LONGITUDINAL CONDUCTION IN CROSS FLOW HEAT EXCHANGER WITH CONDENSATION

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Abstract The deterioration of the performance of a crossflow heat exchanger with condensation due to the longitudinal conduction was investigated. For a relatively small change of wall temperature along the flow directions which was shown to be common in many applications, the energy equations were solved numerically. An iterative method was used to solve the resulting equations of the finite difference form in order to obtain the hot fluid, the cold fluid and the wall temperature distributions and the exchanger thermal effectiveness. A comparison was made between the effects of longitudinal conduction on the exchanger performance, without and with condensation. The results indicated that with the same values of NTU, the capacity ratio and the conduction parameter, λ, the performance deterioration of a crossflow heat exchanger with condensation is not as strong as the case of pure heat transfer.

چکیده تاءثیر نامطلوب هدایت طولی بر عملکرد یک مبدل حرارتی با جریانهای عمود بر هم وقتی که تقطیر در آن صورت می‌گیرد مورد بررسی قرار گرفته است. معادلات انرژی برای تغییر دمای کوچک دیواره مبدل در جهتهای جریان، که برای بسیاری از کاربردها فرض مناسبی است، به روش عددی حل گردیده‌اند. برای حل معادلات بدست آمده بصورت تفاوت محدود از یک روش تکراری استفاده گردیده که در نتیجه توزیع دمای سیال گرم، سیال سرد و دیواره و بازده حرارتی مبدل بدست آمده‌اند. اثرات هدایت طولی بر عملکرد مبدل حرارتی با و بدون وجود تقطیر مورد مقایسه قرار گرفته‌اند. نتایج این مقایسه نشان داد که به ازاء مقادیر مساوی NTU، نسبت ظرفیتهای حرارتی و پارامتر هدایت، λ، اثر هدایت طولی بر عملکرد مبدل حرارتی با جریانهای عمود برهم با وجود تقطیر، خفیف‌تر از وقتی است که فقط انتقال حرارت محسوسی وجود دارد.

INTRODUCTION

Longitudinal conduction through the heat exchanger wall in the flow directions causes a deterioration of the exchanger performance. Many investigations of this effect have been performed for various types of heat exchangers, all under sensible heat transfer condition. The performance deterioration due to longitudinal conduction in counter flow heat exchanger has been studied by Landau and Hlinka [1] and Kroeger [2]. The same analysis has been performed by Chiou [3] for crossflow and by Bahnke and Howard [4] and Schultz [5] for periodic flow heat exchangers.

If the surface temperature is below the dew point temperature of the surrounding air and above the freezing point the vapour present in the air will condense on the surface. Condensation is a common place physical phenomenon which occurs frequently in heat exchangers operating in the summer for air conditioning purposes. When it occurs, droplets or a film of water collects on the surface, depending on the rate of condensation and cleanness of the surface. However, the cleanness of the surface can not last long because of impurities and solid particles present in the air. Thus, the droplets will eventually run off in some direction due to the effect of gravity and/or due to the drag exerted by the air flow. Hence, a film of condensate will finally prevail.

In a crossflow heat exchanger, the wall temperature varies in both the cold stream and hot stream directions. The type of variation is dictated by the inlet temperatures, the transfer coefficients, the wall thermal resistance and the ability of the wall in the longitudinal conduction. The wall temperature may be greater than the local hot air dew point temperature on part of the surface and hence condensation will not take place on this part. However, when condensation occurs only on a portion of the exchanger surface, the exchanger may be positioned such that the condensate will be removed by gravity or drag without flowing over sensible heat transfer portion of the surface. Under certain conditions the condensate will flow on the dry surface and hence will evaporate. The evaporation of the condensate will result in a partial or total regain of the latent heat removed from the hot stream during condensation. In this analysis, however, it is assumed that condensation exists on the entire exchanger surface.

In addition, the following assumptions are made:

a- The mass flow rates, transfer coefficients, thermophysical properties and the entering conditions are independent of time and position.

b- The thermal resistance of the wall and the condensate film in the direction normal to the fluid flows are neglected.

c- The energy exchange between the heat exchanger and the surrounding is negligible.

d- The changes in the wall temperature along the flow directions are small.

ANALYSIS

Consider a crossflow heat exchanger with rectangular channels for both streams. Condensation occurs on the entire surface of the exchanger on the hot air side. There is longitudinal conduction in the wall, in both flow directions.

