FLOW AND PRESSURE DISTRIBUTIONS IN SHORT HEAT EXCHANGER CORES WITH ABRUPT ENTRANCE AND EXIT

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Abstract The typical installation of a heat exchange device usually involves a flow contraction at the core entrance and a flow expansion at the core exit. Repeated flow contraction and expansion are experienced in the flow passages of some compact heat exchangers. The latter refers to the flow passages in the plate-fin type with louvered fins or stripped fins and in the tubular type with dimpled-circular or flat tubes. Similar flow situations are encountered in flat-edged orifices. The majority of these constrictions are characterized by small length-to-hydraulic diameter ratio L^*/D_h^* so that the flow after being disrupted has a little time in developing into an appropriate pattern before it is disrupted again. This work deals with a flow system consisting of a parallel channel with a flow constriction of small L^*/D_h^*. The flows at both ends of the channel are laminar, fully developed. The full Navier-Stokes equations for the steady two-dimensional flow are solved numerically using the finite-difference technique. Some preliminary results are obtained for the distributions of velocity components, streamline and pressure. The result for a flow of small Reynolds number show qualitative agreement with the flow pattern of a similar flow system obtained by visualisation method.

چکیده در ورودی و خروجی یک مبدل حرارتی ، معمولا" جریان منقبض و منبسط میشود در بعضی از مبدلهای فشرده انقباض و انبساط تکراری در مسیرهای جریان به چشم می‌خورد . قسمت اخیر به مسیرهای جریان که درنوع مسطح با پره‌های Louvered یا Stripped و در نوع لوله‌ای Dimpled Circular یا لوله‌های دو پهن شده است مربوط‌می‌شود . موقعیتهائی با جریان مشابه را در سوراخهای با لبه مسطح شده ملاحظه می‌کنیم . بسیاری از این تنگی‌ها و گلوئی‌ها با نسبت کوچک طول به قطرهیدرولیکی یعنی L^*/D_h^* مشخص می‌شود به ترتیبی که جریان بعد از تغییر شکل دادن مجال کمی خواهد داشت که شکل اولیه را بخود بگیرد زیرا بلافاصله مجبور به تغییر شکل دادن بعدی خواهد شد . در این مقاله با سیستم جریانی که شامل کانالی با تنگی یا گلوئی جریان L^*/D_h^* کوچک است سروکار داریم . جریان در هر دو انتهای کانال ، ورقه‌ای و کاملا" توسعه یافته است تمام معادله‌های ناویه ــ استوکس برای جریان پایدار دو بعدی با استفاده از روش تفاوت محدود و بطور عددی حل شده است . چند نتیجه اولیه برای توزیع خطوط جریان بدست آمده‌اند . نتیجه برای یک جریان با عدد رینولدز کوچک با نقش جریان یک سیستم جریان مشابه بدست آمده از روش تجسم فکری مطابقت کیفی دارد .

INTRODUCTION

Flow and heat transfer behavior in long tubes and ducts, as well as at entrance regions, have been well documented. In sharp contrast, little informations is available on the hydrodynamic and thermal characteristics in short tubes and ducts with abrupt entrance and exit.

The ratio of the tube or duct length L^* to its hydrodynamic diameter D_h^* is the dimensionless parameter commonly employed as a measure of length for the development of a boundary layer in the transport phenomenon of momentum, heat or mass. The majority of both the theoretical and experimental studies of transport phenomena in internal flows deal with hydrodynamically smooth entrance and exit. In reality, particularly for industrial applications, internal flows with abrupt entrance and exit are common. For example, the flows into and out of heat exchangers experience abrupt changes in the flow cross-sectional areas at the entrance and exit regions. A flow through a flatedged orifice is another example. When the L^*/D_h^* ratio is large, the fluid being disturbed by an abrupt change in the flow cross-sectional area at entrance has enough down stream distance to readjust its velocity profile, even though vortices may have been generated at the entrance region, as in the case of moderate to high speed flows. Those instruments such as pressure probes or thermocouples can be installed at proper locations to measure the pressure or temperature drop at entrance. However, in case of short L^*/D_h^*, the fluid after entering the tube or duct may not have enough flow length for readjusting its flow patterns before it comes to the exit to undergo another drastic change- a sudden expansion. Under this circumstance, measurements obtained from these instruments, if not meaningless, may not be indications of the potential drop, if not meaningless. Two serious questions arise: One concerns the proper location for installing those instruments. The other is related to the conventional definitions of the Fanning friction factor f and the heat transfer factor j which are employed to measure the magnitude of pressure drop and heat transfer performance, respectively. Are these definitions still meaningful or applicable? If the answer to the latter question is negative, then what alternative is available? Probably, there is no simple answer to both questions. Nevertheless, it is definitely a step forward to investigate the changes in the velocity and temperature profiles inside the tube or duct, following the flow contraction at the entrance and prior to the flow expansion at the exit. It is anticipated that the flow and temperature patterns will be complicated, particularly when vortices are generated in the flow.

