OPTIMUM FLAT GRIDS

A. Behravesh

University of Tabriz,
Tabriz, Iran

Abstract This paper presents the results of the analysis of 56,000 flat grids in a form suitable for minimum weight design.

چکیده در این مقاله نتایج تحلیل ۵٦۰۰ شبکه با ساختارمناسب برای طراحی کمترین وزن ارائه گردیده است.

INTRODUCTION

A grid is defined as any plane network of interconnceted beams with an external loading system composed of forces normal to the plane of network and / or moments whose axes lie in this plane.

According to the above definition, this paper presents an attempt to find answers to the following question: For a prescribed boundary condition and loading system, what are the dimensions of the members and configuration of the grid if the total weight of the grid is to be minimum.

THE OBJECTIVE FUNCTION

The members of a flat grid essentially function as beams and thus the most important type of rigidity encountered in a flat grid is the bending rigidity. There are two other kinds of rigidities that influence the behavior of grids however, these effects in most cases are negligible [1, 2]. For simplicity, in this work, the torsional and shearing rigidities are assumed to be zero and infinite, respectively. Under these assumptions the governing design condition is given by:

$$\sigma_w \geqslant \frac{M}{I} e,$$

or

$$I \geqslant \frac{M}{\sigma_w} e.$$

Where σ_w, M, I, and e are the working stress, maximum bending moment of a typical member, second moment area and tension/compression zone, tespectively.

For the primary design of a grid an arbitrary value for I should be assigned and internal forces should be calculated. The maximum bending moment of the typical member found in this manner does not necessarily satisfy the design condition. In order to satisfy this condition the second moment of area of all members of the grid is multiplied by a non-zero factor, ß. This does not alter the internal forces in the members but the second moment of area is multiplied by ß, thus the design condition becomes:

$$\beta I > \frac{M}{\sigma} e.$$

Furthermore, assuming that the members are to be designed such that at least in one member the maximum tensile or maximum compressive stress or both reach the corresponding working stress, then the value of ß is found to be equal to the maximum value of the ratio $\frac{1}{\sigma_w} \cdot \frac{M}{I} e$ among all the members of the grid. May be represented by:

$$\beta = \frac{1}{\sigma_w} \max \left(\frac{M}{I} e \right)$$

It is interesting to note that an approximation to I in terms of cross - sectional area A may be considered as:

$$I \simeq \alpha_s eHA$$

or

$$A \doteq \frac{I}{\alpha_s eH}.$$

Where H is the overall depth of the cross - section and α_s is a function of the shape of the cross - section. The mean values of α_s for standard cross - sections are given in Table 1:

Table 1. α_s as a function of the shape of cross–section.

cross - section	mean value of I/(eHA)	standard deviation
I	0.3146991	0.0059436
IPE	0.3383929	0.0060963
IPB1	0.3650710	0.0235662
IPB	0.3578176	0.0142061
[	0.2873142	0.0156332
□	0.2609411	0.0120147

Now, if the second moment of area of a typical member is I and all the members of the grid have the same $\alpha_s = F$, then the total volume of the the grid is given by:

$$v = \frac{1}{F\sigma_w} \max\left(\frac{M}{I} e\right) \Sigma \frac{I}{eH} l.$$

Where l is the length of a typical member.

Considering a grid consisting of inner members with the second moment of area I_o, overall depth of the cross-section H_o, $e_o = H_o/2$; the total length of the members L_i, the edge members with the second moment of area I, overall depth H, e = H/2 and the total length of members L_e, then the volume of material reduces to:

$$v = \frac{1}{F\sigma_w} \max\left(\frac{M_o}{I_o} H_o, \frac{M}{I} H\right)\left(\frac{I_o}{H_o^2} Li + \frac{I}{H^2} Le\right)$$

Where M_o and M are the maximum bending moments in inner and edge members, respectively. Furthermore, assuming that P is the total load applied to the grid and L is the width of the grid, the volume of material may be written as:

$$v = \frac{PL^2}{F\sigma_w H_o} \max\left(\frac{M_o}{PL}, \frac{M}{PL} \cdot \frac{H/H_o}{I/I_o}\right)\left(\frac{Li}{L} + \frac{I/I_o}{H^2/H_o^2} \frac{Le}{L}\right) \qquad (1)$$

This equation provides a convenient means for investigating the manner in which the volume of the material can be minimized. Since the ratio PL^2/σ_w is obviously prescribed in any given problem it should be regarded as constant. Also, as far as H_o and F are concerned, their values are normally governed by many other factors besides efficiency in relation to bending stress alone. Consequently, letting $v_o = PL^2/(F\sigma_w H_o)$, the problem of minimization of the volume of material would effectively reduce to the minimization of v/v_o. In other words, v/v_o is a measure of efficiency of a grid layout for resisting the given external load, boundary and boundary conditions.

THE ANALYTICAL RESULTS

This section is concerned with the presentation of the results of the analysis of a large number of grids.

The grid layouts, boundary conditions and types of external loading were selected with care, and the discussion which follows is a description of the cases considered with an attempt to justify the choices.

First of all, it was necessary to select a number of commonly used grid layouts and the question was whether to choose the layouts so as to have various shapes of boundaries or to concentrate on a few families of boundary shapes. The danger of adopting the former was that it could have resulted in a number of isolated pieces of information with the general pattern of behavior remaining obscure, and the disadvantage of the latter suggestion was that the scope of the work would have been limited. It was, however, preferred to adopt the second suggestion, thereby obtaining more reliable information about a few families of grids rather than having scattered pieces of information connected together by guess work. Three families of grids with square boundaries and five families of grids with two types of rectangular boundaries (the ratio, and length of boundary/width of boundary being 4/3 and 2) were chosen for investigation, which are used widely in practice. Altogether 85 layouts were selected and are shown in Figure 1.

