HARMONIC RESPONSE OF PILE GROUPS TO DYNAMIC LOADING

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Received April 1987

Abstract A completely general method of analysis for three-dimensional raked piles under harmonic excitation is discussed. The piles have been represented by a three-dimensional frame structure and the soil has been represented by a boundary element discretization scheme. A computer program has been written which carries out this analysis and produces a group stiffness matrix that can be included as a foundation stiffness matrix in the analysis of a superstructure. Results from this analysis have been compred with those available in the literature.

چکیده در این مقاله یک روش عمومی کامل برای آنالیز شمعهای لاغر چنگکی RAKED PILES تحت نوسانات هارمونیک بحث شده است. شمعها بصورت یک سازه قابی سه‌بعدی و خاک به فرم یک مجموعه متشکل از عناصر مرزی مجزا مدل‌سازی شده‌اند. برنامه کامپیوتری که برای انجام این آنالیز نوشته شده گاما ماتریس سختی گروهی را که میتوان بصورت یک ماتریس سختی پی برای سازه روی خاکی بکار برد بوجود می‌آورد. نتایج حاصل از این آنالیز با دیگر نتایجی که در مقالات قابل دسترسی بوده‌اند مقایسه شده‌اند.

INTRODUCTION

Many structures and their foundations are subjected to dynamic loads resulting from machinery, wave action, blast effects, etc. In many instances these foundations are pile foundations, and in recent years a good deal of attention has been focused on both the static and dynamic analysis of both a single pile and pile groups. The soil-pile interaction plays an important role in the response of the pile group to this loading. The resulting analysis is complex and requires resort to numerical analysis techniques. It is necessary to consider the piles as a structural framework embedded in a three-dimensional continum. In most of the numerical models to date, the soil is not modelled correctly as an infinite half- space, Refs [1-3].. The superiority of the boundary element (integral) method for modelling a half-space is well established. However, it is inefficient to use such a technique to model a long-narrow structural component such as a pile. To overcome this difficulty, a finite difference approximation has been used for axially loaded piles (see Ref. [4]). An explicit expression is used for the transverse displacement of prismatic members under dynamic patch loading. The pile model is then coupled with the boundary elements used to model the soil continum. In the study reported here in the work has been extended from a single pile to a pile group.

THEORY

1. Fundamental Equations

The integral equation for the displacement u_j of the point (ξ) in the soil is given,

$$u_j(\xi,t) = \int_0^t \int_s G_{ij}(x,t,\xi,\tau)\,\phi_i(x,\tau)\,ds\,d\tau$$

$$+ \int_v \rho\,[v_i'(z)\,G_{ij}(x,t,\xi,0)$$

$$+ u_i'(z)\,\frac{\partial G_{ij}}{\partial t}(z,t;\xi,0)]\,dV \qquad (1)$$

In equation (1), v_i' and u_i' are the initial velocity and displacement, respectively. The coordinates (x, z) are points in the soil domain, and (ξ) is a point on the pile-soil interface. The pile-soil interface tractions are ϕ_i, and G_{ij} are the displacement components of the fundamental solution.

For the pile, the equations of motion are:
(a) in the axial direction

$$m\,\frac{\partial^2 u_y}{\partial t^2} + E_p A\,\frac{\partial^2 u_y}{\partial y^2} = \pi d\,\phi_y(t) \qquad (2)$$

(b) in the transverse direction

$$E_p I\,\frac{\partial^4 u_x}{\partial y^4} + m\,\frac{\partial^2 u_x}{\partial t^2} = d\phi_x(t) \qquad (3)$$

In these equations, (u_x, u_y) are the transverse and axial deflections and (ϕ_x, ϕ_y) the corresponding forces. The terms (E_pI, E_pA) are the bending and axial stiffnesses respectively, d is the pile diameter, and m the mass per unit length.

