IJE TRANSACTIONS A: Basics Vol. 37 No. 07, (July 2024) 1369-1383

International Journal of Engineering

Journal Homepage: www.ije.ir

Topology and Thickness Optimization of Concrete Thin Shell Structures Based on Weight, Deflection, and Strain Energy

M. Vatandoost[a], M. Golabchi[a], A. Ekhlassi*[b], M. Rahbar[b]

[a] *Department of Architectural Technology, Faculty of Architecture, College of Fine Arts, University of Tehran, Tehran, Iran*
[b] *School of Architecture and Environmental Design, Iran University of Science and Technology, Tehran, Iran*

P A P E R   I N F O

Paper history:
Received 30 October 2023
Received in revised form 21 December 2023
Accepted 30 December 2023

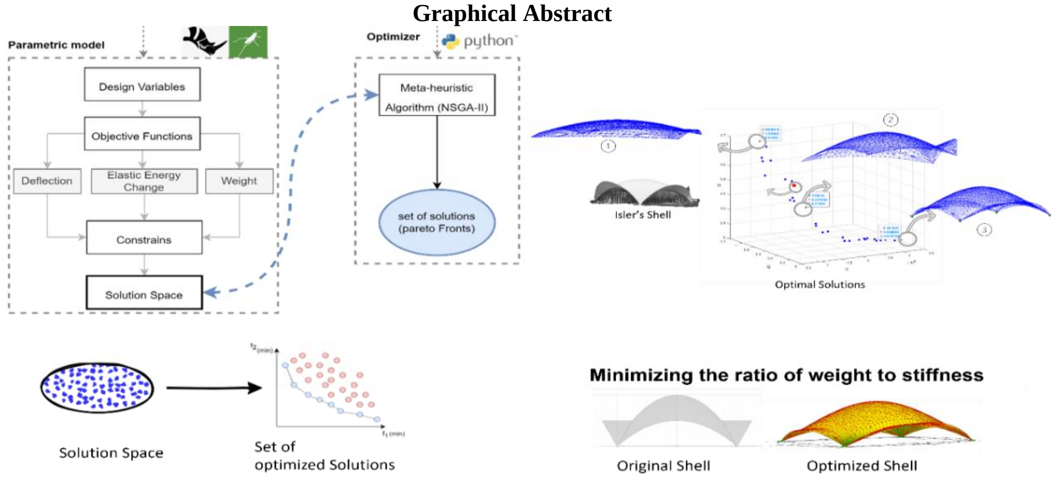
Keywords:
Concrete Shell Structures
Structural Optimization
Shape Optimization
Topology Optimization
Multi-objective Optimization
Non-dominated Sorting

A B S T R A C T

This study presents the optimized shape and thickness of thin continuous concrete shell structures, minimizing their weight, deflection, and elastic energy change while meeting the performance requirements and minimizing material usage. Unlike previous studies that focused on single-objective optimization, this research focuses on multi-objective optimization (MOO) by considering three objective functions. This combination of objective functions has not been reflected in previous research, distinguishing this study. The computational design workflow incorporates a parametric model, multiple components for measuring objective functions in the grasshopper of Rhino, and a metaheuristic algorithm, the non-dominated sorting multi-objective genetic algorithm (NSGA-II), as the search tool, which was coded in Python. This workflow allows us to perform form-finding and optimization simultaneously. To demonstrate the effectiveness of this metaheuristic algorithm in structural optimization, we applied it in a case study of a well-known shell designed using the physical prototyping hanging model technique. Interpretations of samples of optimized results indicate that although solution 1 weighs nearly the same as solution 2, it has less deflection and strain energy. Solution 3, with a three-fold mass, has significantly less deflection and strain energy than solution 1 and solution 2, with deflection reductions of over 50 and 17%, respectively. Solutions 3 and 4 show better deflection and strain energy performance. Furthermore, a comparison of the MOO results with the Isler shell revealed that this method found a solution with less weight and deflection while being stiffer, confirming its practicality. The study found that MOO is a reliable method for form-finding and optimization, generating accurate and reasonable results.

***doi**: 10.5829/ije.2024.37.07a.15*

Graphical Abstract



*Corresponding Author Email: *ekhlassi@iust.ac.ir* (A. Ekhlassi)

Please cite this article as: M. Vatandoost, M. Golabchi, A. Ekhlassi, M. Rahbar. Topology and Thickness Optimization of Concrete Thin Shell Structures Based on Weight, Deflection, and Strain Energy. International Journal of Engineering, Transactions A*:* Basics. 2024;37(07):1369-83.

1. INTRODUCTION

1. 1. Problem Statement

On the one hand, concrete's ubiquitous availability, high strength, durability, and versatility make it an essential and irreplaceable building material, and due to its wide accessibility of necessary raw materials; it is the most often utilized artificially made material (1-3). In addition, concrete shell structures are among the most efficient structural systems (4, 5). On the other hand, Carbon emissions and global warming threaten our environment, according to reports of World Green Building Council (6), and Global status report for buildings and construction (7), and as indicated by Zhong et al. (8) the building sector is responsible for more than 35% of global energy consumption and more than 40% of carbon emissions (9, 10). In response, architects and researchers constantly revise their ways of designing and building structures. If we find ways to design and build our structures more efficiently, less material will be used, and we can reduce and control the environmental impact (11-13).

Since the emergence of computer-aided design in architectural fields, the appearance of complex free forms has materialized. Although computational design has paved the way for the design of complex forms, these structures are required to be designed in their optimum state. Historically, shell structures were form-founded and designed by physical prototyping and hanging models. Conversely, the optimal design for complex free-form thin shell structures requires advanced methods. Structural optimization plays a crucial role in achieving optimal design solutions that meet performance requirements while minimizing material usage and overall weight. In recent years, metaheuristic algorithms have gained prominence as effective tools for solving complex optimization problems. This paper focuses on the application of a metaheuristic algorithm, specifically the non-dominated sorting multi-objective, multi-criteria genetic algorithm (MOGA), in the context of structural optimization. Hence, in this study, we provide shape and thickness optimization of thin concrete shell structures by employing metaheuristic algorithms and by minimizing the shell structure's weight, deflection, and elastic energy change.

1. 2. Related Studies

Historically, shell structures had been designed through physical prototyping, including hanging chains and hanging models (5, 14). Through the introduction of computational methods and advances in Computer-Aided Design (CAD) methods, researchers have developed advanced design methods; one such advanced method is utilizing metaheuristic algorithms in the design process. Metaheuristic algorithms are able to find accurate solutions for complex design problems (15, 16). A vast and diverse solution can be generated using metaheuristic algorithms, known as 'solution space.' These algorithms are able to find the best (fittest) solution(s) considering the defined criteria without evaluating each solution in the solution space (17, 18). Based on knowledge of the domain and simulation of Natural or physical laws, these algorithms can find an accurate answer to a complex problem.

Shell structure optimization is reflected in several previous research that emphasizes this topic's importance. We have classified previous studies based on the objective function of the optimization in two major classes; the objective functions of the related studies are summarized in Table 1.

Pugnale et al. (19) utilized a genetic algorithm to optimize a shell structure, while the objective function was deflection in the Z direction. Additionally, Santhosh et al. (20) optimized a Gridshell structure by genetic algorithm (GA), while objective functions were shell vertical displacement, and they studied different geometry patterns for grids. Besides, the total weight of the structure was considered the optimization criteria in research by Basso et al. (21). A comparable study with the same objective function but a different optimization technique was conducted by Baghdadi et al. (22). Moreover, in the study by Ansola et al. (23) the strain energy was considered as the optimization criteria. Furthermore, In a study by Kimura and Ohmori (24), the topology and thickness distribution of concrete shells were studied by considering strain energy as the objective function. Also, Yang et al. (25) utilized a particle swarm optimization algorithm to minimize the strain energy in a free-form shell. In addition, Tomas and Marti (26) optimized the design of a shell by considering the total structure weight as a single objective optimization, repeated that for stress level, and studied it based on strain energy. Additionally, strain energy change in a Gridshell was considered by Feng and Ge (27) as the objective for optimization. Moreover, in the study by Hassani et al. (28), the objective was structure weight. The total weight was minimized in a similar study by Richardson et al. (29). In addition, maximization of the stiffness was the optimization criteria in the study by Shimoda and Liu (30). Besides, Kaveh et al. (31) provided cross-section optimization for Grid shells, and Tomei et al. (32) developed the method for grid shells constructed by pre-stressed rods. Another example is the research by Ansola et al. (33) which optimize thickness and topology of the shell structures by considering displacement as the objective. Furthermore, Gythiel and Schevenels (34) formulated the optimization of a single layer reticulated shell by minimizing the elements cross section (total mass) of the structure as the objective; In this study, size, shape and topology of the structure were variables while considering multi-criteria in the design. Teimouri and Asgari (35) utilized BESO for topology optimization while maximizing stiffness of the structure. In one of the recent published research, Zhang et al. (36) proposed collapse-resistant optimization for a single-layer grid shell.