Two programs are to be pursued: the theoretical and experimental studies. Only the theoretical study is presented in this paper. A mathematical model is developed to describe the flow behavior in tubes or ducts having abrupt entrance and exit. The flow regions immediately upstream and downstream from the passage are also included. Theoretical results are obtained by means of numerical method with the aid of a digital computer. These results include the distribution of the velocity, streamline and pressure.

Results obtained from this study have applications to the hydrodynamic devices and heat exchangers having short flow passages and compact heat exchangers with louvered, or stripped-fins or dimpled tubes. To achieve high heat transfer performance in compact heat exchangers, louvers are provided to disrupt the boundary layer developed on the heat transfer surface. Another means of disrupting boundary layer is to cut the corrugated surface in the flow direction into short pieces which are then dislocated alternatively. Each section of the flow passage between two consecutive louvers or stripped openings constitutes a flow passage of small L^*/D_h^* ratio. In the case of dimpled tubes of either circular or flattened cross section, the flow inside the tube experiences a periodic change in the flow area, which is precisely the situation under consideration.

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LITERATURE SURVEY

Reference 3 is probably the only important work ever being reported, which deals with the problem of pressure drops at an abrupt contraction and expansion in a flow passage. The expansion and contraction coefficients K_e and K_c are defined as a measure of pressure drops due to abrupt expansion and contraction, respectively, in the flow cross-sectional area:

$$\Delta P^*_{expansion} = K_e \cdot \frac{\rho V^{*2}}{2g_c}$$

$$\Delta P^*_{contraction} = K_c \cdot \frac{\rho V^{*2}}{2g_c}$$

Here ΔP^* denotes the pressure drop; V^*, the mean velocity inside the flow passage; ρ, the fluid density. $\rho V^{*2}/2$ is the so-called dynamic head of the flow. The analysis in reference is intended for a fully-developed flow, in either laminar or turbulent range. Therefore, the results are applicable only to the flow passage of large L^*/D_h^* ratio. No attempt has been made on determining the potential distributions.

ANALYTICAL MODEL

Consider a flow channel with a constriction as shown in Figure 1. The constriction, located at distance L_1^* from the channel entrance with a width a*, has a length L* and width b*. By definition, the hydraulic diameter D_h^* of the constriction is 2b*. The exit of the channel is at a distance L_2^* from the end of the constriction. The flow is laminar, fully developed at both the entance and exit of the channel. In the present study, effort is directed toward the constrictions of a small L^*/D_h^* ratio. It is desired to determine the velocity profile and the streamline and pressure distributions in the entire channel.

a. Formulation

Incompressibility of the fluid, steady two-dimensional flow, constant physical properties, no body force and parabolic velocity profiles at both the channel entrance and exit are assumed in developing the analytical model.

The vorticity and continuity equations in dimensionless form read as:

$$u\frac{\partial \omega}{\partial x} + v\frac{\partial \omega}{\partial y} = c\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \quad (1)$$

and

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\hat{}} = 0 \quad (2)$$

respectively. Here, u and v denote the velocity components in the x and y direction, respectively, ω is the vorticity and C is defined as $(b^*/L^*)^2$. The continuity equation can be satisfied by the use of the stream function defined as

$$u = \frac{\partial \psi}{\partial y} \quad , \quad v = -\frac{\partial \psi}{\partial x} \quad (3)$$

Then, the vorticity is related to the stream function as

$$\omega = c\frac{\partial^2 \psi}{\partial x^2} + \frac{\partial^2 \psi}{\partial y^2} \quad (4)$$

The appropriate boundary conditions are as follows. The velocity components are all zero at the solid surfaces. At the entrance and exit of the channel, $u = \frac{3}{2}\bar{u}[1-(\frac{2y}{a})^2]$ and v = 0, wherein $\bar{u}$ is the average velocity. v = 0 and $\partial u/\partial y = 0$ at y = 0, the centerline of the channel. These boundary conditions read, in terms of the streamline and vorticity,

(i) For stream functions:

$\psi = \frac{3}{2}\,\bar{u}\,y\,[1 - \frac{1}{3}\,(\frac{2y}{a'})^2]$ at the entrance and exit, $\psi = 0$ at y = 0, and constant stream function along the solid surface;

(ii) For vorticities

$\omega = -\frac{12\bar{u}y}{a^2}$ at the entrance and exit, $\omega = 0$ at y = 0 and ω on the solid surface to be determined from

$$\left[c\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2}\right]_{\text{at solid boundary}} = 0.$$

b. Method of Solution

The finite difference technique is employed to solve the governing differential equations, with the appropriate boundary conditions and the aid of a digital computer.