An elemental volume of the wall designated by t dx dy is indicated in Figure 1. Using Lewis relation between the heat and mass transfer coefficients, the energy exchange between the hot air and the solid wall can be written in terms of enthalpy potentials. Therefore, the energy balance for the elemental volume can be expressed as:

$$Kt\left(\frac{\partial^2 T_W}{\partial x^2} + \frac{\partial^2 T_W}{\partial y^2}\right) + h_D (i_H - i_W)$$

$$-h_C (T_W - T_C) = 0 \qquad (1)$$

in which K and t are assumed to be constant. Furthermore, the energy transfer between the wall and the adjacent fluids can be written as:

$$\frac{C'_H}{C_P} \frac{\partial i_H}{\partial x} dx = -2\, h_D\, dxdy\, (i_H - i_W) \qquad (2)$$

$$C'_C \frac{\partial T_C}{\partial y} dy = -2\, h_C\, dxdy\, (T_C - T_W) \qquad (3)$$

where C'_C and C'_H are the thermal capacities of the two fluids contained in the channels of width dx and dy respectively. Assuming uniform flow distribution, these quantities may be replaced by $\frac{C_C}{n_C} \frac{dx}{L}$ and $\frac{C_H}{n_H} \frac{dy}{W}$. On the other hand, the number of hot and cold channels are approximately equal to half

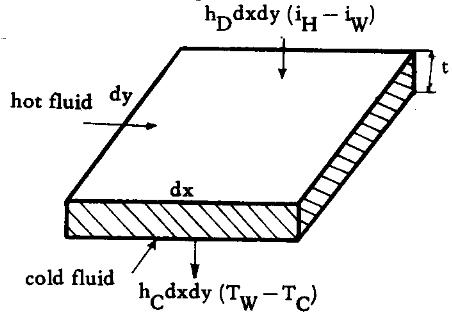

Figure 1. Schematic view of an elemental volume of the exchanger wall

the number of plates. Therefore Eqs. (1), (2) and (3) may be written as:

$$\frac{C_H}{C_P}\frac{\partial i_H}{\partial x} = -h_D\, Wn_P\,(i_H - i_W) \qquad (4)$$

$$C_C\,\frac{\partial T_C}{\partial y} = -h_C\, Ln_P\,(T_C - T_W) \qquad (5)$$

Using Eqs. (4) and (5), Eq. (1) can be written in the following form:

$$-\frac{1}{C_P}\frac{\partial i_H}{\partial x} - \frac{C_C W}{C_H L}\frac{\partial T_C}{\partial y} +$$

$$\frac{KtWn_P}{C_H}\left(\frac{\partial^2 T_W}{\partial x^2} + \frac{\partial^2 T_W}{\partial y^2}\right) = 0 \qquad (6)$$

Now let

$$X = \frac{x}{L} \qquad \beta_H = \frac{h_H A}{C_H}$$

$$Y = \frac{y}{W} \qquad \beta_C = \frac{h_C A}{C_C} \qquad (7)$$

Then Eq. (4), (5) and (6) can be expressed as

$$\left(\frac{\partial}{\partial X} + \beta_H\right) i_H - \beta_H i_W = 0 \qquad (8)$$

$$\left(\frac{\partial}{\partial Y} + \beta_C\right) T_C - \beta_C T_W = 0 \qquad (9)$$

$$\frac{1}{\ }\,\frac{\partial i_H}{\partial X} - \frac{C_C}{\ }\,\frac{\partial T_C}{\partial Y} + \frac{Ktn_P WL}{C_H}$$

$$\left(\frac{1}{L^2}\frac{\partial^2 T_W}{\partial X^2} + \frac{1}{\ }\,\frac{\partial^2 T_W}{\partial Y^2}\right) = 0 \qquad (10)$$

However, at the wall, the hot air is saturated and the following approximate relation between enthalpy and temperature may be used

$$i_W = aT_W + b \qquad (11)$$

in which a and b are functions of T_W. If the wall temperature does not change significantly along the flow directions, a and b may be considered constant. It is shown in Appendix A that the assumption of small change in the wall temperature in a heat exchanger with condensation is not unrealistic. In addition, the longitudinal conduction contributes to smooth the wall temperature variation therefore a and b are considered constant for the present purpose.