Emami et al. (37) optimized a perforated concrete shell concerning daylight and energy performance; research with similar objectives was conducted by Liuti et al. (38) and Emami (39), where the objectives were structure and natural light. In another study, Pugnale (40) considered structure and acoustic as the objective function for optimization. Turrin et al. (41) also provided optimization based on daylight and solar heat gain. Furthermore, In the study by Puppa and Trautz (42), optimization criteria were buckling load, strain energy, and sensitivity to imperfection. Optimization based on the strain energy and total weight of the members is reflected in a study by Nagata and Honma (43) and Wang et al. (44). Henriksson et al. (45) presented the multi-objective optimization considering total mass and deflection. Moreover, in a recognized building, 'NESTHiLo' researchers from the block research group at the ETH Zurich (46, 47) designed and built a thin concrete shell as a roof and optimized it based on the area of glazing, the formwork deviation, and elastic energy change. In another study, Zhao et al. (48) found the optimal design for a Gridshell based on the length and cross-section of elements. Vargas et al. (49) employed The differential evolution (DE) algorithm, and considered penalty function in the workflow to handle constrains in the optimization of different structures, while the objective functions were weight and displacement. Mirra and Pugnale (50) employed a multi-objective genetic algorithm to minimize displacement and shell footprint from the target shape while maximizing the shell height. Nishei and Fujita (51) considered strain energy in combination with the collapse load factor. In Table 1, previous studies are classified based on the objective functions for optimizing the thin shell structures.

TABLE 1. Classifying research based on objective functions

	Objective function	Reference		Objective functions	Reference
Single objective optimization	deflection	Pugnale et al.(19), Gokul et al. (20) Ansola et al. (33)	Multi-objective optimization	f_1: daylight f_2: Energy performance	Emami et al. (37)
	strain energy	Kimura and Ohmori. (24) Feng and Ge (27), Yang et al. (25), Wang (52) Ansola et al. (23)		f_1: structure f_2: acoustic (SPL)	Pugnale (40)
	Material usage (Weight of the structure)	Basso et al. (21), Hassani et al. (28), Baghdadi et al. (22), Kostura et al. (53) Richardson et al. (29)		f_1: daylight f_2: Solar heat gain	Turrin et al. (41)
	Stiffness	Shimoda and Liu (30)		f_1: structure f_2: Natural light	Liuti et al. (38), Emami (39)
	stress level weight, strain energy, (separately)	Tomas and Marti (26)		f_1: strain Energy f_2: surface curvature	Jiang (54)
	Cross-section	Tomei et al. (32), Kaveh et al. (31) Gythiel and Schevenels (34)		f_1: strain energy f_2: Total weight of the members	Nagata and Honma (43), Wang et al. (44)
				f_1: total mass f_2: deflection	Henriksson et al. (45)
				f_1: strain energy f_2: collapse load factor	Nishei and Fujita (51)
				f_1: elements length f_2: cross-section	Zhao et al. (48),

f_1: strain Energy f_2: Geometric index f_3: Economic index	Cao et al. (55)
f_1: displacement f_2: shell height f_3: Footprint deviation	Mirra and Pugnale (50)
f_1: Imperfection sensitivity f_2: buckling load f_3: strain energy	Pappu et al. (42)
f_1: Elastic energy change f_2: Glazing surface area f_3: and formwork deviation	Veenendaal (46), Veenendaal et al. (47)

1. 3. Research Objectives Based on provided literature review, most of these studies considered a single criterion for optimization, a single-objective optimization; however, in some research, two objectives were considered as the multi-objective optimization problem. Nevertheless, considering three or more objective functions to optimize the shell structures is rare. The goal of this research and our contribution is summarized below.

- Finding the optimal design solution for the continuous concrete shell based on structural performance.
- In the design of concrete shell structures, despite most previous research mentioned in the literature review, which only considered one objective, single-objective optimization, or two objectives for the optimization, in this research, we intended to perform multi-objective optimization by considering three objective functions.
- This combination of the three considered objective functions has not been reflected in previous research, distinguishing this study.
- Multi-objective optimization by considering three objective functions: the total weight of the structure, the maximum deflection, and elastic energy change were competing toward the final optimal solution.

Apply the optimization workflow in a case study to demonstrate the effectiveness of the metaheuristic algorithm (NSGA II), and find the optimal solutions compared to the hanging model simulation.

2. METHOD

To conduct a multi-objective optimization by employing metaheuristic algorithms, we need to integrate three parts: a parametric model, functions to measure objectives, and a solver (56, 57). In this study, the parametric model is created in the grasshopper of the Rhino 3D. By altering variables, this parametric model generates many candidate solutions known as ‘solution space.' Moreover, multi-components are added to the model to measure and record each solution rank related to the considered objective functions. In this study, the Karamba 3D plug-in (58) is used to calculate the structural performance of the parametric shell. Lastly, a solver, the MOGA (the metaheuristic algorithm), is employed in the workflow by a series of custom 'Python' programing language codes, which can generate multiple candidate solutions and rank them, and through mutation, crossover and simulation of 'natural selection' law, find the best (fittest) solution(s) (59). The concept of utilizing a metaheuristic algorithm as a search method is depicted in Figure 1. Also, the flowchart of the multi-objective genetic algorithm (the non-dominated sorting genetic algorithm, NSGA-II) is depicted in Figure 1.

2. 1. Optimization Formulation Optimization in engineering is formulated by Mirjalili and Dong (60), Rao (61) as follows:

$$Find\ X = \begin{Bmatrix} x_1 \\ x_2 \\ \vdots \\ x_m \end{Bmatrix}, which\ minimizes\ F(x), \tag{1}$$

X is an array of variables, $X = \{x_1, x_2, \cdots, x_m\}^T$, and in the multi-objective optimization, $F(X)$ is the combination of multiple functions which we intended to minimize simultaneously; therefore, we have:

$$F(X) = \{f_1, f_2, \cdots, f_n\} \tag{2}$$

while satisfying inequality and equality constraints:

$$g_r(x) \leq 0 \tag{3}$$

$$h_r(x) = 0 \text{r = 1 to k} \tag{4}$$

Figure 2 depicts the concept of utilizing metaheuristic algorithms as a search tool.