The physical system is subdivided into a network consisting of a number of nodal points. Two grid sizes, $\Delta x_A \times \Delta y$ and $\Delta x_B \times \Delta y$, are employed as shown in Figure 2. As a result, two different forms of Taylor's series expansion are adopted in the finite difference technique: case A for a nodal point (i, j) located at distance Δx from the left and right neighboring nodal points (i−1, j) and (i+1,j) and Case B for nodal points with $i = k_1, k_2, l_1$ and l_2, the cross sections across which Δx changes from Δx_1 to Δx_2.

One writes, for case A,

$$\left[\frac{\partial f}{\partial x}\right]_{i,j} = \frac{f_{i+1,j} - f_{i,j}}{\Delta x} + 0(\Delta x)\,(\text{forward})$$

$$= \frac{f_{i+1,j} - f_{i-1,j}}{2\Delta x} + 0(\Delta x)^2\,(\text{central})$$

$$= \frac{f_{i,j} - f_{i-1,j}}{\Delta x} + 0(\Delta x)\,(\text{backward})$$

$$\left[\frac{\partial^2 f}{\partial x^2}\right]_{i,j} = \frac{f_{i+1,j} - 2f_{i,j} + f_{i-1,j}}{(\Delta x)^2} + 0(\Delta x)^2$$

and for case B

$$\left[\frac{\partial f}{\partial x}\right]_{i,j} = \frac{\Delta x_1}{\Delta x_2 . \Delta x_{12}} f_{i+1,j} + \frac{\Delta x_2 - \Delta x_1}{\Delta x_1 . \Delta x_2} f_{i,j} \quad \frac{\Delta x_2}{\Delta x_1 . \Delta x_{12}} f_{i-1,j} + R'_{x_{i,j}}$$

$$\left[\frac{\partial^2 f}{\partial x^2}\right]_{i,j} = 2\left[\frac{1}{\Delta x_2 . \Delta x_{12}} f_{i+1,j} \quad \frac{1}{\Delta x_1 . \Delta x_2} f_{i,j} + \frac{1}{\Delta x_1 . \Delta x_{12}} f_{i-1,j}\right] + R''_{x_{i,j}}$$

where $R'_{xi,j}$ and $R''_{xi,j}$ are the remainder terms of the first- and second-order derivative approximations, respectively. Different treatments based on the structure of the grids surrounding the nodal point under consideration results in two expressions for each equation and each boundary condition. The finite-difference expressions of all the governing equations and their appropriate boundary conditions are presented in Tables 1 and 2,

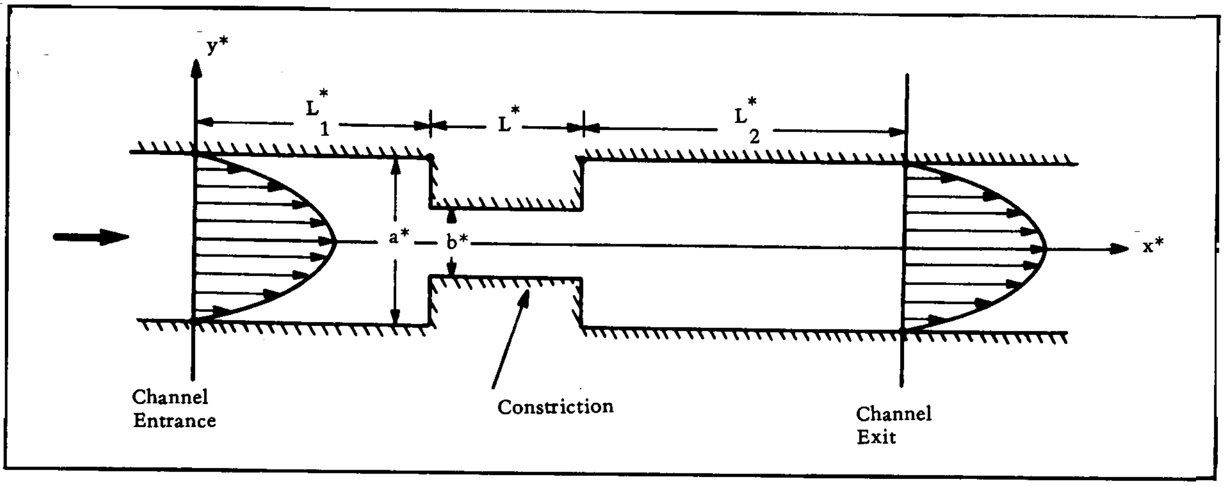

Figure 1. The conduit model configuration and coordinate system.

