A TWO LEVEL APPROXIMATION TECHNIQUE FOR STRUCTURAL OPTIMIZATION

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Abstract This work presents a method for optimum design of structures, where the design variables can be considered as continuous or discrete. The variables are chosen as sizing variables as well as coordinates of joints. The main idea is to reduce the number of structural analyses and the overal cost of optimization. In each design cycle, first the structural response quantities such as forces, displacements, etc. are approximated as functions of the design variables or some intermediate variables. By employing these approximated quantities, an explicit approximate problem will be available, which is in general a nonlinear programming problem. Now, this approximate design task is transformed into a number of second level approximation of separable froms, each of which can be solved by a dual strategy with continuous or discrete variables. The objective of the first level approximation is to reduce the number of structural anlyses required in the optimization problem and that of the second level approximation is to reduce the computational cost of the optimization technique. Two examples are offered to demonstrate the efficiency and reliability of the proposed method.

Key Words Structural Optimization, Approximation Technique, Discrete Variables, Dual Methods

چکیده روشی جهت طرح بهینه سازه ها ارائه شده که متغیرهای طرح می توانند پیوسته یا گسسته باشند. سطح مقاطع اعضاء و مکان هندسی گره های سازه بعنوان متغیرهای طرح انتخاب شده اند. سعی شده است تعداد آنالیز سازه و هزینه کامپیوتری روش بهینه سازی به حداقل ممکنه برسد. درهر سیکل طراحی، ابتدا کمیت های منتج از آنالیز سازه برحسب متغیرهای طرح یا متغیرهای واسطه به صورت روابط تقریبی بیان می شوند. سپس با بکارگیری این روابط تقریبی، مسئله اصلی بهینه سازی تبدیل به تعدادی مسائل تقریبی می شود. مجدداً این مسائل تقریبی، تبدیل به مسائل تقریبی دیگری می شوند که در آنها متغیرهای طرح از یکدیگر جدا شده و توسط روشهای دوگانه با متغیرهای پیوسته یا گسسته به سهولت قابل حل هستند. هدف از ایجاد مسائل تقریبی مرحله اول، کاهش تعدد آنالیز سازه و مرحله دوم، هزینه کلی کامپیوتری را کاهش میدهد. دو مثال جهت مشخص نمودن راندمان و اطمینان روش ارائه شده است.

INTRODUCTION

Optimum design of structures is achieved by combining mathematical programming techniques and finite element analysis. The objective function and the constraint functions are first expressed as functions of the design variables. Then by employing numerical optimization methods, the objective function, which is normally taken as the weight of the structure, is minimized while the constraints are satisfied. The constraints include bounds on member stresses, deflections, frequencies, etc.

In the process of optimiztion, because the constraint functions are implicit functions of the design variables, their precise numerical evaluation requires a complete finite element analysis. In addition, since the solution techniques are iterative, a large number of structural analyses is required to obtain the optimum solution.

In the past, to increase the efficiency of the method of optimiztion, some approximate concepts have been introduced by Schmit and Miura [1], Schmit and Fleury [2] and Salajegheh [3]. the most attractive approach is to linearize the functions under consideration and to solve a sequence of linearized optimization problems. The solution of each approximate problem does not require the analysis of the structure. Some other improvements such as design variable linking, constraints deletion, etc. have increased the robustness of the process.

A second generation approximation techniques was developed by Salajegheh and Vanderplaats [4] and Vanderplaats and Salajegheh [5 , 6] by which the highest quality approximation can be achieved. The implicit structural responses such as forces, displacements, frequencies, etc., appearing in the optimization problem, are first approximated. By substituting these approximate functions into the original problem, a nonlinear explicit problem is created, the solution of which, often requires less than 10 analyses of the structure. This method is very robust and efficient for large structures, where the computational cost of the analysis is high.

Recently, the same approximate technique has been applied by salajegheh and Vanderplaats [7] to the design of structures, where some or all of the design variables are chosen from a prescribed set of values (discrete variables). The discrete optimum design of structures is achieved by combining the approximate techniques and branch and bound method. For practical design problems, where the design variables are linked and the number of independent design variables are chosen reasonably, the method can be used efficiently.

The same approximation concepts are used by Salajegheh and Vanderplaats [8] and Salajegheh [9] to achieve the optimum shape of the structures. In addition to the sizing variables, the coordinates of joints are also considered as design variables. In this case, the numerical results also indicate that the optimum configuration of the structures with discrete shape variables, in conjunction with response approximation, can be achieved at little computational cost.