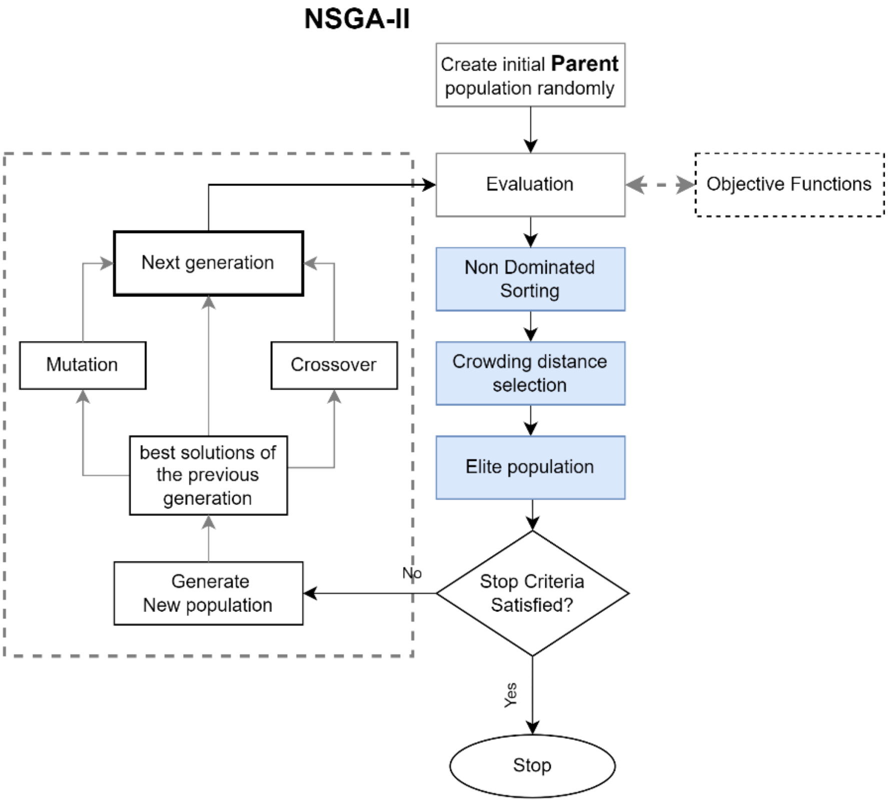


Figure 1. The flowchart of the MOGA-NSGA-II algorithm

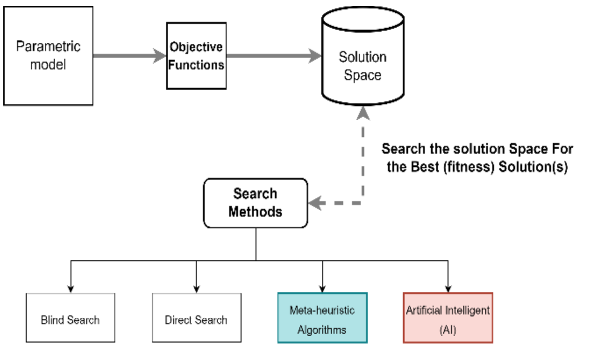


Figure 2. Utilizing metaheuristic algorithms as the search method to find the fittest solution

In multi-objective optimization, we intended to minimize or maximize competing objective functions and find a combination of variables that minimize this objective function, which is a complex and multifaceted problem (62). This is because a set of variables that minimize or maximize one objective function (for instance, f_1) might result in an entirely low-rank solution while considering another objective (for instance, f_2, f_3) (49).

2. 2. Objective Functions In this study, the structure's total weight, the shell's maximum deflection, and the elastic energy change are three competing objectives employed as the main design criteria, summarized in Table 2.

The total weight of the structure is related to the shell surface and thickness, and the deflection is defined as the maximum displacement of the shell in the Z-direction, while the shell is under the load combination of the self-weight and a 1KN equal distribution of the live load. The Strain Energy (Elastic energy change) of the structure is defined as the potential energy that is accumulated within a structural element due to its elastic deformation (63). The strain energy formula is given below:

$$U = \frac{\sigma^2}{2E \times V} \quad (5)$$

where, σ = stress, E = Young's modulus, V = volume of the body (Structure).

Therefore, by measuring Starin Energy, we can compare different structures; the structure with the lowest elastic energy change is stiffer.

These three objectives are competing toward the best solution. At the same time, a shell with a more significant raise-to-span ratio will be stiffer with less deflection and less elastic energy but will have more surface, and more material will be used and, therefore, will weigh more.

TABLE 2. Three objective functions of this study

	Objectives	Unit
f_1	Total weight	Kg
f_2	Deflection	mm
f_3	Strain Energy (Elastic energy change)	KNm

2. 3. MOGA, NSGA-II Multi-objective genetic algorithm, MOGA, is one of the fast and reliable metaheuristic algorithms (64); that has been inspired by nature. The GA is based on the theory of 'natural selection' of Charles Darwin, and Goldberg formalized it to be used in engineering optimization (65). The Non-dominated sorting genetic algorithm (NSGA-II) is the subset of GA, which enables us to find a set of optimal solutions known as the 'Pareto Frontier' by mutation, crossover, and selection functions, based on the defined objective function (criteria).

2. 3. 1. Non-domination Sorting Non-dominated sorting is a technique that selects and stores the solutions not dominated by other solutions in the solution space. Domination is defined as Equations 5 and 6 (66).

Solution 1 dominates Solution 2 if and only if, for all the objective functions (i), the value of the considered objective function for solution 1 is equal to or less than solution 2, or there is a solution in which the value of the considered objective function for that solution is equal to or less than solution 2. Additionally, to select diverse solutions from the Pareto front, a fitness value named 'crowding distance' is defined and assigned to each solution in the Pareto Frontier, which relates the solution density to ranking, which lets the diverse solution be chosen. The solution with the heist crowding distance means the solutions far from each other will be chosen in the Pareto Frontier (67), depicted in Figure 3.

Solution1 dominates, solution 2 if and only if:				
All	For each objective, (i)	$\forall_i: \; f_i^{solution1} \leq f_i^{solution2}$	(Universal quantification)	(6)
Any	There is a solution that:	$\exists_i: \; f_i^{solution1} < f_i^{solution2}$	(Existential quantification)	(7)

Crowding distance is calculated by Equation 7. For boundary solutions, the solutions with the heists and lowest quantities, infinite distance ($d_s = \infty$), are assigned to make sure this is involved in the Pareto frontier (68).

$$d_s = \sum_{i=1}^{l} \frac{f_i(s+1) - f_i(s-1)}{f_i^{max} - f_i^{min}} \tag{8}$$

S = 1 to k

S: Number of solutions in the Pareto frontier

Therefore, NSGA-II, in each cycle, ranks and sorts the solutions in a set called the 'Pareto Frontier.' The solutions in the Pareto Frontier are the solutions that are superior to other solutions but have no advantages over each other, and every solution in the 'Pareto Frontier' founded by NSGA-II can be considered as the final design solution (69).

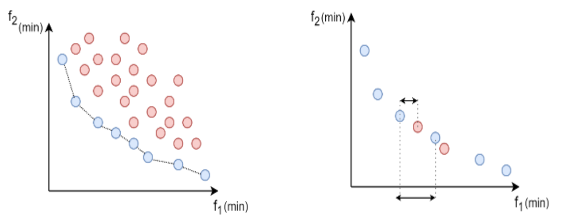


Figure 3. The non-dominated sorting arranges solutions in hierarchies (fronts); the solutions in the same Front are superior to other solutions while not dominated by each other (left). The Crowding distance fitness value enables to elimination of similar solutions and having a more diverse set of solutions in the Pareto front (right)

2. 4. Unified Workflow As described before, the workflow of MOO consists of three parts: a parametric model, objective functions calculator, and a metaheuristic algorithm. This integration workflow is shown in Figure 4.

2. 4. 1. Stopping Criteria In contrast to the Exhaustive search method, which considers all the solutions known as the 'Brut force' method, the metaheuristic algorithms will find the fitness solution without measuring every solution, making it computationally faster. While utilizing metaheuristic algorithms, either it will converge to a satisfactory solution or infinite looping. To avoid infinite looping, a stopping criterion for the search operation must be defined. We have defined the stopping criteria when either of these four occurs (Figure 5): a) an acceptable number of iterations reached, b) algorithm convenient cycles, c) slow or no progress encountered, and d) a solution does not exist.

3. APPLICATION

To show the effectiveness of the MOGA algorithm in structural optimization, we have employed it in a case study. In this case study, we have chosen to optimize one