For harmonic response in the time domain, these equations are replaced by,

$$u_j(\xi\ \omega) = \int G_{ij}(x,\xi\ \omega)\,\phi_i(x,\omega)\,ds \qquad (4)$$

$$-m\omega^2 U_y + E_p A\,\frac{\partial^2 U_y}{\partial y^2} = \pi d\phi_y \qquad (5)$$

$$-m\omega^2 U_x + E_p I\,\frac{\partial^4 U_x}{\partial y^4} = d\phi_x \qquad (6)$$

by making the substitutions,

$$\phi_y = \phi_y\ e^{i\omega t}$$

$$u_y = U_y\,e^{i\omega t},\ \text{etc.}$$

2. Axial Response

For the axially loaded pile, the shaft is subdivided into n cylindrical elements, and the base is represented by a uniformly loaded circular disc (Figures 1 and 2). The equation (5) is now approximated by a finite difference expression so that,

$$[D]\{u\} + \{B\} = \{\phi_a\}_p \qquad (7)$$

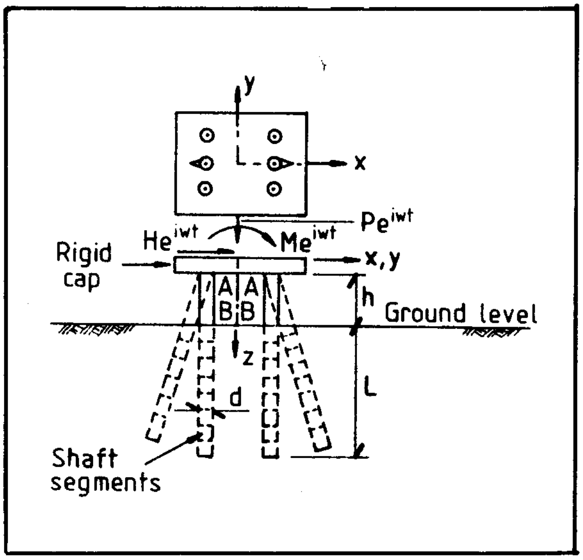

Figure 1. Typical pile group problem.

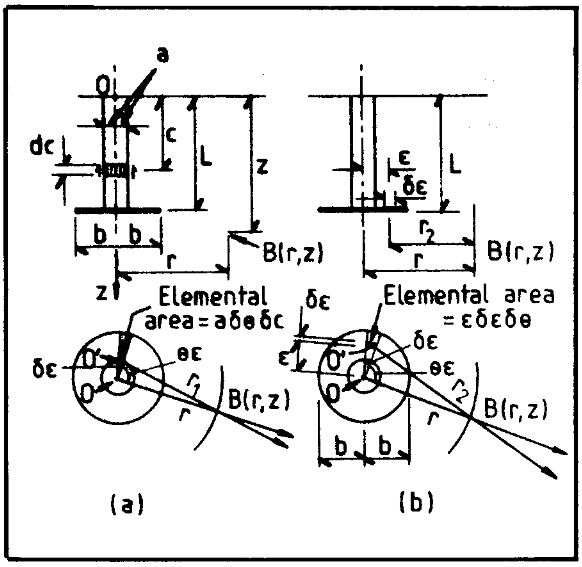

Figure 2. Integration of the kernal function
(a) around the shaft
(b) around the base of the pile

In equation (7), [D] is coefficient matrix and [B] is the pile head boundary condition matrix. The [D] matrix is of the order, (n+1) x (n+1) in the following form,

$$[D] = \frac{E_pA}{\pi dh^2} \cdot \begin{bmatrix} (-3-\omega^2 m) & 1 & & & & & \\ 1 & (-2-\omega^2 m) & 1 & & & \text{ZEROS} & \\ & 1 & (-2-\omega^2 m) & & & & \\ & & & \ddots & & & \\ & \text{ZEROS} & & 1 & (-2-\omega^2 m) & 1 & \\ & & & -0.2 & 2 & (-5-\omega^2 m) & 3.2 \\ & & & & -1.33f & 12f & (-10.67f-\omega^2 m) \end{bmatrix} \qquad (8)$$

and

$$B = \begin{bmatrix} \frac{2E_pAW}{\pi d^2} & 0 & 0 & 0 & \cdot & 0 \end{bmatrix}^T, \qquad (9)$$

is an (n+1) vector.

