# QUADRATURE AMPLITUDE MODULATED DIGITAL RADIO TRANSMISSION MODELING AND SIMULATION

# M. Kavehrad

Department of Electrical Engineering University of Ottawa Ottawa, Ontario

**Abstract** Computer aided design and computer aided modeling tools are becoming increasingly important in the design and performance evaluation of communication systems. In this work, we report on the computer simulation modeling study of terrestrial digital microwave radio transmission, using Block Oriented System Simulator package (BOSS). The work concentrates on semi-analytical error rate evaluation in digital transmission. Simulation results include time and frequency domain representations of various system outputs.

چکیده ابزارهای طراحی و الگوسازی بکمک کامپیوتر در طراحی وارزیابی عملکردسیستم های مخابراتی روز به روز اهمیت بیشتری پیدا میکند. در این مقاله در مورد تهیه الگوی شبیه سازی انتقال امواج مایکرو دبجیتال فیزیکی رادیویی با استفاده از برنامه کامپیوتری BOSS بحث میشود. تأکید مقاله برارزیابی نیمه تحلیلی خطا در انتقال دیجیتال است. نتایج شبیه سازی شامل پاسخ زمان و فرکانس خروجی های مختلف سیستم است.

#### INTRODUCTION

The demand for high capacity digital transmission has brought a surge of interest to the applications of multilevel digital modulation techniques; in particular, M-state quadrature amplitude modulation. The last decade has witnessed applications in terrestrial digital microwave radio transmission and voiceband data modems.

Computer aided design and computer aided modeling tools are becoming increasingly important in the design and performance evaluation of communication systems. In this work, we report on computer simulation modeling of transmission system, using Block Oriented System Simulator (BOSS).

Using generator polynomials in octal Galois fields, we generate octal sequences which are treated as multilevel symbols into a modulator. We describe the procedure in this paper and present the generator polynomials.

Nyquist-type and Butterworth filters are simulated on BOSS for transmitter and receiver filtering. For these we employ frequency domain filters and the existing modules on BOSS which are described in the paper.

A Rummler type [1] line-of-sight channel model is simulated applying primitive modules on BOSS. Simulation of a synchronously-spaced taps least-mean-square adaptive equalizer for 64 - QAM transmission and some preliminary error probability results with semi-analytical QAM error estimator module of BOSS are presented and discussed.

In digital transmission simulation studies that aim at error performance analysis, computing time is one of the more important factors. There are several ways of modeling an end-to-end transmission system for performance evaluation. M. Jeruchim has nicely summarized some of these methods in [2]. The techniques are generally categorized under bit error rate (BER) estimation. Among these are, Monte-Carlo simulation, modified Monte-Carlo simulation with importance sampling, extreme-value theory, tail extrapolation and quasi-analytical (a combined noiseless simulation with an analytical representation of noise). These techniques have a common feature, namely, they are all simulation-based, that is, they emulate a process evolution in time.

In this work we follow a quasi-analytical method, i.e., the noise representation is carried out analytically. This is particularly suited to systems with only additive Gaussian noise. In this case the noise is not carried through the simulation, rather, its effects are incorporated analytically. For communication channels with memory, the simulation duration is determined by the end-to-end system memory, e.g., by intersymbol interference.

In a linear system with Gaussian noise as the only noise source, this method is exact. However, to maintain the integrity of the results, the simulated data source must correctly emulate all possible phase transitions in the data. To ensure this, one may simulate the data source as a PN-sequence generator. This is particularly important when we deal with higher order modulation schemes. To emu-

late all the possible phase transitions, a very long simulated data stream is needed if a random number generator is used instead of a PN-sequence generator. This may cause a lengthy and time-consuming simulation, if not an impractical one.

In the case of a 16-QAM signal, sequence generators in GF(4) are desirable. For 64-QAM signal transmission, octal sequence generators basically simulate the symbols on the in-phase (I) and the quadrature-phase (Q) channels of the modulator.

## SEQUENCE GENERATOR MODEL

Galois field theory and its application to certain aspects of coding in digital communications is well established. The emphasis has always been on the binary regime GF(2) and its extensions GF (2<sup>n</sup>) so that ordinary binary switching circuits may be employed in realizing practical systems. The foundation to studies in these areas is the access to irreducible polynomials and their required exponents over the desired field. Although tables of such polynomials have been in existence for may years in binary fields [3, 4, 5], not many lists are available for other fields, with the exception of those [6] and [7].