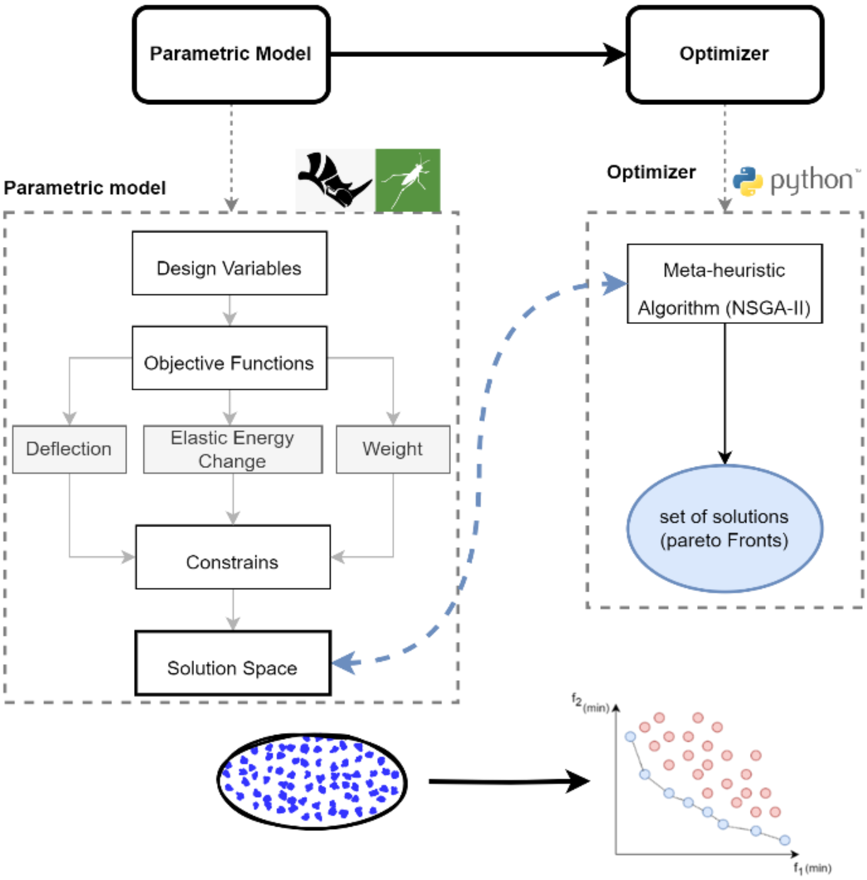


Figure 4. The integrated workflow of multi-objective optimization

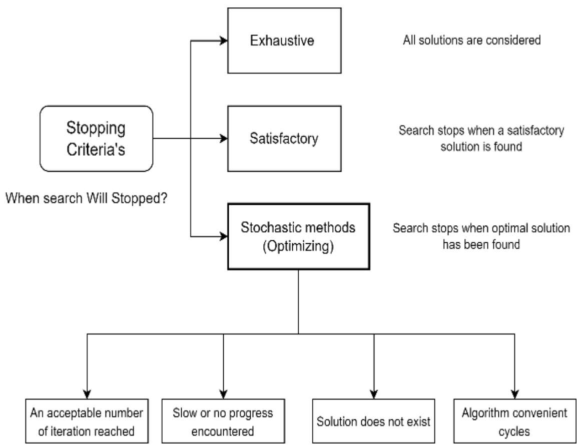


Figure 5. Stopping criteria in the operation of the metaheuristic algorithm

of the well-known previously built concrete thin shell structures. As described before, these shell structures were form-funded and designed by physical modeling and hanging models. We intended to optimize it based on the mentioned objective functions and compare the results.

3. 1. Intro to the Case Study The 'Wyss Garden,' designed and built by Isler in 1962, is selected as the case study to be optimized. The details of the base model are extracted from literature (70) (see Figure 6). According to Chilton and Isler (2), this was the first Isler's 'free-form' shell with 650 m^2 of surfaces generated by circular curves. At that time, CAD modeling and optimization techniques were unavailable, and this shell was designed based on the hanging model. Therefore, we will compare the optimized shell results with Isler's built shell.

Throughout the computational design process, various potential solutions were developed. For each of these solutions, the objective function(s) are calculated, and the resulting data for each solution should be saved. Due to the necessity for repetition across various potential solutions. This procedure is time-consuming and computationally costly. Therefore, it is crucial to establish a parametric model that is both simple and accurate in the first stages of the design (71).

This study employs networks of Non-uniform rational basis splines (NURBS), a widely used method in the field for representing surfaces and curves, to mathematically represent and model shells (72). It provides exceptional adaptability and accuracy when it comes to manipulating modeled and analytical shapes that are defined by standard mathematical formulas.

By considering Isler's 'Wyss Garden' as the base, we have defined the parametric model with a set of eight boundary NURB curves, based on Isler's 'Wyss Garden'. By adjusting the position of the control points on these curves, we can create different topologies. This modification allows for 16 variables to be manipulated, resulting in various shell shapes. Figure 7 illustrates the specific information. Moreover, Table 3 establishes the limits for the variables. In this case study, there are eight control points. The problem variables are the x or y positions of these points, together with the z position that determines the shell topology. In Figure 7, the position of 'n1' can range from zero to 5.00 meters in the y-direction and from zero to 10.00 meters in the Z-direction.

Variables

To have a parametric model and be able to generate

NSGA-II hyper parameters

The algorithm hyper-parameters are summarized in Table 4.

The main material for the shell is lightweight concrete C50/60, and the material properties are summarized in Table 5. Concrete Design Properties is provided according to EN1992-1-1 (γ_c = 1.50, f_{yk} = 500 MPa)[1].

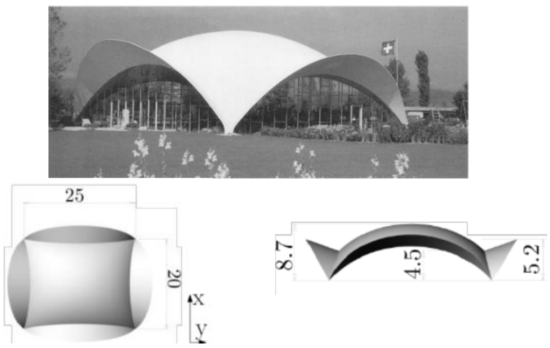


Figure 6. The 'Wyss Garden' by Isler, source: (2, 70, 73)

TABLE 3. Variable bounds

Variable	Lower bound (m)	Upper Bound (m)
x_0, x_2, x_4, x_6	0.	5.00
x_1, x_3, x_5, x_7	-7.00	0.
$z_0, z_1, ..., z_7$	0.	10.00
t	0.08	0.20

[1] f_{yk}: Steel characteristic yield strength; γ_c: Concrete partial material safety factor

TABLE 4. NSGA-II hyperparameters in the case study

NSGA-II hyper parameters	
Number of populations	50
Number of generations	10
Number of Genes	4
Mutation rate %	0.2
Crossover rate %	0.8
Number of Objectives	3

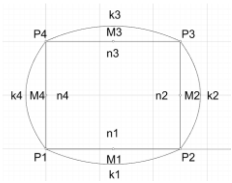
TABLE 5. Lightweight Concrete, Material Properties

Compressive strength (f_c)	50 MPa (~500 kg/cm^2)
Tensile strength	4.07 MPa
Modulus of Elasticity (E)	37278 MPa
Density	2400 kg/m3 (150 lb/ft3)
Poisson's ratio	0.2
Shear strength	6 MPa
Thickness (t)	8 - 20 cm

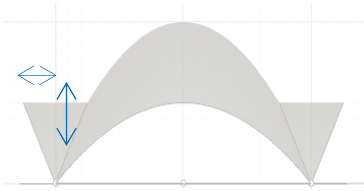
4. RESULTS AND DISCUSSIONS

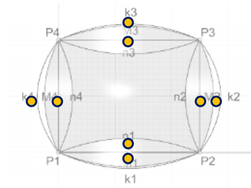
Thin shell concrete structures can adequately withstand forces in compression but are sensitive to tension forces. The hanging model method is based on the form resulting by using fabric and other tensile materials under the self-weight with the defined boundary conditions (loading and supports); the fabric will fall in the equilibrium position in pure tension. By reversing the resulted form, we will have a structured act in pure compression (5). Historically, shell structures were designed based on hanging models. Thus, first, we generate the result by computer simulation of the hanging model using Kangaroo physics (74) in the grasshopper of Rhino to provide a general idea and a good guess about the acceptable result based on the structural performance . In addition, in our parametric model, by setting out the parameters from the built model, we provided the structural performance of the 'Wyss Garden' for Evaluation and compared the results of the Isler shell with the MOO results; these results are summarized in Figure 10, and Table 6.

The result by MOO is provided in Figure 8; the parallel plot of all considered solutions in the optimization search is depicted. Each vertical coordinate represents a variable (17 in this case study, refer to Table 3) and corresponding objective function values (three in this case study, refer to Table 2). In this case study, 498 solutions were considered



a) base geometry


b) Front view


c) plan view

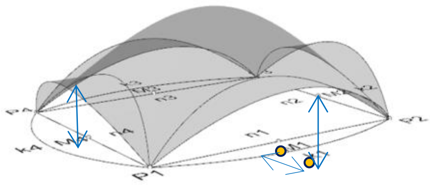


Figure 7. The defined parametric model in this Case study

to find the optimized solutions. The solutions in the Pareto Frontier that are non-dominated and have superiority over other solutions are shown in Figure 9. Each blue dot represents a candidate solution that can be chosen as the final solution.