respectively. Δx_{12} is defined as $\Delta x_1 + \Delta x_2$.

c. Computational Procedure

The values of the vorticity, stream function, velocity components and pressure are computed step-by-step following the procedure:

1. Assume the initial values of u, v, ω, and ψ for all the interior nodal points as well as ω's at the solid surfaces.
2. New values of ω for the interior points are determined using equations (5-A) and (5-B) by Gauss-Seidel iterative scheme [4].
3. Equations (6-A) and (6-B) are employed to compute the new values of ψ corresponding to the new ω's obtained in step 2, by means of the successive row iteration method [2, 5].
4. New values of u and v for all nodal points are obtained using equations (7-A), (8-A) and (8-B) and the equations listed in Table 2 (ii).
5. The equations listed in Table 2 (i) are used to calculate new values of ω for the nodal points on the solid surfaces.
6. Changes in ψ's in percentage between two

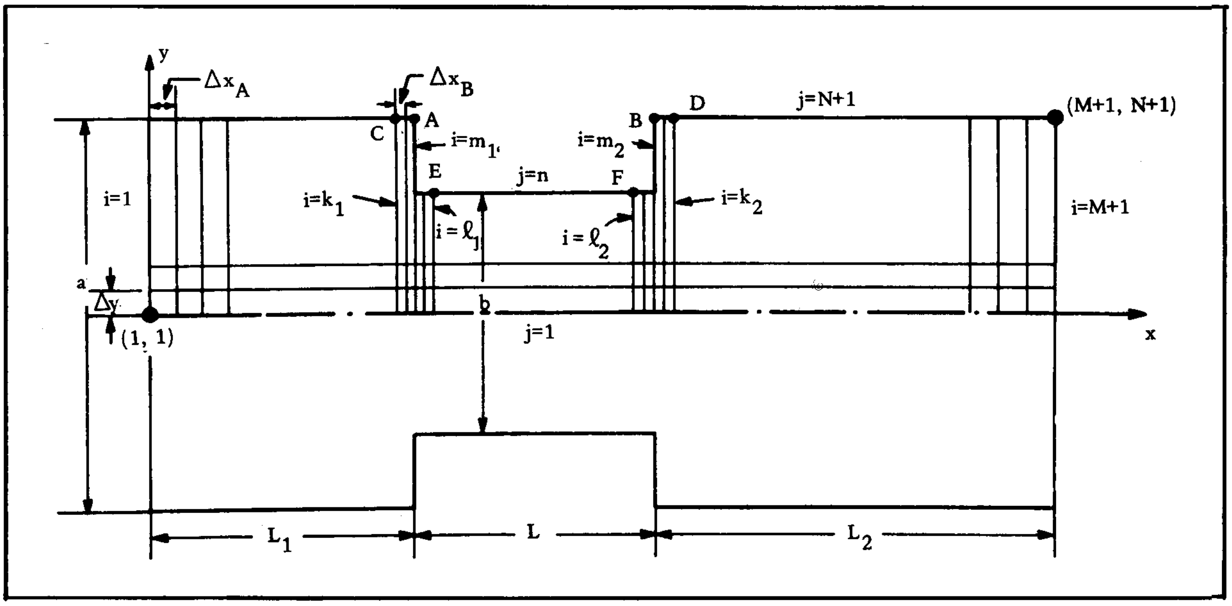

Figure 2. Finite difference network

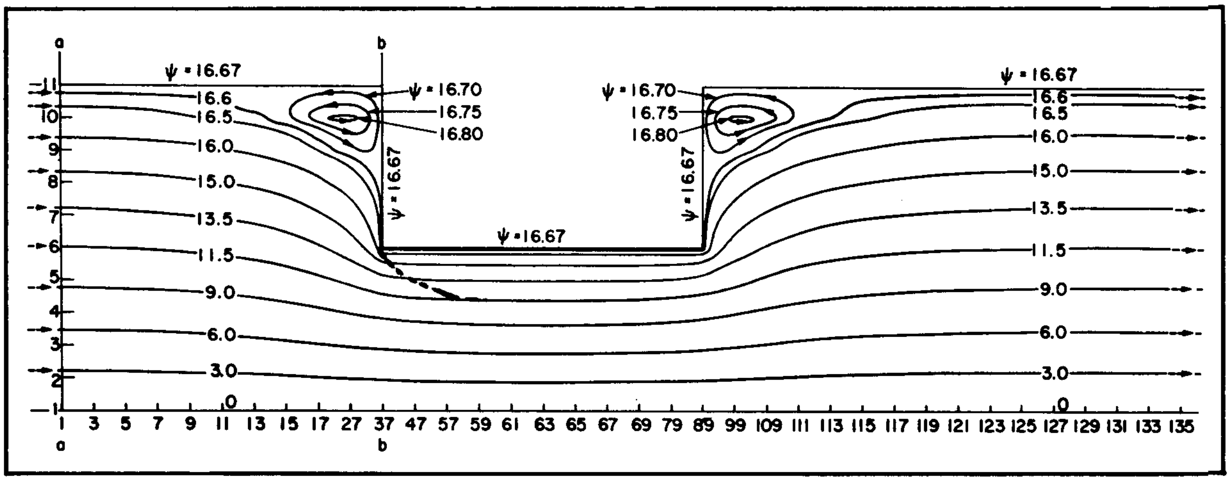

Figure 3. Streamline distribution in a channel with a flow constriction for Re=67, $L^*/D_b^*=0.5$, $L=L_1=L_2/2$.