In order to express the equations in completely non-dimensional, forms, the following quantities are defined:

$$\phi_H = \frac{i_H - b}{a} \qquad \lambda_H = \frac{KtA}{C_H L^2}$$

$$\tau_H = \frac{\phi_H - T_{CI}}{\phi_{HI} - T_{CI}} \qquad \lambda_C = \frac{KtA}{C_C W^2}$$

$$\tau_C = \frac{T_C - T_{CI}}{\phi_{HI} - T_{CI}} \qquad \alpha = \frac{C_P}{a}$$

$$\tau_W = \frac{T_W - T_{CI}}{\phi_{HI} - T_{CI}} \qquad \gamma = \frac{C_C}{C_H}$$

Then Eqs. (8), (9) and (10) can be written in the following forms.

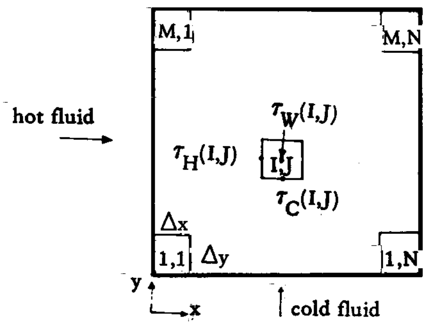

Figure 2. Subdivision scheme of the exchanger wall

$$\left(\frac{\partial}{\partial X} + \beta_H\right)\tau_H - \beta_H \tau_W = 0$$

$$\left(\frac{\partial}{\partial Y} + \beta_C\right)\tau_C - \beta_C \tau_W = 0$$

$$\frac{\partial \tau_H}{\partial X} - \alpha\gamma\frac{\partial \tau_C}{\partial Y} + \alpha\lambda_H\frac{\partial^2 \tau_W}{\partial X^2} +$$

$$\alpha\gamma\lambda_C\frac{\partial^2 \tau_W}{\partial Y^2} = 0$$

These are the basic equations to be solved, subject to the boundary conditions to be discussed now.

The boundary conditions depend on the location where condensation begins. In general, condensation may occur on a part of the exchanger surface only. In that case the regions of sensible and latent heat transfer should be treated separately. However it is assumed, as discussed in the introduction, that condensation begins at the hot air inlet and continuous to exist over the entire exchanger surface. Furthermore, the inlet temperatures of the fluids are supposed to be known and the heat dissipation through the edges of the wall is neglected. Therefore the boundary conditions can be written as

$$\frac{\partial \tau_W(0,Y)}{\partial X} = 0, \qquad \tau_H(0, Y) = 1 \qquad (16\text{-a})$$

$$\frac{\partial \tau_W(1,y)}{\partial X} = 0, \qquad (16\text{-b})$$

$$\frac{\partial \tau_W(X,0)}{\partial Y} = 0, \qquad \tau_C(X, 0) = 0 \qquad (16\text{-c})$$

$$\frac{\partial \tau_W(X,1)}{\partial Y} = 0 \qquad (16\text{-d})$$

It is interesting to note that the differential equations (13), (14) and (15) are similar to the equations derived for sensible heat transfer case in [3], except for the coefficient in which does not appear in the latters. Also the boundary conditions (16) are similar to those obtained for the case of sensible heat transfer as indicated in [3]. A question remains to be answered and that is whether condensation stops somewhere inside the exchanger before it reaches the exit. The answer is no, as indicated in Appendix B.

NUMERICAL SCHEME

The exchanger wall is divided into M x N subdivisions. The subdivisions are numbered from (1, 1) at the two fluid inlet corner, to (M, N) at the outlet corner as shown in Figure 2. For each subdivision, $\tau_W(i, j)$ represents the wall temperature at the center while $\tau_H(i, j)$ and $\tau_C(i, j)$ indicate the values of these quantities at the inlets of the subdivision as shown in Figure 2. Then the differential equations (13), (14) and (15) are written for each subdivision in finite difference forms.

An iterative method is used to solve the resulting equations simultaneously for τ_H, τ_C and τ_W for each subdivision. The solution begins with some arbitrary initial wall temperature distribution. In the present scheme, the initial wall temperature is taken as the midvalue of the inlet temperatures of the two fluids.