In addition, in Figure 9, the objective functions were compared two by two; The solutions distribution in the Pareto frontier indicates that f_1 (total weight) and f_2 (deflection) are competing toward the best solution when f_1 is reduced, f_2 has increased and vice versa. Similarly, f_2 (deflection) and f_3 (strain energy) have an equivalent relation and are in conflict toward finding fitness solutions. However, based on the provided charts, f_2 and f_3 have a linear relation; when f_2 reduces, f_3 reduces based

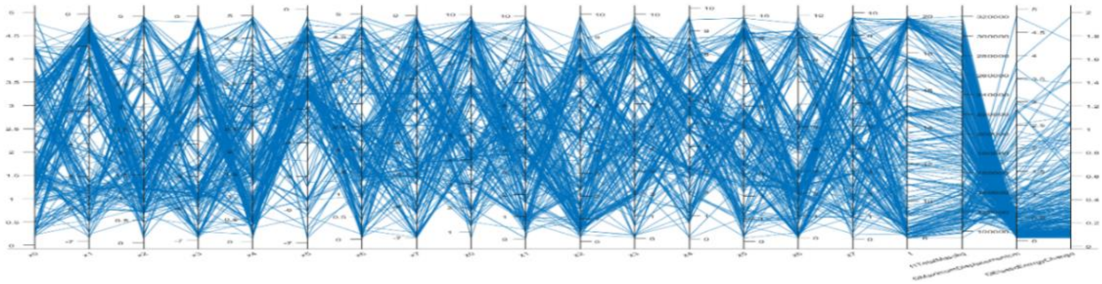
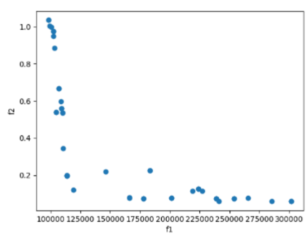
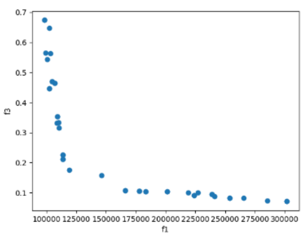
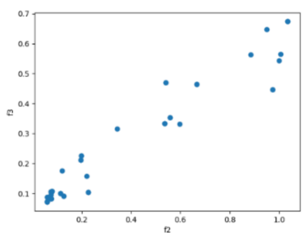
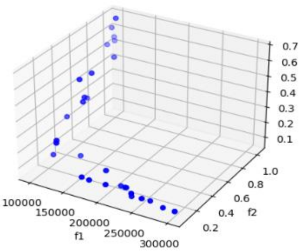


Figure 8. Parallel plot of the solution space; 498 solutions were considered in this case study


a) scatter plot, f_1-f_2


b) scatter plot, f_1-f_3


c) scatter plot, f_2-f_3


d) 3D-scatter plot, f_1-f_2-f_3

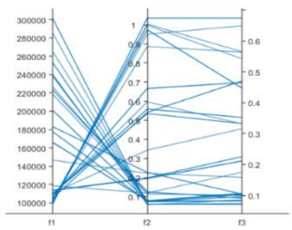

e) Parallel Plot of the objective functions

Figure 9. Pareto Frontier of the Case study I, f1: Total weight, f2: Deflection, f3: Elastic Energy Change

on similarly. In Addition, the Parallel plot is depicted in Figure 9 indicates how these three objectives are competing.

In a similar study conducted by Turrin et al. (41), they utilized genetic algorithm to find the optimal design solution based on structural performance. In their research, the design alternative could be generated by GA. In another study by Sassone and Pugnale (75), the maximum displacement was the objective function for structural. Dispite the above mentioned related studies, in this research we have provided multi-objective optimization of the ligthweigth continious thin shell structures, by combining three objective function, which is scarse in the literature. The provided workflow, enables us to find diverse set of candidate solutions based on the defined criteria. Additionally, Veenendaal (46), Veenendaal et al. (47), in their built research project, which is a roof of a building, provided the Pareto Frontier of a thin shell based on three objective functions, Elastic energy change, glazing surface area and formwork deviation, which validates this method for finding the optimal design solution. The design alternative was chosen based on the head clearance and architectural requirments, and developed.

Furthermore, the results of this study which is depicted in Figures 10 and 11 and Table 6 consist of the resulting shell based on hanging model simulation and the Isler shell remodeling, along with five solutions from the Pareto frontier as samples. Either of these five solutions or any other solutions from the Pareto Frontier (Figure 12) can be chosen by the Decision Maker (architect) as the final solution for further design development. However, a reliable scientific method for this selection is required, and it must be developed.

By comparing two samples of MOO's optimized results with the Isler shell, we can conclude: the total mass of solution 1 is less than Isler shell. At the same time, solution 2 weighs nearly the same as Isler, but the deflection and strain energy of solution 2 is reduced compared to Isler.

Furthermore, based on the provided results in Figure 10 and Table 6, although the total mass of the structure in the solution 3 is about three times the mass of the solutions 1 and 2, the deflection and strain energy of this solution is much less than these two candidate solutions; the deflection in comparison to solutions 1 and 2 is reduced more than 50 and 17%, respectively. Besides, solutions 3 and 4 have better deflection and strain energy performance than solutions1 and 2.

Based on the provided results, and observing that there are some candidate solutions among the best candidate solution in Pareto Frontier that are similar to the Isler model, we can conclude that physical prototyping and hanging model methods are valid methods for form-finding of thin shell structures, and they can give the architecture a good starting point and

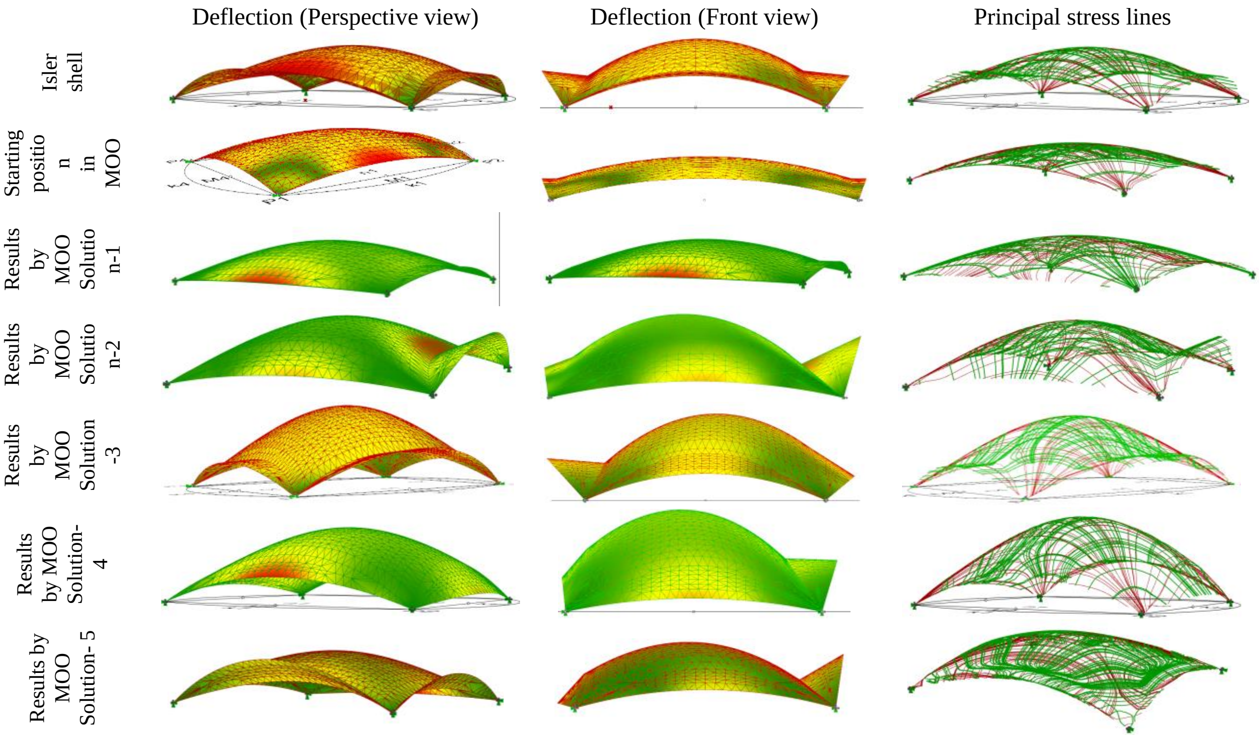


Figure 10. The results of the case study, selected sample of the optimized solutions are depicted