Each of the layouts was to be analyzed with a number of various boundary conditions and among many different possibilities the following two types were selected:

(a) The four corner joints are restrained against translation but can rotate freely. This type of boundary condition will hereafter be referred to by the letter F. In the case of a rectangular boundary with ratio of length / width = 2, at the middle of each longer side of the boundary, an extra joint is restrained as corner joints, and the letter F, in this case, is changed to S.

(b) The four corner joints and four middle joints of the boundary sides are restrained from translation but can rotate freely. This type of boundary condition will hereafter be referred to by the letter E. In the case of a rectangular boundary with ratio length / width = 2, at each longer side two extra joints are restrained as corner joints and the letter E is changed to T.

Of course, none of these boundary conditions can ever be fully realized in practice because they require complete constraints and perfect degrees of freedom at the supports, neither of which can ever be achieved since they require frictionless guides. The practical cases, however, will lie somewhere close to the limiting cases considered.

The next step was the choice of the external loading. Obviously, it was not practical to consider all the possible types of loading and only a few important types had to be selected. An unfortunate restriction was that the loads had to be symmetrical because otherwise the demand on computation time and storage requirements on the computer could not have been met. Of course, the symmetry of loading was of some help if the layouts and boundary conditions were also symmetrical, but these were already chosen to be so. It may be argued that at least a few unsymmetrical cases could have been considered but this was against the general policy of tracing the general patterns of behavior rather than obtaining solutions for isolated cases.

Among the symmetrical types of loading the following two seemed to be the obvious choices:

(a) Uniformly distributed loads over the whole surface of the grid. This type of loading will hereafter be denoted by the letter U.

(b) A single concentrated vertical load applied to the central joint (if the center point is not a joint then the load is divided among the nearest joints to the center point). This type of loading will hereafter be denoted by C. These were the only types of external loading considered.

Summarizing, there were 85 different layouts, each one of which was considered with two different boundary conditions and each one of the resulting 170 grids was analyzed for two types of external loads.

Strictly speaking, if the material cost is the only criterion of design then the relation (1) should be taken as an objective function. In reality, however, it is possible to reduce the cost by grouping the members so that several members of a group are manufactured out of the same cross - section. In the case of flat grids with assumed boundary conditions, practically it is more economical to make the inner members out of one cross - section and edge members out of another. Therefore, it was decided that all the grids considered should have this property.

The cases are identified by case numbers of the form a-bcde, where a is the number of divisions along the width of the grid and b, c, d and e denote the family of the external loading, respectively. For example, 24-DREU is the case number for a grid with a configuration similar to the family of the grid D (see Figure 2.) with 24 bays along the width, rectangular boundary shape, eight supports, and under a uniformly distributed load. Each of the above mentioned 320 cases was analyzed for 210 different modes, the variation being in the ratio I/I_o only. Then for each of the 210 modes, the ratio v/v_o corresponding to the 16 different values of H/H_o was obtained. Altogether, 56,000 grids were analyzed.

The standard stiffness method [3, 4] was used to analyze all cases and to plot the 320 "Design Sheets". Each of the data design sheets represents the ratio of v/v_o for all the 210 variations of I/I_o and the 16 variations of ratio H/H_o. In addition to the case number given in each of the design sheets, there is a small sketch at the bottom lefthand corner which illustrates the layout, the boundary condition and the type of loading of the case under consideration. The legend used in these sketches is described in Figure 2. The "Design Sheets" were prepared in the form of macrofilms.

In all cases considered, the contour levels of H/H_o were plotted in a coordinate system with axes referred to as "RATIO I/I_o and RATIO v/v_o". For a better representation of the contour levels of H/H_o, within the accuracy of the drawing, the contour levels of H/H_o = 0.6, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0 and H/H_o = 4.0, 6.0, 8.0, 12.0, 16.0, 20.0. 30.0, 40.0, were plotted separately in two different coordinate systems. To facilitate the comparison between different cases, the minimum value of each contour level related to v/v_o was marked by a small circle and the coordinates of these points were tabulated on the right hand side of the corresponding contour levels of H/H_o. Furthermore, four symbols, □, □̲, ■, and ■̲ used in these tables have the following meanings:

□ The maximum bending stress in the grid occurs at the edge members

□̲ The maximum bending stress in the gird occurs at the edge and the difference of the maximum bending stress of edge and inner members is less than 2%.

■ The maximum bending stress in the grid occurs at the inner members.

■̲ The maximum bending stress in the grid occurs at the inner members and the difference of the maximum bending stress of inner and edge members is less than 2%.

The significance of these symbols is that, with I/I_o, H/H_o and v/v_o, the maximum bending moment of edge/

inner members is calculated by using relation (1).

CONCLUSION

The results of the analysis are used to establish a number of interesting facts regarding the variation in dimensions of the members and change in configuration of the flat grids. To facilitate the comparison of results, the change in the minimum value of efficiency of the grid layout v/v_0 due to the increase of H/H_0 and the density of the configuration for all the analyzed cases is shown in Table 2. The table classifies the most important pieces of information assessable from the results of analysis and provides a suitable medium from which general patterns of behavior regarding the effect of dimensions of the members and changing configurations can be traced.

A thorough examination of the results given in Table 2 reveals a number of interesting points which are described in the sequal. It is to be noted, however, that the statements are applicable to the minimum value of V/V_0 only, and their validity for values other than minimum cannot be taken for granted.

Some points of particular interest concerned with Table 2 are listed below:

(a) The variation of v/v_0 with changes in I/I_0 for all the cases shows that for small and large values of I/I_0 the value of v/v_0 is large and there will always exist a unique minimum for v/v_0. Furthermore, the function v/v_0 for all cases is convex.

(b) The effect of the configuration of the grid on the value of v/v_0 is very small if there is no restriction in the choice of I/I_0 and H/H_0. However, for the same value of H/H_0, configuration type A (diagonal grids) for all boundary shapes and boundary conditions usually has the least value of v/v_0.