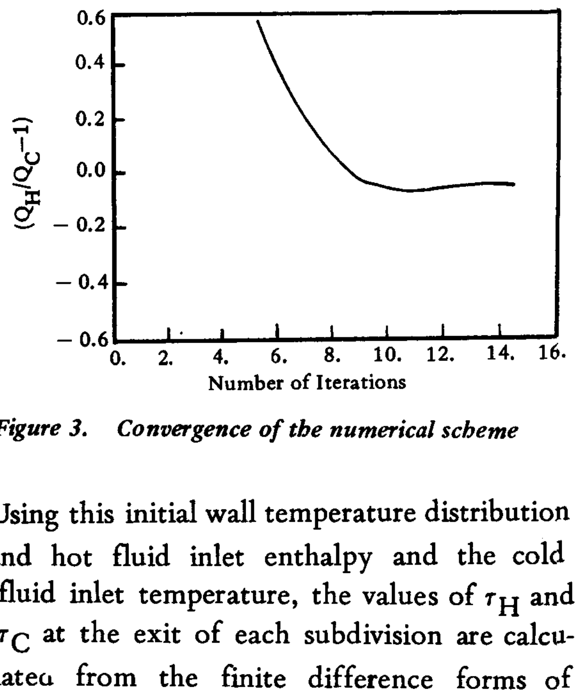

Figure 3. Convergence of the numerical scheme

Using this initial wall temperature distribution and hot fluid inlet enthalpy and the cold fluid inlet temperature, the values of τ_H and τ_C at the exit of each subdivision are calculateu from the finite difference forms of equations (13) and (14). The finite difference form of equation (15) is then used to calculate the new wall temperature distributions. The hot fluid outlet enthalpy and the cold fluid outlet temperature distributions are also calculated. Their mean values are then evaluated by the numerical integration of

$$\bar{i}_{HE} = \int i_{HE}\, dY \quad (17)$$

$$\bar{T}_{CE} = \int T_{CE}\, dX \quad (18)$$

Obviously, the first iteration does not give the correct results. The new wall temperature provides a basis for the second iteration and so on.

The accuracy of the result depends on the number of subdivisions as well as the number of iterations. Given the number of subdivisions, the energy balance between the two fluids is used as a criterion for the accuracy of the results. After each iteration, the rate of energy transfers are calculated from

$$Q_H = m_H (i_{HI} - \bar{i}_{HE}) \quad (19)$$

$$Q_C = C_C (\bar{T}_{CE} - T_{CI}) \quad (20)$$

The desired accuracy is obtained when the absolute value of (Q_H/Q_C-1) remains within certain limits.

The conduction effect on the performance of the heat exchanger is expressed in terms of the "performance deterioration factor", η, which is defined by

$$\eta = 1 - \frac{\epsilon}{\epsilon_0} = \frac{\Delta\epsilon}{\epsilon_0} \quad (21)$$

in which the exchanger effectiveness, ϵ, is defined as

$$\frac{Q_H}{C_{min}(T_{HI} - T_{CI})} = \frac{Q_C}{C_{min}(T_{HI} - T_{CI})} \quad (22)$$

It should be remembered that Q_H includes the latent heat transfer and is calculated from equation (19)

RESULTS AND DISCUSSIONS

The computer program was run for 10 x 10 and 20 x 20 subdivisions. It was found that the calculated values of the performance deterioration factor, η, for these two cases were different in the third digit. Thus a 10 x 10 matrix arrangement was used throughout this study. The number of iterations was the second factor which had a significant effect on the accuracy of the results. Figure 3, which shows the energy difference ratio (Q_H/Q_C-1) as a function of the number of iterations, indicates the relatively fast convergence of the solution. This figure indicates that using less than 10 iterations is enough to give a sufficient accuracy acceptable for most engineering applications.

Although the analysis was made for an exchanger with rectangular flow channels the results are qualitatively applicable for cross-flow heat exchangers with finned surfaces. In fact, with some modifications, such as using total effective heat transfer area for A, the results will be quantitatively applicable for these exchangers.

The effect of conduction parameter, λ, on the exchanger performance for a balanced flow is shown in Figure 4. The performance deterioration factor, η, generally increases as λ increases. From the definition of λ the following conclusions may be drawn.

1- When the flow lengths are short (which is common in heat exchangers designed for high effectiveness) the conduction effects are stronger. It should be noticed that the flow length can be shortened while keeping the heat transfer area, A, constant by adding fins to the surface.