Figure 11. Hanging model simulation

TABLE 6. Comparison of the objective functions

	Objective functions	Isler shell	Starting position	Optimized Solutions				
				Solution-1	Solution-2	Solution-3	Solution-4	Solution-5
t	Thickness (cm)	8	8	8	8	19	17	8
f_1	Total mass (kg)	110022.8248	93214.154691	98080.815803	110570.452284	301837.474533	253604.394806	109828.6
f_2	Deflection (cm)	0.5357	0.711381	1.035417	0.344302	0.058893	0.074598	0.535816
f_3	Strain energy (kNm)	0.39055	0.643162	0.67408	0.31626	0.070759	0.082373	0.333121

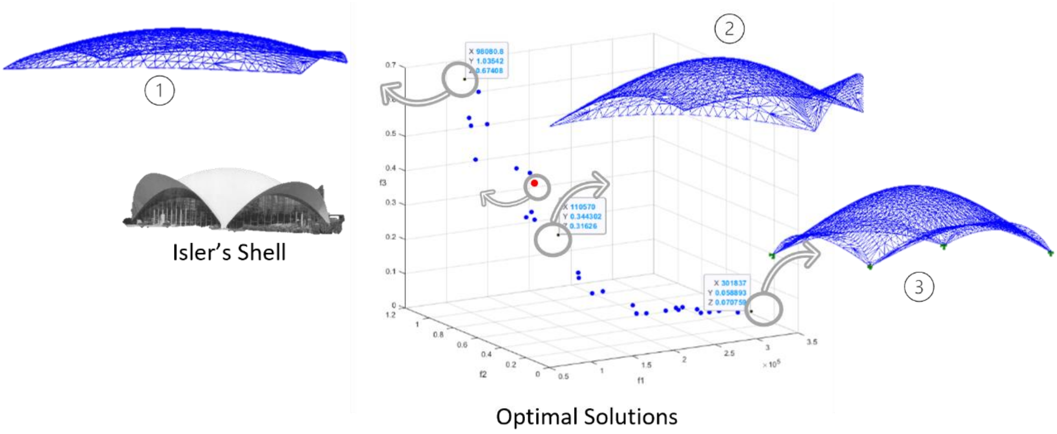


Figure 12. Optimized solution in Pareto Frontier (blue dots) and Isler model (red dot)

basic understanding of the load paths; however, these methods are not able to consider other criteria in the design, such as acoustic, energy, environmental and other criteria.

It is worth note that however, if we consider other criteria in design, such as the structure's acoustic, energy, and environmental performance, the result provided by the hanging method will be different from the fittest solution.

4. CONCLUSION

This study presented the optimal design solution based on structural performance by minimizing three objective functions. The topology and thickness of the thin, lightweight shell structure were optimized by minimizing the structure's total weight, deflection, and strain energy.

We have utilized the non-dominated sorting multi-objective, multi-criteria genetic algorithm in the computational design of thin shell structures. In our workflow, we combined a parametric model, multiple components for measuring defined objectives, and the metaheuristic algorithm (NSGA-II) which was coded in Python. Utilizing metaheuristic algorithms in the design will provide diverse optimal solutions instead of one single solution. To show the effectiveness of this metaheuristic algorithm in structural optimization, we have employed it in a case study for topology and thickness optimization of a shell, which was form-founded based on the hanging model method. By utilizing MOGA, we were able to find a solution with less weight and less deflection while being stiffer, which confirms the practicality of this method. However, a comparison of the results of the Isler concrete thin shell as the base model and the optimized model reveals that since our objective function was the structural performance of the shell, there are a set of diverse solutions in the provided Pareto front that are similar to the Isler solution (for instance, see solutions 2 and 5 in Table 5 and Figure 12). This result indicates that MOO is a reliable method for form-finding and optimization and is able to generate accurate and good results.

5. FUTURE RESEARCH DIRECTION

Based on the existing limitation in current methods, suggestions for the future research direction could be listed as follows:

- Utilizing more criteria and objective functions in the design of concrete thin shell structures, such as 'Embodied Carbon assessment,' 'life cycle assessment,' cost, measuring energy performance, acoustic performance of the structure, and considering fabrication methods as an objective and other criteria.
- Utilizing other metaheuristic algorithms in the design of thin shell structures that have not been employed before, such as Particle swarm optimization algorithm, Graywolf optimization algorithm, Dolphin Echolocation algorithms, and other algorithms, to compare the convergence rate, accuracy, and speed of these algorithms.
- Developing a scientific method to select a final solution from the set of optimal solutions in the Pareto Frontier.
- Utilizing Machine Learning techniques (76) to find the optimal design solutions, constructing reliable Datasets, and training algorithms to find optimal solutions (prediction) even without computing (77), (78).

6. STATEMENTS AND DECLARATIONS

Funding
Not applicable

Declaration of Competing Interest
The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Declaration of Generative AI and AI-assisted Technologies in the writing process
The authors did not use AI or AI-assisted technologies except for the English language grammar and punctuation check's purpose.

7. AUTHORSHIP CONTRIBUTION STATEMENT

MV: Conceptualization, Methodology, Software, Data curation, Validation, Visualization, Writing - original draft. **MG:** Project administration, Supervision, Writing - Review & Editing. **AE:** Supervision, Writing - Review & Editing. **MR:** Software, Validation, Writing - Review & Editing.

8. AVAILABILITY OF DATA AND MATERIALS

The corresponding author, upon responsible request, will provide some or all data, models, or codes that support the findings of this study.

9. ACKNOWLEDGMENT

The authors thank the support provided by the Department of Architectural Technology, Faculty of Architecture, College of Fine Arts, at the University of Tehran, Iran.

10. APPENDIX

TABLE 7. Abbreviations Table

SOO	Simple objective optimization	**DM**	Decision Maker
MOO	Multi-objective optimization	**GA**	Genetic Algorithm
NURBS	Non-Uniform Rational B-spline	**PSO**	Particle Swarm Optimization
NSGA-II	Non-dominated Sorting Genetic Algorithm	**DE**	differential evolution (DE) algorithm
MOGA	Multi-objective Genetic Algorithm	**PF**	Pareto Optimal Front
MOGWO	Multi-objective gray wolf optimizer	**BESO**	Bi-directional Evolutionary Structural Optimization algorithm

11. REFERENCES

1. Adesina A. Recent advances in the concrete industry to reduce its carbon dioxide emissions. Environmental Challenges. 2020;1:100004.
2. Chilton J, Isler H. Heinz isler: Thomas Telford; 2000.
3. Goaiz HA, Shamsaldeen HA, Abdulrehman MA, Al-Gasham T. Evaluation of steel fiber reinforced geopolymer concrete made of recycled materials. International Journal of Engineering. 2022;35(10):2018-26.
4. Farshad M. Design and analysis of shell structures: Springer Science & Business Media; 2013.
5. Adriaenssens S, Block P, Veenendaal D, Williams C. Shell structures for architecture: form finding and optimization: Routledge; 2014.
6. Council WGB. New report: The building and construction sector can reach net zero carbon emissions by 2050. World Green Building Council Toronto, ON, Canada; 2019.