In these equations, $h=(L/n)$, (L=length of pile, $f=(\pi da/4A_b)$, (A_b=area of the base), W is the axial displacement at ground level, and $(\phi_a)_p$ is the axial traction acting on the pile.

The equation (4) for the soil domain is discretized by replacing the continuous pile-soil interface by a number of boundary elements for each of which ϕ is constant. That is, for the soil

$$\{u\} = [G] \quad (\phi_a)_s \qquad (10)$$

In this equation, $(\phi_a)_s$ is the traction on the soil interface. The equations (7) and (10) are now combined by utilizing equilibrium and compatibility $[(\phi_a)_p = -(\phi_a)_s]$, between the soil domain and the pile elements, so that the final system of equations is

$$-[D]\ [G]\}\{(\phi_a)_p\} + \{B\} = \{(\phi_a)\ _p$$

That is

$$\{[D]\ [G] + I\}\ \{(\phi_a)_p\} = \{B\} \qquad (11)$$

The equations (11) are solved for the tractions, and substitution in equation (10) yields the displacements.

3. Lateral Response

The equation (6) can be solved for a pile attached rigidly to a pile headstock, with V_ξ the transverse displacement of the cap and θ_ξ its rotation. It is further assumed that the force conditions at the base of the pile are moment and shear both equal to zero. A cantilever under dynamic patch load and specified displacement components (V_ξ, θ_ξ) at the support is used to model the laterally loaded pile (Figure 3).

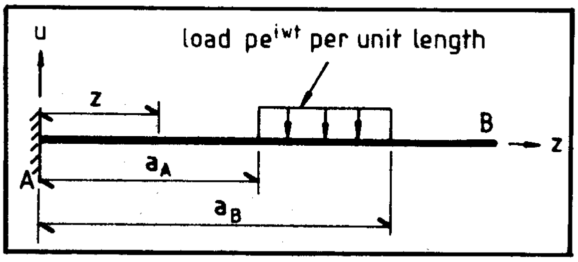

Figure 3. Cantilever under dynamic patch loading

The transverse displacement of an element j due to transverse stress intensity ϕ_t acting on an element i can be obtained from:

$$(u_\xi)_j = V_\xi - \theta_\xi (j-0.5)\, a - \phi_t \,[\, (A_1)_i \cos\eta\,(j-0.5)a + (A_2)_i \sin\eta\,(j-0.5)a + (A_3)_i \cosh\eta\,(j-0.5)a + (A_4)_i \sinh\eta\,(j-0.5)a \,] \quad (\text{for } j \leqslant i) \tag{12a}$$

and

$$(u_\xi)_j = V_\xi - \theta_\xi (j-0.5)a - \phi_t \,[\, (B_1)_i \cos\eta\,(j-0.5)a + (B_2)_i \sin\eta\,(j-0.5)a + (B_3)_i \cosh\eta\,(j-0.5)a + (B_4)_i \sinh\eta\,(j-0.5)a \,] \quad (\text{for all } \geqslant i) \tag{12b}$$

(Coefficients A_1 to B_4) are given in Appendix A.)

Both equations (12a) and (12b) can be written as

$$\{u_\xi\} = -[D_\xi]\,\{\phi_t\} + \{B_\xi\} \tag{13}$$

where $[D_\xi]$ is an $(n \times n)$ matrix formed by varying i=1, 2, ... n and j=1, 2, ... n.

$$\{B_\xi\} = \begin{bmatrix} V_\xi - \theta_\xi\,(0.5a) \\ V \quad (1.5a) \\ \vdots \\ \text{-------------} \\ V_\xi - \theta_\xi\,(n-0.5a) \end{bmatrix}$$

n is the number of boundary elements.