(c) An interesting point revealed is that for the same configuration of the grid and H/H_0, the effect of the density, especially for a uniformly distributed load, is rather small.

(d) For the same value of H/H_0, as the density of a configuration increases, the value of v/v_0 tends to a limit.

(e) When a grid type A is under uniformly distributed loading an increase in density of the configuration causes v/v_0 to decrease. But for all other cases an increase in density of the configuration causes v/v_0 to increase.

Based on the preceding material, it may finally be concluded that:

The variation in the dimensions of the members and change in configuration of the grid have significant effects on the value of the total weight of the grid, but the effect of the density of a configuration is insignificant and hence, for an efficient design, the search for the most favorable configuration of the grid and value of the members' dimensions should be included in any optimization process.

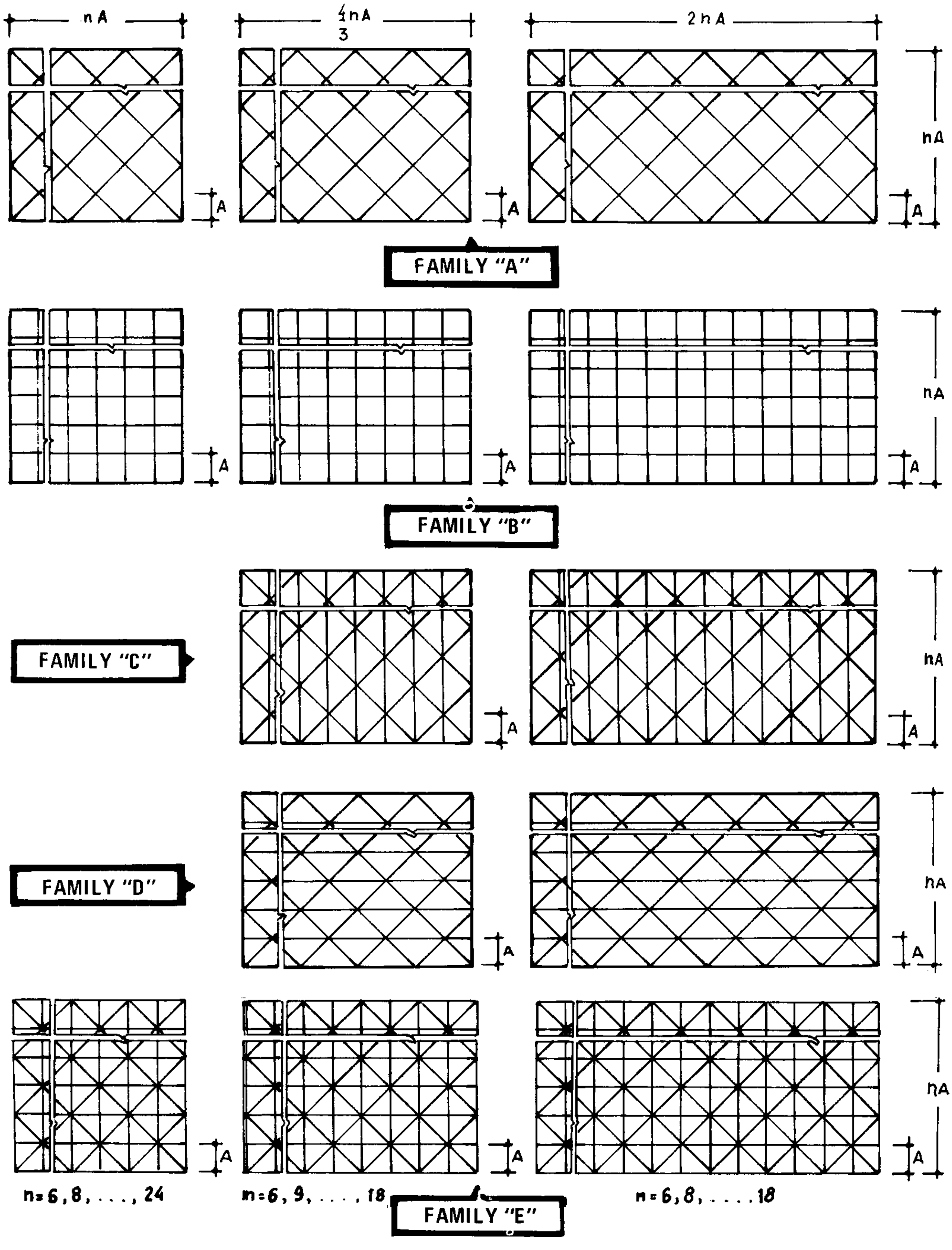
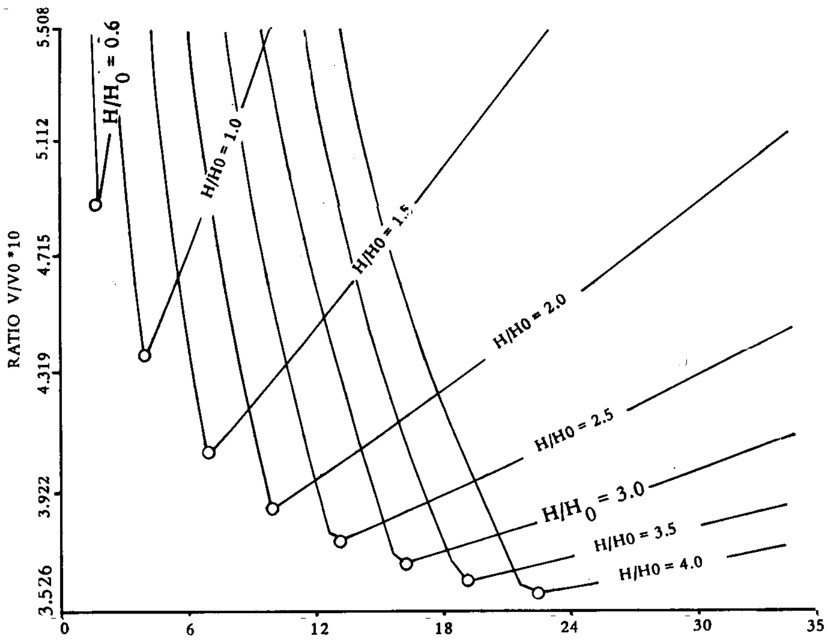
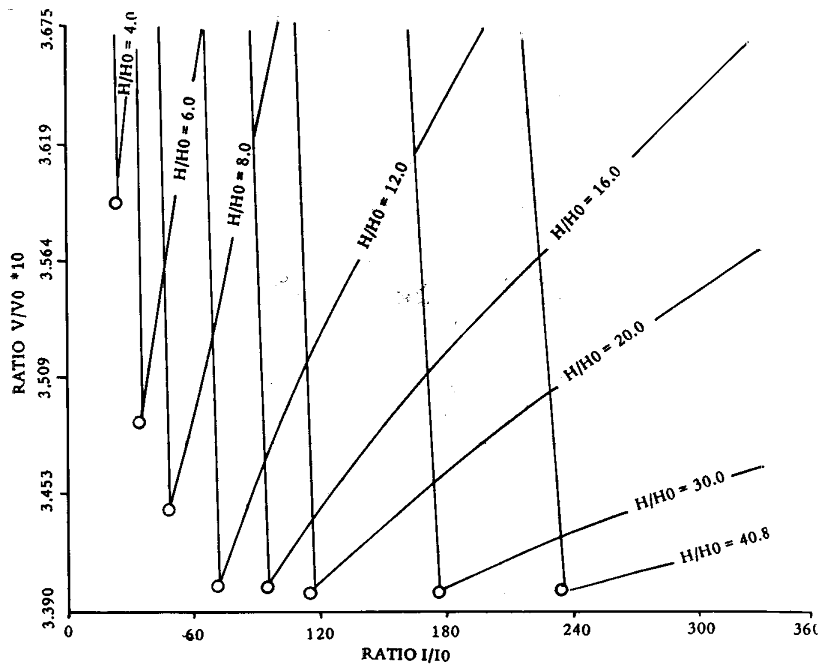