In this work, we are interested in octal sequence generators, so we can simulate the symbols on I and Q rails of a 64-QAM signal. The order of the generator polynomial should match the channel memory in order to properly come up with the statistics of the intersymbol interference at the receiver. Let's assume intersymbol interference (ISI) spans over N symbols. Then there are  $2^N$  possible distinct combinations of symbol patterns and we must examine them all, e.g., for N=3, all the corresponding eight binary words should be studied. For an octal system representing 1 and Q channels of a 64-QAM signal, there are  $8^N$  possible combinations. This is already a very large number of combinations when, for example, channel memory is N=5.

Using PN maximal length sequences, it is possible to generate one sequence of length  $2^N$  in which every combination exists and exists only once. These sequences are generated from primitive polynomials as in coding theory applications. For example, with N=3; the 8-bit sequence 10111000 contains all possible 3-bit combinations. PN-sequences can easily be generated by shift register logic circuits [4]. In non-binary fields the primitive polynomials representing these circuits have to operate with the non-binary elements of the field and should be irreducible in the fields. As an example, to simulate 64-QAM symbols over an octal field GF(8), we may start with a primitive generating polynomial

$$g(x) = x^3 + x + 1 \tag{1}$$

that defines the field.

This polynomial is irreducible, i.e., it cannot be divided by any linear or quatratic polynomial with binary coefficients. The field includes all polynomials of order less than 3;  $q(x) = q_2x^2 + q_1x + q_0$  or, equivalently, all possible threetuples  $q = (q_2, q_1, q_0)$ . The operations in these fields are those of most logical devices. For example, assume a =(0,1,0) and b = (1,1,0). The addition is defined as a bitwise binary addition. To illustrate the product of these numbers consider: a(A) = A and  $b(A) = A^2 + A$ . Then

$$c(A) = a(A), b(A) = A, (A^2 + A) \mod g(A) = (A^3 + A^2), \mod g(a).$$

To evaluate c(A) we have to find the remainder of the polynomial division which is  $A^2 + A + 1$ . Hence,

$$a \cdot b = (0,1,0) \cdot (1,1,0) = (1,1,1).$$

In every GF there is a primitive element  $\alpha$  such that any non-zero element of the field can be expressed as a power of  $\alpha$ . We can illustrate this by using the same example. Let  $\alpha = (0,1,0)$  which corresponds to  $\alpha = A$ . Then,

$$\alpha^{\dagger} = A \sim \tag{0,1,0}$$

$$\alpha^2 = A^2 = B \sim (1.0.0)$$

$$\alpha^3 = A^3 \mod (A^3 + A + 1) = A + 1 = C \sim (0,1,1)$$

$$\alpha^4 = \alpha^3 A = A^2 + A = D \sim$$
 (1.1.0)

$$\alpha^5 = \alpha^4 A \mod (A^3 + A + 1) = A^2 + A + 1 = E \sim (1, 1, 1)$$

$$\alpha^6 = A^2 + 1 = F \sim \tag{1.0.1}$$

$$\alpha^7 = 1 \quad \sim \tag{0.0.1}$$

Basic elements of the field are, therefore:

$$\alpha^{(1)} = 1, \ \alpha = A, \ \alpha^2 = B, \ \alpha^3 = A + 1 = C,$$

$$\alpha^4 = A^2 + A = D, \ \alpha^5 = A^2 + A + 1 = E, \ \alpha^6 = A^2 + 1 = F$$
 (2)

We may now assign various levels of a 64-QAM signal to these elements as follows:

$$+7 = (1,1,1)$$

$$+5 = (1,1,0)$$

$$+3 = (1,0,1)$$

$$+1 = (1,0,0)$$

$$-1 = (0,1,1)$$

$$-3 = (0,1,0)$$

$$-5 = (0,0,1)$$

$$-7 = (0,0,0)$$

This mapping completes the theory of multilevel generation. A sample of an octal sequence generator function for a memory-3 channel is:

$$f(x) = x^3 + x + \alpha \sim (101A)$$
 (3)

The difference equation corresponding to this function is:

$$N(j+3) = N(j+1) \oplus \alpha_0 N(j)$$
 (4)

where  $\oplus$  and $\Theta$  are addition and multiplication operators over octal field, respectively. The polynomial equation (3) is denoted by 101A. An octal shift register sequence generator circuit for this case is shown in Figure 1. To make the I and Q channel sequences fairly uncorrelated, we may apply to the Q channel a sequence generated by another memory-3 irreducible polynomial over GF(8), represented by 101B. A sample of the octal sequence output of the above shift register is shown in Figure 2.