Once again, for the soil domain

$$\{u\} = -[G]\,\{\phi_t\} \tag{14}$$

that is, soil tractions are of opposite sign to pile tractions. Combining equations (13) and (14), as in the axial load case, give

$$\{[D] + [G]\}\,\{\phi_t\} = \{B_\xi\} \tag{15}$$

Equation (15) can be solved to obtain tractions on soil-pile interface. Substitution in equation (14) yields the displacements.

The deflection u at a point on the cantilever (Figure 3) can be written as

$$u = A_1 \cos\eta z + A_2 \sin\eta z + B_3 \cosh\eta z + A_4 \sinh\eta z \,(0 \leqslant z \leqslant a_A) \tag{I}$$

$$u = B_1 \cos\eta z + B_2 \sin\eta z + B_3 \cosh\eta z + B_4 \sinh\eta z \,(a_B \leqslant z \leqslant \ell) \tag{II}$$

For points $(a_A < z < a_B)$ either equation (I) or equation (II) can be used.

The constants A_1 to B_4 can be solved using boundary conditions

(i) u = 0, and
(ii) (du/dz) = 0 at point A;
(iii) $(d^2u/dz^2) = 0$, and
(iv) $(d^3u/dz^3) = 0$ at point B.
(v) Displacement and rotation compatibility and equilibrium under the load.

For a laterally loaded pile, it is possible to write

$$\begin{Bmatrix} V \\ \theta \end{Bmatrix} = \begin{bmatrix} \dfrac{(I_{VH})_{ST}}{E_s L} & \dfrac{(I_{VM})_{ST}}{E_s L^2} \\ \dfrac{(I_{\theta H})_{ST}}{E_s L^2} & \dfrac{(I_{\theta M})_{ST}}{E_s L^3} \end{bmatrix} \begin{Bmatrix} H \\ M \end{Bmatrix} \tag{16}$$

where

V = applied lateral displacement
θ = applied rotation

E_s = soil modulus

L = length of the pile

$(I_{VH})_{ST}$, $(I_{VM})_{ST}$ = elastic influence factors for displacement caused by horizontal load and moment respectively for constant E_s

$(I_{\theta H})_{ST}$, $(I_{\theta M})_{ST}$ = elastic influence factors for rotation caused by horizontal load and moment respectively for constant E_s.

The subscript ST indicates static values of the influence factors.

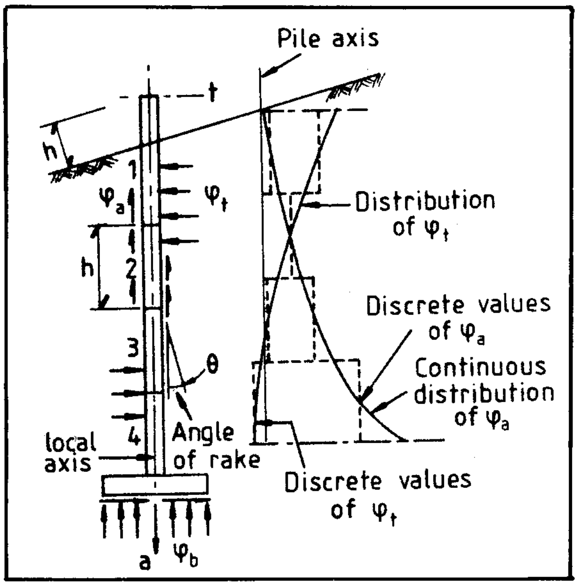

Figure 4. Single pile stresses.

When a pile is subjected to a steady state lateral force and moment, the motion of the pile head can be described by the general equation:

$$\begin{Bmatrix} Ve^{i\omega t} \\ \theta e^{i\omega t} \end{Bmatrix} = \begin{bmatrix} \dfrac{(I_{VH})_{DYN}}{E_s L} & \dfrac{(I_{VM})_{DYN}}{E_s L} \\ \dfrac{(I_{\theta H})_{DYN}}{E_s L} & \dfrac{(I_{\theta M})_{DYN}}{E_s L} \end{bmatrix} \begin{Bmatrix} He^{i\omega t} \\ Me^{i\omega t} \end{Bmatrix} \quad (17)$$

Equation (17) is essentially the dynamic counterpart of equation (16), where

ω = frequency of excitation, and

t = time.