nA
4/3 nA
2nA
nA
A
FAMILY "A"
FAMILY "B"
FAMILY "C"
FAMILY "D"
FAMILY "E"
n=6,8, . . . , 24
n=6,9, . . . , 18
n=6,8, ... 18

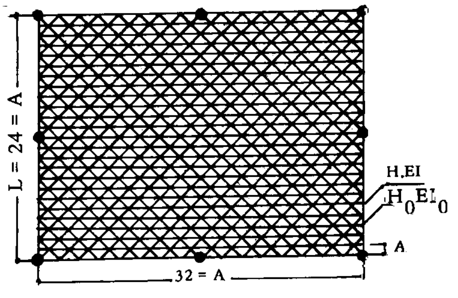
Figure 1



OPTIMUM POINTS		
H/HO	I/IO	V/VO
0.6 ■	1.90E+00	4.91E–01
1.0 ■	4.15E+00	4.39E–01
1.5 ■	7.39E+00	4.07E–01
2.0 ■	1.07E+01	3.88E–01
2.5 ■	1.43E+01	3.78E–01
3.0 ■	1.75E+01	3.69E–01
3.5 ■	2.07E+01	3.63E–01
4.0 ■	2.44 E% 0.1	3.59E–01



OPTIMUM POINTS		
H/H0	I/I0	V/V0
4.0 ■	2.44E+01	3.59 E–01
5.0 ■	3.68E+01	3.49E–01
8.0 ■	5.11E+01	3.45E–01
12.0 ■	7.71E+01	3.41 E–01
16.0 ■	1.03E+02	3.41E–01
20.0 ■	1.26E+02	3.41E–01
30.0 ■	1.91E+02	3.41E–01
40.0 ■	2.54E+02	3.41E–01



L IS THE WIDTH OF THE CRIO.

F IS THE BENDING EFFICIENCY FACTOR.

EI IS THE FLEXURAL RIGIOITY.

H IS THE OVERALL DEPTH OF THE CROSS–SECTION.

6* IS THE WORKING STRESS.

Q IS THE INTENSITY OF THE U.D.E.LOAD

V IS THE VOLUME OF THE MATERIAL OF THE GRID.

$V_D=Q=L^4/[F=6\omega=H_0]$

CASE NO. 24–DREU.