To further increase the efficiency of the technique for problems with great number of discrete variables, a dual strategy is used by Vanderplaats and Salajegheh [10]. The discrete variable optimization is achieved after the completion of the continuous variable optimization.

In the present approach, a two level approximation concept is used to solve the continuous or discrete sizing and shape variable problem in each design cycle. Ths first level is the creation of a high quality explicit approximate problem. In the second phase, this problem is converted into a sequence of problems, in separable froms, which can be solved by dual methods. The solution of the second level approximate problems, does not require the analysis of the structure under consideration.

PROBLEM FORMULATION

The general optimization problem involving discrete variables, can be mathematically stated as follows:

Minimize $F\ (X)$ (1)

Subject to;

$$g_j\ (X) \leq 0 \qquad i = 1, m \qquad (2)$$

$$X_i^l \leq X_i \leq X_i^u \qquad i = 1, n \qquad (3)$$

$$X_i \in D_i \qquad (4)$$

where $F(X)$ and $g_j(X)$ are the objective function and constraints, respectively. X is the vector of design variables which includes s sizing variables and n - s shape variables where n is the total number of variables as;

$$X^T = \{X_1, X_2, \ldots, X_i, \ldots, X_s, X_{s+1}, \ldots, X_n\} \qquad (6)$$

m is The total number of constraints. X_i^l and X_i^u are the lower and upper bounds (side constraints) on the design variables respectively. D_i is the set of discrete values, which may be different for each variable. Here, the objective function is taken to be the structural weight and constraints include stress, buckling and displacement limits.

The problem stated by Equations 1 to 3 is a general nonlinear programming task, and a variety of methods and softwares are available to solve the problem. To solve the problem efficiently, first all the structural responses are approximated. Of course, the quality of approximation can be increased if the responses are approximated with respect to some intermediate variables. For example, in frame structures, the intermediate variables can be chosen as the cross-sectional properties (areas and moments of inertia), while the design variables may be selected as the physical member cross section dimensions. The intermediate variables can be easily obtained from design variables.

Let P_k represent the k-th structural response, such as a component of internal force in an element in one of the loading conditions, and Z represent the vector of intermediate variables, then the following approximation relation can be created:

$$P_k(Z) = P_k(Z^0) + \sum_{i=1}^{n} B_i \frac{\partial P_k(Z^0)}{\partial Z_i} \qquad (6)$$

where

$$B_i = \begin{cases} Z_i - Z_i^0 & \text{for direct approximation} \\ \left(\dfrac{Z_i - Z_i^0}{Z_i}\right) Z_i^0 & \text{for reciporcal approximation} \end{cases}$$

Z^0 is the current initial vector of intermediate variables and $\frac{\partial P_k(Z^0)}{\partial Z_i}$ represents the gradient of P_k with respect to Z_i.

Usually, move limits are imposed to control the quality of the approximation, although the relations (6) and (7) represent an accurate estimation of the responses.

Now, by substituting the approximate relations into the original design problem, given by Equations 1 to 4, an explicit nonlinear problem is obtained which can be solved for the values of the continuous or discrete variables, subject to move limits. This is one design cycle, which does not require the analyses of the structure and of which the solution is a starting point for the next design iteration.

To reduce the cost of gradient calculation in each design cycle, as most constraints may be far from critical, it is reasonable to ignore many of the constraints for this cycle. The simplest approach would be to just retain only those that are critical or potentially critical for the current design cycle. Thus, we first sort all constraints and then retain all those that are within a specified tolerence of being critical. Assuming all constraints are normalized, we may retain those with a numerical value greater than, say, -0.3. Typically, the number of retained constraints may be two to three times the number of independent design variables.

Let $\bar{g}_j$ represent the approximated form of the jth constraint and J_r indicate the set of retained constraints. Then the general form of the approximate problem, in each design cycle, can be expressed as;

Minimize $F(X)$ (8)

Subject to;

$$\bar{g}_j\,[X, P_k(Z)] \leq 0 \quad j \in J_r \tag{9}$$

$$X_i^l \leq X_i \leq X_i^u \quad i = 1, n \tag{10}$$

$$Z_i^l \leq Z_i \leq Z_i^u \quad i = 1, r \tag{11}$$

$$X_i \in D_i \tag{12}$$

where P_k given by Equations 6 and 7 represents any quantity which is the output of the structural analysis. Z_i^l and Z_i^u are the current lower and upper limits on Z_i (move limits) and r is the number of intermediate variables.