The influence factor, flexibility coefficients, $(I_{VH})_{DYN}$, etc. are complex numbers with their real part representing pile flexibility and the imaginary part representing damping in the piles. $I_{VM} = I_{\theta M}$, by virtue of symmetry and it will be useful to define

$$\begin{aligned} (I_{VH})_{DYN} / (I_{VH})_{ST} &= (I_{11} + i\, J_{11}) \\ (I_{VM})_{DYN} / (I_{VM})_{ST} &= (I_{12} + i\, J_{12}) \\ (I_{\theta M})_{DYN} / (I_{\theta M})_{ST} &= (I_{22} + i\, J_{22}) \end{aligned} \quad (18)$$

4. Pile Group Analysis

The analysis for a single pile can readily be extended to a pile group by summation of the interaction factors for each pile in a group resulting from all the other piles in the group; the displacement of each pile may be written in terms of the loads on every pile in the group. It is assumed that the interaction factor for axial displacement caused by axial load equals that for vertical displacement caused by vertical load on a vertical pile. Similarly, the rotaiton and lateral displacement interaction factors caused by lateral load and moment are identical with those for horizontal displacement and ratation caused by horizontal load and moment on a vertical pile. On the basis of these assumptions, the resulting equations for vertical and horizontal displacement and rotation may be written conveniently in matrix form as follows:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{24} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{Bmatrix} We^{i\omega t} \\ Ve^{i\omega t} \\ \theta e^{i\omega t} \end{Bmatrix} \begin{Bmatrix} Pe^{i\omega t} \\ He^{i\omega t} \\ Me^{i\omega t} \end{Bmatrix} \quad (19)$$

In equation (19), $Pe^{i\omega t}$ = vertical load, $He^{i\omega t}$= horizontal load and $Me^{i\omega t}$=moment acting on the pile cap (Figure 1). The (3 x 3) matrix in equation (19) is the global foundation stiffness matrix and can be included as boundary conditions for the analysis of super - structures. This matrix is obtained by successively applying unit vertical displacement, unit horizontal displacement and unit rotation to the pile cap and calculating the system of vertical loads, horizontal loads and moments required to equilibrate the system of stresses developed. The flexibility matrix is obtained by inverting equation (19)

$$\begin{Bmatrix} We^{i\omega t} \\ Ve^{i\omega t} \\ \theta e^{i\omega t} \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}^{-1} \begin{Bmatrix} Pe^{i\omega t} \\ He^{i\omega t} \\ Me^{i\omega t} \end{Bmatrix} \quad (20)$$

Equation (20) may be solved for any combination of (P, H, M).

5. Computer Program

A computer program (PDYNA) has been written that incorporates the analysis described above. This program can be used to solve both single pile and pile group problems. Symmetry in a pile group is taken into account to reduce the computational effort (Figure 5). The program can analyze raked piles and piles projected above the ground level. If the pile cap is in contact with the ground, then the interaction between the pile cap and piles has to be taken into account, and this facility is also present in the program.

6. Results and Discussion

Though the analysis described here is for pile groups under periodic excitation, the results converge to static solutions for small frequencies ('ω' should not be made less than 0.01 for stability in numerical integration). Static results obtained from single pile analysis have been compared with poulos [5] and Davis [6]. Dynamic results from laterally loaded single pile analysis have been compared with those by Kuhlemeyer [1].