Figure 2

TABLE 2

CASE NUMBER	CHANGE IN THE MINIMUM VALUE OF $V/V_o \times 10$ $H/H_o=0.6$	$H/H_o=1.5$	$H/H_o=2.5$	$H/H_o=3.5$	$H/H_o=6.0$	$H/H_o=12$
6-ASFU	4.04	2.25	1.65	1.47	1.28	1.13
8-ASFU	3.93	2.29	1.68	1.46	1.26	1.12
10-ASFU	3.90	2.24	1.65	1.42	1.24	1.11
12-ASFU	3.81	2.27	1.66	1.43	1.22	1.10
14-ASFU	3.79	2.24	1.66	1.42	1.22	1.09
16-ASFU	3.76	2.26	1.66	1.43	1.21	1.09
18-ASFU	3.27	2.25	1.65	1.43	1.21	1.08
20-ASFU	3.72	2.26	1.66	1.42	1.21	1.08
22-ASFU	3.69	2.24	1.65	1.42	1.21	1.08
24-ASFU	3.68	2.26	1.65	1.42	1.21	1.08
6-BSFU	3.99	2.11	1.78	1.63	1.47	1.37
8-BSFU	4.39	2.14	1.83	1.69	1.55	1.44
10-BSFU	4.58	2.16	1.86	1.72	1.58	1.48
12-BSFU	4.68	2.17	1.87	1.75	1.61	1.51
14-BSFU	4.73	2.17	1.89	1.76	1.63	1.53
16-BSFU	4.76	2.18	1.90	1.78	1.64	1.54
18-BSFU	4.78	2.18	1.91	1.78	1.65	1.56
20-BSFU	4.79	2.19	1.92	1.79	1.66	1.56
22-BSFU	4.80	2.19	1.93	1.80	1.66	1.57
24-BSFU	4.80	2.21	1.93	1.80	1.67	1.58
6-ESFU	4.35	2.16	1.72	1.53	1.37	1.27
8-ESFU	4.22	2.21	1.75	1.56	1.40	1.30
10-ESFU	4.53	2.18	1.75	1.56	1.40	1.31
12-ESFU	4.47	2.21	1.77	1.57	1.41	1.32
14-ESFU	4.61	2.19	1.76	1.58	1.41	1.32
16-ESFU	4.59	2.21	1.78	1.59	1.41	1.33
18-ESFU	4.67	2.20	1.78	1.59	1.42	1.33
20-ESFU	4.66	2.21	1.78	1.59	1.42	1.33
22-ESFU	4.70	2.20	1.77	1.59	1.42	1.34
24-ESFU	4.69	2.21	1.79	1.59	1.43	1.33
6-ARFU	8.82	4.51	3.33	2.86	2.37	2.04
12-ARFU	8.67	4.53	3.23	2.74	2.29	1.99
18-ARFU	8.59	4.49	3.24	2.75	2.28	1.96
24-ARFU	8.56	4.49	3.24	2.74	2.27	1.96
6-BRFU	8.47	4.45	3.79	3.50	3.16	2.92
12-BRFU	9.29	4.52	3.94	3.70	3.38	3.17
18-BRFU	9.50	4.55	4.01	3.76	3.45	3.25
24-BRFU	9.60	4.58	4.05	3.78	3.50	3.28

Continued in the next page.

TABLE 2 CONTINUED

CASE NUMBER	CHANGE IN THE MINIMUM VALUE OF $V/V_0 \times 10$					
	$H/H_0=0.4$	$H/H_0=1.0$	$H/H_0=2.0$	$H/H_0=3.0$	$H/H_0=4.0$	$H/H_0=8.0$
6-ASFC		8.90	7.33	6.55	6.13	5.46
8-ASFC		9.66	8.06	7.26	6.83	6.14
10-ASFC		10.20	8.64	7.83	7.38	6.67
12-ASFC		10.60	9.11	8.29	7.83	7.10
14-ASFC		10.90	9.50	8.68	8.21	7.47
16-ASFC		11.30	9.84	9.01	8.54	7.78
18-ASFC		11.50	10.10	9.31	8.83	8.06
20-ASFC		11.80	10.40	9.57	9.09	8.31
22-ASFC		12.00	10.60	9.81	9.32	8.54
24-ASFC		12.20	10.90	10.00	9.54	8.74
6-BSFC	7.82	6.11	6.27	6.29	6.28	6.26
8-BSFC	7.71	6.51	6.93	7.03	7.06	7.10
10-BSFC	7.58	6.85	7.44	7.60	7.67	7.72
12-BSFC	7.52	7.17	7.97	8.06	8.14	8.23
14-BSFC	7.52	7.44	8.20	8.42	8.52	8.64
16-BSFC	7.82	7.72	8.49	8.74	8.87	8.99
18-BSFC	8.08	7.95	8.73	9.03	9.16	9.29
20-BSFC	8.30	8.19	8.96	9.28	9.41	9.57
22-BSFC	8.51	8.40	9.15	9.50	9.65	9.82
24-BSFC	8.72	8.59	9.34	9.69	9.85	10.00
6-ESFC		7.46	6.86	6.48	6.25	5.86
8-ESFC		7.93	7.56	7.21	6.99	6.60
10-ESFC		8.33	8.06	7.75	7.54	7.16
12-ESFC		8.68	8.49	8.19	7.99	7.61
14-ESFC		8.93	8.84	8.56	8.36	7.99
16-ESFC		9.28	9.15	8.88	8.68	8.31
18-ESFC		9.52	9.42	9.16	8.96	8.59
20-ESFC		9.75	9.66	9.41	9.21	8.84
22-ESFC		9.95	9.88	9.63	9.44	9.06
24-ESFC		10.10	10.10	9.83	9.64	9.27
6-ARFC		8.74	6.07	5.09	4.58	3.82
12-ARFC		15.4	12.80	11.60	10.90	9.93
18-ARFC		12.3	9.66	8.56	7.98	7.06
24-ARFC		17.50	15.00	13.90	13.2	12.10
6-BRFC	13.0	11.20	10.60	10.00	9.56	9.28
12-BRFC	14.20	13.40	13.10	12.70	12.30	11.80
18-BRFC	15.20	14.60	14.50	14.20	13.80	13.10
24-BRFC	16.00	15.60	15.40	15.20	14.90	14.10

Continued in the next page.