The generator function in the case of a memory-4 channel corresponds to

$$f(x) = x^4 + x + \alpha^3 \tag{5}$$

This is also denoted by 1001C and can be applied, e.g., to the I channel. We can apply 1001E to the Q channel. We found the latter by a computer search, and ensured a uniform number of levels at the corresponding generator outputs. These sequences are sufficiently uncorrelated, hence proper representation of symbols on the I and Q channel is possible.

For one of the memory-5 channel symbol streams we may use the following sequence generator polynomial:

$$f(x) = X^5 + X^2 + X + \alpha^3$$
 (6)

alternatively denoted by 10011C. For the second channel symbol stream we may employ 10011 E which again we found by simple computer search.

In point-to-point terrestrial microwave transmission, channel memory values 3 and 4 suffice in studying the performance under normal fading multi Path conditions. Value 5 is occasionally needed to represent extremely deep fades that occur rarely.

For applications involving 16-QAM transmission, the primitive generator polynomials are selected from GF(4)

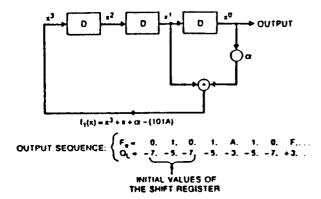


Figure 1. Octal PN-Sequence Generator.

to represent the I and Q channels. The polynomial over this field is [7]:

$$g(x) = x^2 + x + 1 \tag{7}$$

If  $\alpha$  is a root of this, the field elements are defined by (0,1,A, B), where the non-zero elements can be identified as:

0; 
$$\alpha^0 = 1$$
;  $\alpha^1 = A$ ; and  $\alpha^2 = \alpha + 1 = B$  (8)

and a PN sequence generating function is, e.g., 11A.

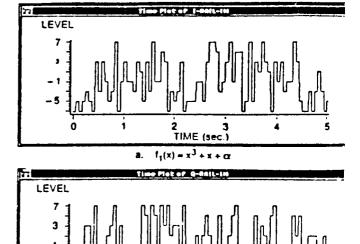
With this background on source simulator, we will describe other subsystems simulations in the following sections.

## TRANSMISSION SUBSYSTEMS SIMULATION

The overall block diagram of the transmission system is shown in Figure 3. Because the transmitter sends square pulses, the overall transfer function characteristic for ISIfree pulse transmission [8] is defined as:

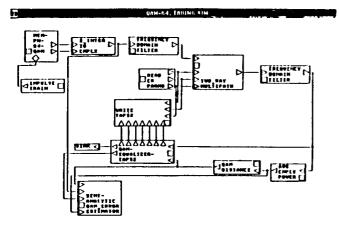
$$H(\omega) = \begin{cases} \frac{\omega \frac{T_{s}}{2}}{\sin(\omega \frac{T_{s}}{2})}; & 0 \le \omega \le \frac{\pi}{T_{s}} (1-\beta) \\ \frac{\omega \frac{T_{s}}{2}}{\sin[\omega \frac{T_{s}}{2}]} & \cos^{2}\left(\frac{T_{s}}{4\beta} \left[\omega - \frac{\pi}{T_{s}} (1-\beta)\right]\right); & \frac{\pi(1-\beta)}{T_{s}} \le \omega \le \frac{\pi(1+\beta)}{T_{s}} \end{cases}$$

$$0 \le \omega \le \frac{\pi}{T_{s}} (1-\beta)$$



b.  $f_2(x) = x^3 + x + \alpha^2$ Figure 2. Sample Outputs of Octal PN-Sequence Generators.

TIME (sec.)