Agreement with static results of Davis (Figure 6) is good because he used the corresponding static kernel function G(s, ξ) for

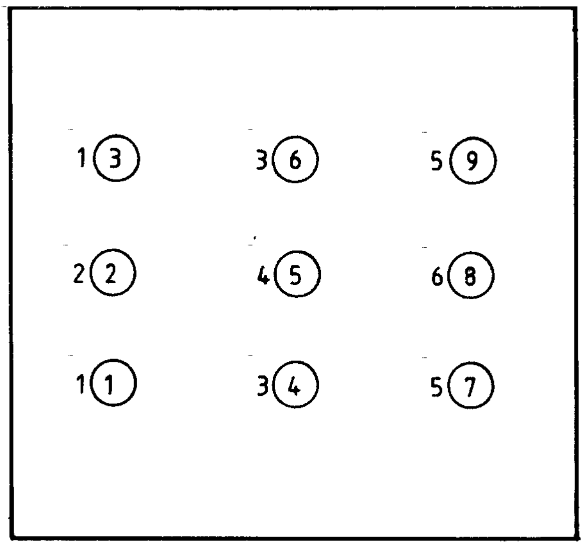

Figure 5. Numbering of piles in a group utilising geometric symmetry.

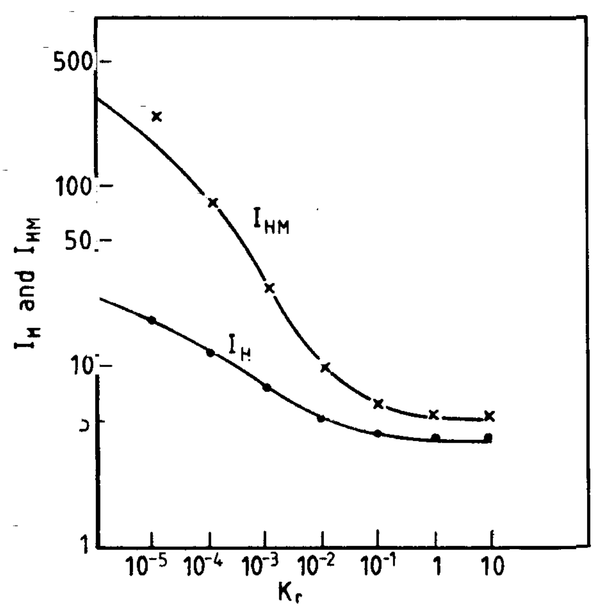

Figure 6. Comparisons with Davis (Ref. 3)

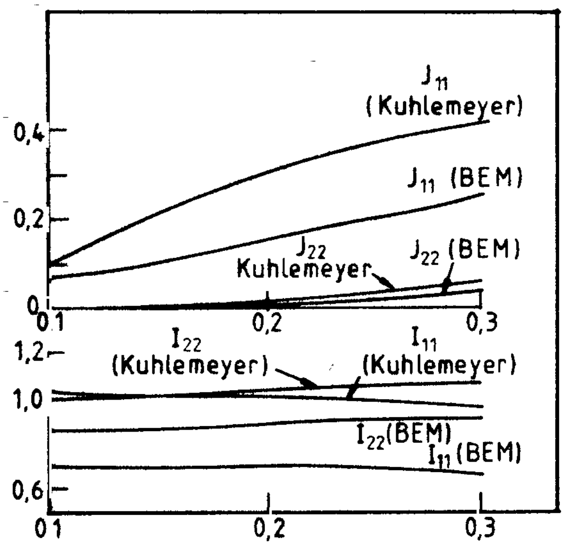

Figure 7 Dynamic magnification factors comparisons with kuhlemeyer [Ref 1]

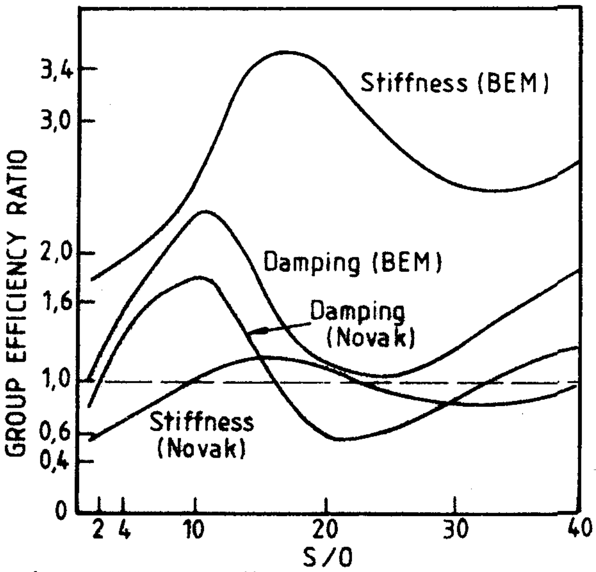

Figure 8 Group efficiency ratio for two floating piles in homogeneous soil. [Ref 7]