2- The conduction effect is stronger in heat exchanger made of materials of high conductivity. However, this conclusion does not justify the use of low conductivity materials for heat exchanger core, because such materials enhance the thermal resistance of the wall normal to the flow directions, this has a significant adverse effect on the exchanger performance.

3- The performance deterioration in heat exchanger with thinner wall structures is less significant.

4- Among the operating conditions, the flow rates and the inlet temperatures are particularly important. Increasing the flow rates reduces the conduction effect on the exchanger performance. On the other hand, when the wall temperature is raised as a result of increased inlet temperatures the conduction effect becomes less important. This latter is due to the appearance of α=cp/a in equation (15).

The performance deterioration of a crossflow heat exchanger with pure sensible heat transfer for two values of λ_H is also shown in Figure 4. This result was obtained from [3]. A comparison indicates the conduction effect, for the case of sensible heat transfer is much stronger than the case of condensation. The difference is due to the smaller gradient of the wall temperature in the latter, as shown in Appendix A.

The last observation from Figure 4 is concerned with the effect of NTU on η. The performance deterioration factor reaches a maximum for a value of NTU less than 10 for small values of λ_H. As λ_H increases the point of maximum moves towards higher values of NTU. This result is not observed in the case of pure sensible heat transfer for the capacity ratio equal to 1.0.

Figures 5 and 6 show the effects of conduction parameters ratio and the heat transfer coefficients ratio on η respectively. As the ratio λ_C/λ_H increases the conduction effect becomes greater. The same trend is observed when h_H/h_C is raised.

Finally Figure 7 indicates the effect of the capacity ratio on η. The maximum deterioration in the performance occurs for C_H/C_C less than 1.0. The value of capacity ratio C_H/C_C for maximum η depends on the wall temperature and thus on the inlet temperatures. The location of maximum η moves towards C_H/C_H= 1.0 as the inlet temperatures are lowered. For the typical case used in this analysis the maximum deterioration occurs at C_H/C_C equal to 0.4.

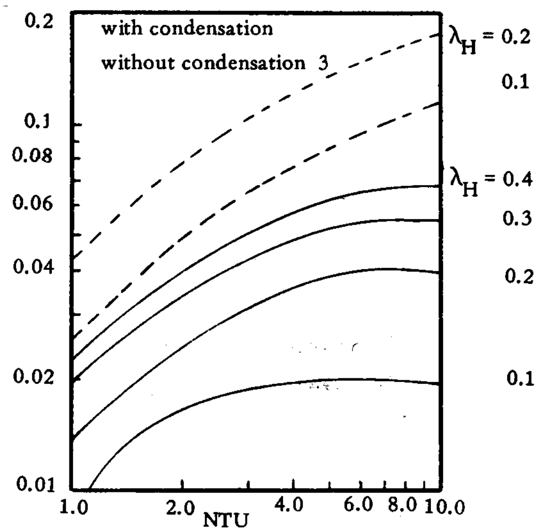

Figure 4. Effect of conduction parameter on η, conditions: $C_H/C_C=1.0$, $\lambda_H/\lambda_C=1.0$, $b_H/b_C=1.0$

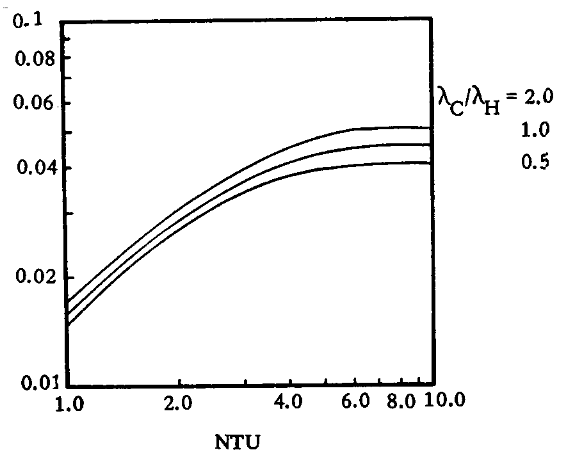

Figure 5. Effect of conduction ratio on η, conditions: $C_H/C_C=1.0$, $b_H/b_C=1.0$, $\lambda_H=0.25$