The explicit nonlinear problem given by Equations 8 to 12 is again approximated, using conservative approximation (Fleury and Braibant [11]). In fact, a second level approximation is used to convert the first level approximation into a problem of separable form. Now the problem can be stated as follows;

$$\text{Minimize} \quad \bar{F}(X) \approx F(X^0) + \sum_{i=1}^{n} B_i \frac{\partial F(X^0)}{\partial X_i} \tag{13}$$

Subject to;

$$\tilde{g}_j(X) \approx \bar{g}_j(X^0) + \sum_{i=1}^{n} B_{ji} \frac{\partial \bar{g}_j(X^0)}{\partial X_i} \leq 0 \quad j \in J_r \tag{14}$$

$$X_i^l \leq X_i \leq X_i^u \quad i = 1, n \tag{15}$$

$$X_i \in D_i \tag{16}$$

where

$$B_i = \begin{cases} X_i - X_i^0 & \text{if } X_i^0 \dfrac{\partial F(X^0)}{\partial X_i} \geq 0 \\ \left(\dfrac{X_i - X_i^0}{X_i}\right) X_i^0 & \text{if } X_i^0 \dfrac{\partial F(X^0)}{\partial X_i} < 0 \end{cases} \tag{17}$$

and

$$B_{ji} = \begin{cases} X_i - X_i^0 & \text{if } X_i^0 \dfrac{\partial \bar{g}_j(X^0)}{\partial X_i} \geq 0 \\ \left(\dfrac{X_i - X_i^0}{X_i}\right) X_i^0 & \text{if } X_i^0 \dfrac{\partial \bar{g}_j(X^0)}{\partial X_i} < 0 \end{cases} \tag{18}$$

The Lagrangian function can now be written as follows:

$$L(X, \lambda) = \bar{F}(X) + \sum_{j \in J_r} \lambda_j . \tilde{g}_j(X) \tag{19}$$

where λ represents the vector of dual variables.

Using duality theory, $L(X, \lambda)$ is minimized with respect to X and then maximized with respect to λ, subject to non-negativity constraints on the dual variabls. Noting that $L(X, \lambda)$ after the substitution of $\bar{F}(X)$ and $\tilde{g}_j(X)$ from Equations 13 and 14 is a separable function in terms of X. Nothing further that using the property that the minimums of a separable function is the sum of the minimums of the individual parts the minimization of $L(X,\lambda)$ with respect to X is performed by minimizing a number of one dimensional functions of X, subject to side constraints. If the variables are discrete, then minimization of the one dimensional functions is carried out with discrete values. In fact, first the continuous solution of the one dimensional problems are obtained, then the next lower and upper discrete values to the continuous solution problems are obtained, then the next lower and upper discrete values to the continuous solution are found. Finally, whichever minimizes the one dimensional function, will be the discrete solution.

The main steps in the process of continuous or discrete variable optimization using the duality theory can be summarized as follows:

1. At the current design point, establish the approximate relations of the responses and construct the first level approximation problem with move limits.
2. Construct a separable problem, by re-approximating the first level approximate problem, and using a conservative type approximation (second level approximation).
3. Solve this problem by the continuous (discrete) dual method with move limits and repeat steps 2 and 3 to converge.
4. Check the overall convergence. If converged, terminate; otherwise, repeat from step 1.

The method has been applied to a number of problems for continuous and discrete sizing and shape variable optimizations and the numerical results indicate that the approach is very efficient and the results are reasonable, compared with those obtained by branch and bound with approximation.

NUMERICAL RESULTS

Here two examples are offered to demonstrate the reliability of the method. The DOT optimizer [12] was used to maximize the dual problem. The gradients of the retained responses are calculated by finite difference, while gradients in the approximate primal and dual problems are calculated analytically.

Problem 1: 47-Bar Planar Tower

The 47-bar planar towar shown in Figure 1 is designed for optimum geometry with continuous and discrete sizing and shape variables, subject to three independent load conditions and material properties given in Table 1.