TABLE 2 CONTINUED

CASE NUMBER	CHANGE IN THE MINIMUM VALUE OF $V/V_0 \times 10$ $H/H_0=0.4$	$H/H_0=1.0$	$H/H_0=2.0$	$H/H_0=3.0$	$H/H_0=4.0$	$H/H_0=8.0$
6-CRFC		9.73	6.70	5.67	5.12	4.42
12-CRFC		18.40	14.80	13.40	12.70	11.60
18-CRFC		14.70	11.20	9.95	9.30	8.32
24-CRFC		21.40	17.70	16.20	15.40	14.20
6-DRFC		7.53	6.28	5.65	5.31	4.74
12-DRFC		15.20	13.90	13.10	12.70	11.90
18-DRFC		12.00	10.50	9.78	9.38	8.73
24-DRFC		17.80	16.70	15.80	25.40	14.60
6-ERFC		9.68	6.60	5.66	5.39	4.95
12-ERFC		15.40	13.70	12.40	11.60	10.80
18-ERFC		12.60	10.60	9.34	8.66	8.29
24-ERFC			16.00	14.70	14.00	12.90
6-ARSC		7.20	6.57	6.23	6.03	5.68
8-ARSC		13.60	13.60	13.30	13.10	12.70
10-ARSC		9.15	8.68	8.36	8.15	7.79
12-ARSC		15.60	15.40	15.10	14.90	14.50
14-ARSC		10.60	10.10	9.83	9.62	9.26
16-ARSC		17.00	16.70	16.40	16.20	15.80
18-ARSC		11.70	11.20	10.90	10.70	10.40
6-BRSC		17.10	16.00	15.40	15.10	14.60
8-BRSC		18.60	17.70	17.10	16.80	16.30
10-BRSC		19.80	18.90	18.40	18.10	17.50
12-BRSC		20.70	19.90	19.40	19.10	18.50
14-BRSC		21.40	20.70	20.30	20.00	19.40
16-BRSC		22.10	21.40	21.00	20.70	20.10
18-BRSC		22.60	22.00	21.60	21.30	20.70
6-CRSC		7.78	7.19	6.94	6.81	6.58
8-CRSC		13.80	13.80	13.80	13.70	13.70
10-CRSC		9.74	9.19	8.93	8.79	8.54
12-CRSC		15.90	15.90	15.90	15.90	15.90
14-CRSC		11.00	10.50	10.20	10.10	9.85
16-CRSC		17.30	17.40	17.40	17.40	17.40
18-CRSC		11.90	11.40	11.20	11.20	11.20
6-DRSC		9.73	9.08	8.64	8.39	7.96
8-DRSC		18.10	17.70	17.20	16.90	16.40
10-DRSC		12.60	12.00	11.50	11.20	10.80
12-DRSC		20.40	20.00	19.60	19.30	18.70

Continued in the next page.

TABLE 2 CONTINUED

CASE NUMBER	CHANGE IN THE MINIMUM VALUE OF $V/V_0 \times 10$ $H/H_0=0.6$	$H/H_0=1.5$	$H/H_0=2.5$	$H/H_0=3.5$	$H/H_0=6.0$	$H/H_0=12$
6-CRFU	13.30	6.88	5.23	4.51	3.75	3.21
12-CRFU	13.80	6.97	5.29	4.56	3.76	3.21
18-CRFU	13.90	7.05	5.34	4.61	3.82	3.27
24-CRFU	14.00	7.07	5.35	4.61	3.83	3.27
6-DRFU	10.2	6.11	4.85	4.43	3.96	3.63
12-DRFU	10.4	6.34	5.03	4.44	3.92	3.65
18-DRFU	10.5	6.36	5.08	4.50	3.95	3.64
24-DRFU	10.6	6.39	5.08	4.52	3.99	3.64
6-ERFU	9.31	4.57	3.67	3.28	2.84	2.52
12-ERFU	9.88	4.51	3.69	3.29	2.85	2.53
18-ERFU	9.97	4.54	3.71	3.33	2.90	2.60
24-ERFU	9.99	4.55	3.73	3.35	2.91	2.60
6-ARSU	8.38	5.69	4.69	4.26	3.87	3.56
8-ARSU	8.49	5.88	4.78	4.27	3.83	3.54
10-ARSU	8.27	5.81	4.76	4.25	3.75	3.50
12-ARSU	8.24	5.88	4.75	4.28	3.74	3.49
14-ARSU	8.24	5.82	4.74	4.25	3.74	3.47
16-ARSU	8.23	5.84	4.74	4.26	3.76	3.47
18-ARSU	8.21	5.84	4.76	4.25	3.74	3.44
6-BRSU	10.70	7.47	6.48	6.07	5.61	5.29
8-BRSU	10.80	7.64	6.68	6.25	5.80	5.48
10-BRSU	10.70	7.75	6.79	6.36	5.91	5.59
12-BRSU	10.70	7.80	6.87	6.45	5.99	5.66
14-BRSU	10.70	7.85	6.92	6.49	6.04	5.71
16-BRSU	10.70	7.89	6.95	6.54	6.08	5.76
18-BRSU	10.70	7.90	6.98	6.57	6.11	5.79
6-CRSU	13.3	8.95	7.61	7.00	6.36	5.89
8-CRSU	13.5	9.10	7.67	7.06	6.39	5.90
10-CRSU	13.7	9.22	7.76	7.15	6.46	5.98
12-CRSU	13.8	9.21	7.80	7.15	6.48	5.98
14-CRSU	13.8	9.28	7.85	7.19	6.52	6.02
16-CRSU	13.8	9.27	7.84	7.21	6.51	6.02
18-CRSU	13.9	9.32	7.88	7.22	6.54	6.04
6-DRSU	14.9	1.11	9.50	8.73	7.87	7.27
8-DRSU	15.7	1.14	9.60	9.13	7.97	7.36
10-DRSU	15.4	1.14	9.70	8.85	7.96	7.41
12-DRSU	15.8	1.16	9.78	8.98	8.04	7.43

Continued in the next page.