successive computations are determined for all nodal points. Steps 2 through 5 are repeated until the changes in ψ's at all nodal points become less than a specified percentage value.

7. The final step is to determine pressure distribution in the flow system using equations (9-A) and (9-B) and those listed in Table 2 (iii).

RESULTS AND DISCUSSION

Numerical results for ω, ψ, u, v and P are obtained for Re=67, $L^*/D_h^*=0.5$ $L=L_1=L_2/2$, i = 145, j = 11 and 1.0% specified maximum change in ψ's. Smaller grid size was used in the regions near the entrance and exit of the constriction. The streamline distribution is plotted in Figure 3. Two stationary eddies are seen in the Figure, one at the front

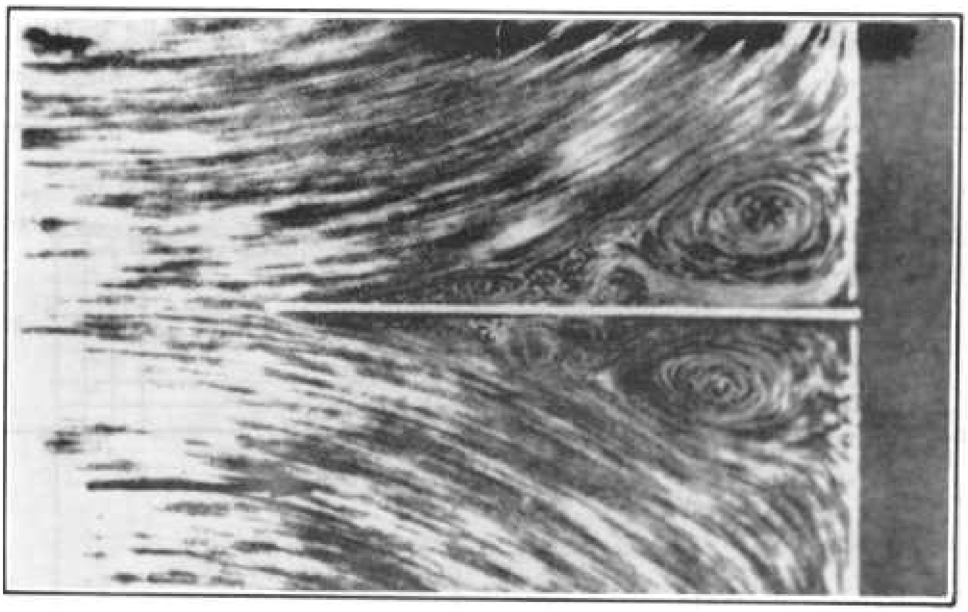

Figure 4. Streamlines observed in a flow toward a stagnation point at a wall with a thin plate at the plane of symmetry [5]

Figure 5. Streamlines observed in a back-step flow at Re=47 [3]

step and the other at the back step. The streamline distribution and the eddy downstream from the constriction resemble those observed in the case of a flow over a back step in reference [3]. The dimpling of the stream lines adjacent to the eddies is clearly observed in typical back-step flows [6].