Both member areas and coordinates are linked such that the symmetry is maintained about the vertical y-axis. Joints 15, 16, 17 and 22 are fixed in space and joints 1 and 2 are required to lie on the x-axis ($y = 0$). There are a total of 27 independent sizing variables and 17 independent shape variables. The constraints include stress and Euler buckling. The Euler buckling compressive stress limit, σ_{bi} is taken as;

$$\sigma_{bi}=\frac{-K_i E_i A_i}{L_i^2},$$

where K_i is a constant determined from the cross-sectional geometry and E_i is the modulus of elasticity. A_i is the member area. L_i is the member length, which is a function of shape variables. The set of available discrete values are:

$$X_i \in \{0.1, 0.2, 0.3, 0.4, 0.5, \ldots\} \times 6.4516\ (\text{cm}^2),\ i=1, 27$$

TABLE 1. Load Conditions and Material Properties of the Tower

	Joint	F_x	F_y
(a) Load condition 1 (N):			
	17	26688.0	-62272.0
	22	26688.0	-62272.0
(b) Load condition 2 (N):			
	17	26688.0	-62272.0
	22	0.0	0.0
c) Load condition 3 (N):			
	17	0.0	0.0
	22	26688.0	-62272.0
(d) Material properties:			
Modulus of elasticity	20.68E + 7 KPa		
Allowable tensile stress	+137, 900 KPa		
Allowable compressive stress	-103, 425 KPa		
Weight density	0.008 kg/cm^3		
Minimum area	0.645 cm^2		
Buckling constant, K_i	3.96		

$X_i \in \{1.0, 2.0, 3.0, 4.0, 5.0, ...\} \times 2.54$ (cm2), $i = 28, 44$

A total of 10 analyses was used to obtain the continuous and discrete solutions. The results are presented in Table 2. The results are similar to those of branch and bound method (Salajegheh and Vanderplaats [8]). However, the computer time required to complete the optimization process was less than 1/10 of the time required by branch and bound.

Problem 2: 25-Bar Space Truss

The 25-bar truss, shown in Figure 2, is designed to support two independent load conditions. The load conditions and material properties are given in Table 3. The member areas are linked in the following groups to maintain symmetry: $A1$; $A2 = A3 = A4 = A5$; $A6 = A7 = A8 = A9$; $A10 = A11$; $A12 = A13$; $A14 = A15 = A16 = A17$; $A18 = A19 = A20 = A21$ and $A22 = A23 = A24 = A25$. The coordinate variables are also linked to maintain symmetry. The independent shape variables are the following coordinates: $X4$, $Y4$, $Z4$, $X8$ and $Y8$. There are then a total of eight independent sizing variables and five independent shape variables. The discrete values for the design