his analysis. Next, dynamic magnification factors have been compared with those obtained by Kulhemeyer (Figure 7); there is some difference because Kuhlemeyer did not use a kernel function for modelling the half-space. Novak [7] has published results of vertical vibration of pile groups in homogeneous and non-homogeneous soil. His results of group efficiency ratio for two floating piles in homogeneous soil have been compared with present analysis (Figure 8). There is a fair deal of agreement between the damping part of the group efficiency ratio obtained by the two methods. It seems however that Novak's method underestimates the stiffness part of the group efficiency ratio. This may be attributed to the approximate soil model used by Novak which may underestimate the contribution by the soil block trapped between the two piles and the rigid pile cap. Interestingly, curves suggested by Novak and those obtained by the present method follow the same pattern for both stiffness and damping terms. The present analysis cannot handle layered soil structure. However, the analysis is being extended to incorporate this capability by using the kernel function $G(x, \xi)$ for multilayered soil as suggested by Kausel [8].

APPENDIXA

$$A_1 = \frac{p}{4EI\eta^4T_o}[T_3(\sin\eta a_B - \sin\eta a_A) - T_3(\sinh\eta a_B - \sinh\eta a_A) + (T_4-1)(\cosh\eta a_B - \cosh\eta a_A) + (T_1+1)\cos\eta\ a_B - \cos\eta a_A)]$$

$$A_2 = \frac{-p}{4EI\eta^4T_o}[(T_4-1)(\sin\eta a_B - \sin\eta a_A) - (T_1+1)\sinh\eta a_B - \sinh\eta a_A) + T_2(\cosh\eta a_B - \cosh\eta a_A) + T_2(\cos\eta a_B - \cos\eta a_A)]$$

$$B_1 = = \frac{p}{4EI\eta^4T_o}[T_3(\sin\eta a_B - \sin\eta a_A) - T_3(\sinh\eta a_B - \sinh\eta a_A) + (T_4-1)(\cosh\eta a_B - \cosh\eta a_A) + (T_4-1)(\cos\eta a_B - \cos\eta a_A)]$$

$$B_2 = \frac{-p}{4EI\eta^4 T_o}[(T_1+1)(\sin\eta\, a_B - \sin\eta\, a_A)$$

$$-(T_1+1)(\sinh\eta\, a_B - \sinh\eta\, a_A)$$

$$+T_2(\cosh\eta\, a_B - \cosh\eta\, a_A)$$

$$+T_2(\cos\eta\, a_B - \cos\eta\, a_A)]$$

$$A_3 = -A_1$$

$$A_4 = -A_2$$

$$B_3 = B_1T_1 + B_2T_3$$

$$B_4 = -(B_1T_2 + B_2T_4)$$

where $\eta = [\frac{m\omega^2}{EI}]^{1/4}$

ℓ = length of the cantilever

m = mass per unit weight

E = Young's modulus

I = second moment of area

$T_o = (1 + \cos\eta\ell \cosh\eta\ell)$

$T_1 = (\sin\eta\ell \sinh\eta\ell + \cos\eta\ell \cosh\eta\ell)$

$T_2 = (\cos\eta\ell \sinh\eta\ell + \cosh\eta\ell \sin\eta\ell)$

$T_3 = (\cosh\eta\ell \sin\eta\ell - \cos\eta\ell \sinh\eta\ell)$

$T_4 = (\sin\eta\ell \sinh\eta\ell - \cos\eta\ell \cosh\eta\ell)$

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