7. IEA U, editor Global status report for buildings and construction 20192019: IEA Paris, France.
8. Zhong X, Hu M, Deetman S, Steubing B, Lin HX, Hernandez GA, et al. Global greenhouse gas emissions from residential and commercial building materials and mitigation strategies to 2060. Nat Commun. 2021;12(1):6126. 10.1038/s41467-021-26212-z https://www.ncbi.nlm.nih.gov/pubmed/34675192
9. Block P, Schlueter A, Veenendaal D, Bakker J, Begle M, Hischier I, et al. NEST HiLo: Investigating lightweight construction and adaptive energy systems. J Build Eng. 2017;12:332-41. 10.1016/j.jobe.2017.06.013
10. Hosseinian SM, Faghani M. Assessing the effect of structural parameters and building site in achieving low carbon building materialization using a life-cycle assessment approach. J Build Eng. 2021;44:103318. 10.1016/j.jobe.2021.103318
11. Pawar AJ, Suryawanshi SR. Comprehensive Analysis of Stress-strain Relationships for Recycled Aggregate Concrete. International Journal of Engineering. 2022;35(11):1-10. 10.5829/ije.2022.35.11b.05
12. Sreeman D, Kumar Roy B. Optimization study of isolated building using shape memory alloy with friction pendulum system under near-fault excitations. International Journal of Engineering. 2022;35(11):2176-85.
13. Talib HY, Al-Salim NHA. Improving Punching Shear in Flat Slab by Replacing Punching Shear Reinforcement by Ultrahigh Performance Concrete. International Journal of Engineering. 2022;35(8):1619-28. 10.5829/ije.2022.35.08b.18
14. Kotnik T, Schwartz J. The architecture of heinz isler. Journal of the international association for shell and spatial structures. 2011;52(3):185-90.
15. Kaveh A. Advances in metaheuristic algorithms for optimal design of structures: Springer; 2014.
16. Talbi E-G. Metaheuristics: from design to implementation: John Wiley & Sons; 2009.
17. Kaveh A, Ghazaan MI. Meta-heuristic algorithms for optimal design of real-size structures: Springer; 2018.
18. Mirjalili S, Faris H, Aljarah I. Evolutionary machine learning techniques: Springer; 2019.
19. Pugnale A, Sassone M. Morphogenesis and structural optimization of shell structures with the aid of a genetic algorithm. Journal of the International Association for Shell and Spatial Structures. 2007;48(3):161-6.
20. Gokul Santhosh S, Singh A, Abdul Akbar M. Parametric Studies and Optimization of Grid Shell Structures Using Genetic Algorithm. Recent Trends in Civil Engineering: Select Proceedings of ICRACE 2021: Springer; 2022. p. 109-22.
21. Basso P, Del Grosso AE, Pugnale A, Sassone M. Computational morphogenesis in architecture: Cost optimization of free-form grid shells. Journal of the international association for shell and spatial structures. 2009;50(3):143-50.
22. Baghdadi A, Doerrie R, Kloft H, editors. Application of different optimisation techniques for material minimisation of individual structural elements. Proceedings of IASS Annual Symposia; 2020: International Association for Shell and Spatial Structures (IASS).
23. Ansola R, Canales J, Tarrago JA, Rasmussen J. Combined shape and reinforcement layout optimization of shell structures. Struct Multidiscip O. 2004;27(4):219-27. 10.1007/s00158-004-0399-7
24. Kimura T, Ohmori H. Computational morphogenesis of free form shells. Journal of the International Association for Shell and Spatial structures. 2008;49(3):175-80.
25. Yang F, Yu T, Bui TQ, editors. Morphogenesis of free-form surfaces by an effective approach based on isogeometric analysis and particle swarm optimization. Structures; 2023: Elsevier.
26. Tomás A, Martí P. Shape and size optimisation of concrete shells. Engineering Structures. 2010;32(6):1650-8.
27. Feng RQ, Ge JM. Shape optimization method of free-form cable-braced grid shells based on the translational surfaces technique. Int J Steel Struct. 2013;13(3):435-44. 10.1007/s13296-013-3004-3
28. Hassani B, Tavakkoli SM, Ghasemnejad H. Simultaneous shape and topology optimization of shell structures. Struct Multidiscip O. 2013;48(1):221-33. 10.1007/s00158-013-0894-9
29. Richardson JN, Adriaenssens S, Coelho RF, Bouillard P. Coupled form-finding and grid optimization approach for single layer grid shells. Engineering Structures. 2013;52:230-9. 10.1016/j.engstruct.2013.02.017
30. Shimoda M, Liu Y. A non-parametric free-form optimization method for shell structures. Struct Multidiscip O. 2014;50(3):409-23. 10.1007/s00158-014-1059-1
31. Kaveh A, Amirsoleimani P, Eslamlou AD, Rahmani P, editors. Frequency-constrained optimization of large-scale dome-shaped trusses using chaotic water strider algorithm. Structures; 2021: Elsevier.
32. Tomei V, Grande E, Imbimbo M. Design optimization of gridshells equipped with pre-tensioned rods. J Build Eng. 2022;52:104407. 10.1016/j.jobe.2022.104407
33. Ansola R, Canales J, Tarrago J, Rasmussen J. On simultaneous shape and material layout optimization of shell structures. Struct Multidiscip O. 2002;24:175-84.
34. Gythiel W, Schevenels M. Gradient-based size, shape, and topology optimization of single-layer reticulated shells subject to distributed loads. Struct Multidiscip O. 2022;65(5):144. 10.1007/s00158-022-03225-w
35. Teimouri M, Asgari M. Developing an efficient coupled function-based topology optimization code for designing lightweight compliant structures using the BESO algorithm. Optim Eng. 2023:1-29. 10.1007/s11081-023-09808-w
36. Zhang ZJ, Kanno Y, Feng RQ. Collapse-resistance optimization of fabricated single-layer grid shell based on sequential approximate optimization. Struct Multidiscip O. 2023;66(7):153. 10.1007/s00158-023-03604-x
37. Emami N, Giles H, von Buelow P. Structural, daylighting, and energy performance of perforated concrete shell structures. Automat Constr. 2020;117:103249. 10.1016/j.autcon.2020.103249
38. Liuti A, Pugnale A, Erioli A, editors. Computational morphogenesis in architecture: Structure and light as a multi-objective design/optimization problem. Proceedings of the 2nd International Conference on Structures and Architecture (ICSA 2013), Guimaraes, Portugal; 2013.
39. Emami N, editor Structure and daylighting performance comparisons of Heinz Isler's roof shell based on variations in parametrically derived multi opening topologies. Proceedings of IASS Annual Symposia; 2015: International Association for Shell and Spatial Structures (IASS).
40. Pugnale A. Integrating Form, Structure and Acoustics: A Computational Reinterpretation of Frei Otto's Design Method and Vision. Journal of the International Association for Shell and Spatial Structures. 2018;59(1):75-86. 10.20898/j.iass.2018.195.901.
41. Turrin M, von Buelow P, Stouffs R. Design explorations of performance driven geometry in architectural design using parametric modeling and genetic algorithms. Adv Eng Inform. 2011;25(4):656-75. 10.1016/j.aei.2011.07.009
42. Puppa GD, Trautz M, editors. Multi-objective shape optimization of shell structures based on buckling forms. Proceedings of IASS Annual Symposia; 2015: International Association for Shell and Spatial Structures (IASS).