TABLE 2 CONTINUED

CASE NUMBER	CHANGE IN THE MINIMUM VALUE OF $V/V_0 \times 10$ $H/H_0=0.4$	$H/H_0=1.0$	$H/H_0=2.0$	$H/H_0=3.0$	$H/H_0=4.0$	$H/H_0=8.0$
14-DRSC		14.50	13.90	13.40	13.10	12.60
16-DRSC		22.00	21.70	21.30	20.90	20.40
18-DRSC		15.80	15.30	14.80	14.50	14.00
6-ERSC		9.44	8.81	8.51	8.35	8.06
8-ERSC		16.60	16.20	15.90	15.70	15.40
10-ERSC		11.90	11.40	11.10	10.90	10.60
12-ERSC		18.60	18.20	17.90	17.70	17.30
14-ERSC		13.50	13.00	12.70	12.50	12.10
16-ERSC		19.80	19.50	19.30	19.10	18.70
18-ERSC		14.60	14.10	13.80	13.60	13.30
8-ASEC		5.45	5.55	5.53	5.51	5.47
12-ASEC		6.35	6.40	6.38	6.37	6.35
16-ASEC		7.00	7.03	7.02	7.01	6.99
20-ASEC	-	7.49	7.51	7.50	7.50	7.49
24-ASEC	-	7.89	7.91	7.91	7.90	7.89
6-BSEC	7.18	6.68	6.47	6.40	6.36	6.30
8-BSEC	7.81	7.47	7.32	7.26	7.22	7.17
10-BSEC	8.34	8.07	7.96	7.90	7.87	7.82
12-BSEC	8.77	8.55	8.46	8.42	8.39	8.34
14-BSEC	9.14	8.95	8.88	8.84	8.82	8.77
16-BSEC	9.46	9.29	9.24	9.20	9.18	9.14
18-BSEC	9.74	9.60	9.55	9.52	9.49	9.45
20-BSEC	9.99	9.86	9.82	9.79	9.77	9.73
22-BSEC	10.20	10.10	10.10	10.00	10.00	9.98
24-BSEC	10.40	10.30	10.30	10.30	10.20	10.20
6-ESEC		5.85	5.67	5.60	5.56	5.49
8-ESEC		6.36	6.35	6.31	6.29	6.23
10-ESEC		7.04	6.93	6.88	6.85	6.79
12-ESEC		7.37	7.35	7.31	7.29	1.23
14-ESEC		7.80	7.73	7.68	7.66	7.61
16-ESEC		8.05	8.03	8.00	7.97	7.92
18-ESEC		8.36	8.31	8.27	8.25	8.20
20-ESEC		8.56	8.55	8.52	8.50	8.45
22-ESEC		8.80	8.76	8.74	8.71	8.67
24-ESEC		8.97	8.96	8.94	8.91	8.87
12-AREC		8.97	8.96	8.93	8.92	8.89
24-AREC		10.80	11.00	11.00	11.00	10.90

Continued in the next page.

TABLE 2 CONTINUED

CASE NUMBER	CHANGE IN MINIMUM VALUE OF $V/V_0 \times 10$ $H/H_0=0.6$	$H/H_0=1.5$	$H/H_0=2.5$	$H/H_0=3.5$	$H/H_0=6.0$	$H/H_0=12$
14-DRSU	15.6	1.16	9.79	8.96	8.08	7.45
16-DRSU	15.8	1.17	9.83	8.98	8.11	7.49
18-DRSU	15.7	1.16	9.87	9.04	8.11	7.49
6-ERSU	10.50	7.67	6.64	6.18	5.67	5.31
8-ERSU	10.20	7.56	6.59	6.14	5.65	5.29
10-ERSU	10.30	7.73	6.74	6.28	5.77	5.41
12-ERSU	10.20	7.69	6.70	6.25	5.76	5.40
14-ERSU	10.30	7.75	6.78	6.32	5.82	5.46
16-ERSU	10.20	7.75	6.76	6.31	5.82	5.46
18-ERSU	10.20	7.78	6.82	6.54	5.85	5.49
8-ASEU	1.61	1.31	1.20	1.15	1.08	1.03
12-ASEU	1.66	1.25	1.14	1.10	1.05	1.01
16-ASEU	1.65	1.27	1.13	1.07	1.03	1.00
20-ASEU	1.65	1.26	1.13	1.07	1.02	0.99
24-ASEU	1.65	1.26	1.13	1.07	1.02	0.99
6-BSEU	1.78	1.48	1.39	1.36	1.31	1.29
8-BSEU	1.83	1.56	1.47	1.43	1.39	1.36
10-BSEU	1.86	1.60	1.51	1.48	1.44	1.41
12-BSEU	1.87	1.62	1.54	1.51	1.46	1.44
14-BSEU	1.88	1.64	1.56	1.53	1.49	1.46
16-BSEU	1.88	1.65	1.58	1.54	1.50	1.47
18-BSEU	1.88	1.67	1.59	1.55	1.51	1.48
20-BSEU	1.89	1.67	1.60	1.56	1.52	1.49
22-BSEU	1.89	1.68	1.60	1.57	1.53	1.50
24-BSEU	1.89	1.69	1.61	1.57	1.53	1.51
6-ESEU	1.73	1.44	1.35	1.29	1.24	1.20
8-ESEU	1.92	1.40	1.32	1.29	1.26	1.23
10-ESEU	1.83	1.42	1.33	1.29	1.26	1.25
12-ESEU	1.74	1.43	1.33	1.29	1.27	1.25
14-ESEU	1.70	1.45	1.35	1.30	1.27	1.26
16-ESEU	1.74	1.44	1.35	1.31	1.27	1.26
18-ESEU	1.71	1.45	1.35	1.31	1.27	1.26
20-ESEU	1.71	1.45	1.36	1.32	1.28	1.26
22-ESEU	1.69	1.46	1.36	1.32	1.28	1.26
24-ESEU	1.71	1.46	1.36	1.32	1.28	1.27
12-AREU	3.20	2.29	2.03	1.94	1.83	1.76
24-AREU	3.17	2.31	2.03	1.91	1.79	1.73

Continued in the next page.