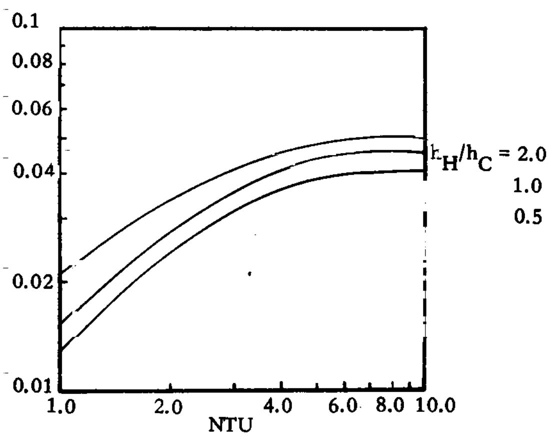

Figure 6. Effect of heat transfer coefficients ratio on η, conditions: C_H/C_C=1.0 λ_H=0.25, λ_H/λ_C=1.0

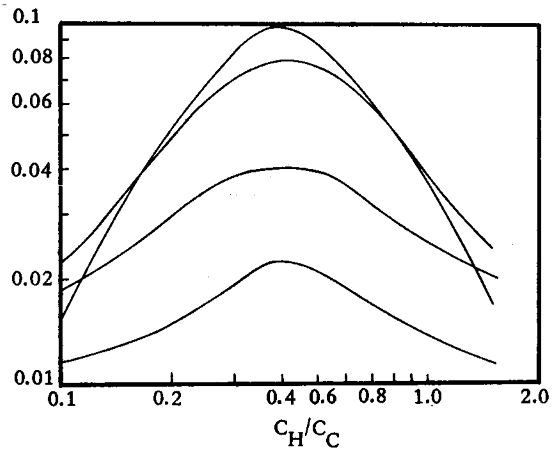

Figure 7. Effect of flow capacity ratio on η, conditions: λ_H=0.2, λ_H/λ_C=1.0, b_H/b_C=1.0

CONCLUSIONS

The differential equations of energy transfer through the wall, and between the wall, and the fluids for a crossflow heat exchanger with condensation were written in finite difference forms and solved by an iterative method. In this study, the effect of two dimensional longitudinal heat conduction through the exchanger wall on the exchanger performance was analysed. The results were compared with those of pure sensible heat transfer case [3] and it was found that the longitudinal conduction effect is less significant, when there is condensation. It was also observed from this study that the performance deterioration is smaller for balanced flow than for unbalanced flow. In addition, the performance deterioration due to longitudinal conduction seemed to be stronger in heat exchanger of short flow lengths and thick wall particularly when the capacity flow rates and the inlet temperature are low.

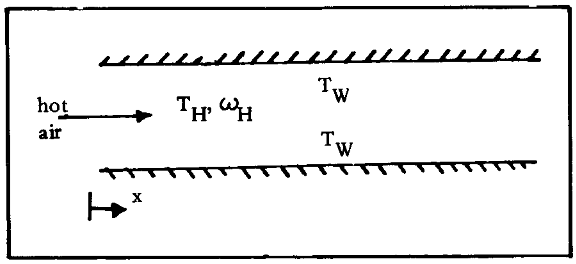

Figure 8. Humid air flowing between isothermal plates

NUMENCLATURE

a	defined by Eq. (11), KJ/Kg C
A	heat transfer surface area, m^2
b	defined by Eq. (11), KJ/Kg
C	capacity flow rate, KW/C
Cp	specific heat at constant pressure, KJ/KgC
h	heat or mass transfer coefficient, KW/m^2 C or Kg/m^2
i	specific enthalpy, KJ/Kg
$\bar{i}$	average specific enthalpy, KJ/Kg
K	thermal conductivity of the wall material, KW/m C
L	flow length of the hot stream, m
$\dot{m}$	mass flow rate, Kg/S
M	number of subdivisions in the hot stream
n	number of flow channels or plates
N	number of subdivisions in the cold stream
Q	rate of heat transfer, KW
t	thickness of the wall, m
T	temperature, C
$\bar{T}$	average temperature, C
W	flow length of the cold stream, m

x coordinate in the hot stream direction, m
y coordinate in the cold stream direction, m
α defined by Eq. (12), dimensionless
β defined by Eq. (11), dimensionless
$\mathcal{E}$ thermal effectiveness defined by Eq. (22)
η performance deterioration factor, defined by Eq. (21)
γ capacity ratio defined by Eq. (12)
λ conduction parameter defined by Eq. (12)
ω specific humidity
defined by Eq. (12), dimensionless