**Figure 3.** Simulation Block Diagram of A 64-QAM Transmission System with Fading Channel

where  $\beta$  is the roll-off factor and  $T_s$  is a symbol period. To meet the zero-ISI condition in the simulation model, we assume the transmit/receive filter functions follow:

$$H_T(\omega) = \begin{cases} \frac{\omega \frac{T_s}{2}}{\sin(\omega \frac{T_s}{2})} \\ \frac{\omega \frac{T_s}{2}}{\sin[\omega \frac{T_s}{2}]} \cos\left\{\frac{T_s}{4\beta} \left[\omega - \frac{\pi}{T_s}(1-\beta)\right]\right\} \end{cases} (10)$$

and

$$H_{R}(\omega) \left\{ \begin{array}{c} 1 \\ Cos \\ 0 \end{array} \left\{ \frac{T_{s}}{4\beta} \left[ \omega - \frac{\pi}{T_{s}} (1-\beta) \right] \right\} \right. \tag{11}$$

over the same trequency regions as in equation (9). Other blocks in the overall system diagram, as shown

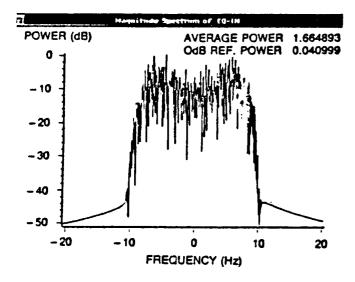


Figure 4. Spectrum of Faded Signal

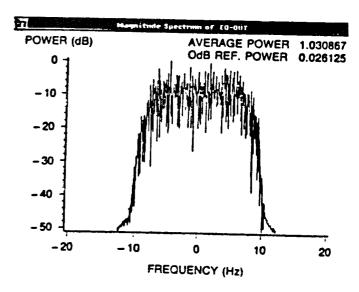


Figure 5. Spectrum Equalized Signal

in Figure 3, are standard BOSS modules. The noise band impulse inject module enables measuring the receiver band by sending an isolated impulse through the receive filter. The semi-analytic QAM error estimator module as described in [9], based on the received signal symbols and with reference to the transmitted signal will make decisions on the data symbols and computes the error probability.

In Figure 3, we have the simulation block diagram with an adaptive equalizer working on the received signal under fading multipath conditions. The power spectra of the signal before and after equalization are shown in Figures 4 and 5. The eye diagrams on the in-phase channel before and after equalization are shown in Figures 6 and 7, respectively.

We will discuss some simulation results in the next section and we will present BOSS post-processor displays of various subsystem outputs.

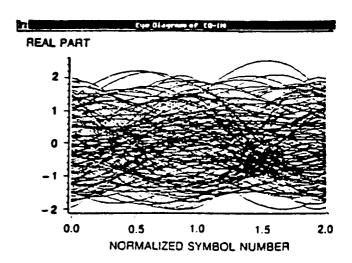


Figure 6. EYE. Diagram of Faded Signal

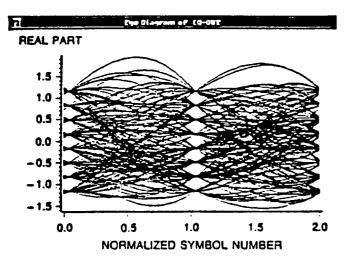


Figure 7: EYE. Diagram of Equalized Signal

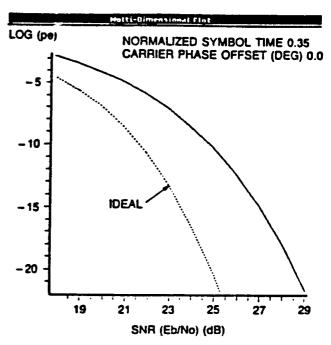
#### SIMULATION RESULTS

In this section, using a block diagram similar to Figure 3, we present error probability performance for a 96 Mbps 64-QAM system using two 2-pole Butterworth filters as transmitter and receiver filters. The equalizer module was not used in this simulation. The 3 dB bandwidth of these filters is at 12 MHz and no channel fading was assumed.

The optimum timing point, normalized to a symbol time, for  $\frac{E_b}{N_o}$  = 23 dB is equal to 0.35. The optimum carrier phase offset is zero degree in this linear system, as expected. We have plotted the logarithm of symbol error probability versus  $\frac{E_b}{N_o}$  in Figure 8. The degradation is solely due to filtering of the 64-QAM signal.

## CONCLUSION

In this paper, we presented the basics of simulating multilevel quadrature amplitude modulated signals used for microwave radio transmission. Simulations included PNsequence generation for source symbols, channel and transmit/receive filtering, equalization, and error prob-



**Figure 8.** Probability of Error for A 96 Mbps 64-QAM System with Butterworth Filters

ability evaluation. For the most part, the modeling and simulation were done by using software package BOSS.

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