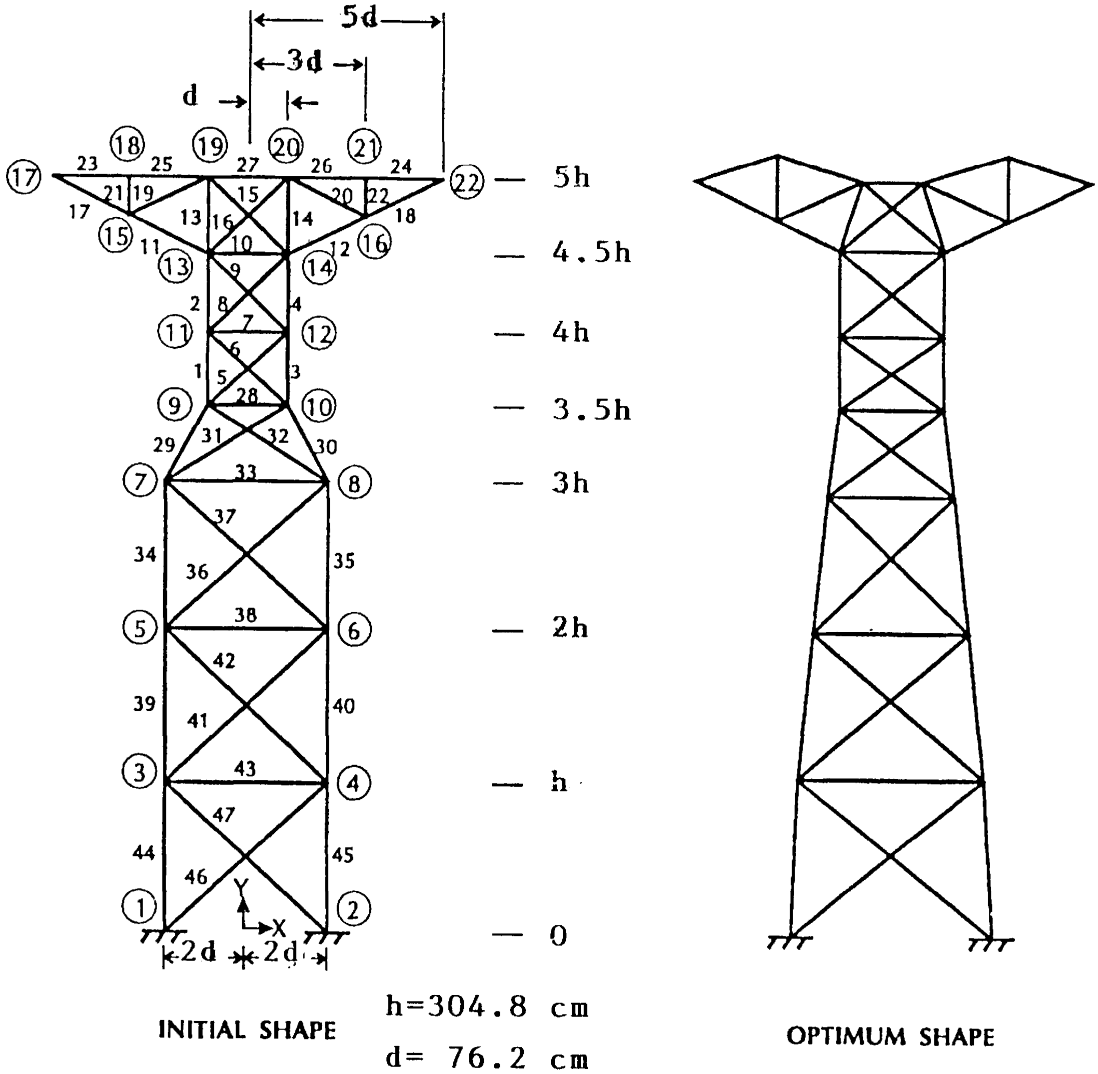

Figure 1. 47-bar planar tower

TABLE 2. Reults for Tower (A: cm^2; X, Y : cm)

Variable number	Variable	Initial value*	Continuous solution*	Discrete solution*
1	$A3$	3.8	2.61	2.7
2	$A4$	3.4	2.56	2.6
3	$A5$	0.8	0.69	0.7
4	$A7$	0.9	0.47	0.4
5	$A8$	0.9	0.80	0.8
6	$A10$	1.8	1.13	1.2
7	$A12$	2.1	1.71	1.7
8	$A14$	1.2	0.77	0.8
9	$A15$	1.6	1.09	1.1
10	$A18$	2.1	1.34	1.4
11	$A20$	0.7	0.36	0.4
12	$A22$	0.9	0.97	1.0
13	$A24$	1.7	1.00	1.0
14	$A26$	1.7	1.03	1.1
15	$A27$	1.4	0.88	0.8
16	$A28$	0.9	0.55	0.6
17	$A30$	3.7	2.59	2.7
18	$A31$	1.5	0.84	0.9
19	$A33$	0.7	0.25	0.1
20	$A35$	2.9	2.86	2.9
21	$A36$	0.7	0.92	1.0
22	$A38$	1.6	0.67	0.5
23	$A40$	3.7	3.06	3.1
24	$A41$	1.6	1.04	1.1
25	$A43$	0.7	0.10	0.1
26	$A45$	4.5	3.13	3.2
27	$A46$	1.6	1.12	1.1
28	$X2$	60.0	107.76	106.0
29	$X4$	60.0	89.15	89.0
30	$Y4$	120.0	137.98	136.0
31	$X6$	60.0	66.75	66.0
32	$Y6$	240.0	254.47	255.0
33	$X8$	60.0	57.38	57.0
34	$Y8$	360.0	342.16	342.0
35	$X10$	30.0	49.85	50.0
36	$Y10$	420.0	417.17	415.0
37	$X12$	30.0	44.66	45.0
38	$Y12$	480.0	475.35	475.0
39	$X14$	30.0	41.09	40.0
40	$Y14$	540.0	513.15	513.0
41	$X20$	30.0	17.90	17.0
42	$Y20$	600.0	597.92	598.0
43	$X21$	90.0	93.54	93.0
44	$Y21$	600.0	623.94	624.0
Weight (kg)		1109.8	861.8	877.3

* Areas should be multiplied by 6.4516.