It is worth noting that the successive row iterative scheme employed in the present study gives results with only a few iterations, much faster convergence than other methods [7, 8].

CONCLUSION

At numerical method has been developed to analytically investigate the distributions of streamline, velocity components, vorticity and pressure for laminar flows through a parallel channel with a flow constriction. The constriction is characterized by a small value of the flow length to hydraulic diameter ratio. The streamline distribution and the eddies generated in the flow system compare qualitatively well with the results obtained by both the numerical and visualization studies on back-step flows. Figures 4 & 5.

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Table 1. Governing equations in finite difference form

Equation	CASE A
Vorticity	$\left[\frac{u_{i,j}}{\Delta x} + \frac{v_{i,j}}{\Delta y} + \frac{2c}{\Delta x^2} + \frac{2}{\Delta y^2}\right]\omega_{i,j} = \left[\frac{c}{\Delta x^2}\right]\omega_{i+1,j} + \left[\frac{c}{\Delta x^2} + \frac{u_{i,j}}{\Delta x}\right]\omega_{i-1,j} + \left[\frac{1}{\Delta y^2}\right]\omega_{i,j+1} + \left[\frac{1}{\Delta y^2} + \frac{v_{i,j}}{\Delta y}\right]\omega_{i,j-1}$ (5-A)
Stream Function Vorticity	$\psi_{i,j} = \frac{1}{2\left[\frac{c}{\Delta x^2} + \frac{1}{\Delta y^2}\right]}\left[\frac{c}{\Delta y^2}[\psi_{i+1,j} + \psi_{i-1,j}] +\right.$

Table 1 (continued)

$$\left[\frac{1}{\Delta y^2}\left[\psi_{i,j+1}+\psi_{i,j-1}\right]-\omega_{i,j}\right]] \qquad (6\text{-A})$$

Velocity $$u_{i,j}=\frac{\psi_{i,j-2}-8\psi_{i,j-1}+8\psi_{i,j+1}-\psi_{i,j+2}}{12\Delta y} \qquad (7\text{-A})$$

$$v_{i,j}=\frac{-\psi_{i-2,j}+8\psi_{i-1,j}-8\psi_{i+1,j}+\psi_{i+2,j}}{12\Delta x} \qquad (8\text{-A})$$

Pressure $$P_{i,j}=P_{i-1,j}+\frac{1}{\Delta x}\left[u_{i+1,j}-2u_{i,j}+u_{i-1,j}\right]+\frac{\Delta x}{c\Delta y^2}\left[u_{i,j+1}-2u_{i,j}+u_{i,j-1}\right]-\frac{u_{i,j}}{2c}\left[u_{i+1,j}-u_{i-1,j}\right]-\frac{\Delta x\cdot v_{i,j}}{2c\Delta y}\left[u_{i,j+1}-u_{i,j-1}\right] \qquad (9\text{-A})$$

Equation — Case B

Vorticity $$\left[u_{i,j}\frac{\Delta x_2-\Delta x_1}{\Delta x_1\Delta x_2}+\frac{v_{i,j}}{\Delta y}+\frac{2c}{\Delta x_1\Delta x_2}+\frac{2}{\Delta y^2}\right]\omega_{i,j}=\left[\frac{2c-u_{i,j}\Delta x_1}{\Delta x_2\Delta x_{12}}\right]\omega_{i+1,j}+\left[\frac{2c+u_{i,j}\Delta x_2}{\Delta x_1\Delta x_{12}}\right]\omega_{i-1,j}+\left[\frac{1}{\Delta y^2}+\frac{v_{i,j}}{\Delta y}\right]\omega_{i,j-1}+\left[\frac{1}{\Delta y^2}\right]\omega_{i,j+1} \qquad (5\text{-B})$$

Stream Function-Vorticity $$\psi_{i,j}=\frac{1}{2\left[\frac{c}{\Delta x_1\Delta x_2}+\frac{1}{\Delta y^2}\right]}\left[\frac{2c}{\Delta x_2\cdot\Delta x_{12}}\psi_{i+1,j}+\frac{2c}{\Delta x_1\Delta x_{12}}\psi_{i-1,j}+\left[\frac{1}{\Delta y^2}\left[\psi_{i,j+1}+\psi_{i,j-1}\right]-\omega_{i,j}\right]\right] \qquad (6\text{-B})$$

