unsupervised fuzzy classification technique has been used for fingerprint ridge detection. A new membership function is introduced, by using the property of oriented pattern in the fingerprints, which is fuzzy in nature. Also, the structural information of finger patterns, as a grammar rule, has been incorporated in the process to enhance the segmented image. The procedure is on line, unsupervised and adaptive. Examples show a good ridge enhancement and noise reduction. It is revealed that, the segmented image has additional information about the ridges' direction, which can be used as a new feature for the fingerprint identification system.

The ridge detection is of course a first, but important, step in a fingerprint recognition system. Nevertheless, the other steps, such as feature selection and classifier design, as well as ridge detection, can improve the cost and performance of the AFIS [25].

REFERENCES

1. Benjamin Miller, "Biometrics: Vital Signs of Identity," *IEEE Spectrum*, (1994), 22-30 .
2. B. C. Bridges, "Practical Fingerprinting,"Furank & Wagnalls Co. (1942).
3. B. Moayer and K. S. Fu, " A Tree System Approach for Fingerprint Pattern Recognition," *IEEE Trans. PAMI*, Vol. 8, No. 3, (May 1986) , 376-387.
4. G. T. Candella and R. Chellappa, " Comparative Performance of Classification Methods for Fingerprints," National Institute of Standards and Technology, (April 1993) .
5. M. H. Ghassemian, "Automatic Singular Points Detection in the Fingerprints," Proc. of ICEE-94, Vol. 5, (1994) 286-294 .
6. R. M. Haralick and L. G. Shapiro, " Image Segmentation Techniques," *Computer Vision, Graphics an Image Processing*, Vol. 29, (1986) 100-132.
7. T. N. Pappas, "An Adaptive Clustering Algorithm for Image Segmentation," *IEEE Trans. on Signal Processing*, Vol. 40, No. 4, (1992), 901-914.
8. P. W. Palumba, et al., "Document Image Binarization, Evaluation Algorithms," *SPIE*, Vol. 697, Application of Digital Image Processing IX, (1986), 278-285.
9. W. K. Pratt, "Digital Image Processing," Second Edition, Wiley, (1991).
10. David Marr, "Vision," Freeman, (1982).
11. P. Danielsson and Q. Ye, " A New Procedure for Line Enhancement Applied to Fingerprints, " Pattern Recognition and Artificial Intelligence, Science Publishers B. V., (1988) 49-61.
12. L. O'Gorman and J. V. Nickerson, "An Approach to Fingerprint Filter Design, " *Pattern Recognition*, Vol. 22, No. 4, (1989), 29-38.
13. T. F. Krile and J. F. Walkup, "Enhancement of Fingerprints Using Digital and Optical Techniques", Image Analysis Application, Marcel Dekker, ch. 10, (1990), 343-371.
14. M. R. Verma, et al., "Edge Detection in Fingerprints," Pattern Recognition, Vol. 20, No. 5,(1987), 513-523.
15. S. Ghosal and R. Mehrotra, "Detection of Composite Edges," *IEEE Trans. Image Processing*, Vol. 3, No. 1, (Jan. 1994).
16. C. C. Chue and J. K. Aggarwal, "The Integration of Image Segmentation Maps Using Region and Edge Information" *IEEE Tran. PAMI*, Vol. 15, No. 12, (Dec. 1993).
17. O. Nakamura, " A Restoration Algorithm of Fingerprint Images," *System and Computer in Japan*, Vol. 17, No.

6, (1986), 31-39.

18. R. Kasturi and J. F. Walkup, "Nonlinear Image Restoration in Signal Dependent Noise," Advances in Computer Vision and Image Processing, Vol. 2, JAI Press, (1986), 167-212,.

19. B. M. Mehtre and B. Chatterjee, "Segmentation of Fingerprint Images A Composite Method," Pattern Recognition, Vol. 22, No. 4, (1989), 381-385.

20. M. H. Ghassemian and D. A. Landgrebe " Object-Oriented Feature Extraction Method for Image Data Compaction," *IEEE Control System Magazine*, Vol. 8, No. 3, (1988) 42-48.

21. K. S. Fu, "Syntactic Pattern Recognition and Application", Prentice Hall, (1982).

22. J. Bezdek, "Pattern Recognition with Fuzzy Objective Function Algorithms, " Ple. Pess, (1981).

23. A. Kandel, "Fuzzy Techniques in Pattern Recognition," Wiley, (1982).

24. L. Lam, et al., "Thinning Methodologies a Comprehensive Survey, " *IEEE Tran. on PAMI*, Vol. 14, No. 9. (1992), 869-885.

25. M. H. Ghassemian, "Automatic Fingerprint Classification System," Technical Reports, Vol. one and Vol. two, Intelligent Signal Processing Research Center and EE. Dept., Tarbiat Modarres University, (1994 and 1996).