* Coordinates should be multiplied by 2.54.

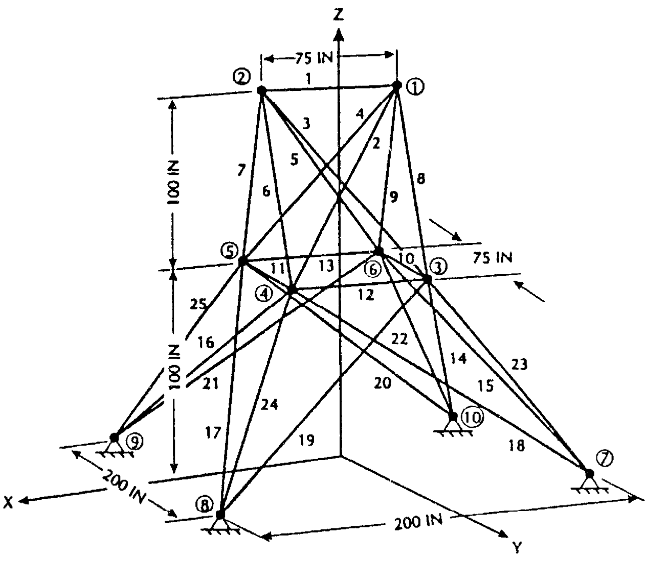

Figure 2. 25-bar space truss

TABLE 3. Load Conditions and Material Properties for 25-Bar Truss

(a) Load condition 1 (1bs):

Joint	F_x	F_y	F_z
1	0.0	20,000.0	-5000.0
2	0.0	-20,000.0	-5000.0
3	0.0	0.0	0.0
6	0.0	0.0	0.0

(b) Load condition 2 (1bs):

Joint	F_x	F_y	F_z
1	1000.0	10,000.0	-5000.0
2	0.0	10,000.0	-5000.0
3	500.0	0.0	0.0
6	500.0	0.0	0.0

(c)Material properties:

Modulus of elasticity	1.0E+7
Allowable stress	±40,000 psi
Weight density	0.1 lb/in^3
Buckling coefficient, K_i	39.274
Minimum area	0.1 in^2

variables are considered to be:

$$X_i \in \{0.1, 0.2, 0.3, 0.4, 0.5, ...\} \quad i = 1, 8$$

$$X_i \in \{0.1, 0.2, 0.3, 0.4, 0.5, ...\} \quad i = 9, 13$$

This problem, converged with 7 analyses for continuous design optimization. One extra analysis was required to complete the discrete variable problem, which was only used for convergence check. The results are presented in Table 4. Again the results are similar to those of branch and bound with less computational efforts.

CONCLUSIONS

The efficiency of a method of optimum design of structures depends on the number of continuous and discrete variables, number of constraints and the number of structural analyses required in the process of optimization. To reduce the number of analyses, all the quantities that are obtained from the analysis are approximated as functions of the design variables or some intermediate variables in each design iteration. The idea of constraint deletion is employed in each design cycle to reduce the cost of gradient calculation. Finally, the implementation of the dual methods in conjunction with approximation concepts is very effective in the design problems with great number of variables. Thus, it is concluded that the combination of response approximation, constraint deletion and dual methods form the basis of an efficient technique for optimum design of practical problems.

TABLE 4. Reults for 25-Bar Space Truss

Variable number	Variable	Initial value	Continuous solution	Discrete solution
1	*A1*	0.009	0.100^L	0.1^L
2	*A2*	0.782	0.434	0.5
3	*A6*	0.754	0.951	1.0
4	*A10*	0.001	0.100^L	0.1^L
5	*A12*	0.130	0.100^L	0.1^L
6	*A14*	0.558	0.154	0.2
7	*A18*	0.982	0.705	0.7
8	*A22*	0.801	0.544	0.6
9	*X4*	37.5	19.922	19.0
10	*Y4*	37.5	42.522	42.0
11	*Z4*	100.0	92.777	90.0
12	*X8*	100.0	13.239	10.0
13	*Y8*	100.0	79.057	76.0
Weight		229.41	130.76	140.20

L: Lower Bound

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