SUBSCRIPTS

O no longitudinal conduction
C cold side
D mass transfer, hot side
E exit
H hot side
I inlet
P plates
S sensible heat transfer alone

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APPENDIX A

It is desired to specify the relative variation of the wall temperature of a crossflow heat exchanger, with and without condensation. To do this an exchanger of short flow lenghts is considered so that the wall temperature is within a small range and a and b remain unchanged. Obviously, the results obtained for the relative magnitude of the gradient of the wall temperature with and without condensation would be qualitatively applicable for any flow length. It is also assumed that longitudinal conduction through the walls is negligible.

For condensation the energy balance may be expressed as

$$\frac{h_H}{C_P}(i_H - i_W) = h_C(T_W - T_C) \tag{23}$$

Using Eq. (11) the following relation for the wall temperature is obtained

$$T_W = \frac{\frac{h_H}{C_P h_C}(i_H - b) + T_C}{1 + \frac{a}{C_P}\frac{h_H}{h_C}}$$

Differentiating with respect to x gives

$$\frac{\partial T_W}{\partial x} = \frac{\frac{h_H}{\;}\frac{\partial i_H}{\partial x} + \frac{\partial T_C}{\partial x}}{1 + \frac{a}{C_P}\frac{h_H}{h_C}}$$

On the other hand, the gradient of i_H and T_C are related through

$$\frac{C_H}{\;}\frac{\partial i_H}{\;} = -C_C\frac{\partial T_C}{\partial y}$$

Substituting for $\frac{\partial i_H}{\partial x}$ in Eq. (25) one can get

$$\frac{\partial T_W}{\partial x} = \frac{-\frac{h_H}{h_C}\frac{C_C}{C_H}\frac{\partial T_C}{\partial y} + \frac{\partial T_C}{\partial x}}{1 + \frac{a}{C_P}\frac{h_H}{h_C}}$$

For $C_C = C_H$ and $h_H = h_C$ the above equation is simplified to

$$\frac{\partial T_W}{\partial x} = \left(\frac{1}{1 + a/C_p}\right)\left(\frac{\partial T_C}{\partial x} \quad \frac{\partial T_C}{\partial y}\right) \tag{27}$$

By taking similar steps for heat transfer along the following equation may be derived

$$\left(\frac{\partial T_W}{\partial x}\right)_S = \left(\frac{\partial T_C}{\partial x} - \frac{\partial T_C}{\partial y}\right)_S \tag{28}$$

The gradients of the cold fluid temperature for condensation and heat transfer alone are almost the same. Thus dividing Eq. (27) by Eq. (28) gives:

$$\frac{\partial T_W/\partial x}{(\partial T_W/\partial x)_S} = \frac{1}{1 + a/C_p}$$

This ratio for a 5°C interval of the wall temperature from 0°C to 35°C changes from 0.35 to 0.15. The same result can be obtained for ratio of the gradient of the wall temperature in the y direction

Therefore, for a crossflow heat exchanger, the variation of the wall temperature is much smaller for the condensation case than it is for the case of heat transfer alone.

APPENDIX B

Does condensation stop somewhere inside the exchanger once it begins at the hot stream inlet? In other words, will the wall temperature exceed the local dew point temperature of the hot air? In fact, the wall temperature decreases in x direction. But in certain conditions, the wall remains isotherm and this is the most probable case for which condensation might stop before the exit.

Consider a flow of humid air between two cold isothermal parallel plates as indicated in Figure 8. The surface temperatures are below the dew point temperature at the inlet and condensation begins at this point. Since the air at the interface with the liquid is saturated, its humidity ratio ω_W, remains unchanged along the flow direction. The vapor mass balance can be expressed as:

$$m \frac{d\omega_H}{dx} = -2h_D W (\omega_H - \omega_W) \tag{29}$$

which can be integrated to give

$$\frac{\omega_H - \omega_W}{(\omega_H - \omega_W)_{x=0}} = e^{\frac{-2h_D W}{\dot{m}} x} \tag{30}$$

The right hand side is always positive. This means the humidity ratio of the hot stream is always greater than the humidity ratio at the wall. In other words, the wall temperature remains below the local bulk air dew point temperature in the entire exchanger length.