TABLE 2 CONTINUED

CASE NUMBER	CHANGE IN THE MINIMUM VALUES $V/V_O \times 10$					
	H/H_O=0.6	H/H_O=1.5	H/H_O=2.5	H/H_O=3.5	H/H_O=6.0	H/H_O=12
6-BREU	3.92	3.25	3.04	2.94	2.83	2.75
12-BREU	4.08	3.51	3.29	3.19	3.09	3.01
18-BREU	4.08	3.57	3.37	3.28	3.17	3.09
24-BREU	4.09	3.60	3.41	3.31	3.21	3.13
12-CREC	5.07	3.70	3.29	3.11	2.93	2.82
24-CREC	5.17	3.79	3.83	3.18	2.98	2.84
6-DREU	6.05	4.04	3.80	3.67	3.52	3.40
12-DREU	4.93	4.00	3.68	3.54	3.47	3.42
18-DREU	4.83	4.05	3.76	3.61	3.44	3.41
24-DREU	4.91	4.07	3.78	3.63	3.49	3.41
6-EREU	3.84	2.83	2.60	2.49	2.37	2.29
12-EREU	3.45	2.84	2.61	2.50	2.38	2.30
18-EREU	3.45	2.89	2.67	2.56	2.45	2.36
24-EREU	3.43	2.88	2.67	2.57	2.45	2.36
8-ARTU	4.96	4.08	3.83	3.68	3.52	3.39
12-ARTU	4.78	3.87	3.59	3.51	3.41	3.33
16-ARTU	4.77	3.92	3.60	3.46	3.34	3.29
6-BRTU	5.70	5.28	5.16	5.10	5.05	5.00
8-BRTU	5.86	5.48	5.38	5.30	5.24	5.19
10-BRTU	5.94	5.60	5.47	5.41	5.35	5.31
12-BRTU	6.01	5.66	5.55	5.49	5.43	5.38
14-BRTU	6.04	5.72	5.60	5.54	5.48	5.43
16-BRTU	6.05	5.76	5.64	5.58	5.52	5.47
18-BRTU	6.06	5.78	5.67	5.61	5.55	5.50
8-CRTU	6.75	6.12	5.86	5.75	5.62	5.52
12-CRTU	6.88	6.18	5.94	5.81	5.68	5.58
16-CRTU	6.89	6.20	5.96	5.84	5.71	5.61
8-DRTU	11.30	8.73	7.91	7.59	7.33	7.13
12-DRTU	10.40	8.19	7.73	7.52	7.27	7.13
16-DRTU	9.62	8.29	7.81	7.58	7.32	7.15
6-ERTU	5.82	5.37	5.22	5.15	5.07	5.00
8-ERTU	5.80	5.36	5.22	5.15	5.06	5.00
10-ERTU	5.74	5.43	5.31	5.24	5.16	5.10
12-ERTU	5.71	5.45	5.31	5.24	5.16	5.10
14-ERTU	5.70	5.46	5.35	5.28	5.21	5.15
16-ERTU	5.74	5.48	5.37	5.29	5.21	5.15
18-ERTU	5.68	5.48		5.31	5.24	5.18

Continued in the next page.

TABLE 2 CONTINUED

CASE NUMBER	CHANGE IN THE MINIMUM VALUES OF $V/V_0 \times 10$					
	$H/H_0=0.4$	$H/H_0=1.0$	$H/H_0=2.0$	$H/H_0=3.0$	$H/H_0=4.0$	$H/H_0=8.0$
6-BREC	10.30	10.00	9.80	9.68	9.61	9.50
12-BREC	12.60	12.60	12.50	12.40	12.30	12.20
18-BREC	13.90	14.00	13.90	13.80	13.80	13.70
24-BREC	14.90	14.90	14.90	14.80	14.80	14.70
12-CREC		10.40	10.60	10.50	10.50	10.50
24-CREC		13.00	13.00	13.00	13.00	13.00
6-DREC		4.68	4.47	4.37	4.32	4.22
12-DREC		6.27	6.20	6.16	6.13	6.08
18-DREC		8.29	8.21	8.17	8.14	8.09
24-DREC		10.30	10.23	10.19	10.16	10.11
6-EREC		5.01	4.83	4.73	4.67	4.57
12-EREC		11.10	11.00	10.90	10.80	10.70
18-EREC		8.46	8.35	8.26	8.21	8.10
24-EREC		13.10	13.10	13.10	13.00	12.90
8-ARTC		12.30	12.40	12.40	12.30	12.30
12-ARTC		14.10	14.20	14.20	14.10	14.10
16-ARTC		15.40	15.50	15.40	15.40	15.40
6-BRTC		13.90	14.00	14.00	14.00	13.90
8-BRTC		15.60	15.60	15.60	15.60	15.60
10-BRTC		16.80	16.90	16.90	16.90	16.90
12-BRTC		17.80	17.90	17.90	17.90	17.80
14-BRTC		18.60	18.70	18.70	18.70	18.70
16-BRTC		19.30	19.40	19.40	19.40	19.40
18-BRTC		19.90	20.00	20.00	20.00	20.00
8-CRTC		13.80	13.80	13.80	13.80	13.70
12-CRTC		16.00	16.00	16.00	16.00	15.90
16-CRTC		17.60	17.60	17.50	17.50	11.50
8-DRTC		-	16.00	16.00	15.90	15.80
12-DRTC			18.30	18.20	18.20	18.10
16-DRTC			19.90	19.80	19.8	19.70
6-ERTC		8.05	7.97	7.92	7.88	7.81
8-ERTC		15.00	15.10	15.10	15.10	15.00
10-ERTC		15.50	10.40	10.40	10.30	10.30
12-ERTC		17.00	17.10	17.0	17.0	17.0
14-ERTC		12.0	12.0	11.90	11.90	11.80
16-ERTC		18.40	18.40	18.40	18.40	18.30
18-ERTC		13.10	13.10	13.00	13.00	13.00

REFERENCES

1. Nooshin, H., and Makowski, Z. S., "Some aspects of the behavior of square diagonal grids", Build. Sci. 9: 1974, 269-276.
2. Nooshin, H., "Diagonal grids with members having torsional and shear rigidity", Ph. D. Thesis University of London 1967.
3. Cope, R. J., et al., Computer methods for civil engineers, McGRAW-HILL book company, 1982.
4. Ronald, L. Sack, Structural analysis, McGRAW-HILL book company, 1984.