Velocity $u_{i,j}$ same as (7–A)

$$v_{i,j}=\frac{\Delta x_2}{\Delta x_1\cdot\Delta x_{12}}\psi_{i-1,j}-\frac{\Delta x_1}{\Delta x_2\cdot\Delta x_{12}}\psi_{i+1,j}-\frac{\Delta x_2-\Delta x_1}{\Delta x_1\Delta x_2}\psi_{i,j} \qquad (8\text{-B})$$

Pressure $$P_{i,j}=P_{i-1,j}+\frac{\Delta x_1}{c}\left[\frac{2c}{\Delta x_2\cdot\Delta x_{12}}\cdot u_{i+1,j}-\frac{2c}{\Delta x_1\Delta x_2}\cdot u_{i,j}+\frac{2c}{\Delta x_1\cdot\Delta x_{12}}\cdot u_{i-1,j}+\frac{u_{i,j+1}-2u_{i,j}+u_{i,j-1}}{\Delta y^2}-u_{i,j}\left[\frac{\Delta x_2}{\Delta x_2\cdot\Delta x_{12}}\cdot u_{i+1,j}+\frac{\Delta x_2-\Delta x_1}{\Delta x_1\cdot\Delta x_2}\cdot u_{i,j}-\frac{\Delta x_2}{\Delta x_1\cdot\Delta x_{12}}\cdot u_{i-1,j}\right]-v_{i,j}\left[\frac{u_{i,j+1}-u_{i,j-1}}{2\Delta y}\right]\right] \qquad (9\text{-B})$$

Table 2. Boundary conditions in finite difference form

(i) For vorticity equation

Solid Surfaces $\omega_{i,j}$

Left vertical $X\omega_{m_1,j} = \frac{c}{\Delta x^2}[\omega_{m_1-1,j} - \frac{1}{2}\omega_{m_1-2,j}] + \frac{1}{\Delta y^2}[\omega_{m_1,j+1} + \omega_{m_1,j-1}]$

Horizontal $X\omega_{i,j} = \frac{c}{\Delta x^2}[\omega_{i+1,j} + \omega_{i-1,j}] + \frac{1}{\Delta y^2}[\omega_{i,j-1} - \frac{1}{2}\omega_{i,j-2}]$

$[i=N+1, n]$

Right vertical $X\omega_{m_2,j} = \frac{c}{\Delta x^2}[\omega_{m_2+1,j} - \frac{1}{2}\omega_{m_2+2,j}] + \frac{1}{\Delta y^2}[\omega_{m_2,j+1} + \omega_{m_2,j-1}]$

Point

A

$$Y\omega_{m_1,N+1} = \frac{c}{\Delta x^2}[\omega_{m_1-1,N+1} - \frac{1}{2}\omega_{m_1-2,N+1}] + \frac{1}{\Delta y^2}[\omega_{m_1,N} - \frac{1}{2}\omega_{m_1,N-1}]$$

B

$$Y\omega_{m_2,N+1} = \frac{c}{\Delta x^2}[\omega_{m_2+1,N+1} - \frac{1}{2}\omega_{m_2+2,N+1}] + \frac{1}{\Delta y^2}[\omega_{m_2,N} - \frac{1}{2}\omega_{m_2,N-1}]$$

C, D, E, F

$$[\frac{2c}{\Delta x_1 . \Delta x_2} + \frac{1}{2\Delta y^2}]\omega_{i,N+1} = \frac{2c}{\Delta x_2 \Delta x_{12}}\omega_{i+1,N+1} + \frac{2c}{\Delta x_1 . \Delta x_{12}}\omega_{i-1,N+1} + \frac{1}{\Delta y^2}[\omega_{i,N} - \frac{1}{2}\omega_{i,N-1}]$$

$[i = k_1, k_2, l_1, l_2]$

where $X = \frac{2c}{\Delta x^2} + \frac{1}{2\Delta y^2}$; $Y = \frac{1}{2}[\frac{c}{\Delta x^2} + \frac{1}{\Delta y^2}]$