43. Nagata K, Honma T, editors. Multi-objective Optimization for Free Form Surface Shell Using Swarm Intelligence with Decent Solutions Search. Proceedings of IASS Annual Symposia; 2016: International Association for Shell and Spatial Structures (IASS).
44. Wang XY, Zhu SJ, Zeng Q, Guo XN. Improved multi-objective Hybrid Genetic Algorithm for Shape and Size Optimization of Free-form latticed structures. J Build Eng. 2021;43:102902. https://doi.org/10.1016/j.jobe.2021.102902
45. Henriksson V, Lomax S, Richardson JN, editors. Multi-Objective Optimization of a Concrete Shell. Proceedings of IASS Annual Symposia; 2019: International Association for Shell and Spatial Structures (IASS).
46. Veenendaal D. DESIGN AND FORM FINDING OF FLEXIBLY FORMED CONCRETE SHELL STRUCTURES: ETH Zuric; 2017.
47. Veenendaal D, Bakker J, Block P. Structural Design of the Flexibly Formed, Mesh-Reinforced Concrete Sandwich Shell Roof of Nest Hilo. Journal of the International Association for Shell and Spatial Structures. 2017;58(1):23-38. 10.20898/j.iass.2017.191.847
48. Zhao ZW, Wu JJ, Qin Y, Liang B. The strong coupled form-finding and optimization algorithm for optimization of reticulated structures. Adv Eng Softw. 2020;140:102765. 10.1016/j.advengsoft.2019.102765
49. Vargas DEC, Lemonge ACC, Barbosa HJC, Bernardino HS. Differential evolution with the adaptive penalty method for structural multi-objective optimization. Optim Eng. 2019;20(1):65-88. 10.1007/s11081-018-9395-4
50. Mirra G, Pugnale A, editors. Comparison between human-defined and AI-generated design spaces for the optimisation of shell structures. Structures; 2021: Elsevier.
51. Nishei T, Fujita S, editors. Structural optimization of latticed shells considering strain energy and collapse load factor. Proceedings of IASS Annual Symposia; 2020: International Association for Shell and Spatial Structures (IASS).
52. Wang H, Li PC, Wang J. Shape optimization and buckling analysis of novel two-way aluminum alloy latticed shells. J Build Eng. 2021;36:102100. 10.1016/j.jobe.2020.102100
53. Kostura Z, Clark MC, Olson JR, editors. Design of a 500,000 m2 Iconic Roof for the New International Airport for Mexico City. Proceedings of IASS Annual Symposia; 2018: International Association for Shell and Spatial Structures (IASS).
54. Jiang B, Zhang J, Ohsaki M, editors. Shape optimization of free-form shell structures combining static and dynamic behaviors. Structures; 2021: Elsevier.
55. Cao Z, Wang Z, Zhao L, Fan F, Sun Y. Multi-constraint and multi-objective optimization of free-form reticulated shells using improved optimization algorithm. Engineering Structures. 2022;250:113442.
56. Desai R. Structural Optimization Using Genetic Algorithm. Handbook of AI-based Metaheuristics: CRC Press; 2021. p. 73-116.
57. Reisinger J, Zahlbruckner MA, Kovacic I, Kán P, Wang-Sukalia X, Kaufmann H. Integrated multi-objective evolutionary optimization of production layout scenarios for parametric structural design of flexible industrial buildings. J Build Eng. 2022;46:103766.
58. Preisinger C. Karamba3D. Vienna: Bollinger und Grohmann ZT GmbH. 2018.
59. Vatandoost M, Yazdanfar S, Ekhlassi A. Computer-Aided Design: Classification of Design problems in Architecture. Journal of Information and Computational Science. 2019;9(10):605-25.
60. Mirjalili S, Dong JS. Multi-objective optimization using artificial intelligence techniques: Springer; 2020.
61. Rao SS. Engineering optimization: theory and practice: John Wiley & Sons; 2019.
62. Yi YK, Tariq A, Park J, Barakat D. Multi-objective optimization (MOO) of a skylight roof system for structure integrity, daylight, and material cost. J Build Eng. 2021;34:102056. 10.1016/j.jobe.2020.102056
63. Smith G, Rivlin R. The strain-energy function for anisotropic elastic materials. Collected Papers of RS Rivlin: Volume I and II. 1997:541-59.
64. Al-Tashi Q, Mirjalili S, Wu J, Abdulkadir SJ, Shami TM, Khodadadi N, et al. Moth-Flame Optimization Algorithm for Feature Selection: A Review and Future Trends. Handbook of Moth-Flame Optimization Algorithm. 2022:11-34.
65. Vatandoost M, Ekhlassi A, Yazdanfar S. Computer-aided architectural design: classification and application of optimization algorithms. Journal of Information and Computational Science. 2020;10(6):18-43.
66. Chatterjee S, Sarkar S, Dey N, Ashour AS, Sen S. Hybrid non-dominated sorting genetic algorithm: II-neural network approach. Advancements in Applied Metaheuristic Computing: IGI Global; 2018. p. 264-86.
67. Sayyaadi H. Modeling, assessment, and optimization of energy systems: Academic Press; 2020.
68. Deb K, Agrawal S, Pratap A, Meyarivan T, editors. A fast elitist non-dominated sorting genetic algorithm for multi-objective optimization: NSGA-II. Parallel Problem Solving from Nature PPSN VI: 6th International Conference Paris, France, September 18–20, 2000 Proceedings 6; 2000: Springer.
69. Yang X-S. Nature-inspired optimization algorithms: Academic Press; 2020.
70. Baghdadi A, Heristchian M, Kloft H, editors. Displacement-based seismic assessment of Heinz Isler's shell structures. Proceedings of IASS Annual Symposia; 2020: International Association for Shell and Spatial Structures (IASS).
71. Vu-Bac N, Duong TX, Lahmer T, Zhuang X, Sauer RA, Park HS, et al. A NURBS-based inverse analysis for reconstruction of nonlinear deformations of thin shell structures. Comput Method Appl M. 2018;331:427-55. 10.1016/j.cma.2017.09.034
72. Zhang XK, Jin CN, Hu P, Zhu XF, Hou WB, Xu JT, et al. NURBS modeling and isogeometric shell analysis for complex tubular engineering structures. Comput Appl Math. 2017;36(4):1659-79. 10.1007/s40314-016-0312-1
73. Baghdadi A, Heristchian M, Kloft H. Structural assessment of remodelled shells of Heinz Isler. International Journal of Advanced Structural Engineering. 2019;11(4):491-502.
74. Piker D. Kangaroo Form Finding with Computational Physics. Archit Design. 2013;83(2):136-7. 10.1002/ad.1569
75. Sassone M, Pugnale A, editors. Evolutionary Structural Optimization in Shell Design. Advanced Numerical Analysis of Shell-like Structures: Special Workshop, Zagreb, 26-28 September 2007; 2007.
76. Ramavath S, Suryawanshi S. Optimal Prediction of Shear Properties in Beam-Column Joints Using Machine Learning Approach. International Journal of Engineering. 2024;37(1):67-82.
77. El Alaoui M, Chahidi LO, Rougui M, Lamrani A, Mechaqrane A. Prediction of Energy Consumption of an Administrative Building using Machine Learning and Statistical Methods. Civil Engineering Journal. 2023;9(5):1007-22.
78. Bessa M, Pellegrino S. Design of ultra-thin shell structures in the stochastic post-buckling range using Bayesian machine learning and optimization. International Journal of Solids and Structures. 2018;139:174-88.

Persian Abstract

چکیده

در این مطالعه بهینه‌سازی شکل و ضخامت سازه‌های پوسته بتنی پیوسته نازک ارائه شده است که وزن، انحراف قائم و تغییر انرژی الاستیک آنها را به حداقل می‌رساند در حالی که الزامات عملکردی را برآورده می‌کند و مصرف مصالح را به حداقل می‌رساند. برخلاف مطالعات قبلی که بیشتر بهینه سازی تک هدفه مورد توجه قرار گرفته است، این تحقیق با در نظر گرفتن سه تابع هدف، بر بهینه‌سازی چندهدفه تمرکز دارد. این ترکیب از توابع هدف در تحقیقات قبلی منعکس نشده است که این مطالعه را متمایز می‌کند. گردش کار طراحی محاسباتی شامل یک مدل پارامتریک، مؤلفه های متعدد برای اندازه گیری توابع هدف در *گرس‌هاپر راینو* و یک الگوریتم فراابتکاری، الگوریتم ژنتیک چندهدفه مرتب‌سازی غیرغالب (NSGA-II)، به عنوان ابزار جستجو است که در زبان برنامه نویسی *پایتون* کدنویسی شده است. این گردش کار به ما امکان می‌دهد تا فرم‌یابی و بهینه‌سازی را به طور همزمان انجام دهیم. برای نشان دادن موثر بودن این الگوریتم فراابتکاری در بهینه‌سازی سازه‌ها، ما آن را در یک مطالعه موردی، در یک پوسته شناخته شده طراحی شده با استفاده از تکنیک شبیه‌سازی فیزیکی مدل‌آویخته به کار بردیم. تفاسیر چند نمونه از نتایج بهینه شده نشان می‌دهد که اگرچه وزن پاسخ-۱ تقریباً مشابه پاسخ-۲ است، اما انرژی کرنشی و انحراف قائم کمتری دارد. پاسخ-۳، با جرم سه برابر، انحراف قائم و انرژی کرنشی به طور قابل توجهی کمتری از پاسخ ۱ و پاسخ ۲ دارد که به ترتیب بیش از ۵۰٪ و ۱۷٪ کمتر است. پاسخ-۳ و پاسخ-٤ عملکرد سازه‌ای بهتری در انحراف قائم و انرژی کرنشی نشان می دهند. علاوه بر این، مقایسه نتایج بهینه سازی چندهدفه با پوسته طرح شده *ایسلر* Isler نشان داد که این روش پاسخ سازه‌ای با وزن و انحراف قائم کمتر و در عین حال سازه‌ای سخت‌تر پیدا کرد و کاربردی بودن این روش را تأیید کرد. این مطالعه نشان داد که بهینه‌سازی چندهدفه روشی قابل اعتماد برای فرم‌یابی و بهینه‌سازی پوسته‌ها است که نتایج دقیق و قابل قبولی ایجاد می‌کند.