(ii) For velocity equations

j	u	i	v
N and n−1	$\frac{2\psi_{i,j+1}+3\psi_{i,j}-6\psi_{i,j-1}+\psi_{i,j-2}}{6\Delta y}$	2	$\frac{2\psi_{1,j}+3\psi_{2,j}-6\psi_{3,j}+\psi_{4,j}}{6\Delta X_A}$
2	$\frac{-\psi_{i,4}+6\psi_{i,3}-3\psi_{i,2}-2\psi_{i,1}}{6\Delta y}$	M	$\frac{-2\psi_{M+1,j}-3\psi_{M,j}+6\psi_{M-1,j}-\psi_{M-2,j}}{6\Delta X_A}$
1	$\frac{-7\psi_{i,1}+8\psi_{i,2}-\psi_{i,3}}{6\Delta y}$	m_1-1	$\frac{-\psi_{m_1-3,j}+6\psi_{m_1-2,j}-3\psi_{m_1-1,j}-2\psi_{m_1,j}}{6\Delta X_B}$
		m_2+1	$\frac{\psi_{m_2+3,j}-6\psi_{m_2+2,j}+3\psi_{m_2+1,j}+2\psi_{m_2,j}}{6\Delta X_B}$

(iii) For pressure equation

Boundary	$P_{i,j}$
Horizontal solid surfaces	$P_{i-1,j}+\frac{\Delta x}{C\Delta y^2}\left[u_{i,j-2}-2u_{i,j-1}\right]$; $j = N+1, n$
Left vertical solid surface	$P_{m_1,j+1}-\frac{c\Delta y}{\Delta x^2}\left[v_{m_1-2,j}-2v_{m_1-1,j}\right]$
Right vertical solid surface	$P_{m_2,j-1}+\frac{c\Delta y}{\Delta x^2}\left[v_{m_2+2,j}-2v_{m_2+1,j}\right]$
Center line	$P_{i,2}-\frac{1}{\Delta y}\left[v_{i,3}-2v_{i,2}\right]$
Entrance	0
Exit	$P_{M,j}+\frac{1}{\Delta x}\left[2u_{M+1,j}-5u_{M,j}+4u_{M+1,j}-u_{M+2,j}\right]-\frac{u_{M+1,j}}{c}\left[u_{m+1,j}-u_{M,j}\right]+\frac{\Delta x}{c\Delta y^2}\left[u_{M+1,j+1}-2u_{M+1,j}+u_{M+1,j-1}\right]$

NOMENCLATURE

a	a^* / b^*
a^*	channel width, m
b	equal to unity ($=b^* / b^*$)
b^*	constriction width, m
C	$(b^* / L^*)^2$
D_h^*	hydraulic diameter, $=2b^*$, m
f	Fanning friction factor
g_c	conversion factor
i	integer, for x-direction position in the space grid
j	integer, for y-direction position in the space grid, or heat transfer factor
k_1, k_2	integer
K_c	contraction coefficient
K_e	expansion coefficient
L	equal to unity ($= L^* / L^*$)
L^*	constriction length, m
l_1, l_2	integer
L_1	L_1^* / L^*
L_1^*	distance from channel entrance to constriction entrance, m
L_2	L_2^*/L^*
L_2^*	distance from constriction exit to channel exit, m
m_1, m_2, M, N	integer
P	$P^* g_c (b^*)^2 / (\nu^2 \rho)$
P^*	static pressure, N/m^2
ΔP	$\Delta P^* g_c (b^*)^2 / (\nu^2 \rho)$
ΔP^*	pressure drop, N/m^2
Re	Reynolds number $= D_h^* u^*_{constriction} / \nu$
$R'_{x_{i,j}}, R''_{x_{i,j}}$	remainder terms of the first- and second-order derivative, approximations, respectively
u	velocity component in x-direction, $= \frac{\partial \psi}{\partial y}$
u	average velocity in channel, $-\bar{u}^* (b^*)^2 / (\nu L^*)$
$\bar{u}^*$ constriction	average velocity in constriction, m/s
V^*	bulk velocity inside flow passage, m/s
v	velocity component in y-direction, $= - \frac{\partial \psi}{\partial x}$
x	x^* / L^*
x^*	distance measured from channel entrance in flow direction
Δx	grid size in x-direction and in the region where grid size does not change
$\Delta x_1, \Delta x_2$	grid sizes at upstream and downstream of nodal point, respectively
Δx_{12}	defined as $\Delta x_1 + \Delta x_2$
$\Delta x_A, \Delta x_B$	larger and smaller grid sizes in x-direction, respectively
y	y^* / b^*
y^*	distance measured from channel center, m
Δy	grid size in y-direction
ν	kinematic viscosity, m^2/s
ρ	fluid density, kg/m^3
ψ	stream function
ω	vorticity $= C \cdot \frac{\partial^2 \psi}{\partial^2 x} + \frac{\partial^2 \psi}{\partial^2 y}$

Subscripts

c	contraction
e	expansion
i, j	denotes position in the space grid

Superscripts

*	quantity with dimension